Fundamentals of Physics

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2 Motion Along a Straight Line

2.1 Position, displacement, and average velocity

$$\text{Distance: } d = v \times t \quad \text{Time: } t = \frac{d}{v} \quad \text{Velocity: } v = \frac{d}{t}$$

$$\text{Magnitude} \Rightarrow \Delta x_1 - x_2$$

$$\text{Average velocity} \Rightarrow v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{Magnitude} \Rightarrow \frac{\text{total distance}}{\Delta t}$$

2.2 Instantaneous velocity and speed

$$\text{Instantaneous velocity} \Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

2.3 Acceleration

$$\text{Average acceleration} \Rightarrow a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

$$\text{Acceleration at a point} \Rightarrow a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

⋆ If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

2.4 Constant acceleration

The following five equations describe the motion of a particle with constant acceleration.

Nr	Equation	Missing quantity
1	$v = v_0 + at$	$x - x_0$
2	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
3	$v^2 = v_0^2 + 2a(x - x_0)$	t
4	$x - x_0 = \frac{1}{2}(v_0 + v_t)t$	a
5	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

2.5 Free-fall acceleration

⋆ The free-fall acceleration near Earth's surface is $a = -g = -9.8$ m/s^2, and the magnitude of the acceleration is $g = 9.8$ m/s^2. Do not substitute -9.8 m/s^2 for g.

2.6 Graphical integration in motion analysis

• On a graph of acceleration a versus time t, the change in the velocity is given by:

$$v_1 - v_0 = \int_{t_0}^{t_1} a\,dt.$$

• On a graph of velocity v versus time t, the change in the position is given by:

$$x_1 - x_0 = \int_{t_0}^{t_1} v\,dt,$$

3 Vectors

3.1 Vectors and their components

- **Vector** - has magnitude and direction.
- **Scalar** - quantities that can are fully described by a magnitude (a numerical value alone), without any direction.
- **Vector sum (resoultant)** - are the product from adding two or more vecotrs.

$$\vec{s} = \vec{a} + \vec{b},$$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(commutative law)}$$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad \text{(associative law)}$$

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \qquad \text{(vector subtraction)}$$

A component of a vector is the projection of a vector on an axis.

Finding the components:

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta$$

If we know a vectors a_x and a_y and want magnitude or angle we can use:

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \theta = \tan^{-1}(\tfrac{a_y}{a_x})$$

3.2 Unit vectors, adding vectors by components

Unit vector - is a vector with magnitude of exactly 1.

$$\vec{r} = \vec{a} + \vec{b} \quad \begin{aligned} r_x &= a_x + b_x \\ r_y &= a_y + b_y \\ r_z &= a_z + b_z \end{aligned}$$

We can write a vector $\vec{a}$ in terms of unit vectors as: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

3.3 Multiplying vectors

There are two ways of multiplying vectors, one way produces a scalar (scalar product) and the other way produces a new vector (vector product):

Feature	Scalar product (dot)	Vector product (cross)
Symbol	$\vec{A} \cdot \vec{B}$	$\vec{A} \times \vec{B}$
Result	Scalar (number)	Vector
Formula	$AB\cos\theta$	$AB\sin\theta$
Component form	$A_xB_x + A_yB_y + A_zB_z$	$(A_yB_z - A_zB_y,\ A_zB_x - A_xB_z,\ A_xB_y - A_yB_x)$

4 Motion in two and three dimentions

4.1 Position and displacment

Position vector - a vector that extends from a reference point (usually the origin) to the particle.

- **Unit vector notation:** $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
- **Particle's displacement:** $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$
- **Alternative form:**
$$\begin{aligned}\Delta\vec{r} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}\end{aligned}$$

4.2 Average velocity and instantaneous velocity

- If a particle undergoes a displacment $\Delta\vec{r}$ in time interval Δt, it's average velocity $\vec{v}_{avg}$ for that time interval is: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$.
- As Δt is shrank to 0, $\vec{v_{avg}}$ reaches a limit called either the velocity or the instantenious velocity $\vec{v}$:
$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

$\star$ The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particles path at the particles position.

