Part I

Literature review

Dragert *et al.* (2001 [?]) carefully analyzed data from 14 GPS sites in Cascadia. They detected a reversed direction of motion, lasting for a few days, at several sites located landward from the locked seismogenic zone. The observed signal propagated parallel to the strike of the subducting slab at around 6 km per day. They fitted the observed displacements with the displacements produced by a fault located at the plate boundary in an elastic half-space, and concluded that the slip must have occurred downdip of the locked and transition zones. The total moment derived from their numerical modelling was equivalent to a $M_w = 6.7$ earthquake. They suggested that deep-slip events like this could play a key role in the stress loading of the seismogenic zone, and therefore trigger megathrust earthquakes.

Obara (2002 [?]) computed the cross correlations of envelope seismograms between pairs of stations of the Hi-net network in southwestern Japan, while moving the time lag between the two traces. It allowed him to identify long coherent signals, with a predominant frequency of 1 to 10 Hz. A rough estimate of the propagation velocity of these tremors led him to conclude that they were propagated by S-wave velocity. The tremors were located in the western part of Shikoku and in the Kii peninsula, near the Mohorovičić discontinuity. Obara speculated that the observed tremors may be a continuous sequence of low frequency earthquakes. Due to the long duration of the tremors and the mobility of the tremor activity, he suggested that the occurrence of tremors may be related to the release of fluids in the subduction zone.

Preston *et al.* (2003 [?]) analyzed first and second arrivals travel times from marine active sources, and first arrival travel times from local earthquakes in Cascadia. They inverted their data for 3D P-wave velocity structure, earthquake locations, and geometry of the reflector. They interpreted the reflector as the Moho of the subducting Juan de Fuca plate. The earthquakes were separated into two groups. The earthquakes in the first group were located in the oceanic mantle, up-dip of the Moho's 45km depth contour, and might be caused by serpentinite dehydration. The earthquakes in the second group occurred in the subducted oceanic crust, down-dip of the Moho's 45km depth contour, and might be caused by basalt-to-eclogite transformation. No other reflector that could correspond to the plate boundary and the upper limit of a low velocity zone was detected.

Rogers and Dragert (2003 [?]) examined a number of seismograph signals in Vancouver Island together and observed tremorlike signals identified by the similarities in their envelopes. These signals were characterized by frequencies between 1 and 5 Hz, pulses of energy, and a duration from a few minutes to several days. Their occurrence was correlated temporally and spatially with slip events on the subduction zone interface. The source depth of the tremors was comprised between 25 and 45 km, which is compatible with a location on the plate interface where the slip was supposed to occur. They named this phenomenon Episodic Tremor and Slip (ETS). They believed that the cause of these tremors could be a shearing source and that fluids may play an important role in their generation.

Obara *et al.* (2004 [?]) observed tiltmeters data in boreholes in western Shikoku, and compared them with tremor activity. They noticed the simultaneous occurrence and migration of slow slip and tremors, with a recurrence interval of approximately six months. They divided the phenomena into two steps, modelled the slow slip by a dislocation on two reverse faults, and found a good agreement between predicted slip and tilt observations. The updip limit of the slip corresponded to the location of the tremors, whereas the downdip limit of the slip corresponded to the junction between the top of the subducting oceanic plate and the continental Moho. They concluded that the coupling phenomenon producing both tremors and slow slip should be located on the plate boundary and be related to the stress accumulation of the locked zone.

Shelly *et al.* (2006 [?]) used a combination of waveform cross-correlation and double-difference tomography to get a precise location of the source of low-frequency earthquakes (LFE), and a high-resolution velocity structure, in western Shikoku. They observed that the LFEs are located on a plane 5-8 km above the dipping plane of regular seismicity within the subducting plate, and concluded that the LFEs must occur at the plate interface, while regular seismicity begins within the lower part of the crust at shallower depths, and expand upwards the crust as the slab subducts. They also found a zone of high V_P/V_S ratio in the vicinity of the LFEs, which they interpret as high pore-fluid pressure. They hypothesize that the LFEs may be generated by local shear slip accelerations due to local heterogeneities during large slow slip events at the plate interface, and that the high pore-fluid pressure might enable slip by reducing normal effective stress.

Ide *et al.* (2007 [?]) studied the scaling and spectral behaviour of slow earthquakes (e.g. silent earthquakes, low-frequency earthquakes, low-frequency tremor, slow slip events, and very-low-frequency earthquakes). They observed that their seismic moment is proportional to their duration (and not the cube of the duration, as is the case for regular earthquakes), and that their seismic moment rate is proportional to the inverse of the frequency (and not the inverse of the square of the frequency, as is the case for regular earthquake). They proposed two models to explain this behaviour. In the constant low-stress drop model, the propagation velocity is proportional to the square of the characteristic length. In the diffusional model, the slip is constant and the propagation velocity is proportional of the characteristic length. They conclude that all slow earthquakes are different manifestations of the same physical phenomenon, and constitute a new earthquake category.

