

RootFindingMethods

Comparison of different functions

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$$f(x) = x^2 - 3$$
$$g(x) = x - 1/4(x^2 - 3)$$

Bisection Method Iteration Table:

Iteration	a	b	c	f(c)	Error	$ x_{n+1} - x_n $
1	1.000000	2.000000	1.500000	-0.750000		0.250000
2	1.500000	2.000000	1.750000	0.062500		0.125000
3	1.500000	1.750000	1.625000	-0.359375		0.062500
4	1.625000	1.750000	1.687500	-0.152344		0.031250
5	1.687500	1.750000	1.718750	-0.045898		0.015625
6	1.718750	1.750000	1.734375	0.008057		0.007812
7	1.718750	1.734375	1.726562	-0.018982		0.003906
8	1.726562	1.734375	1.730469	-0.005478		0.001953
9	1.730469	1.734375	1.732422	0.001286		0.000977
10	1.730469	1.732422	1.731445	-0.002097		0.000488

Stopped at iteration 10 after reaching max iterations (10)

Bisection Method with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	1.666667	-0.222222	0.065384
2	1.666667	-0.222222	0.065384
3	1.750000	0.062500	0.017949
4	1.750000	0.062500	0.017949
5	1.729167	-0.009983	0.002884
6	1.729167	-0.009983	0.002884
7	1.734375	0.008057	0.002324
8	1.731771	-0.000970	0.000280

Newton-Raphson Method Iteration Table:

Iteration	x_n	f(x_n)	f'(x_n)	Error	$ x_{n+1} - x_n $
1	1.500000	-0.750000	3.000000		0.250000
2	1.750000	0.062500	3.500000		0.017857
3	1.732143	0.000319	3.464286		0.000092
4	1.732051	0.000000	3.464102		0.000000

Stopped at iteration 4 because $|x_{n+1} - x_n| = 2.445850e-09 < \epsilon = 1.000000e-06$

Newton-Raphson Method with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	1.733333	0.004444	0.001283
2	1.732050	-0.000002	0.000000

Fixed Point Iteration Method Table:			
Iteration	x_n	g(x_n)	Error x_{n+1} - x_n
1	1.500000	1.687500	0.187500
2	1.687500	1.725586	0.038086
3	1.725586	1.731174	0.005588
4	1.731174	1.731933	0.000759
5	1.731933	1.732035	0.000102
6	1.732035	1.732049	0.000014
7	1.732049	1.732051	0.000002
8	1.732051	1.732051	0.000000

Stopped at iteration 8 because $|x_{n+1} - x_n| = 2.450361e-07 < \epsilon = 1.000000e-06$

Fixed Point Method with Aitken Acceleration:			
Iteration	x_hat	f(x_hat)	Error
1	1.735294	0.011246	0.003243
2	1.732135	0.000292	0.000094
3	1.732052	0.000006	0.000002
4	1.732051	0.000000	0.000000
5	1.732051	0.000000	0.000000
6	1.732051	0.000000	0.000000

False Position Method Iteration Table:

Iteration	a	b	c	f(c)	Error x_{n+1} - x_n
1	1.000000	2.000000	1.666667	-0.222222	0.060606
2	1.666667	2.000000	1.727273	-0.016529	0.004435
3	1.727273	2.000000	1.731707	-0.001190	0.000319
4	1.731707	2.000000	1.732026	-0.000085	0.000023
5	1.732026	2.000000	1.732049	-0.000006	0.000002
6	1.732049	2.000000	1.732051	-0.000000	0.000000

Stopped at iteration 6 because $|x_{n+1} - x_n| = 1.180091e-07 < \epsilon = 1.000000e-06$

False Position with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	1.732057	0.000023	0.000007
2	1.732051	0.000000	0.000000
3	1.732051	0.000000	0.000000
4	1.732051	0.000000	0.000000

Secant Method Iteration Table:

Iteration	x_n	f(x_n)	Error x_{n+1} - x_n
1	2.000000	1.000000	0.285714
2	1.714286	-0.061224	0.016484
3	1.730769	-0.004438	0.001288
4	1.732057	0.000023	0.000007
5	1.732051	-0.000000	0.000000

Stopped at iteration 5 because $|x_{n+1} - x_n| = 2.445846e-09 < \epsilon = 1.000000e-06$

Secant Method with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	1.729870	-0.007549	0.002181
2	1.732167	0.000401	0.000116
3	1.732051	0.000000	0.000000

Comparison of Root-Finding Methods:

Method	Iterations	Root	Abs Error x - x_true	Stopping Error	Time (s)
Bisection	10	1.7314453125000000000	0.0006054950688725302	0.0004882812500000000	0.000010
Newton-Raphson	4	1.7320508100147276043	0.0000000024458550740	0.0000000024458504111	0.005084
Fixed Point	8	1.7320505246255208931	0.0000002829433516371	0.0000002450361145101	0.000005
False Position	6	1.7320506804317221672	0.0000001271371503631	0.0000001180091178110	0.000010
Secant	5	1.7320508051230270041	0.0000000024458455261	0.0000000024458457482	0.000005

Secant Method with Last Two Newton-Raphson Iterations ($x_0 = 1.732143$, $x_1 = 1.732051$):

Iteration	x_n	f(x_n)	Error x_{n+1} - x_n
1	1.732051	0.000000	0.000000

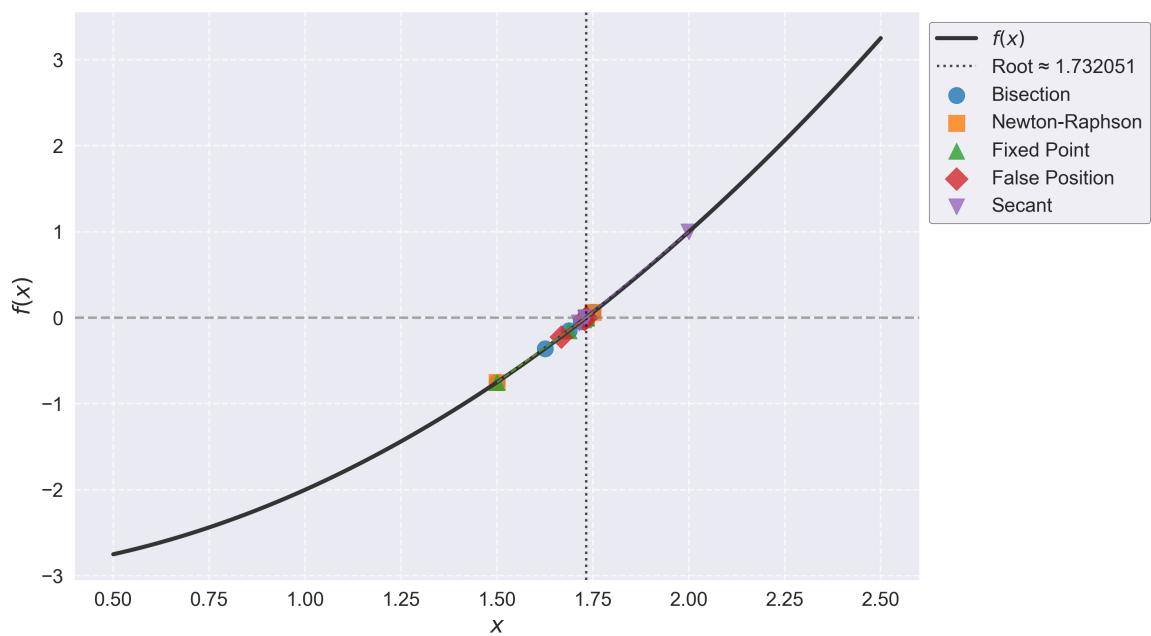
Stopped at iteration 1 because $|x_{n+1} - x_n| = 2.445785e-09 < \epsilon = 1.000000e-06$

Secant Method with Last Two Bisection Iterations ($x_0 = 1.732422$, $x_1 = 1.731445$):