4.3 Average acceleration and instantanious acceleration

- If a particle's velocity changes from $\vec{v_1}$ to $\vec{v_2}$ in time interval Δt, it's average acceleration during Δt is:
$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v_1}}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$

4.4 Projectile motion

When you throw an object at an angle:

- It moves forward at a constant horizontal speed.
- It moves up and then down because of gravity pulling it downward.
- The path it follows is a parabola.

⋆ In projectile motion,the horizontal motionand the vertical motion are independent of each other; that is, neither motion affects the other.
The vertical motion of a projectile is governed by the following kinematic equations:

$$v_y = v_0 \sin\theta_0 - gt, \qquad v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

The trajectory (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}, \quad \text{if } x_0 \text{ and } y_0 \text{ are zero.}$$

The equations of motion for the particle (while in flight) can be written as

$$x - x_0 = (v_0\cos\theta_0)t, \qquad y - y_0 = (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2,$$

The particle's horizontal range R, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g}\sin 2\theta_0.$$

⋆ The horizontal range R is maximum for a launch angle of 45°.

4.5 Uniform circular motion

$$a = \frac{v^2}{r} \quad \text{(centripetal acceleration)}, \qquad T = \frac{2\pi r}{v} \quad \text{(period)}.$$

4.6 Relative motion in one dimention

In relative motion, two reference frames moving at constant velocity measure different velocities for the same particle but the same acceleration. The relationship between the measured velocities is

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

and since the frames move at constant velocity,

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

⋆ Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

4.7 Relative motion in two dimentions

In two-dimensional relative motion, two reference frames moving at constant velocity measure different velocities for a particle but the same acceleration. The relationship between the measured velocities is

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

and since the frames move at constant velocity,

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

5 Force and motion - I

5.1 Newton's first and second laws

⋆ **Newton's First Law:** If no *net* force acts on a body ($\vec{F}_{\text{net}} = 0$), the body's velocity cannot change; that is, the body cannot accelerate.

⋆ An inertial reference frame is one in which Newton's laws hold.

⋆ **Newton's Second Law:** The net force on a body is equal to the product of the body's mass and its acceleration.

- The law can be written as:

$$\vec{F}_{\text{net}} = m\vec{a} \quad \text{(Newton's second law)}.$$

- In component form:

$$F_{\text{net},x} = ma_x, \qquad F_{\text{net},y} = ma_y, \qquad F_{\text{net},z} = ma_z.$$

⋆ The acceleration component along a given axis is caused *only by* the sum of the force components along that *same* axis, and not by force components along any other axis.

5.2 Some particular forces

$$F_g = mg.$$

⋆ The weight W of a body is equal to the magnitude F_g of the gravitational force on the body.

$$W = mg \quad \text{(weight)},$$

⋆ When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force $\vec{F}_N$ that is perpendicular to the surface.

5.3 Applying Newton's laws

⋆ **Newton's Third Law:** When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

$$F_{BC} = F_{CB} \quad \text{(equal magnitudes)}$$

6 Force and motion - II

6.1 Friction

Two types of friction:

- Static frictional force - Prevents motion when a body is at rest. Is written as $\vec{f_s}$.
- Kinetic frictional force - What friction becomes when sliding begins. Is written as $\vec{f_k}$.

Properties of friction:

1. If a body does not move, the static frictional force $\vec{f_s}$ balances the component of the applied force that is parallel to the surface.
2. The maximum static friction is given by

   $$f_{s,\max} = \mu_s F_N,$$

   where μ_s is the coefficient of static friction and F_N is the normal force.
3. Once the body starts sliding, the frictional force becomes kinetic and is given by

   $$f_k = \mu_k F_N,$$

   where μ_k is the coefficient of kinetic friction.

6.2 The drag force and terminal speed

When a body moves through a fluid, it experiences a drag force $\vec{D}$ opposing its motion. The drag magnitude depends on the relative speed v and is given by

$$D = \tfrac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, ρ is the fluid density, and A is the cross-sectional area.

At terminal velocity, the drag force equals the gravitational force, and the speed becomes constant:

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

6.3 Uniform Circular Motion

⋆ A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.
A particle moving in a circle of radius R at constant speed v has a centripetal acceleration directed toward the center:

$$a = \frac{v^2}{R}.$$

This acceleration is caused by a net centripetal force of magnitude

$$F = \frac{mv^2}{R},$$

where m is the particle's mass.