Ide *et al.* (2007 [?]) computed cross correlation of low-frequency earthquakes (LFE) waveforms in western Shikoku, and concluded from their predominantly positive distribution that mechanisms of LFEs are very similar. Then, they compared the polarity of regular intraplate earthquake waveforms and stacked LFE waveforms in order to identify the nodal planes of P-wave radiation. Finally, they inverted the moment tensor of a reference LFE using S-waveforms and intraplate earthquakes waveforms as empirical Green's function. The focal mechanism was a shallow dipping thrust fault, consistent with the subduction of the Philippine plate. The fact that this mechanism is consistent with fault models for slow slip events led them to conclude that slow slip, LFEs and tremors must be different manifestation of the same process.

Shelly *et al.* (2007 [**?**]) observed that tremor and low-frequency earthquakes (LFE) had similar frequency content, distinct of the spectra of background noise and regular earthquakes. They used well recorded LFEs in Shikoku as template events, and cross correlated their waveforms with tremor waveforms recorded in the same area. They stacked the corresponding correlation coefficients for the three components of several stations, and identify the tremor waveform as a small LFE if the summed correlation coefficient was higher than a threshold. They found that the events thus detected had a good spatial coherence, and that most of the tremor signal could be explained by a swarm of small LFEs. They concluded that the tremors were generated by the same mechanism that causes LFEs and slow slip, that is fluid enabled shear slip on the plate boundary.

Wech and Creager (2008 [**?**]) computed the cross correlations of envelope seismograms for a set of 20 stations in western Washington and southern Vancouver Island. They performed a grid search over all possible source locations to determine which one minimizes the difference between the maximum cross correlation and the value of the correlogram at the lag time corresponding to the S-wave travel time between two stations. This location method is automated (and thus, less labor intensive), makes use of near real-time regional data, and is less computationally intensive than previously proposed methods. They applied their method to the 2007 and 2008 Episodic Tremor and Slip (ETS) events, and located the epicentres in the region where the plate interface is 30-45 km deep and found a sharp updip boundary of the tremor 75km east of the downdip edge of the megathrust zone. Moreover, they identified between the two ETS events geodetically undetectable tremor that represents nearly half of the total amount of tremor. They concluded that their location method could help mapping the transition and locked zones of the plate boundary.

Audet *et al.* (2009 [**?**]) computed receiver functions of teleseismic waves in Vancouver Island, and analyzed the delay times between the forward-scattered P-to-S, and back-scattered P-to-S and S-to-S conversions at two seismic reflectors identified as the top and bottom of the oceanic crust. It allowed them to compute the P-to-S velocity ratio (V_P/V_S) of the layer and the S-wave velocity contrast at both interfaces. The very low Poisson's ratio of the layer could not be explained by the composition, and they interpreted it as evidence for high pore-fluid pressure. They explained the sharp velocity contrast on top of the layer as a low permeability boundary between the oceanic plate and the overriding continental crust. They concluded that the high pore-fluid pressures in the oceanic crust could explain the recurrence of Episodic Tremor and Slip (ETS) events, either by hydrofracturing, either by extending the region of conditionally stable slip.

La Rocca *et al.* (2009 [**?**]) stacked seismograms over all stations of the array for each component, and for three arrays in Cascadia. They then computed the cross-correlation between the horizontal and the vertical component, and found a distinct and persistent peak at a positive lag time, corresponding to the time between P-wave and S-wave arrivals. Using a standard layered Earth model, and horizontal slowness estimated from array analysis, they computed the depths of the tremor sources. They located the sources near or at the plate interface, with a much better depth resolution than previous methods based on seismic signal envelopes, source scanning algorithm, or small-aperture arrays. They concluded that at least some of the tremor consisted in the repetition of low-frequency events as was the case in Shikoku. A drawback of the method was that it could be applied only to tremor located beneath an array, and coming from only one place for an extended period of time.

Ghosh *et al.* (2010 [**?**]) used a beam backprojection (BBP) method to detect and locate tremor from seismic recordings of a small-aperture array in the Olympic Peninsula. They observed tremor propagating near-continuously in the slip-parallel direction, at velocities between 30 and 200 km/h, and for distances up to 40 km. They proposed two mechanisms to explain these tremor streaks. In the first model, the slip and the plate dip direction differ by up to $\theta = 35°$. The tremor propagates along slip-parallel striations, corrugations, and ridge-and-groove structures on the fault surface. The long-term front velocity V_L and the short-term streak velocity V_S are related by $\sin \theta = \frac{V_L}{V_S}$. In the second model, periodic breaking of the impermeable caprock increases the pore pressure and creates a pressure gradient that will in turn induce fluid flow along a conduit made available by striations and grooves on the fault plane. However, this last model requires long continuous conduits that seem unlikely. Tremor distribution thus varies over different time scales: along-strike migration of the front at about 10 km/day, rapid tremor reversals at about 10 km/h, and along-slip tremor streaks at about 30-200 km/h. Moreover, the moment release of tremors is distributed among patches.

Wech and Creager (2011 [**?**]) studied the variations of slip size and periodicity of slow slip with increasing depth in the Cascadia subduction zone. They used the waveform envelope correlation and clustering (WECC) method developed in their previous work (Wech and Creager, 2008 [**?**]) to detect and locate tremor epicentres, and assumed that slow slip happens at the same time and location as tremor. They then divided the tremor region into four 20-km-wide strike-perpendicular bins, and found evidence of small and frequent slip on the downdip size of the tremor zone, and larger and less frequent slip on the updip size. They speculate that higher temperatures at higher depths would produce lower frictional strength and a weaker fault. Each small slip event would thus transfer stress updip to a stronger portion of the fault, with a higher stress threshold. When enough stress has been transferred, this updip portion would slip and transfer slip further updip of the fault.

