Polarisation Basis

Qcamp 2018, Experimental Session 01/06/2018

1 Introduction

In Mission 2, you construct a quantum communication channel, where the information is encoded in light polarisation. As a result, you are able to construct symmetric secure key by comparing the measurement basis. However, we might have skipped some of the technical aspect. In this handout, we will review some of them and hopefully you will come to appreciate this implementation more.

1.1 Polarisation

Malus's law states that the intensity of a linear polariser output is:

$$I = I_0 \cos^2 \theta \tag{1}$$

when the incoming light is linearly polarised with intensity I_0, and θ is the angle between the polarisation axis of the incoming light and the polariser.

1.2 Basis

In the experiment, we define two types of measurement basis: (H) horizontal and (D) diagonal. In the H basis, the $|H\rangle$ (horizontally polarised) light will pass through, while the $|V\rangle$ (vertically polarised) light will be blocked completely. In the D basis however, the $|D\rangle$ (diagonally polarised) light will pass through, while the $|A\rangle$ (anti-diagonally polarised) light will be blocked completely.

From this, we infer that the polarisation axis angle between $|H\rangle$ and $|V\rangle$ is $90°$. If we then define $|D\rangle$ to be halfway in between $|H\rangle$ and $|V\rangle$, i.e. $|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$, then we can also infer that the polarisation axis angle between $|H\rangle$ and $|D\rangle$ is $45°$. Thus, we define the angle of $|H\rangle$, $|D\rangle$, $|V\rangle$, and $|A\rangle$ as $0°$, $45°$, $90°$, and $135°$ respectively.

2 Assignment [10 pts]

Task 1 [2 pts] Verify that if you set the measurement basis to be H, and you measure $|D\rangle$ polarised light, the light intensity will be half of the original $|D\rangle$ light.

Task 2 [2 pts] In Mission 2, you measured the intensity matrix of the 4 incoming light polarisation ($|H\rangle$, $|D\rangle$, $|V\rangle$, and $|A\rangle$) and 4 measurement basis (H, D, V, and A). Assuming that the incoming light intensity is I, construct the matrix and write down the expected values (in terms of I).

Task 3 [2 pts] Out of those 16 entries in the matrix, there are only a few entries that matter, i.e. important to construct the key. Identify those entries. *Hint: There are 4.*

Hint: Bob only measures in two basis: (H) horizontal and (D) diagonal. If say, Bob measures in (H) basis and Alice sends $|V\rangle$, then Bob should measure ~ 0. If Alice sends $|H\rangle$ instead, then Bob should measure ~ 1. If Alice sends $|D\rangle$ or $|A\rangle$, it does not matter anyway because that measurement result will be discarded. So, one basically can eliminate those "measurements" that do not matter, as the result will not matter to the final key.

Task 4 [2 pts] As a sender, let's say you want to run a sequence of 8 states as follows:

$$|H\rangle, |D\rangle, |A\rangle, |H\rangle, |A\rangle, |V\rangle, |D\rangle, |A\rangle$$

Now, to save time, you only allow a movement of $90°$ maximum. Write the list of angles that you need to set to run the sequence, provided that the first two angles are $0°$ and $45°$.

Task 5 [2 pts] Another issue to think about is the synchronisation between Alice and Bob. During the experiment, Alice's and Bob's stepper motor can be rotated in sync. This is due to a synchronisation pulse at the start of the sequence. However, Arduino's crystal oscillator (the device that generates the ticking of clock) is only accurate up to 10^{-4}, i.e. in 10000 seconds, the oscillator might drift around $\sim$ 1 second. This might pose a problem, as the movement of Alice and Bob will go out of sync after some time.

If each bit takes around 1.5 s to transmit (as in the experiment), and between Alice and Bob there can only be at most 50 ms of timing difference, estimate how many Alice can send to Bob before doing another synchronisation procedure!