Population Shape Regression from Random Design data

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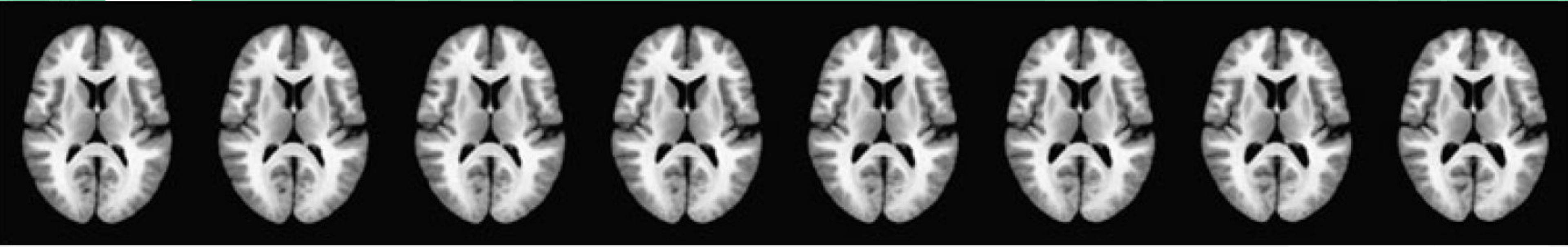

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MATHÉMATIQUES
VISION
APPRENTISSAGE

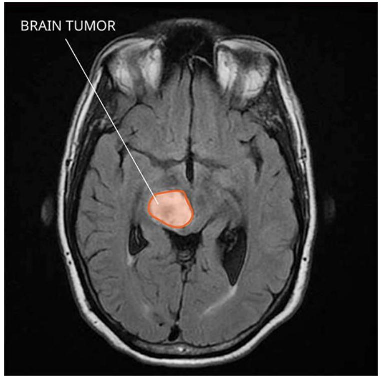
Introduction

Why is this an important subject ?

- We observe a deformation of the brain with aging
- Classical methods are focused on volume change which is less informative

→ Understanding healthy aging and shape change of the brain could help diagnose abnormal deformation linked to diseases like Alzheimer’s

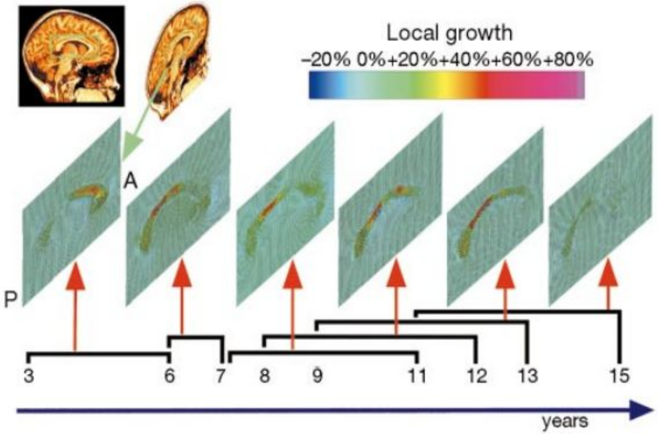
State of the art before publication of the article



Glial tumor

Clatz et al. 2005
Swanson et al. 2000

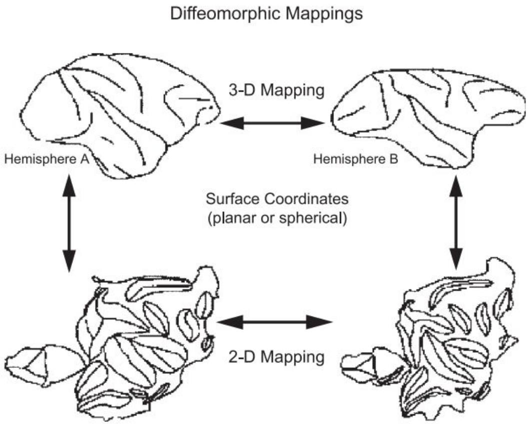
Based on volume estimation



Continuum mechanical tensor maps

Thompson et al. 2000

Focus on children brain development



Anatomy shape, growth, atrophy comparison via diffeomorphisms

Miller 2004

Longitudinal approach only

The algorithm - Step 1

Compute a representative anatomical configuration, for each time t

Observations of the form : age of the patient, three-dimensional image

The algorithm - Step 1

Compute a representative anatomical configuration, for each time t

Observations of the form : age of the patient, three-dimensional image

Use the manifold Ω of brain shape configurations instead of R^3

Shape change is described as a **diffeomorphism**

The algorithm - Step 1

Action of a diffeomorphism on an image : $I_\phi(x) = I(\phi^{-1}(x))$

Relationship between Phi and v : $\frac{d}{ds}\phi_s(x) = v_s(\phi_s(x))$

The algorithm - Step 1

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Relationship between Phi and v : $\frac{d}{ds}\phi_s(x) = v_s(\phi_s(x))$