Bostock *et al.* (2012 [**?**]) looked for low-frequency earthquakes (LFE) by computing autocorrelations of 6-second long windows for each component of 7 stations in Vancouver Island. They then classified their LFE detections into 140 families. By stacking all waveforms of a given family, they obtained an LFE template for each family. They extended their templates by adding more stations and computing cross correlations between station data and template waveforms. They used P- and S-traveltime picks to obtain an hypocentre for each LFE template and concluded that the LFEs were located on the plate boundary and that their downdip extension coincided with the seaward extrapolation of the continental Moho. By observing the polarizations of the P- and S-waveforms of the LFE templates, they computed focal mechanisms and obtained a mixture of strike slip and thrust mechanisms, corresponding to a compressive stress field consistent with thrust faulting parallel to the plate interface.

Ghosh *et al.* (2012 [**?**]) used multibeam-backprojection (MBBP) to detect and locate tremor with much higher resolution. They used data recorded by 8 small-aperture seismic arrays in the Olympic peninsula during the large August 2010 Episodic Tremor and Slip (ETS) event and an entire inter-ETS cycle. They observed that the tremors were located near the plate boundary, on a layer parallel to and a few kilometres above the layer of regular earthquakes. Distinct patches, tens of kilometres of dimension, were found to produce the majority of the tremor. The propagation velocity varied from 4 to 20 km/day at large time scale (days), and up to 100 km/h at small time scale (minutes). They interpreted their observations with a model made of patches of asperities surrounded by regions slipping aseismically. Propagation velocity was supposed to be slow in the asperities area, and fast outside of the asperities area.

Bostock (2013 [**?**]) proposed a new model to explain the nature of a landward-dipping, low velocity zone (LVZ) that was detected in most subduction zones. Previous models in Cascadia interpreted the LVZ as the entire oceanic crust, an extended plate boundary, serpentinized material above the plate boundary, or a fluid-rich layer in the overriding continental crust. In the new model, the LVZ is interpreted as upper oceanic crust. The upper oceanic crust is hydrated by hydrothermal circulation at the ridge. The free water is the incorporated into hydrous minerals. As subduction begins, prograde metamorphic reactions release hydrous fluid in the upper oceanic crust. They stayed trapped by an impermeable upper plate boundary and the impermeable gabbroic lower oceanic crust. The high pore-fluid pressure explains the low shear wave velocity and the high Poisson's ratio. At about 45km depth, the onset of eclogitization liberates additional fluids and causes volumetric changes that break the plate boundary seal. The penetration of hydrous fluids in the mantle wedge leads to serpentinization of the mantle wedge material and erasure of the Moho's seismic contrast. By 100 km depth, the eclogitization is largely completed and the LVZ disappears.

Nowack and Bostock (2013 [**?**]) used a set of 140 low-frequency earthquakes (LFE) waveform templates in southern Vancouver Island as a record of empirical Green's functions. They used a regional 3D tomographic model, and inserted a low velocity zone under the plate boundary. They computed synthetic waveforms of a pulse using 3D ray-tracing for different source locations corresponding to the locations of the LFE waveforms templates. They then compared their synthetics to the data from the LFE templates, and carried out a grid search to check which values of P-wave velocity, ratio of P- and S-wave velocities and thickness of the low velocity zone gave the better fit. Their estimates of the thickness of the low velocity zone, the velocity contrast and the ratio between P- and S-wave velocities were consistent with the results from previous teleseismic studies.

Armbruster *et al.* (2014 [**?**]) proposed a new method to accurately locate tremor sources. They started with seismic data from two stations and computed the cross correlation of the seismic signals on 150 seconds time windows. They carried out a grid search on the polarization angles of each station and the offset time between both stations, and look for the greatest cross correlation value. They assumed that a tremor event occurred when the polarization angles and the offset times are consistent for several consecutive time windows. They extended the method for three stations, and looked for the consistency between the polarization angles and offsets found for the three possible pairs of stations, without imposing a duration criterion. Finally, they used the waveforms containing tremors from the three-stations detections to look for S-wave and P-wave at additional stations. With four S-wave detections and one P-wave detection, they were able to retrieve the location and depth of the tremor source with a 1 to 2 kilometres accuracy. They noticed that the polarization of the waveforms were consistent with a shear mechanism on the plate boundary. They also found out a similarity of pattern of the locations of the tremor sources for the three main Episodic Tremor and Slip (ETS) events for which they analyzed seismic recordings.

Royer and Bostock (2014 [**?**]) generated low-frequency earthquake (LFE) templates in northern Cascadia using the same processing steps (network autocorrelation, waveform correlation cluster analysis and network cross correlation) as in Bostock *et al.* (2012). They identified their LFE templates as empirical Green's functions, which justifies their subsequent use in waveform inversions. They computed template locations using standard linearized inversion and double difference algorithm, and concluded that LFE templates parallel the plate boundary. They carried out a moment tensor inversion for each LFE template and found out that a majority of the focal mechanisms were consistent with shallow thrust faulting, although there is more variability in northern Washington state due to poorer station coverage and lower signal-to-noise ratio.