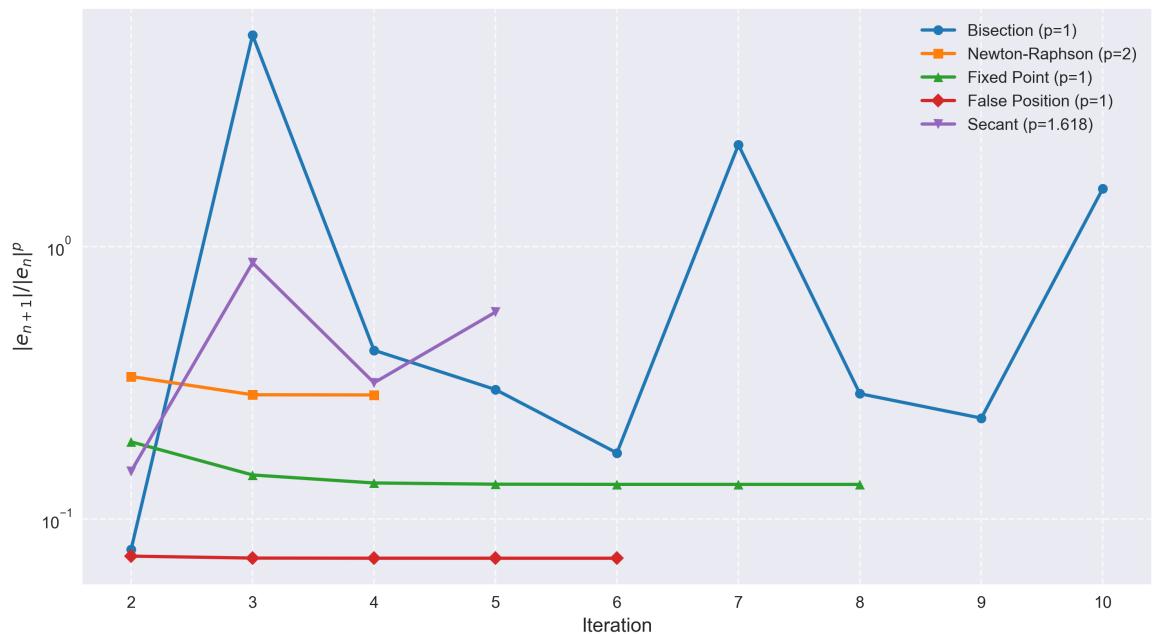
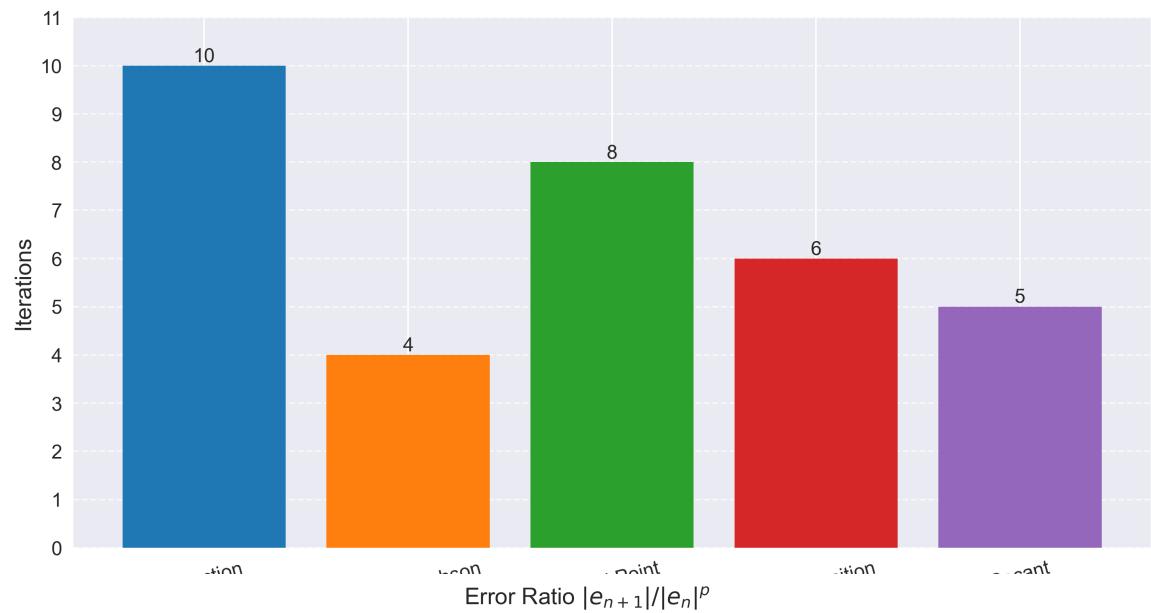
Iteration	x_n	f(x_n)	Error x_{n+1} - x_n
1	1.731445	-0.002097	0.000605
2	1.732051	-0.000000	0.000000

Stopped at iteration 2 because $|x_{n+1} - x_n| = 6.487511e-08 < \epsilon = 1.000000e-06$

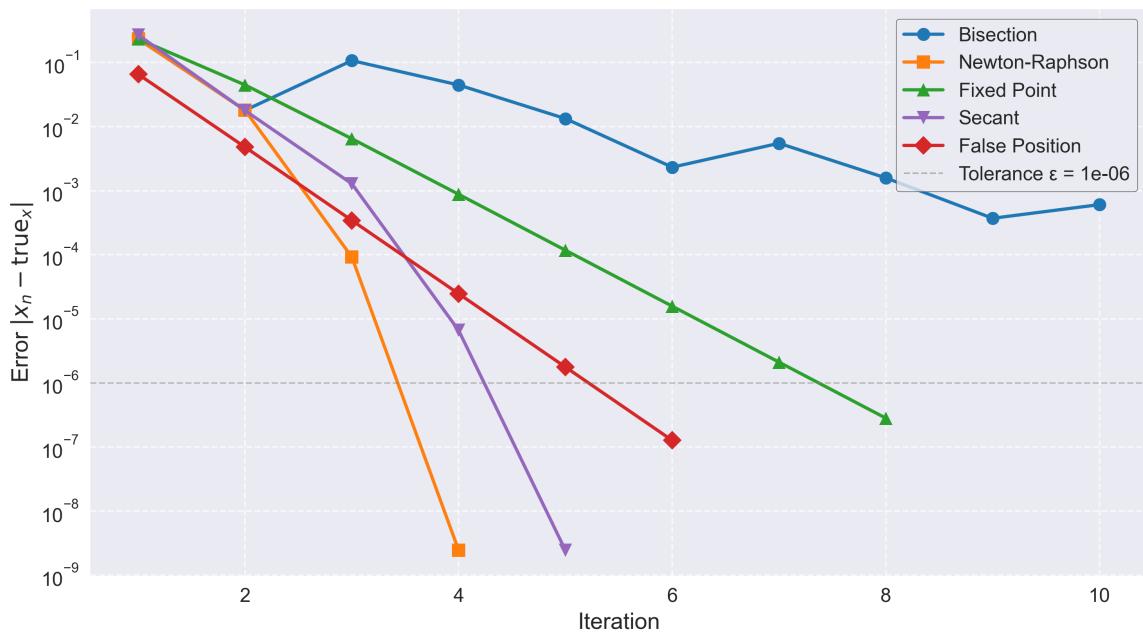
Trajectory of Root-Finding Methods for $f(x)$



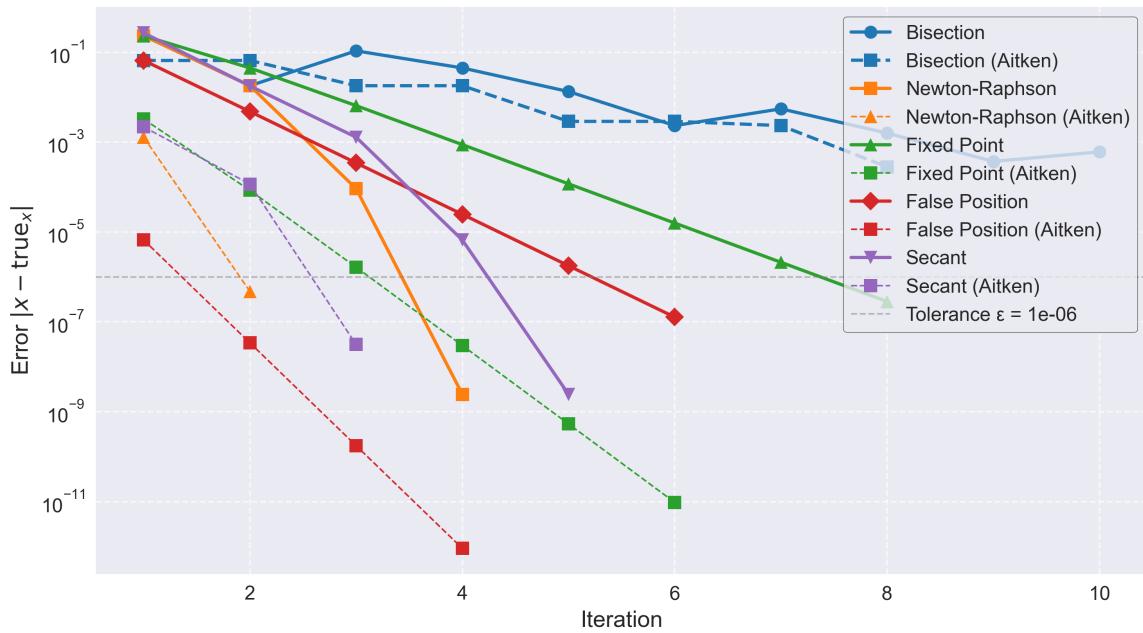
Number of Iterations for Root-Finding Methods



Convergence of Root-Finding Methods for $f(x)$



Aitken's Acceleration Error Reduction for $f(x)$





$$f(x) = x^{**4} - 2*x^{**3} + x - 1$$
$$g(x) = (2*x^{**3} - x + 1)^{**}(1/4)$$

Bisection Method Iteration Table:

