Fundamentals of Physics

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3 Vectors

3.1 Vectors and their components

- **Vector -** has magnitude and direction.
- **Scalar -** quantities that can are fully described by a magnitude (a numerical value alone), without any direction.
- **Vector sum (resoultant) -** are the product from adding two or more vecotrs.

$$\vec{s} = \vec{a} + \vec{b},$$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(commutative law)}$$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad \text{(associative law)}$$

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \qquad \text{(vector subtraction)}$$

A component of a vector is the projection of a vector on an axis.

Finding the components:

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta$$

If we know a vectors a_x and a_y and want magnitude or angle we can use:

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \theta = \tan^{-1}(\tfrac{a_y}{a_x})$$

3.2 Unit vectors, adding vectors by components

Unit vector - is a vector with magnitude of exactly 1.

$$\vec{r} = \vec{a} + \vec{b} \quad \begin{aligned} r_x &= a_x + b_x \\ r_y &= a_y + b_y \\ r_z &= a_z + b_z \end{aligned}$$

We can write a vector $\vec{a}$ in terms of unit vectors as: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

3.3 Multiplying vectors

There are two ways of multiplying vectors, one way produces a scalar (scalar product) and the other way produces a new vector (vector product):

Feature	Scalar product (dot)	Vector product (cross)
Symbol	$\vec{A} \cdot \vec{B}$	$\vec{A} \times \vec{B}$
Result	Scalar (number)	Vector
Formula	$AB \cos \theta$	$AB \sin \theta$
Component form	$A_x B_x + A_y B_y + A_z B_z$	$(A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x)$