Thurber *et al.* (2014 [?]) compared the efficiency of linear and phase-weighted stacking for picking low-frequency earthquakes (LFEs) arrivals. Once initial templates have been identified using the cross-station method of Savard and Bostock (2013), the signal is stacked using linear or phase-weighted stacking. The author then used an iterative procedure in which, at each iteration, they cross-correlate the stack with the continuous seismic signal, detecting new LFEs. At the end of each iteration, all the LFE waveforms are stacked to produce a new template with a higher SNR. The phase-weighted stack produced faster a little more detections than the linear stack, and a final template with a much better SNR than the linear stack.

Bostock *et al.* (2015 [?]) studied the magnitudes of low-frequency earthquakes (LFE) templates below southern Vancouver Island. They computed the magnitudes from the waveforms using the ray approximation, and observed that the magnitude-frequency distribution was better represented by a power law, with a b-value (~ 6.3) much higher than what is observed for regular earthquakes. They assumed that the source pulse duration is measured by the reciprocal of the instantaneous frequency, and observed a weak scaling between seismic moment and duration. They observed that the ratio of slip between two template waveforms is much higher than the ratio of pulse duration (7.36 and 1.29) , and concluded that there is no self-similarity for LFE and that larger moment events appear to be the result of increased slip. To reconcile the scaling between magnitude and frequency, and the scaling between seismic moment and slip, they proposed that multiple independently slipping sources are present within the same LFE template. The scaling of LFE would thus be different from both large scale slow slip events (SSE) and regular earthquakes.

Houston (2015 [?]) studied the sensitivity of tremor to tidal stress. She divided tremor into two groups: tremors arriving before 1.5 days after the tremor front, and tremors arriving after 1.5 days after the tremor front. She computed the evolution of tidal stress within the tremor region, and computed for each point in the regional grid the ratio of tremors occurring at a given level of tidal stress divided by the total number of tremors recorded at this grid point. She noticed a much stronger correlation between tremor activity and tidal stress changes after the passage of the tremor front. She interpreted this phenomenon with a stress threshold failure model. There is a big stress increase on the fault with the arrival of the tremor front, such that the stress stays much higher than the fault strength even when the tidal stress varies. It generates a lot of tremor, but a weak influence of tides on tremor activity. After the passage of the tremor front, tides cause small variations of stress on a weaker fault, such that there is an alternance of states with fault strength higher than stress on fault, and fault strength lower than stress on fault with each tidal cycle. Thus, there are less tremors, but a stronger influence of tides on tremor activity.

Hyndman *et al.* (2015) investigated the processes that control the position of Episodic Tremor and Slip (ETS) in the Cascadia subduction zone. They noticed that the high temperatures in the young subducting oceanic plate, the geodetic data, and the recordings of coseismic subsidence in buried coastal marshes during past great earthquakes, all point out to a downdip limit of the seismogenic zone located offshore. The position of the slow slip and the tremor is well known, although the depths have some uncertainty. The slip may extend seaward of the tremor, but there is a clear separation between the seismogenic zone and the ETS zone, with the ETS zone being located about 70 km east of the downdip of the seismogenic zone, and the volcanic arc being located about 100 km east of the ETS zone. A previous study showed that the position of the subduction zone ETS does not coincide with a specific temperature or dehydration reaction. The authors pointed out that ETS has been related to high pore fluid pressures close to the plate boundary. They argued that the bending of the subducting plate at the ocean trench may introduce a large amount of water in the upper oceanic mantle, resulting in extensive serpentinization. Moreover, the serpentinization of the fore-arc mantle corner may increase its vertical impermeability, while keeping a high permeability parallel to the fault, thus channelling all the fluid updip in the subducting oceanic crust. The dehydration of the serpentinite from the upper oceanic mantle, and the focusing of rising fluids along the plate boundary should result in large amounts of fluids available at the fore-arc mantle corner. Additionally, there seems to be a good coincidence between the location of the fore-arc mantle corner, and the location of ETS. The authors then observed that the deep fore-arc crust has a very low Poisson's ratio (less than 0.22), and that the only mineral with a very low Poisson's ratio is quartz (about 0.1), which led them to conclude that there may be a significant amount of quartz (about 10 % in volume) in the deep fore-arc crust above the fore-arc mantle. Moreover, as the solubility of silica increases with temperature, fluids generated at depth and rising up the subduction channel should be rich in silica. The authors concluded that there may be a relation between quartz veins formation in the deep fore-arc crust and ETS. However, several constraints as the magnitude and mechanism of the low-frequency earthquakes, and the vertical extent of the tremor should be explained.