Iteration	a	b	c	f(c)	Error	$ x_{n+1} - x_n $
1	1.000000	2.000000	1.500000	-1.187500		0.250000
2	1.500000	2.000000	1.750000	-0.589844		0.125000
3	1.750000	2.000000	1.875000	0.051025		0.062500
4	1.750000	1.875000	1.812500	-0.303940		0.031250
5	1.812500	1.875000	1.843750	-0.135573		0.015625
6	1.843750	1.875000	1.859375	-0.044615		0.007812
7	1.859375	1.875000	1.867188	0.002612		0.003906
8	1.859375	1.867188	1.863281	-0.021148		0.001953
9	1.863281	1.867188	1.865234	-0.009305		0.000977
10	1.865234	1.867188	1.866211	-0.003356		0.000488

Stopped at iteration 10 after reaching max iterations (10)

Bisection Method with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	2.000000	1.000000	0.133240
2	1.833333	-0.193673	0.033427
3	1.833333	-0.193673	0.033427
4	1.875000	0.051025	0.008240
5	1.875000	0.051025	0.008240
6	1.864583	-0.013261	0.002177
7	1.864583	-0.013261	0.002177
8	1.867188	0.002612	0.000427

Newton-Raphson Method Iteration Table:

Iteration	x_n	$f(x_n)$	$f'(x_n)$	Error	$ x_{n+1} - x_n $
1	1.800000	-0.366400	4.888000		0.074959
2	1.874959	0.050769	6.272632		0.008094
3	1.866865	0.000642	6.114377		0.000105
4	1.866760	0.000000	6.112339		0.000000

Stopped at iteration 4 because $|x_{n+1} - x_n| = 1.750630e-08 < \varepsilon = 1.000000e-06$

Newton-Raphson Method with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	1.867654	0.005471	0.000894
2	1.866759	-0.000008	0.000001

Fixed Point Iteration Method Table:				
Iteration	x_n	g(x_n)	Error x_{n+1} - x_n	
1	1.800000	1.815505	0.015505	
2	1.815505	1.827440	0.011935	
3	1.827440	1.836615	0.009175	
4	1.836615	1.843660	0.007045	
5	1.843660	1.849065	0.005405	
6	1.849065	1.853209	0.004144	
7	1.853209	1.856385	0.003176	
8	1.856385	1.858818	0.002433	
9	1.858818	1.860681	0.001863	
10	1.860681	1.862108	0.001427	

Stopped at iteration 10 after reaching max iterations (10)

Fixed Point Method with Aitken Acceleration:				
Iteration	x_hat	f(x_hat)	Error	
1	1.867348	0.003594	0.000587	
2	1.867106	0.002112	0.000345	
3	1.866963	0.001240	0.000203	
4	1.866879	0.000727	0.000119	
5	1.866830	0.000426	0.000070	
6	1.866801	0.000250	0.000041	
7	1.866784	0.000146	0.000024	
8	1.866774	0.000086	0.000014	

False Position Method Iteration Table:

False Position Method Iteration Table:					
Iteration	a	b	c	f(c)	Error x_{n+1} - x_n
1	1.000000	2.000000	1.500000	-1.187500	0.271429
2	1.500000	2.000000	1.771429	-0.499125	0.076102
3	1.771429	2.000000	1.847530	-0.113991	0.015602
4	1.847530	2.000000	1.863132	-0.022052	0.002953
5	1.863132	2.000000	1.866085	-0.004125	0.000550
6	1.866085	2.000000	1.866635	-0.000767	0.000102
7	1.866635	2.000000	1.866737	-0.000142	0.000019
8	1.866737	2.000000	1.866756	-0.000026	0.000004
9	1.866756	2.000000	1.866760	-0.000005	0.000001

Stopped at iteration 9 because $|x_{n+1} - x_n| = 6.532294e-07 < \epsilon = 1.000000e-06$

False Position with Aitken Acceleration:

False Position with Aitken Acceleration:			
Iteration	x_hat	f(x_hat)	Error
1	1.877180	0.064749	0.010420
2	1.867155	0.002415	0.000395
3	1.866774	0.000085	0.000014
4	1.866761	0.000003	0.000000
5	1.866760	0.000000	0.000000
6	1.866760	0.000000	0.000000
7	1.866760	0.000000	0.000000

Secant Method Iteration Table:

Secant Method Iteration Table:				
Iteration	x_n	f(x_n)	Error x_{n+1} - x_n	
1	2.000000	1.000000	0.146370	
2	1.853630	-0.078596	0.010666	
3	1.864296	-0.015006	0.002517	
4	1.866813	0.000319	0.000052	
5	1.866760	-0.000001	0.000000	

Stopped at iteration 5 because $|x_{n+1} - x_n| = 2.049552e-07 < \epsilon = 1.000000e-06$

Secant Method with Aitken Acceleration:

Secant Method with Aitken Acceleration:			
Iteration	x_hat	f(x_hat)	Error
1	1.863571	-0.019394	0.003189
2	1.867590	0.005077	0.000830
3	1.866761	0.000005	0.000001

Comparison of Root-Finding Methods:

Method	Iterations	Root	Abs Error x - x_true	Stopping Error	Time (s)
Bisection	10	1.866210937500000000	0.0005494616738583868	0.0004882812500000000	0.000012
Newton-Raphson	4	1.8667604166801643473	0.0000000175063059604	0.0000000175063017416	0.008677
Fixed Point	10	1.8621076010350379182	0.0046527981388204687	0.0014265621219990976	0.000007
False Position	9	1.8667595970801815763	0.00000008020936768105	0.0000006532294172246	0.000017
Secant	5	1.8667601942017035821	0.0000002049721548048	0.0000002049551548478	0.000005

Secant Method with Last Two Newton-Raphson Iterations ($x_0 = 1.866865$, $x_1 = 1.866760$):

Secant Method with Last Two Newton-Raphson Iterations ($x_0 = 1.866865$, $x_1 = 1.866760$):			
Iteration	x_n	f(x_n)	Error x_{n+1} - x_n
1	1.866760	0.000000	0.000000

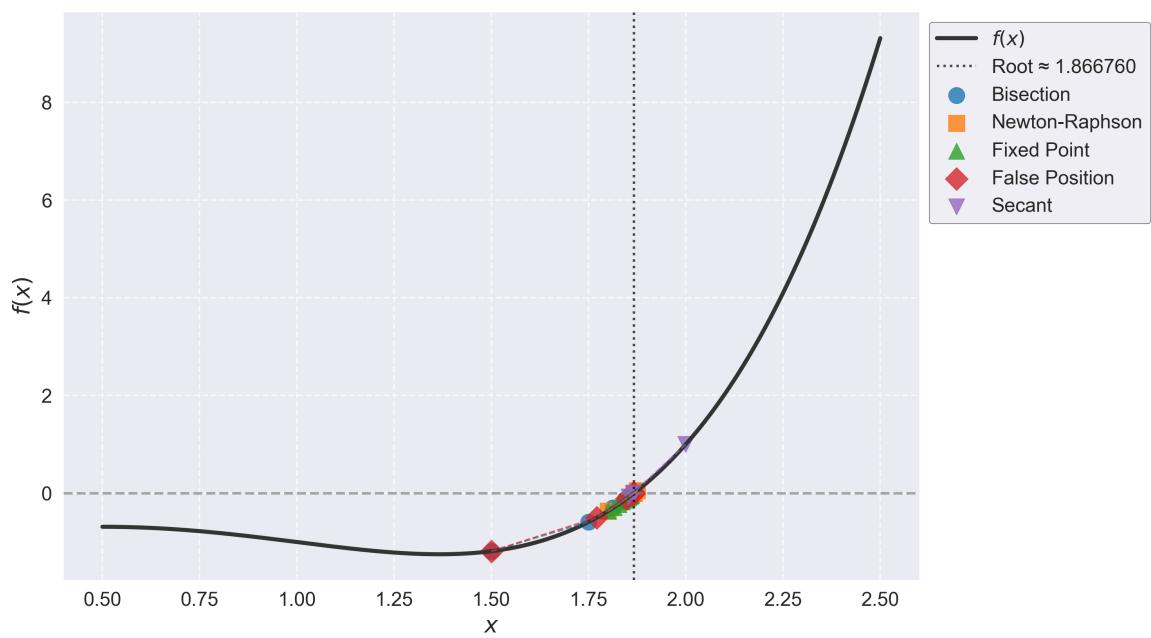
Stopped at iteration 1 because $|x_{n+1} - x_n| = 1.750338e-08 < \epsilon = 1.000000e-06$

Secant Method with Last Two Bisection Iterations ($x_0 = 1.865234$, $x_1 = 1.866211$):

