Fundamentals of Physics

Arian DK

October 11, 2025

4 Motion in two and three dimentions

4.1 Position and displacment

Position vector - a vector that extends from a reference point (usually the origin) to the particle.

- **Unit vector notation:** $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
- **Particle's displacement:** $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$
- **Alternative form:** $\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

4.2 Average velocity and instantaneous velocity

- If a particle undergoes a displacment $\Delta\vec{r}$ in time interval Δt, it's average velocity $\vec{v}_{avg}$ for that time interval is: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$.
- As Δt is shrank to 0, $\vec{v_{avg}}$ reaches a limit called either the velocity or the instantenious velocity $\vec{v}$: $\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

$\star$ The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particles path at the particles position.

4.3 Average acceleration and instantanious acceleration

- If a particle's velocity changes from $\vec{v_1}$ to $\vec{v_2}$ in time interval Δt, it's average acceleration during Δt is: $\vec{a}_{avg} = \frac{\vec{v_2} - \vec{v_1}}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$

4.4 Projectile motion

When you throw an object at an angle:

- It moves forward at a constant horizontal speed.
- It moves up and then down because of gravity pulling it downward.
- The path it follows is a parabola.

$\star$ In projectile motion,the horizontal motionand the vertical motion are independent of each other; that is, neither motion affects the other.
The vertical motion of a projectile is governed by the following kinematic equations:

$$v_y = v_0 \sin\theta_0 - gt, \qquad v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

The trajectory (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}, \quad \text{if } x_0 \text{ and } y_0 \text{ are zero.}$$

The equations of motion for the particle (while in flight) can be written as

$$x - x_0 = (v_0\cos\theta_0)t, \qquad y - y_0 = (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2,$$

The particle's horizontal range R, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g} \sin 2\theta_0.$$

$\star$ The horizontal range R is maximum for a launch angle of 45°.

4.5 Uniform circular motion

$$a = \frac{v^2}{r} \quad \text{(centripetal acceleration)}, \qquad T = \frac{2\pi r}{v} \quad \text{(period)}.$$

4.6 Relative motion in one dimention

In relative motion, two reference frames moving at constant velocity measure different velocities for the same particle but the same acceleration. The relationship between the measured velocities is

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

and since the frames move at constant velocity,

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

$\star$ Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

4.7 Relative motion in two dimentions

In two-dimensional relative motion, two reference frames moving at constant velocity measure different velocities for a particle but the same acceleration. The relationship between the measured velocities is

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

and since the frames move at constant velocity,

$$\vec{a}_{PA} = \vec{a}_{PB}.$$