Plourde *et al.* (2015 [?]) have detected low-frequency earthquakes (LFEs) in Northern California during the April 2008 Episodic Tremor and Slip (ETS) event using seismic data from the EarthScope Flexible Array Mendocino Experiment

(FAME). They used a combination of autodetection methods and visual identification to obtain the initial templates. Then, they recovered higher signal-to-noise (SNR) LFE signals using iterative network cross correlation. They found that the LFE families were located above the plate boundary, with a large distribution of depths (28-47 km). Three additional families were found on the Maacama and Bucknell Creek faults. On these faults, LFEs tend to occur in bursts, while repeating earthquakes occur as single events or in small groups. LFEs and earthquakes have also different frequency contents. They conclude that dehydration of the mantle and further upward migration of water through the deep crustal fault system could explain the generation of both tremor and regular seismicity on these two faults.

Gomberg *et al.* (2016 [**?**]) studied the relationship between seismic moment and duration for fast and slow earthquakes population. They used GPS data and tremor catalogs in Japan and Cascadia for slow slip events, and crustal earthquakes from the SRCMOD database for fast slip events. They distinguished between unbounded events, for which fault growth is two-dimensional and moment is proportional to the cube of duration, and bounded events, for which fault growth is one dimensional and moment is proportional to duration. The proposed dislocation model does not require different scaling between fast and slow earthquakes. Instead, there is a continuous but bimodal distribution of slip modes: elastic, velocity-weakening patches generate fast slip, while viscous, velocity-strengthening background generates slow, aseismic slip. The size and distribution of patches on a fault determinate the dominant mode.

Part II

Episodic Tremor and Slip (ETS)

Chapter 1

Slow slip

Slow slip on the plate boundary is inferred to happen when there is a reversal of the direction of motion at GPS stations, compared to the secular motion of the surface displacement.

The amplitude of the horizontal displacement measured by the GPS stations at the surface is a few millimetres. Dragert *et al.* (2001 [?]) found displacements ranging from 2 to 4 millimetres. Dragert *et al.* (2004 [?]) found an average displacement of 5 millimetres. Szeliga *et al.* (2008 [?]) found a displacement consistently lower than 6 millimetres. This should be compared to a secular velocity of 5.6 millimetres per year on average, and an inter-slip velocity of 9.7 millimetres per year on average (Dragert *et al.*, 2004 [?]).

The reversal of the direction of motion is observed during a few weeks at each GPS station. Dragert *et al* (2001 [?]) observed a reversal lasting about 6 to 15 days depending on the GPS station for the summer 1999 event. Miller *et al.* (2002 [?]) observed a reversal lasting on average 2 to 4 weeks for the eight events between 1992 and 2001. Dragert *et al.* (2004 [?]) observed reversals lasting 1 to 3 weeks. The average dislocation risetime was found to be 14 days with a maximum of about 30 days (Schmidt and Gao, 2010 [?]).

The reversal of the direction of motion does not occur at the same time for each GPS station. Dragert *et al.* (2001 [?]) observed a 35 days time lag between the beginning of the reverse displacement at the most southeastern station and beginning of the reverse displacement at the most northwestern station for the summer 1999 event. Miller *et al.* (2002 [?]) observed an average time lag of 3 weeks between the beginning of the event at the first station and the beginning of the event at the last station for the eight events between 1992 and 2001. The overall duration of an event is 2 to 7 weeks (Gao *et al.*, 2012 [?]). This corresponds to a propagation velocity along the strike of the plate boundary of about 6 kilometres per day for the summer 1999 event (Dragert *et al.*, 2001 [?]). Dragert *et al.* (2004 [?] found a propagation velocity varying from 5 to 15 kilometres per day. Schmidt and Gao (2010 [?]) found an average propagation rate for the slip initiation of 5.9 kilometres per day, although some fault elements showed a rate as high as 17 kilometres per day.

The recurrence interval of the eight slow slip events between 1992 and 2001 was on average 14.5 months according to Miller *et al.* (2002 [?]), or between 13 and 16 months according to Dragert *et al.* (2004 [?]).

Numerical simulation of faulting in an elastic half-space have been carried out by several authors in order to retrieve the corresponding slip at the plate interface. Dragert *et al.* (2001 [?]) found a slip of about 2 centimetres between 30 and 40 kilometres depth, and a smaller slip updip of 30 kilometres for the summer 1999 event. Numerical modelling carried out by Miller *et al.* (2002, [?]) suggests that the eight events from 1992 to 2001 were evidence of a creep of a few centimetres along the plate interface at depths of 30 to 50 kilometres. Dragert *et al.* (2004 [?]) found a slip of 2 to 4 centimetres on the plate interface bounded by the 25 and 45 kilometres depth contours. Melbourne *et al.* (2005 [?]) found a maximum slip of 3.8 centimetres centered at 28 kilometres depth with most of the slip located above 38 kilometres depth. Szeliga *et al.* (2008 [?]) found an average slip of 2 to 3 centimetres. The total area where this reversal of the direction of motion was observed was about 50 * 300 kilometres for the summer 1999 event (Dragert *et al.*, 2001 [?]). Dragert *et al.* (2001 [?]) found that the surface displacement was largest at the sites located more than 100 kilometres landward of the locked zone. Wech and Creager (2008 [?]) observed that the western boundary of the area where reversal of the direction of motion occurs is located 75 km east of the downdip edge of the seismogenic zone. The strain release from slow slip was not uniform along strike, and the greater amount of slip is centered around Port Angeles (Schmidt and Gao, 2010 [?]).

