Fundamentals of Physics

Arian DK

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2 Motion Along a Straight Line

2.1 Position, displacement, and average velocity

$$\text{Magnitude} \Rightarrow \Delta x_1 - x_2$$

$$\text{Average velocity} \Rightarrow v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{Magnitude} \Rightarrow \frac{\text{total distance}}{\Delta t}$$

2.2 Instantaneous velocity and speed

$$\text{Instantaneous velocity} \Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

2.3 Acceleration

$$\text{Average acceleration} \Rightarrow a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

$$\text{Acceleration at a point} \Rightarrow a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

⋆ If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

2.4 Constant acceleration

The following five equations describe the motion of a particle with constant acceleration.

Nr	Equation	Missing quantity
1	$v = v_0 + at$	$x - x_0$
2	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
3	$v^2 = v_0^2 + 2a(x - x_0)$	t
4	$x - x_0 = \frac{1}{2}(v_0 + v_t)t$	a
5	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

2.5 Free-fall acceleration

⋆ The free-fall acceleration near Earth's surface is $a = -g = -9.8\ \text{m/s}^2$, and the magnitude of the acceleration is $g = 9.8\ \text{m/s}^2$. Do not substitute $-9.8\ \text{m/s}^2$ for g.

2.6 Graphical integration in motion analysis

• On a graph of acceleration a versus time t, the change in the velocity is given by:

$$v_1 - v_0 = \int_{t_0}^{t_1} a\,dt.$$

• On a graph of velocity v versus time t, the change in the position is given by:

$$x_1 - x_0 = \int_{t_0}^{t_1} v\,dt,$$

3 Vectors

3.1 Vectors and their components

- **Vector** - has magnitude and direction.
- **Scalar** - quantities that can are fully described by a magnitude (a numerical value alone), without any direction.
- **Vector sum (resoultant)** - are the product from adding two or more vecotrs.

$$\vec{s} = \vec{a} + \vec{b},$$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(commutative law)}$$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad \text{(associative law)}$$

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \qquad \text{(vector subtraction)}$$

A component of a vector is the projection of a vector on an axis.

Finding the components:
$a_x = a\cos\theta$ and $a_y = a\sin\theta$

If we know a vectors a_x and a_y and want magnitude or angle we can use:

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \theta = \tan^{-1}(\tfrac{a_y}{a_x})$$

3.2 Unit vectors, adding vectors by components

Unit vector - is a vector with magnitude of exactly 1.

$$\vec{r} = \vec{a} + \vec{b} \quad \begin{aligned} r_x &= a_x + b_x \\ r_y &= a_y + b_y \\ r_z &= a_z + b_z \end{aligned}$$

We can write a vector $\vec{a}$ in terms of unit vectors as: $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

3.3 Multiplying vectors

There are two ways of multiplying vectors, one way produces a scalar (scalar product) and the other way produces a new vector (vector product):

Feature	Scalar product (dot)	Vector product (cross)
Symbol	$\vec{A} \cdot \vec{B}$	$\vec{A} \times \vec{B}$
Result	Scalar (number)	Vector
Formula	$AB\cos\theta$	$AB\sin\theta$
Component form	$A_xB_x + A_yB_y + A_zB_z$	$(A_yB_z - A_zB_y,\ A_zB_x - A_xB_z,\ A_xB_y - A_yB_x)$

4 Motion in two and three dimentions

4.1 Position and displacment

Position vector - a vector that extends from a reference point (usually the origin) to the particle.

- **Unit vector notation:** $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
- **Particle's displacement:** $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$
- **Alternative form:**
$$\begin{aligned}\Delta\vec{r} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}\\ &= \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}\end{aligned}$$

4.2 Average velocity and instantaneous velocity

- If a particle undergoes a displacment $\Delta\vec{r}$ in time interval Δt, it's average velocity $\vec{v}_{avg}$ for that time interval is: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$.
- As Δt is shrank to 0, $\vec{v_{avg}}$ reaches a limit called either the velocity or the instantenious velocity $\vec{v}$:
$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

$\star$ The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particles path at the particles position.

4.3 Average acceleration and instantanious acceleration

- If a particle's velocity changes from $\vec{v_1}$ to $\vec{v_2}$ in time interval Δt, it's average acceleration during Δt is:
$\vec{a}_{avg} = \frac{\vec{v_2} - \vec{v_1}}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$

4.4 Projectile motion

When you throw an object at an angle:

- It moves forward at a constant horizontal speed.
- It moves up and then down because of gravity pulling it downward.
- The path it follows is a parabola.

$\star$ In projectile motion,the horizontal motionand the vertical motion are independent of each other; that is, neither motion affects the other.
The vertical motion of a projectile is governed by the following kinematic equations:

$$v_y = v_0 \sin\theta_0 - gt, \qquad v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

The trajectory (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}, \quad \text{if } x_0 \text{ and } y_0 \text{ are zero.}$$

The equations of motion for the particle (while in flight) can be written as

$$x - x_0 = (v_0\cos\theta_0)t, \qquad y - y_0 = (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2,$$

The particle's horizontal range R, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g}\sin 2\theta_0.$$

$\star$ The horizontal range R is maximum for a launch angle of 45°.

4.5 Uniform circular motion

$$a = \frac{v^2}{r} \quad \text{(centripetal acceleration)}, \qquad T = \frac{2\pi r}{v} \quad \text{(period)}.$$

4.6 Relative motion in one dimention

In relative motion, two reference frames moving at constant velocity measure different velocities for the same particle but the same acceleration. The relationship between the measured velocities is

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

and since the frames move at constant velocity,

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

$\star$ Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

4.7 Relative motion in two dimentions

In two-dimensional relative motion, two reference frames moving at constant velocity measure different velocities for a particle but the same acceleration. The relationship between the measured velocities is

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

and since the frames move at constant velocity,

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

5 Force and motion - I

5.1 Newton's first and second laws

⋆ **Newton's First Law:** If no *net* force acts on a body ($\vec{F}_{\text{net}} = 0$), the body's velocity cannot change; that is, the body cannot accelerate.