7 Kinetic energy and work

7.1 Kinetic Energy

The kinetic energy K associated with the motion of a particle of mass m and speed v, where v is well below the speed of light, is

$$K = \tfrac{1}{2}mv^2 \quad \text{(kinetic energy)}.$$

7.2 Work and kinetic energy

⋆ Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.
Work W is the energy transferred to or from an object by a force acting over a displacement $\vec{d}$. For a constant force,

$$W = Fd\cos\phi = \vec{F}\cdot\vec{d},$$

where ϕ is the angle between $\vec{F}$ and $\vec{d}$.
⋆ To calculate the work a force does on an object as the object movesthrough some displacement, we use only the force component along the object's displacement. The force component perpendicularto the displacement does zero work.
⋆ A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

Only the component of $\vec{F}$ along $\vec{d}$ does work. The net work equals the change in kinetic energy:

$$\Delta K = K_f - K_i = W,$$

or equivalently,

$$K_f = K_i + W.$$

7.3 Work done by the gravitational force

The work done by the gravitational force on an object of mass m over displacement $\vec{d}$ is

$$W_g = mgd\cos\phi,$$

where ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

The work done by an applied force W_a and gravity are related by

$$\Delta K = K_f - K_i = W_a + W_g.$$

If $K_f = K_i$, then

$$W_a = -W_g,$$

meaning the applied force does as much work on the object as gravity removes.

7.4 Work done by a spring force

The force from a spring is given by Hooke's law:

$$\vec{F}_s = -k\vec{d}, \quad \text{or} \quad F_x = -kx,$$

where k is the spring constant and $\vec{d}$ (or x) is the displacement from equilibrium.

The work done by a spring when moving from x_i to x_f is

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2.$$

⋆ Work W_s is positive if the block ends up closer to the relaxed position (x = 0) than it was initially.It is negative if the block ends up farther away from x = 0.It is zero if the block ends up at the same distance from x = 0.

If $x_i = 0$ and $x_f = x$, then

$$W_s = -\tfrac{1}{2}kx^2.$$

⋆ If a block that is attached to a spring isstationary before and after a displacement,then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

7.5 Work done by a general variable force

If the force $\vec{F}$ on an object depends on position, the total work is found by integrating the force over the path:

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

If the force acts only along the x-axis, this simplifies to

$$W = \int_{x_i}^{x_f} F(x)\,dx.$$

7.6 Power

Power is the rate at which a force does work.

The *average power* over a time interval Δt is

$$P_{\text{avg}} = \frac{W}{\Delta t}.$$

The *instantaneous power* is

$$P = \frac{dW}{dt}.$$

For a force $\vec{F}$ making an angle ϕ with velocity $\vec{v}$,

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v}.$$

8 Potential energy and conservation of energy

8.1 Potential energy

⋆ The net work done by a conservative force on a particle moving around any closed path is zero.
⋆ The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.
⋆ The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position y = 0, not on the horizontal position.

A *conservative force* does zero net work around a closed path and depends only on position (e.g., gravity, springs).

The change in potential energy is related to work by

$$\Delta U = -W = -\int_{x_i}^{x_f} F(x)\,dx.$$

For gravity:

$$\Delta U = mg(y_f - y_i) = mg\Delta y, \quad \text{and if } y_i = 0,\ U(y) = mgy.$$

For a spring (elastic potential energy):

$$U(x) = \tfrac{1}{2}kx^2,$$

where $U = 0$ when the spring is at its relaxed length.

8.2 Conservation of mechanical energy

⋆ In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy Emec of the system, cannot change.
⋆ When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant without considering the intermediate motion and without finding the work done by the forces involved.

The total mechanical energy of a system is

$$E_{\text{mec}} = K + U,$$

where K is kinetic energy and U is potential energy.

In an isolated system (no external forces), mechanical energy is conserved:

$$K_1 + U_1 = K_2 + U_2,$$

or equivalently,

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.$$

8.3 Reading a potential energy curve

For a one-dimensional system, the force is related to the potential energy by

$$F(x) = -\frac{dU(x)}{dx}.$$

The kinetic energy is

$$K(x) = E_{\text{mec}} - U(x),$$

where E_{mec} is the total mechanical energy.