These values of slip and area correspond to earthquakes of moment magnitude 6.7 for the summer 1999 event (Dragert *et al.*, 2001 [?]), 6.8 for the July 1998 event, 6.7 for the August 1999 event, 6.7 for the December 2000 event, 6.5 for the February 2002 event (Dragert *et al.*, 2004 [?]), 6.6 for the February 2003 event (Melbourne *et al.*, 2005 [?]), 6.3 to 6.8 for the events studied by Szeliga *et al.* (2008 [?]), and 6.1 to 6.7 for the events studied by Schmidt and Gao (2010 [?]).

The average stress drop is about 0.01 to 0.10 MPa (Schmidt and Gao, 2010 [?]).

Chapter 2

Tremor

The predominant frequency of tremors ranges from 1 to 10 Hz and is lower than that of ordinary earthquakes of similar size (10 to 20 Hz). The envelopes of tremors have gradual rise times and differ from those of a normal earthquake, which has a spike-like envelope shape (Obara, 2002 [?]). The frequency content is mainly between 1 and 5 Hz, whereas most of the energy in small earthquakes is above 10 Hz. A tremor onset is usually emergent and the signal consists of pulses of energy, often about a minute in duration. A continuous signal may last from a few minutes to several days (Rogers and Dragert, 2003 [?]).

It is only when a number of seismograph signals are viewed together that the similarity in the envelope of the seismic signal at each site identifies the signal as ETS (Rogers and Dragert, 2003 [?])

Characteristics: low amplitude, lack of energy at high frequency, emergent onsets, absence of clear impulsive phases (La Rocca *et al.*, 2009)

Depth = 30 km, near the Mohorovičić discontinuity (southwest Japan, Obara, 2002 [?]). 20 to 40 km (Rogers and Dragert, 2003 [?]).

correlate temporally and spatially with six deep slip events that have occurred over the past 7 years (Rogers and Dragert, 2003 [?])

Spatially clustered (Obara *et al., 2004*). Belt-like distribution Patches tens kilometers of dimension (Ghosh *et al*, 2012).

small-amplitude tremors that lasted from a few minutes to a few days (Obara, 2002 [?]). 1 min (Rogers and Dragert, 2003)

Duration of tremor activity = 10 to 20 days in any one region (Rogers and Dragert, 2003 [?]). Several days to a few weeks (Obara *et al.*, 2004)

Frequency = rom 1 to 10 Hz (Obara, 2002 [?]). Time windows of 35 to 50 min. 1-8 Hz (Ide et al, 2007) The frequency content is mainly between 1 and 5 Hz, whereas most of the energy in small earthquakes is above 10 Hz (Rogers and Dragert, 2003 [?]).

Propagation = along strike 5 to 15 km / day (Rogers and Dragert, 2003 [?]). Along-strike 5-17 km / day (Shelly *et al.*, 2007)

Short-trem 15 km up-dip in 20 min (Nankai, Shelly *et al.*, 2007) = 45 km / h

Recurrence interval = 2 to 3 months (eastern Shikoku)

Propagated with a velocity of 4 km/s, that is the source of the tremors was located at a deep portion and the envelopes were propagated by S-wave velocity (Obara, 2002 [?]).

no impulsive body wave arrivals → Difficult to locate

Chapter 3

LFEs

Depth = 30-35 km (nankai, Ide *et al.*, 2007), 7km above regular intraplate earthquakes

Location = spatially clustered, at the plate boundary, 25 to 37 km depth plate boundary contour, between two bands of seismicity (crustal and intraslab earthquakes)

Magnitude= ¡ 2

Mechanism = shear slip on low-angle thrust fault. Point-source, double-couple excitation ; combination of strike-slip and thrust faulting (Bostosck *et al.*, 2012)

Frequency = 1-10 Hz 1-8 Hz (Ide et al, 2007)

Chapter 4

Slow earthquakes

Moment / duration : $M_0 = T \times 10^{12-13}$ (slow) versus $M_0 = T^3 \times 10^{15-16}$ (regular)
Moment rate / frequency : $\dot{M}_0 \propto f^{-1}$ (slow) versus $\dot{M}_0 \propto f^{-2}$ (regular)

Chapter 5

Subduction

Plate convergence (Juan de Fuca) = 4 cm / year Age 10 million year

up-dip of 45km depth, earthquakes below the reflector (serpentinite dehydration of the mantle), down-dip within subducted crust (basalt-to-eclogite transformation) (Preston *et al.*, 2003) down to 60 km depth

Physical mechanism intraslab earthquakes: dehydration embrittlement, metamorphic dehydration (prograde metamorphism)

Low velocity layer = 3-4km thin, Vs=2-3 km/s Poisson's ratio = 0.4, depth 20-40 km (Nowack and Bostock, 2013).