⋆ An inertial reference frame is one in which Newton's laws hold.

⋆ **Newton's Second Law:** The net force on a body is equal to the product of the body's mass and its acceleration.

- The law can be written as:

$$\vec{F}_{\text{net}} = m\vec{a} \quad \text{(Newton's second law)}.$$

- In component form:

$$F_{\text{net},x} = ma_x, \qquad F_{\text{net},y} = ma_y, \qquad F_{\text{net},z} = ma_z.$$

⋆ The acceleration component along a given axis is caused *only by* the sum of the force components along that *same* axis, and not by force components along any other axis.

5.2 Some particular forces

$$F_g = mg.$$

⋆ The weight W of a body is equal to the magnitude F_g of the gravitational force on the body.

$$W = mg \quad \text{(weight)},$$

⋆ When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force $\vec{F}_N$ that is perpendicular to the surface.

5.3 Applying Newton's laws

$\star$ **Newton's Third Law:** When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

$$F_{BC} = F_{CB} \quad \text{(equal magnitudes)}$$

6 Force and motion - II

6.1 Friction

Two types of friction:

- Static frictional force - Prevents motion when a body is at rest. Is written as $\vec{f}_s$.
- Kinetic frictional force - What friction becomes when sliding begins. Is written as $\vec{f}_k$.

Properties of friction:

1. If a body does not move, the static frictional force $\vec{f}_s$ balances the component of the applied force that is parallel to the surface.
2. The maximum static friction is given by
$$f_{s,\max} = \mu_s F_N,$$
where μ_s is the coefficient of static friction and F_N is the normal force.
3. Once the body starts sliding, the frictional force becomes kinetic and is given by
$$f_k = \mu_k F_N,$$
where μ_k is the coefficient of kinetic friction.

6.2 The drag force and terminal speed

When a body moves through a fluid, it experiences a drag force $\vec{D}$ opposing its motion. The drag magnitude depends on the relative speed v and is given by

$$D = \tfrac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, ρ is the fluid density, and A is the cross-sectional area.

At terminal velocity, the drag force equals the gravitational force, and the speed becomes constant:

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

6.3 Uniform Circular Motion

⋆ A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.
A particle moving in a circle of radius R at constant speed v has a centripetal acceleration directed toward the center:

$$a = \frac{v^2}{R}.$$

This acceleration is caused by a net centripetal force of magnitude

$$F = \frac{mv^2}{R},$$

where m is the particle's mass.

7 Kinetic energy and work

7.1 Kinetic Energy

The kinetic energy K associated with the motion of a particle of mass m and speed v, where v is well below the speed of light, is

$$K = \tfrac{1}{2}mv^2 \quad \text{(kinetic energy)}.$$

7.2 Work and kinetic energy

⋆ Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.
Work W is the energy transferred to or from an object by a force acting over a displacement $\vec{d}$. For a constant force,

$$W = Fd\cos\phi = \vec{F} \cdot \vec{d},$$

where ϕ is the angle between $\vec{F}$ and $\vec{d}$.
⋆ To calculate the work a force does on an object as the object movesthrough some displacement, we use only the force component along the object's displacement. The force component perpendicularto the displacement does zero work.
⋆ A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

Only the component of $\vec{F}$ along $\vec{d}$ does work. The net work equals the change in kinetic energy:

$$\Delta K = K_f - K_i = W,$$

or equivalently,

$$K_f = K_i + W.$$

7.3 Work done by the gravitational force

The work done by the gravitational force on an object of mass m over displacement $\vec{d}$ is

$$W_g = mgd\cos\phi,$$

where ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

The work done by an applied force W_a and gravity are related by

$$\Delta K = K_f - K_i = W_a + W_g.$$

If $K_f = K_i$, then

$$W_a = -W_g,$$

meaning the applied force does as much work on the object as gravity removes.

7.4 Work done by a spring force

The force from a spring is given by Hooke's law:

$$\vec{F}_s = -k\vec{d}, \quad \text{or} \quad F_x = -kx,$$

where k is the spring constant and $\vec{d}$ (or x) is the displacement from equilibrium.

The work done by a spring when moving from x_i to x_f is

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2.$$

$\star$ Work W_s is positive if the block ends up closer to the relaxed position (x = 0) than it was initially.It is negative if the block ends up farther away from x = 0.It is zero if the block ends up at the same distance from x = 0.

If $x_i = 0$ and $x_f = x$, then

$$W_s = -\tfrac{1}{2}kx^2.$$

$\star$ If a block that is attached to a spring isstationary before and after a displacement,then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

7.5 Work done by a general variable force

If the force $\vec{F}$ on an object depends on position, the total work is found by integrating the force over the path:

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

If the force acts only along the x-axis, this simplifies to

$$W = \int_{x_i}^{x_f} F(x)\,dx.$$

7.6 Power

Power is the rate at which a force does work.

The *average power* over a time interval Δt is

$$P_{\text{avg}} = \frac{W}{\Delta t}.$$

The *instantaneous power* is

$$P = \frac{dW}{dt}.$$

For a force $\vec{F}$ making an angle ϕ with velocity $\vec{v}$,

$$P = Fv\cos\phi = \vec{F} \cdot \vec{v}.$$