Metrics :

$$||v_s||_V^2 = \int_\Omega Av_s \cdot v_s dx$$

$$d_{\mathcal{H}}(e,\phi)^2 = \min_{v:\frac{d}{ds}\phi_s = v_s(\phi_s)} \int_0^1 ||v_s||_V^2 ds$$

subject to $\phi(x) = x + \int_0^1 v_s(\phi_s(x))ds$ for all $x \in \Omega$.

$$d_{\mathcal{H}}(\phi_1,\phi_2)^2 = d_{\mathcal{H}}(e,\phi_1^{-1}\circ\phi_2)^2.$$

The algorithm - Step 1

Distance between two images :

$$d_{\mathcal{I}}(I_1, I_2)^2 = \min_{v: \frac{d}{ds}\phi_s = v_s(\phi_s)} \left[\int_0^1 ||v_s||_V^2 ds + \frac{1}{\sigma^2} ||I_1(\phi^{-1}) - I_2||_{L^2}^2 \right]$$

The algorithm - Step 1

Distance between two images :

$$d_{\mathcal{I}}(I_1, I_2)^2 = \min_{v:\frac{d}{ds}\phi_s = v_s(\phi_s)} \left[\int_0^1 ||v_s||_V^2 ds + \frac{1}{\sigma^2} ||I_1(\phi^{-1}) - I_2||_{L^2}^2 \right]$$

Representative image for all patients at a given time :

$$\hat{I}_h(t) = \operatorname*{argmin}_{I \in \mathcal{I}} \left(\frac{\sum_{i=1}^n K_h(t - t_i) d_{\mathcal{I}}(I, I_i)^2}{\sum_{i=1}^n K_h(t - t_i)} \right)$$

Solved by an iterative greedy algorithm

The algorithm - Step 2

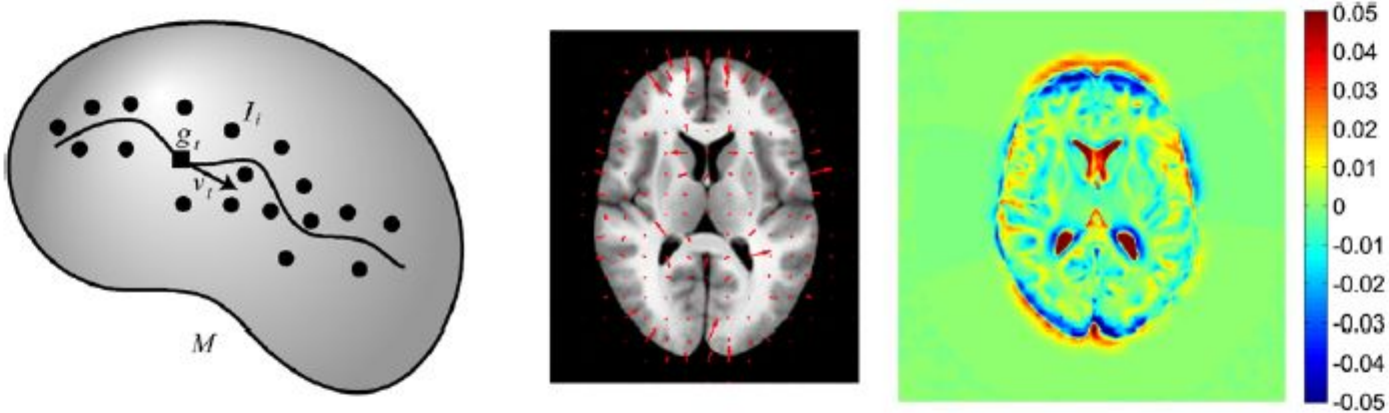
Growth model for a population :

Given several image observations J_t at different times , we now seek the diffeomorphic flow g_t that flows through these images as time increases

$$\underset{v:\dot{g}_t=v_t(g_t)}{\operatorname{argmin}} \int_0^1 \|v_t\|_V^2\, dt + \frac{1}{\sigma^2}\int_0^1 \Bigg\| I_\alpha(g_t^{-1}) - \underset{I\in\mathcal{I}}{\operatorname{argmin}}\left(\frac{\sum_{i=1}^N K_h(t-t_i)d_{\mathcal{I}}(I,I_i)^2}{\sum_{i=1}^N K_h(t-t_i)}\right)\Bigg\|_{L_2}^2 dt.$$

