

Calculus II

Arian DK

January 14, 2026

Contents

6	Logarithmic Exponential and Other Transcendental Functions	2
6.1	The Natural Logarithmic Function: Differentiation	2

6 Logarithmic Exponential and Other Transcendental Functions

6.1 The Natural Logarithmic Function: Differentiation

Definition of the Natural Logarithmic Function

The **natural logarithmic function** is defined by

$$\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0.$$

The domain of the natural logarithmic function is the set of all positive real numbers.

Properties of the Natural Logarithmic Function

The natural logarithmic function has three important properties.

1. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.
2. The function is continuous, increasing, and one-to-one.
3. The graph is concave downward.

Logarithmic Properties

If a and b are positive numbers and n is rational, then the four properties below are true.

- | | |
|------------------------------|--|
| 1. $\ln 1 = 0$ | 3. $\ln(a^n) = n \ln a$ |
| 2. $\ln(ab) = \ln a + \ln b$ | 4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ |

Definition of e

The letter e denotes the positive real number such that

$$\ln e = \int_1^e \frac{1}{t} dt = 1.$$

Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x .

1.
$$\frac{d}{dx} [\ln x] = \frac{1}{x}, \quad x > 0$$
2.
$$\frac{d}{dx} [\ln u] = \frac{1}{u} \frac{du}{dx} = \frac{u'}{u}, \quad u > 0$$

Derivative Involving Absolute Value

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx} [\ln |u|] = \frac{u'}{u}.$$