A *turning point* occurs when $K = 0$, and the particle reverses direction. An *equilibrium point* occurs where $\frac{dU(x)}{dx} = 0$, meaning $F(x) = 0$.

8.4 Work done on a system by an external force

Work is energy transferred to or from a system by means of an external force acting on that system.

If no friction is involved:

$$W = \Delta E_{\text{mec}} = \Delta K + \Delta U.$$

When friction is present, thermal energy changes as well:

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}},$$

where the thermal energy increase is

$$\Delta E_{\text{th}} = f_k d,$$

with f_k being the kinetic friction force and d the displacement.

8.5 Conservation of energy

⋆ The total energy E of a system can change only by amounts of energy that are transferred to orfrom the system.
⋆ The total energy E of an isolated system cannot change.
⋆ In an isolated system, we can relate the total energy at one instant to the total energy at anotherinstant without considering the energies at intermediate times.

The total energy of a system (mechanical + thermal + internal) can only change due to work done on it:

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}.$$

For an isolated system ($W = 0$):

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0.$$

Power is the rate of energy transfer. Average power:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}.$$

Instantaneous power:

$$P = \frac{dE}{dt}.$$

On an energy–time graph, power equals the slope at a given point.

9 Center of mass and linear momentum

9.1 Center of mass

⋆ The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

The center of mass of a system of n particles is the weighted average of their positions:

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i,$$

or in vector form,

$$\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i,$$

where M is the total mass of the system.

9.2 Newton's second law for a system of particles

The motion of a system's center of mass follows Newton's second law:

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}},$$

where $\vec{F}_{\text{net}}$ is the total external force, M is the total mass, and $\vec{a}_{\text{com}}$ is the acceleration of the center of mass.

9.3 Linear momentum

⋆ The time rate of change of the momentumof a particle is equal to the net force acting on the particle and is in the direction of that force.
⋆ The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

For a single particle, linear momentum is defined as

$$\vec{p} = m\vec{v},$$

a vector in the same direction as velocity. Newton's second law can be written as

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

For a system of particles,

$$\vec{p} = M\vec{v}_{\text{com}}, \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

9.4 Collision and impulse

Newton's second law in momentum form gives the impulse–momentum theorem:

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J},$$

where impulse is

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt.$$

For constant average force,

$$J = F_{\text{avg}}\Delta t.$$

When multiple bodies of mass m collide steadily with a fixed object,

$$F_{\text{avg}} = -\frac{n}{\Delta t} m\Delta v = -\frac{\Delta m}{\Delta t}\Delta v.$$

9.5 Conservation of linear momentum

⋆ If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.
⋆ If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

In a closed and isolated system (no external forces), the total momentum remains constant:

$$\vec{P} = \text{constant}.$$

This can also be written as

$$\vec{P}_i = \vec{P}_f.$$

9.6 Momentum and kinetic energy in collisions

In an inelastic collision, kinetic energy is not conserved, but total momentum is:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}.$$

For motion along one axis:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

If the bodies stick together, the collision is *completely inelastic* and they share a common final velocity V. The velocity of the system's center of mass remains unchanged during the collision.

9.7 Elastic collisions in one dimention

$\star$ In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

In an elastic collision, both kinetic energy and momentum are conserved. For a one-dimensional collision between two bodies (1 and 2), the final velocities are:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}, \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

Elastic collisions conserve total energy and momentum within the system.

9.8 Collisions in two dimentions

If two bodies collide and their motion is not along a single axis, the collision is *two-dimensional*. For a closed and isolated system, momentum is conserved in both directions:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}.$$

In component form, this gives two equations (for x and y directions). If the collision is also elastic, kinetic energy is conserved:

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}.$$

9.9 Systems with various mass: A rocket

In the absence of external forces, a rocket's acceleration is governed by the thrust equation:

$$R v_{\text{rel}} = M a,$$

where M is the rocket's instantaneous mass, R is the fuel consumption rate, and v_{rel} is the exhaust speed of the fuel relative to the rocket.

For a rocket with constant R and v_{rel}, whose mass changes from M_i to M_f while its velocity changes from v_i to v_f, the velocity change is given by:

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f}.$$