Solved by gradient descent

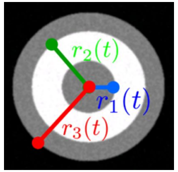
The algorithm - Step 2



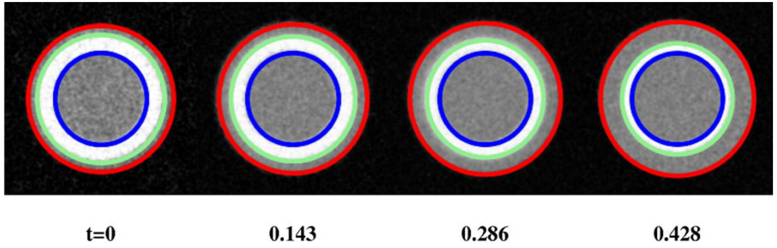
(a) (b) (c)

Results

Synthetic data

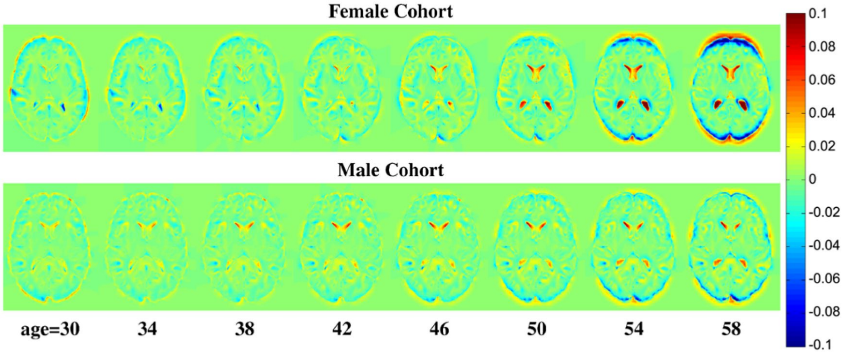


$$r_1(t_i) = f_1(t_i) + \epsilon_i + \epsilon_{i,1}$$
$$r_2(t_i) = f_2(t_i) + \epsilon_i + \epsilon_{i,2}$$
$$r_3(t_i) = f_3(t_i) + \epsilon_i + \epsilon_{i,3}$$



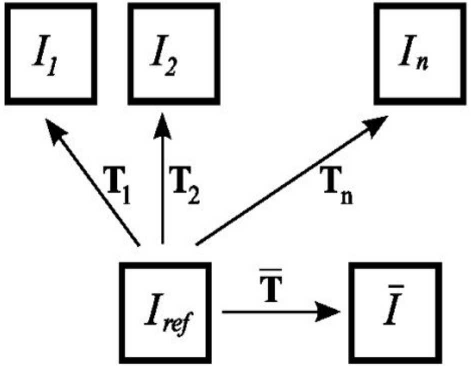
- Excellent results (grey = prediction, colors = ground truth)
- But simplistic (no shape variability)

Brain MRIs



- Illustration of the local brain shape change as a function of age
- Only issue: small dataset (97 MRIs) 1000+ images datasets nowadays

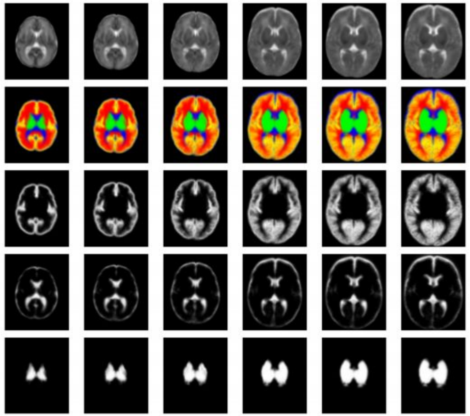
Further use of the paper



Patient-specic atlas of the brain

Ericsson et Al. 2008

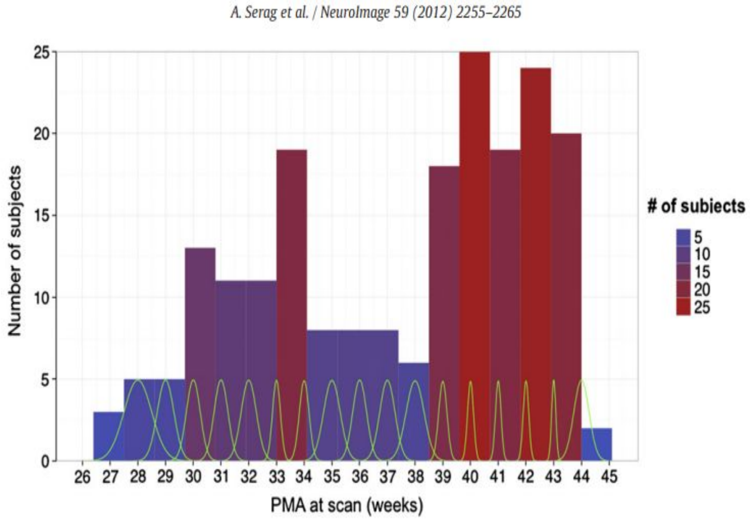
Sex, age, ethnicity, medical history as similarity measure