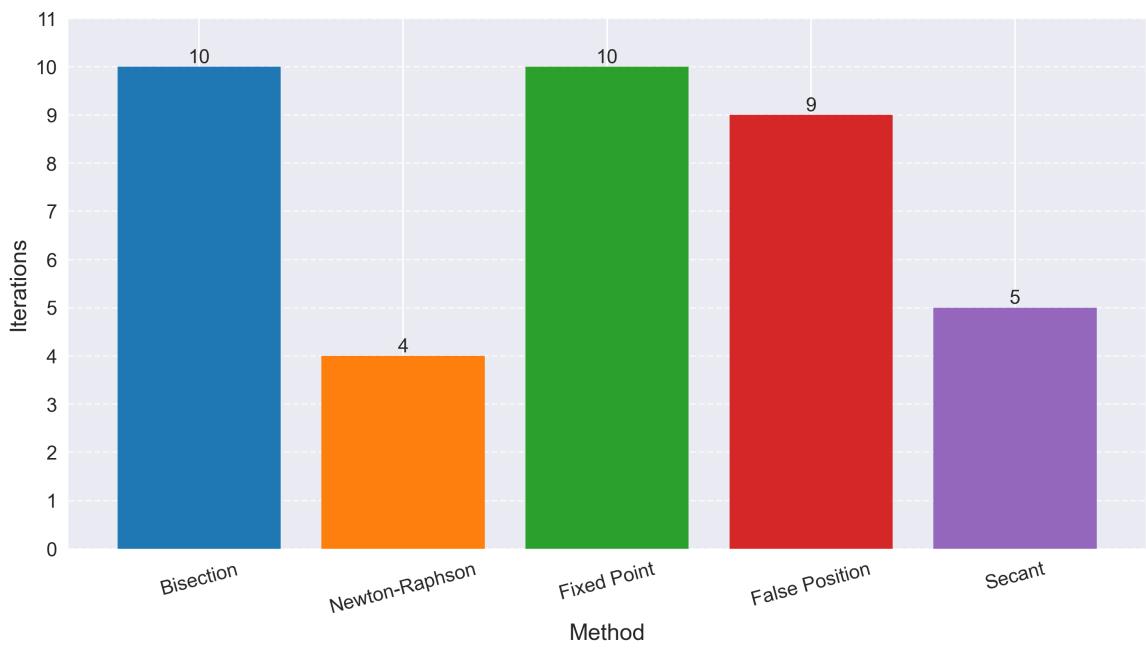
Secant Method with Last Two Bisection Iterations ($x_0 = 1.865234$, $x_1 = 1.866211$):			
Iteration	x_n	f(x_n)	Error x_{n+1} - x_n
1	1.866211	-0.003356	0.000551
2	1.866762	0.000008	0.000001
3	1.866760	-0.000000	0.000000

Stopped at iteration 3 because $|x_{n+1} - x_n| = 1.165379e-09 < \epsilon = 1.000000e-06$

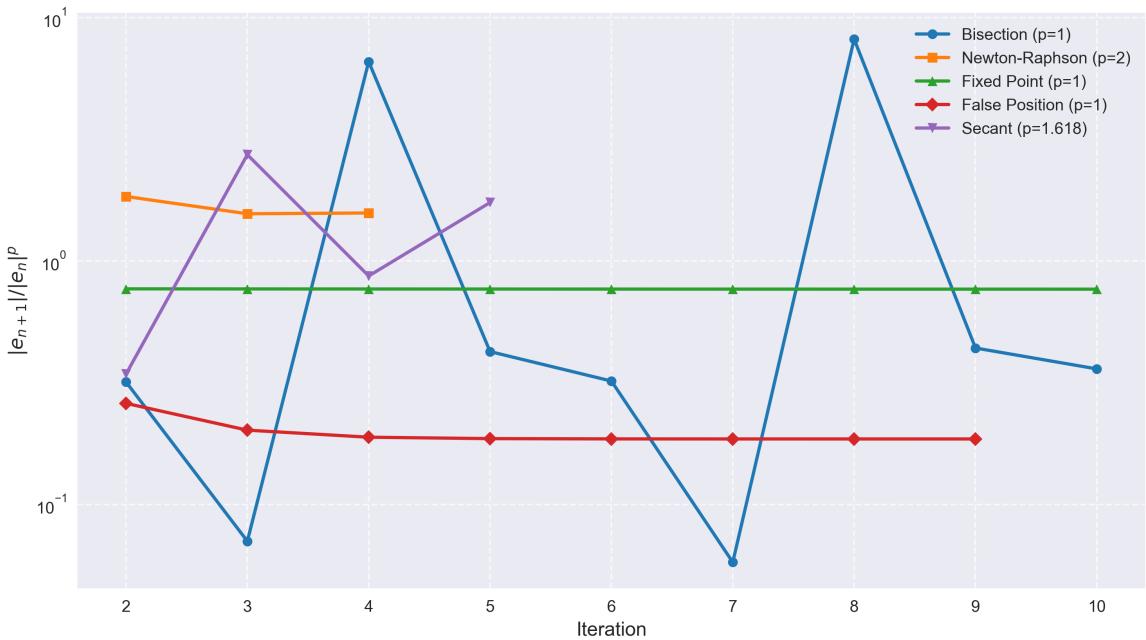
Trajectory of Root-Finding Methods for $f(x)$



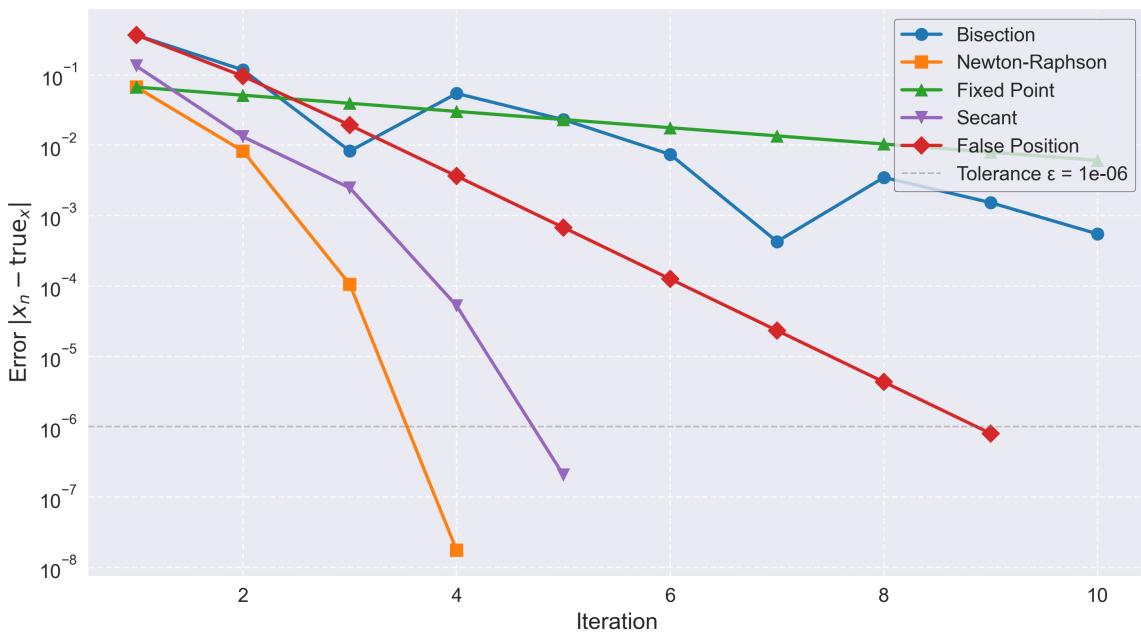
Number of Iterations for Root-Finding Methods



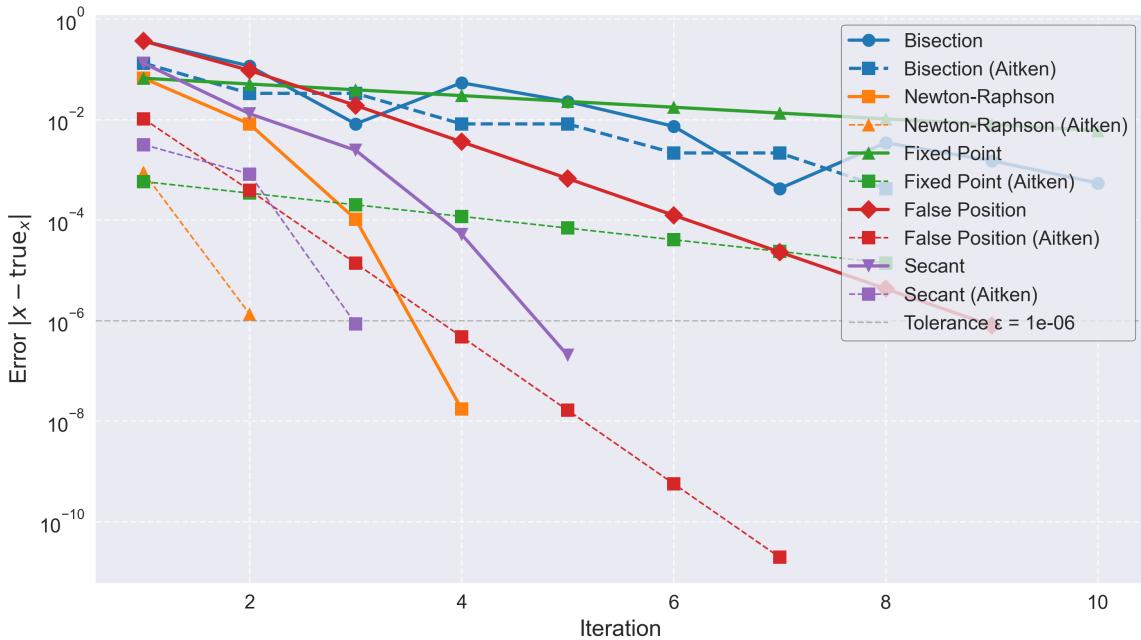
Error Ratio $|e_{n+1}|/|e_n|^p$



Convergence of Root-Finding Methods for $f(x)$



Aitken's Acceleration Error Reduction for $f(x)$



```
f(x) = np.exp(x) - x - 2
g(x) = np.log(x + 2)
```