Part III

Data

Chapter 6

Time lags

6.1 Description of dataset

Bla bla about Ghosh et al. (2010)

6.2 Websites to access the data

International Federation of Digital Seismograph Networks

This website gives a list of the network codes, and the corresponding map, with the names and locations of stations. The selected experiments are:

- XG (2009-2011): Cascadia Array of Arrays
- XU (2006-2012): Collaborative Research: Earthscope integrated investigations of Cascadia subduction zone tremor, structure and process

The stations are the following:

- Port Angeles XG - PA01 to PA13
- Danz Ranch XG - DR01 to DR10, and DR12
- Lost Cause XG - LC01 to LC14
- Three Bumps XG - TB01 toTB14
- Burnt Hill XG - BH01 to BH11
- Cat Lake XG - CL01 to CL20
- Gold Creek XG - GC01 to GC14
- Blyn XU - BS01 to BS06, BS11, BS20 to BS27

IRIS DMC MetaData Aggregator

This website gives for each station:

- Location and time of recording
- Epoch (effective periods of recording during the time that the station was installed)
- Type of instrument
- Channels

Pacific Northwest Seismic Network tremor catalog

This website gives the dates and locations of tremor activity in Cascadia. Following Ghosh *et al.* (2012), the selected periods of tremors are:

- From November 9th 2009 to November 13th 2009,
- From March 16th 2010 to March 19th 2010,
- From August 16th 2010 to August 20th 2010.

Chapter 7

LFE catalog

7.1 Description of dataset

Bla bla about Plourde et al. (2015)

7.2 Websites to access the data

Chapter 8

Slow slip

Bla bla about PANGA

8.1 Time lags

8.1.1 Method

Some bla bla about cross correlaion and stacking

8.1.2 Cross-correlation of tremor recordings

Definitions

We define the Fourier transform $\hat{f}$ of the function f by:

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \tag{8.1}$$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega t} d\omega \tag{8.2}$$

We define the convolution product by:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau \tag{8.3}$$

We have:

$$(f \hat{*} g)(\omega) = \sqrt{2\pi} \hat{f}(\omega) \hat{g}(\omega) \tag{8.4}$$

and:

$$(\hat{fg})(\omega) = \frac{1}{\sqrt{2\pi}} \hat{f}(\omega) * \hat{g}(\omega) \tag{8.5}$$

We define the cross correlation by:

$$(f \otimes g)(t) = \int_{-\infty}^{\infty} f^*(\tau) g(t + \tau) d\tau \tag{8.6}$$

where f^* is the complex conjugate of f. We have:

$$(f \hat{\otimes} g)(\omega) = \sqrt{2\pi} \hat{f}^*(\omega) \hat{g}(\omega) \tag{8.7}$$

and:

$$(\hat{fg})(\omega) = \frac{1}{\sqrt{2\pi}} \hat{f}^*(\omega) \otimes \hat{g}(\omega) \tag{8.8}$$

Finally, we have:

$$(f(t) * g(t)) \otimes (f(t) * g(t)) = (f(t) \otimes f(t)) * (g(t) \otimes g(t)) \tag{8.9}$$

If we have a point source at ξ with a source function $f_k(t)$ for $k = 1, 2, 3$, then we can express the displacement $u(x,t)$ with the Green's function:

$$u_i(x,t) = \int_{-\infty}^{\infty} G_{ik}(x,\xi,t-\tau) f_k(\xi,\tau) d\tau \tag{8.10}$$

In the case of a moment tensor, we have:

$$u_i(x,t) = \int_{-\infty}^{\infty} \frac{\partial G_{ip}}{\partial \xi_q}(x,\xi,t-\tau) M_{pq}(\xi,\tau) d\tau \tag{8.11}$$

In the Fourier domain, we have:

$$\hat{u_i}(x,\omega) = \sqrt{2\pi} \hat{G_{ik}}(x,\xi,\omega) \hat{f_k}(\xi,\omega) \tag{8.12}$$

or:

$$\hat{u_i}(x,\omega) = \sqrt{2\pi} \frac{\partial \hat{G}_{ip}}{\partial \xi_q}(x,\xi,\omega) \hat{M_{pq}}(\xi,\omega) \tag{8.13}$$

When we compute the cross correlation between two components of the displacements, we have:

$$(u_i \otimes u_j)(x,t) = \int_{-\infty}^{\infty} u_i^*(x,\tau) u_j(x,t+\tau) d\tau \tag{8.14}$$

In the Fourier domain, we have:

$$\begin{aligned}
(u_i \hat{\otimes} u_j)(x,\omega) &= \sqrt{2\pi} \hat{u_i}^*(x,\omega) \hat{u_j}(x,\omega) \\
&= (2\pi)^{\frac{3}{2}} [\hat{G_{ik}}^*(x,\xi,\omega) \hat{f_k}^*(\xi,\omega)][\hat{G_{jl}}(x,\xi,\omega) \hat{f_l}(\xi,\omega)] \\
&= (2\pi)^{\frac{3}{2}} [\frac{\partial \hat{G}_{ip}}{\partial \xi_q}^*(x,\xi,\omega) \hat{M_{pq}}^*(\xi,\omega)][\frac{\partial \hat{G}_{jr}}{\partial \xi_s}(x,\xi,\omega) \hat{M_{rs}}(\xi,\omega)]
\end{aligned} \tag{8.15}$$