Probabilistic atlas of the developing brain

Kuklisova-Murgasova et Al. 2010

Tissue probability as target



Anatomy shape, growth, atrophy comparison via diffeomorphisms

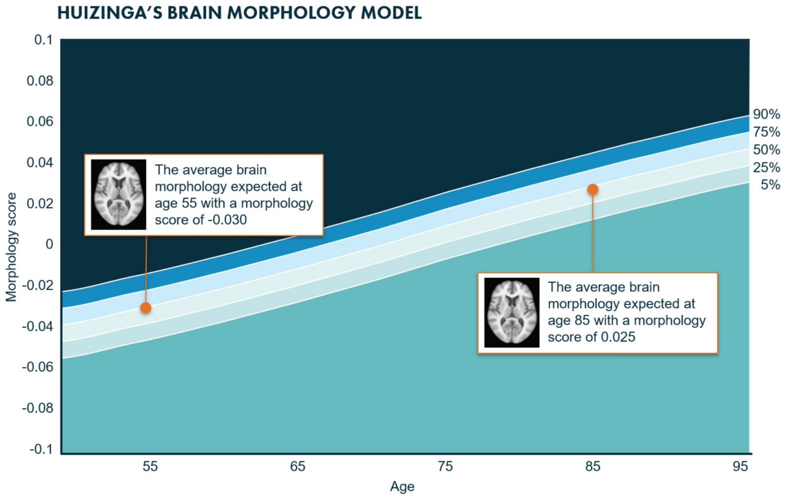
Serag et Al. 2012

Adaptive kernel band-width

Example of similar recent method

Quantib (company)

- Same principle as in the paper, except :
    - global average brain instead of regressive average with time,
    - “morphology score”: curves for which score is to be expected at a certain age

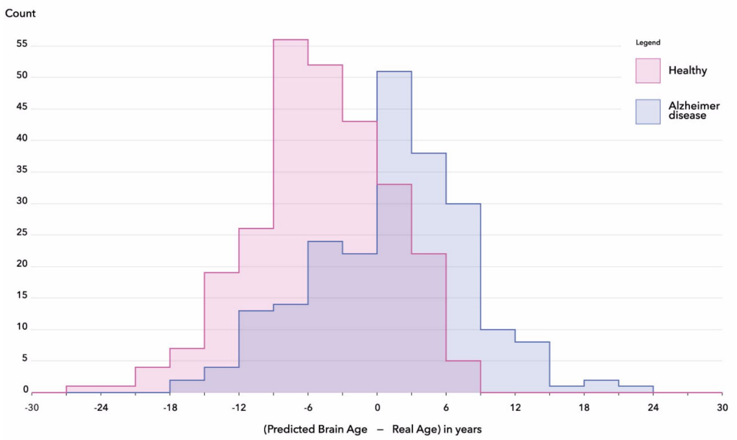


https://www.quantib.com/blog/how-to-measure-the-changing-shape-of-the-aging-brain; https://www.researchgate.net/publication/321588351_A_spatio-temporal_reference_model_of_the_aging_brain

Answers the question in the conclusion of Davis’s paper about the quantification of the descriptive trends he had + helps medical diagnosis

Possible applications for shape regression

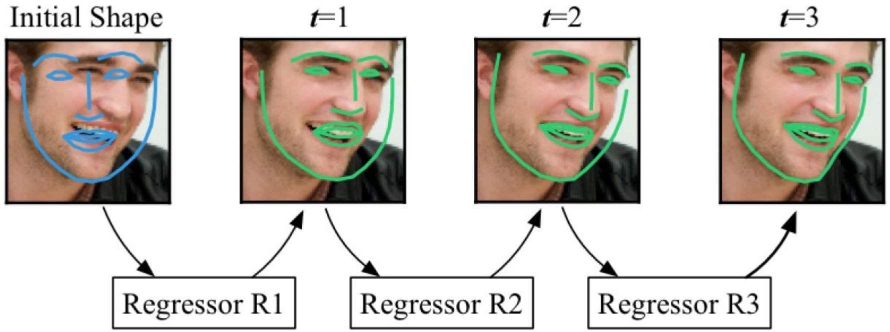
Medical: Alzheimer's diagnosis



- By comparing the patient brain shape to the theoretical one.

(Example done with Neural Nets + Linear regression on volume; https://medium.com/thelaunchpad/how-to-estimate-the-age-of-your-brain-with-mri-data-c60df60da95d)

Non-medical: facial recognition



- Instead of brain deformation with age, facial deformation with pose
- Based on Recurrent Neural Network

(https://www.researchgate.net/publication/323502242_Recurrent_Convolutional_Shape_Regression)

Thank you!