

# Calculus II

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## Contents

<b>6</b>	<b>Logarithmic Exponential and Other Transcendental Functions</b>	<b>2</b>
6.1	The Natural Logarithmic Function: Differentiation . . . . .	2

## 6 Logarithmic Exponential and Other Transcendental Functions

### 6.1 The Natural Logarithmic Function: Differentiation

#### Definition of the Natural Logarithmic Function

The **natural logarithmic function** is defined by (fill in the function and bound)

$$\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0.$$

The domain of the natural logarithmic function is the set of all positive real numbers.

#### Properties of the Natural Logarithmic Function

The natural logarithmic function has three important properties (cross out the word that is incorrect).

1. The domain is  $(-\infty, \infty)$  and the range is  $(-\infty, \infty)$ .
2. The function is discrete/continuous, decreasing/increasing, and one-to-one.
3. The graph is convex/concave downward.

#### Logarithmic Properties

If  $a$  and  $b$  are positive numbers and  $n$  is rational, then the four properties below are true.

- |                |                                    |
|----------------|------------------------------------|
| 1. $\ln 1 =$   | 3. $\ln(a^n) =$                    |
| 2. $\ln(ab) =$ | 4. $\ln\left(\frac{a}{b}\right) =$ |

#### Definition of $e$

The letter  $e$  denotes the positive real number such that

$$\ln e = \int_1^e \frac{1}{t} dt = 1.$$

#### Derivative of the Natural Logarithmic Function

Let  $u$  be a differentiable function of  $x$ .

1. 
$$\frac{d}{dx} [\ln x] = \frac{1}{x}, \quad x > 0$$
2. 
$$\frac{d}{dx} [\ln u] = \frac{1}{u} \frac{du}{dx} = \frac{du}{u dx}, \quad u > 0$$

## Derivative Involving Absolute Value

If  $u$  is a differentiable function of  $x$  such that  $u \neq 0$ , then

$$\frac{d}{dx} [\ln |u|] = \frac{u'}{u}.$$