Bisection Method Iteration Table:

Iteration	a	b	c	f(c)	Error	$ x_{n+1} - x_n $
1	1.000000	2.000000	1.500000	0.981689	0.250000	
2	1.000000	1.500000	1.250000	0.240343	0.125000	
3	1.000000	1.250000	1.125000	-0.044783	0.062500	
4	1.125000	1.250000	1.187500	0.091374	0.031250	
5	1.125000	1.187500	1.156250	0.021743	0.015625	
6	1.125000	1.156250	1.140625	-0.011902	0.007812	
7	1.140625	1.156250	1.148438	0.004825	0.003906	
8	1.140625	1.148438	1.144531	-0.003563	0.001953	
9	1.144531	1.148438	1.146484	0.000625	0.000977	
10	1.144531	1.146484	1.145508	-0.001470	0.000488	

Stopped at iteration 10 after reaching max iterations (10)

Bisection Method with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	1.000000	-0.281718	0.146193
2	1.166667	0.044604	0.020473
3	1.166667	0.044604	0.020473
4	1.125000	-0.044783	0.021193
5	1.145833	-0.000772	0.000360
6	1.145833	-0.000772	0.000360
7	1.145833	-0.000772	0.000360
8	1.145833	-0.000772	0.000360

Newton-Raphson Method Iteration Table:

Iteration	x_n	f(x_n)	f'(x_n)	Error	$ x_{n+1} - x_n $
1	1.800000	2.249647	5.049647	0.445506	
2	1.354494	0.520306	2.874800	0.180989	
3	1.173506	0.059802	2.233307	0.026777	
4	1.146728	0.001149	2.147877	0.000535	
5	1.146193	0.000000	2.146194	0.000000	

Stopped at iteration 5 because $|x_{n+1} - x_n| = 2.097877e-07 < \epsilon = 1.000000e-06$

Newton-Raphson Method with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	1.049669	-0.192964	0.096524
2	1.142079	-0.008804	0.004114
3	1.146183	-0.000023	0.000011

Fixed Point Iteration Method Table:

Iteration	x_n	$g(x_n)$	Error $ x_{n+1} - x_n $
1	1.800000	1.335001	0.464999
2	1.335001	1.204473	0.130528
3	1.204473	1.164548	0.039925
4	1.164548	1.152010	0.012538
5	1.152010	1.148040	0.003970
6	1.148040	1.146780	0.001260
7	1.146780	1.146380	0.000400
8	1.146380	1.146253	0.000127
9	1.146253	1.146212	0.000040
10	1.146212	1.146199	0.000013

Stopped at iteration 10 after reaching max iterations (10)

Fixed Point Method with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	1.153534	0.015840	0.007341
2	1.146954	0.001634	0.000761
3	1.146271	0.000166	0.000078
4	1.146201	0.000017	0.000008
5	1.146194	0.000002	0.000001
6	1.146193	0.000000	0.000000
7	1.146193	0.000000	0.000000
8	1.146193	0.000000	0.000000

False Position Method Iteration Table:

Iteration	a	b	c	$f(c)$	Error $ x_{n+1} - x_n $
1	1.000000	2.000000	1.076746	-0.141632	0.037036
2	1.076746	2.000000	1.113782	-0.067925	0.017413
3	1.113782	2.000000	1.131195	-0.031836	0.008085
4	1.131195	2.000000	1.139281	-0.014760	0.003732
5	1.139281	2.000000	1.143013	-0.006809	0.001718
6	1.143013	2.000000	1.144732	-0.003134	0.000798
7	1.144732	2.000000	1.145522	-0.001441	0.000363
8	1.145522	2.000000	1.145885	-0.000662	0.000167
9	1.145885	2.000000	1.146052	-0.000304	0.000077
10	1.146052	2.000000	1.146128	-0.000140	0.000035

Stopped at iteration 10 after reaching max iterations (10)

False Position with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	1.146647	0.000975	0.000454
2	1.146290	0.000207	0.000096
3	1.146214	0.000044	0.000020
4	1.146198	0.000009	0.000004
5	1.146194	0.000002	0.000001
6	1.146193	0.000000	0.000000
7	1.146193	0.000000	0.000000
8	1.146193	0.000000	0.000000

Secant Method Iteration Table:

Iteration	x_n	$f(x_n)$	Error $ x_{n+1} - x_n $
1	2.000000	3.389056	0.594880
2	1.405120	0.670896	0.146828
3	1.258292	0.261113	0.093559
4	1.164733	0.040335	0.017093
5	1.147641	0.003110	0.001428
6	1.146213	0.000042	0.000019
7	1.146193	0.000000	0.000000

Stopped at iteration 7 because $|x_{n+1} - x_n| = 2.069384e-08 < \epsilon = 1.000000e-06$

Secant Method with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	1.210176	0.143898	0.063982
2	1.0000414	-0.281006	0.145779
3	1.143820	-0.005084	0.002373
4	1.146083	-0.000238	0.000111
5	1.146193	-0.000001	0.000000

Comparison of Root-Finding Methods:

Method	Iterations	Root	Abs Error $ x - x_{true} $	Stopping Error	Time (s)
Bisection	10	1.145507812500000000	0.0006854081206209006	0.0004882812500000000	0.000025
Newton-Raphson	5	1.1461934304826704438	0.000002097876461438	0.000002097876523610	0.007834
Fixed Point	10	1.1461992101392239896	0.0000059895186030889	0.0000128547207367546	0.000012
False Position	10	1.1461281297699881243	0.00000650908506327763	0.0000351936324169966	0.000054
Secant	7	1.1461932413147208898	0.0000000206940999892	0.0000000206938421954	0.000013

Secant Method with Last Two Newton-Raphson Iterations ($x_0 = 1.146728$, $x_1 = 1.146193$):

Iteration	x_n	$f(x_n)$	Error $ x_{n+1} - x_n $
1	1.146193	0.000000	0.000000

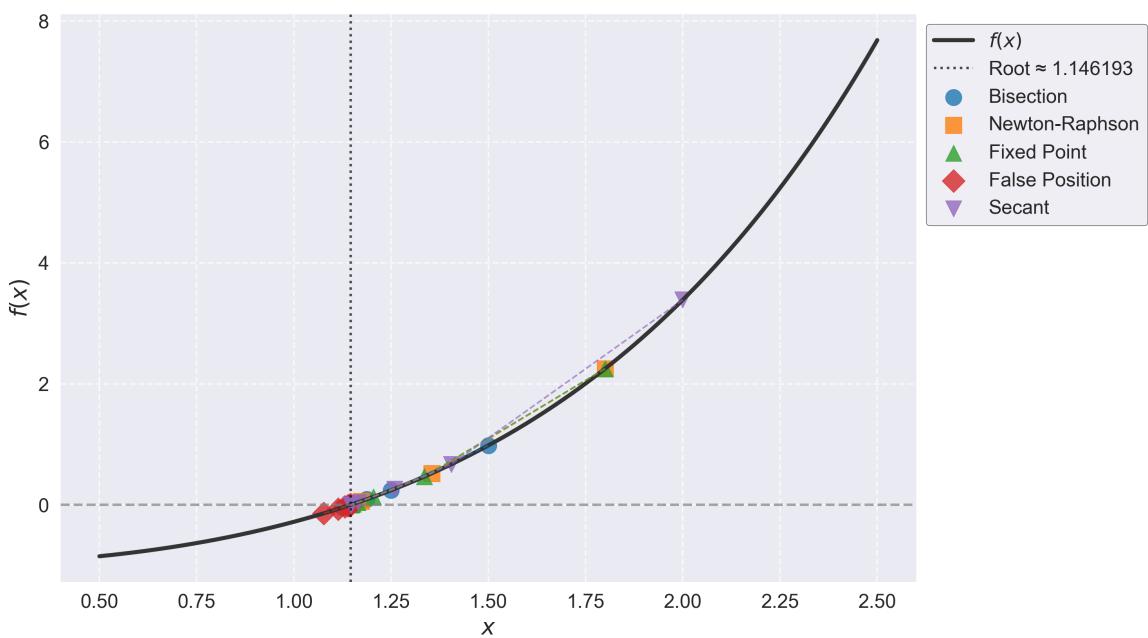
Stopped at iteration 1 because $|x_{n+1} - x_n| = 2.097054e-07 < \epsilon = 1.000000e-06$

