Calculus II

Arian DK

January 18, 2026

Contents

5 Logarithmic Exponential and Other Transcedental Functions

5.1 The Natural Logarithmic Function: Differentiation

Definition of the Natural Logarithmic Function

The **natural logarithmic function** is defined by

$$\ln x = \int_1^x \frac{1}{t}\,dt, \qquad x > 0.$$

The domain of the natural logarithmic function is the set of all positive real numbers.

Properties of the Natural Logarithmic Function

The natural logarithmic function has three important properties.

1. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.
2. The function is continuous, increasing, and one-to-one.
3. The graph is concave downward.

Logarithmic Properties

If a and b are positive numbers and n is rational, then the four properties below are true.

1. $\ln 1 = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Definition of e

The letter e denotes the positive real number such that

$$\ln e = \int_1^e \frac{1}{t}\,dt = 1.$$

Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x.

1. $$\frac{d}{dx}\big[\ln x\big] = \frac{1}{x}, \qquad x > 0$$
2. $$\frac{d}{dx}\big[\ln u\big] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}, \qquad u > 0$$

Derivative Involving Absolute Value

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}\left[\ln|u|\right] = \frac{u'}{u}.$$

5.2 The Natural Logarithmic Function: Integration

Log Rule for Integration

Let u be a differentiable function of x.

1. $$\int \frac{1}{x}\,dx = \ln|x| + C$$

2. $$\int \frac{1}{u}\,du = \ln|u| + C$$

Alternative Form of the Log Rule

Since $\dfrac{du}{dx} = u'$, we can write the Log Rule in the following useful form:

$$\int \frac{u'}{u}\,dx = \ln|u| + C.$$

Guidelines for Integration

1. Learn a basic list of integration formulas.
2. Find an integration formula that resembles all or part of the integrand and, by trial and error, find a choice of u that will make the integrand conform to the formula.
3. When you cannot find a u-substitution that works, try altering the integrand.

   You might try a trigonometric identity, multiplication and division by the same quantity, addition and subtraction of the same quantity, or long division.
4. When a graphing utility that finds antiderivatives symbolically is available, use it.
5. Check your result by differentiating to obtain the original integrand.

Integrals of the Six Basic Trigonometric Functions

$$\int \sin u \, du = -\cos u + C \qquad \int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C \qquad \int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C \qquad \int \csc u \, du = -\ln|\csc u + \cot u| + C$$