Secant Method with Last Two Bisection Iterations ($x_0 = 1.146484$, $x_1 = 1.145508$):

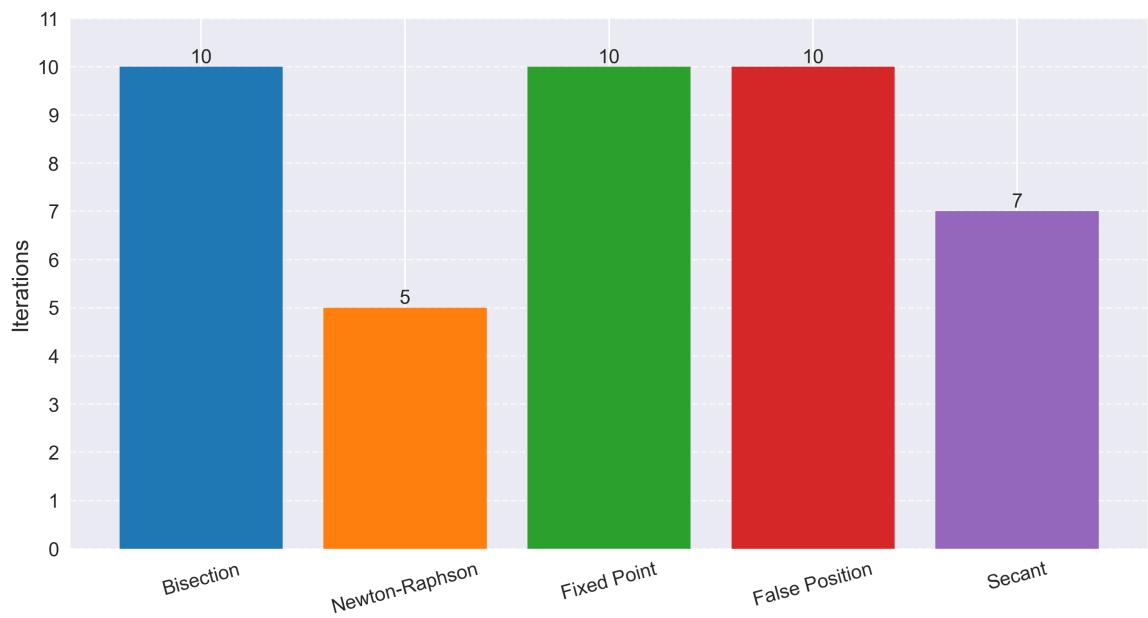
Iteration	x_n	$f(x_n)$	Error $ x_{n+1} - x_n $
1	1.145508	-0.001470	0.000685
2	1.146193	-0.000000	0.000000

Stopped at iteration 2 because $|x_{n+1} - x_n| = 1.463679e-07 < \epsilon = 1.000000e-06$

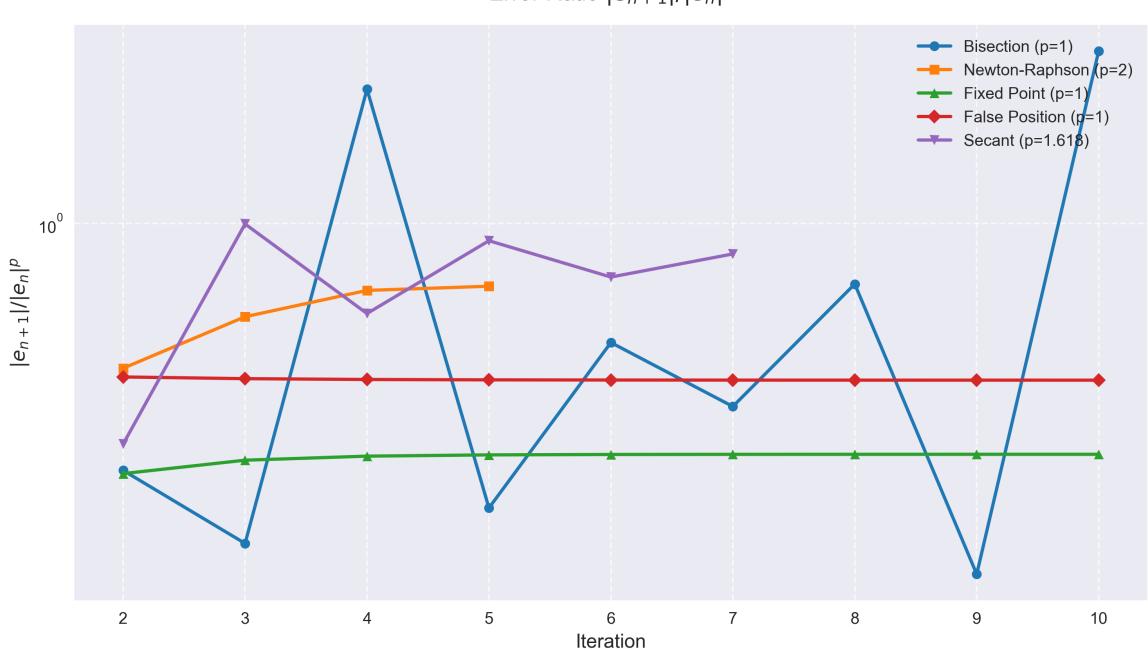
Trajectory of Root-Finding Methods for $f(x)$



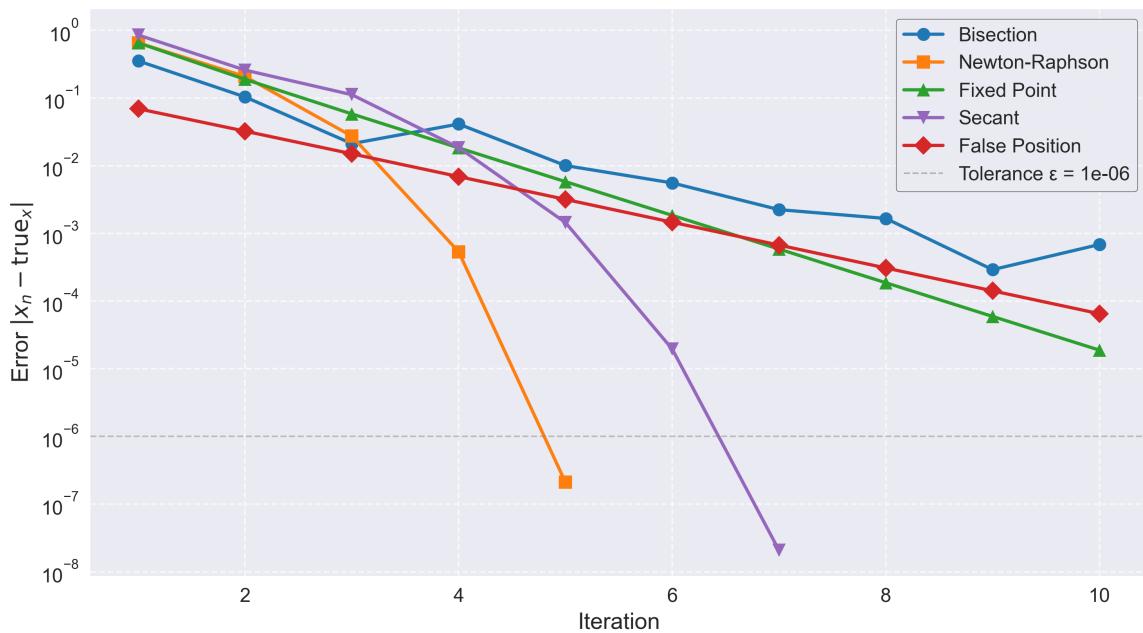
Number of Iterations for Root-Finding Methods



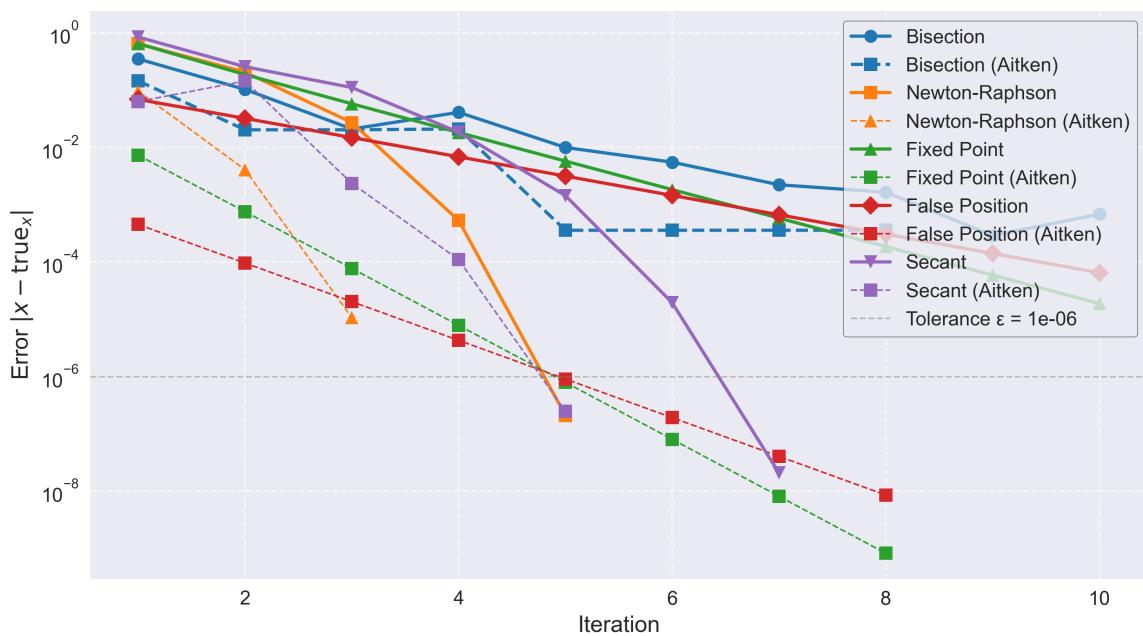
Error Ratio $|e_{n+1}|/|e_n|^p$



Convergence of Root-Finding Methods for $f(x)$



Aitken's Acceleration Error Reduction for $f(x)$





$$f(x) = x^{**} 3 - np.cos(x)$$
$$g(x) = np.cos(x)^{**} (1 / 3)$$

Bisection Method Iteration Table:

Iteration	a	b	c	f(c)	Error	$ x_{n+1} - x_n $
1	0.000000	1.000000	0.500000	-0.752583	0.250000	
2	0.500000	1.000000	0.750000	-0.309814	0.125000	
3	0.750000	1.000000	0.875000	0.028925	0.062500	
4	0.750000	0.875000	0.812500	-0.151309	0.031250	
5	0.812500	0.875000	0.843750	-0.063988	0.015625	
6	0.843750	0.875000	0.859375	-0.018241	0.007812	
7	0.859375	0.875000	0.867188	0.005164	0.003906	
8	0.859375	0.867188	0.863281	-0.006583	0.001953	
9	0.863281	0.867188	0.865234	-0.000721	0.000977	
10	0.865234	0.867188	0.866211	0.002219	0.000488	

Stopped at iteration 10 after reaching max iterations (10)

Bisection Method with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	1.000000	0.459698	0.134526
2	0.833333	-0.093709	0.032141
3	0.833333	-0.093709	0.032141
4	0.875000	0.028925	0.009526
5	0.875000	0.028925	0.009526
6	0.864583	-0.002677	0.000891
7	0.864583	-0.002677	0.000891
8	0.867188	0.005164	0.001713

Newton-Raphson Method Iteration Table:

Iteration	x_n	f(x_n)	f'(x_n)	Error	$ x_{n+1} - x_n $
1	0.900000	0.107390	3.213327	0.033420	
2	0.866580	0.003330	3.015001	0.001105	
3	0.865475	0.000004	3.008545	0.000001	
4	0.865474	0.000000	3.008539	0.000000	

Stopped at iteration 4 because $|x_{n+1} - x_n| = 1.363687e-12 < \epsilon = 1.000000e-06$

Newton-Raphson Method with Aitken Acceleration:

Iteration	x_hat	f(x_hat)	Error
1	0.865437	-0.000110	0.000037
2	0.865474	-0.000000	0.000000

Fixed Point Iteration Method Table:

Iteration	x_n	$g(x_n)$	Error $ x_{n+1} - x_n $
1	0.900000	0.853439	0.046561
2	0.853439	0.869512	0.016073
3	0.869512	0.864101	0.005411
4	0.864101	0.865939	0.001837
5	0.865939	0.865317	0.000622
6	0.865317	0.865527	0.000211
7	0.865527	0.865456	0.000071
8	0.865456	0.865480	0.000024
9	0.865480	0.865472	0.000008
10	0.865472	0.865475	0.000003

Stopped at iteration 10 after reaching max iterations (10)

Fixed Point Method with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	0.865387	-0.000260	0.000087
2	0.865464	-0.000030	0.000010
3	0.865473	-0.000003	0.000001
4	0.865474	-0.000000	0.000000
5	0.865474	-0.000000	0.000000
6	0.865474	-0.000000	0.000000
7	0.865474	-0.000000	0.000000
8	0.865474	-0.000000	0.000000

False Position Method Iteration Table:

Iteration	a	b	c	$f(c)$	Error $ x_{n+1} - x_n $
1	0.000000	1.000000	0.685073	-0.452850	0.156282
2	0.685073	1.000000	0.841355	-0.070876	0.021192
3	0.841355	1.000000	0.862547	-0.008780	0.002576
4	0.862547	1.000000	0.865123	-0.001054	0.000309
5	0.865123	1.000000	0.865432	-0.000126	0.000037
6	0.865432	1.000000	0.865469	-0.000015	0.000004
7	0.865469	1.000000	0.865473	-0.000002	0.000001

Stopped at iteration 7 because $|x_{n+1} - x_n| = 5.279156e-07 < \epsilon = 1.000000e-06$

False Position with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	0.865872	0.001198	0.000398
2	0.865480	0.000018	0.000006
3	0.865474	0.000000	0.000000
4	0.865474	0.000000	0.000000
5	0.865474	0.000000	0.000000

Secant Method Iteration Table:

Iteration	x_n	$f(x_n)$	Error $ x_{n+1} - x_n $
1	1.000000	0.459698	0.130482
2	0.869518	0.012215	0.003562
3	0.865956	0.001452	0.000481
4	0.865476	0.000006	0.000002
5	0.865474	0.000000	0.000000

Stopped at iteration 5 because $|x_{n+1} - x_n| = 8.839505e-10 < \epsilon = 1.000000e-06$

Secant Method with Aitken Acceleration:

Iteration	$x_{\hat{n}}$	$f(x_{\hat{n}})$	Error
1	0.865857	0.001151	0.000382
2	0.865401	-0.000220	0.000073
3	0.865474	-0.000000	0.000000

Comparison of Root-Finding Methods:

Method	Iterations	Root	Abs Error $ x - x_{true} $	Stopping Error	Time (s)
Bisection	10	0.86621093750000000000	0.0007369043983856960	0.0004882812500000000	0.000029
Newton-Raphson	4	0.8654740331029781020	0.000000000000000013637980	0.000000000000000013636869	0.006856
Fixed Point	10	0.8654747363426771312	0.0000007032410628272	0.0000027787275512781	0.000011
False Position	7	0.8654734334829765618	0.0000005996186377422	0.0000005279155826932	0.000047
Secant	5	0.865474033985665487	0.0000000008839522447	0.0000000008839504684	0.000011

Secant Method with Last Two Newton-Raphson Iterations ($x_0 = 0.865475$, $x_1 = 0.865474$):

Iteration	x_n	$f(x_n)$	Error $ x_{n+1} - x_n $
1	0.865474	0.000000	0.000000

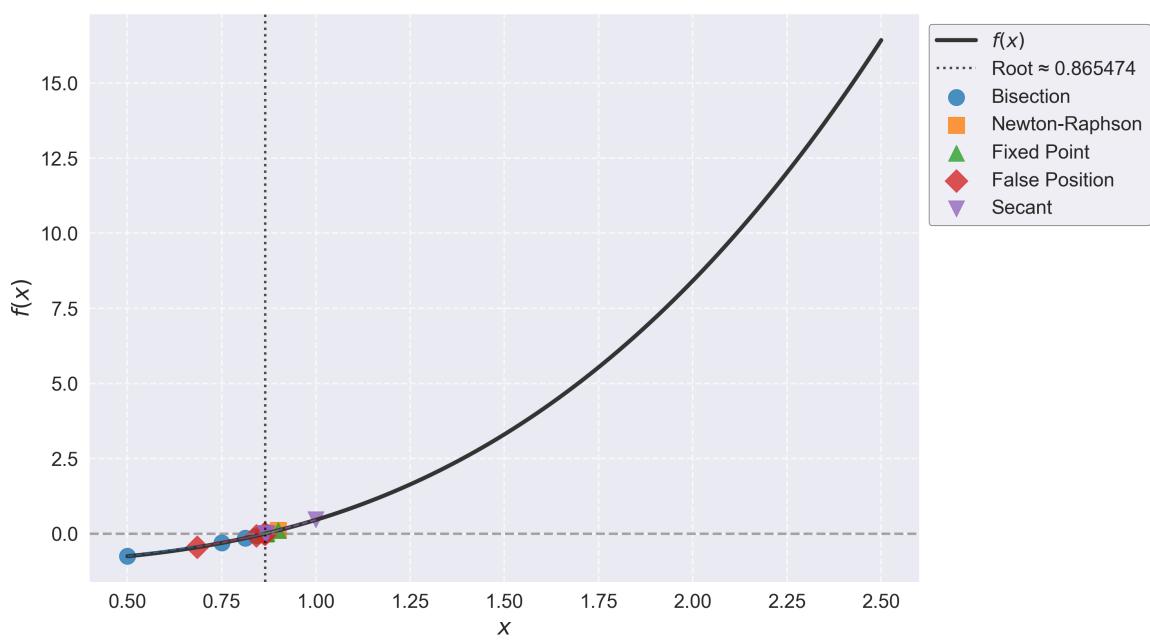
Stopped at iteration 1 because $|x_{n+1} - x_n| = 1.363687e-12 < \epsilon = 1.000000e-06$

Secant Method with Last Two Bisection Iterations ($x_0 = 0.865234$, $x_1 = 0.866211$):

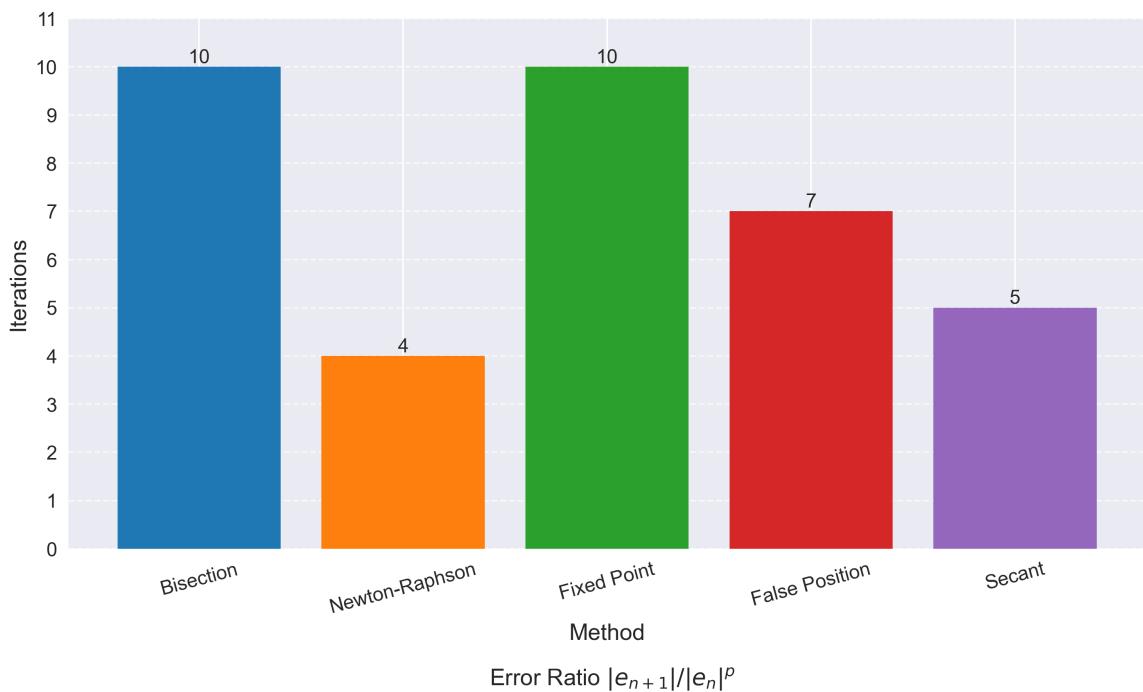
Iteration	x_n	$f(x_n)$	Error $ x_{n+1} - x_n $
1	0.866211	0.002219	0.000737
2	0.865474	-0.000001	0.000000

Stopped at iteration 2 because $|x_{n+1} - x_n| = 1.712610e-07 < \epsilon = 1.000000e-06$

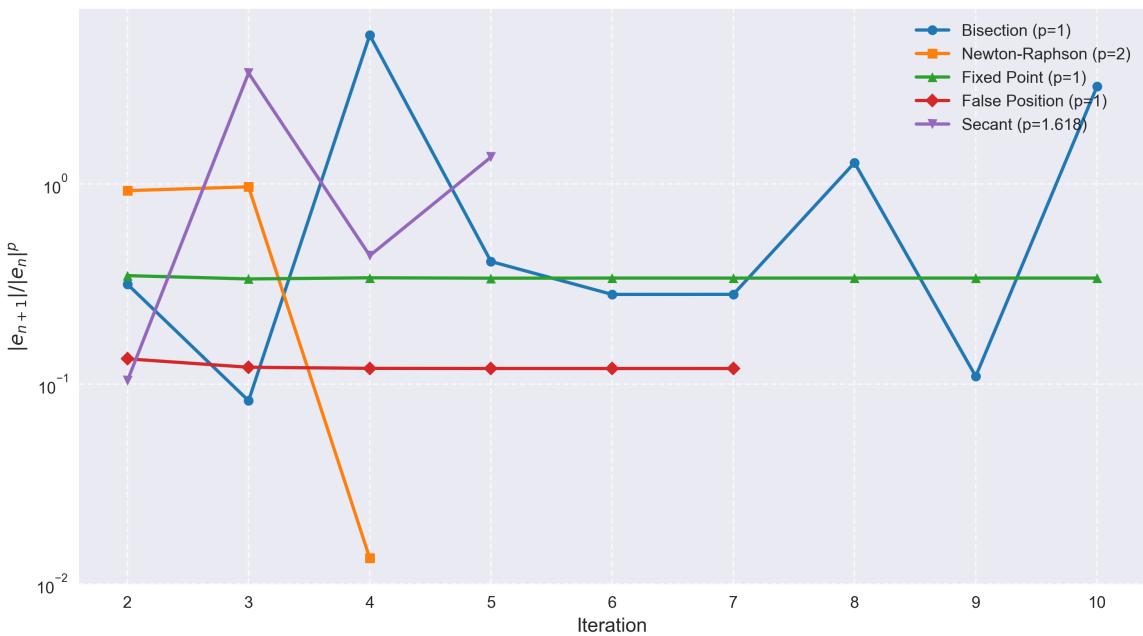
Trajectory of Root-Finding Methods for $f(x)$



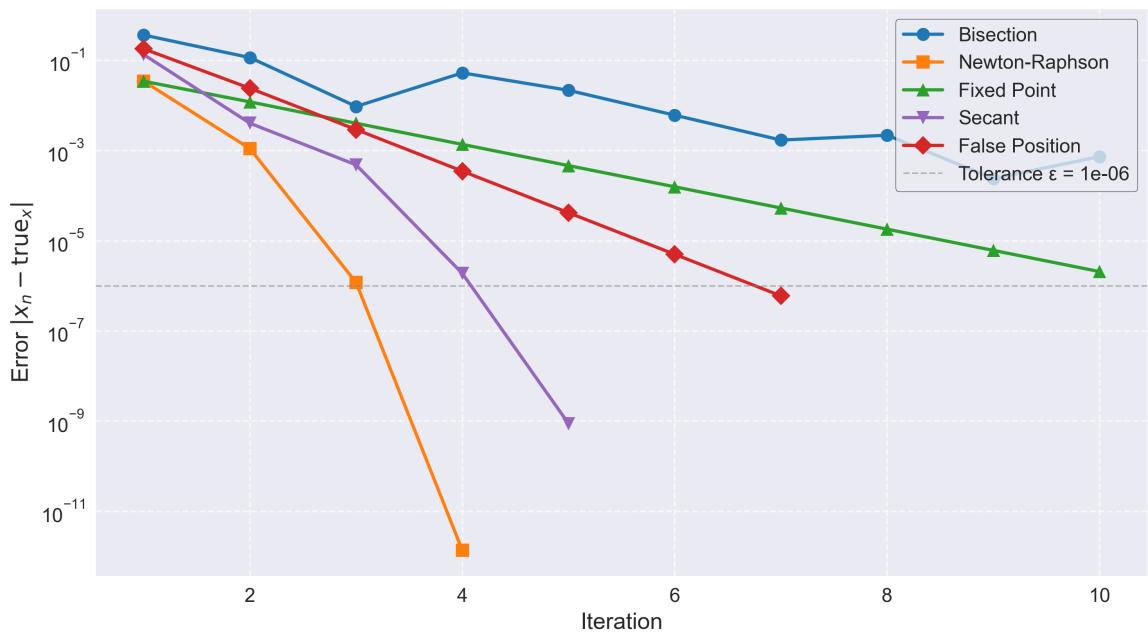
Number of Iterations for Root-Finding Methods



Error Ratio $|e_{n+1}|/|e_n|^p$



Convergence of Root-Finding Methods for $f(x)$



Aitken's Acceleration Error Reduction for $f(x)$