Change of coordinates

Point source Equation (15) can be written as:

$$\begin{pmatrix} (u_1 \hat{\otimes} u_1) & (u_1 \hat{\otimes} u_2) & (u_1 \hat{\otimes} u_3) \\ (u_2 \hat{\otimes} u_1) & (u_2 \hat{\otimes} u_2) & (u_2 \hat{\otimes} u_3) \\ (u_3 \hat{\otimes} u_1) & (u_3 \hat{\otimes} u_2) & (u_3 \hat{\otimes} u_3) \end{pmatrix} = (2\pi)^{\frac{3}{2}} \begin{pmatrix} \hat{G_{11}}^* & \hat{G_{12}}^* & \hat{G_{13}}^* \\ \hat{G_{21}}^* & \hat{G_{22}}^* & \hat{G_{23}}^* \\ \hat{G_{31}}^* & \hat{G_{32}}^* & \hat{G_{33}}^* \end{pmatrix} \begin{pmatrix} \hat{f_1}^* \\ \hat{f_2}^* \\ \hat{f_3}^* \end{pmatrix} \begin{pmatrix} \hat{f_1} & \hat{f_2} & \hat{f_3} \end{pmatrix} \begin{pmatrix} \hat{G_{11}} & \hat{G_{21}} & \hat{G_{31}} \\ \hat{G_{12}} & \hat{G_{22}} & \hat{G_{32}} \\ \hat{G_{13}} & \hat{G_{23}} & \hat{G_{33}} \end{pmatrix} \tag{8.16}$$

that is:

$$\hat{U} = (2\pi)^{\frac{3}{2}} \hat{G}^* \hat{f}^* \hat{f}^T \hat{G}^T \tag{8.17}$$

We define a new coordinate system with the unit vectors $n^{(1)}$, $n^{(2)}$ and $n^{(3)}$, and the matrix N by:

$$N = \begin{pmatrix} n_1^{(1)} & n_1^{(2)} & n_1^{(3)} \\ n_2^{(1)} & n_2^{(2)} & n_2^{(3)} \\ n_3^{(1)} & n_3^{(2)} & n_3^{(3)} \end{pmatrix} \tag{8.18}$$

In the new coordinate system, the Green's function is equal to $G' = N^T G N$, thus we have:

$$N^T \hat{U} N = (2\pi)^{\frac{3}{2}} \hat{G'}^* N^T \hat{f}^* \hat{f}^T N \hat{G'}^T \tag{8.19}$$

with:

$$N^T \hat{U} N = \begin{pmatrix} (u.n^{(1)} \hat{\otimes} u.n^{(1)}) & (u.n^{(1)} \hat{\otimes} u.n^{(2)}) & (u.n^{(1)} \hat{\otimes} u.n^{(3)}) \\ (u.n^{(2)} \hat{\otimes} u.n^{(1)}) & (u.n^{(2)} \hat{\otimes} u.n^{(2)}) & (u.n^{(2)} \hat{\otimes} u.n^{(3)}) \\ (u.n^{(3)} \hat{\otimes} u.n^{(1)}) & (u.n^{(3)} \hat{\otimes} u.n^{(2)}) & (u.n^{(3)} \hat{\otimes} u.n^{(3)}) \end{pmatrix} \tag{8.20}$$

If we choose N_1 such that:

$$N_1^T \hat{f} = \begin{pmatrix} \hat{F} \\ 0 \\ 0 \end{pmatrix} \tag{8.21}$$

we get:

$$N_1^T \hat{U} N_1 = (2\pi)^{\frac{3}{2}} \hat{F}^* \hat{F} \begin{pmatrix} \hat{G'_{11}}^* \hat{G'_{11}} & \hat{G'_{11}}^* \hat{G'_{21}} & \hat{G'_{11}}^* \hat{G'_{31}} \\ \hat{G'_{21}}^* \hat{G'_{11}} & \hat{G'_{21}}^* \hat{G'_{21}} & \hat{G'_{21}}^* \hat{G'_{31}} \\ \hat{G'_{31}}^* \hat{G'_{11}} & \hat{G'_{31}}^* \hat{G'_{21}} & \hat{G'_{31}}^* \hat{G'_{31}} \end{pmatrix} \tag{8.22}$$

If we choose N_2 such that:

$$N_2^T \hat{f} = \begin{pmatrix} 0 \\ \hat{F} \\ 0 \end{pmatrix} \tag{8.23}$$

we get:

$$N_2^T \hat{U} N_2 = (2\pi)^{\frac{3}{2}} \hat{F}^* \hat{F} \begin{pmatrix} \hat{G'_{12}}^* \hat{G'_{12}} & \hat{G'_{12}}^* \hat{G'_{22}} & \hat{G'_{12}}^* \hat{G'_{32}} \\ \hat{G'_{22}}^* \hat{G'_{12}} & \hat{G'_{22}}^* \hat{G'_{22}} & \hat{G'_{22}}^* \hat{G'_{32}} \\ \hat{G'_{32}}^* \hat{G'_{12}} & \hat{G'_{32}}^* \hat{G'_{22}} & \hat{G'_{32}}^* \hat{G'_{32}} \end{pmatrix} \tag{8.24}$$

If we choose N_3 such that:

$$N_3^T \hat{f} = \begin{pmatrix} 0 \\ 0 \\ \hat{F} \end{pmatrix} \tag{8.25}$$

we get:

$$N_3^T \hat{U} N_3 = (2\pi)^{\frac{3}{2}} \hat{F}^* \hat{F} \begin{pmatrix} \hat{G'_{13}}^* \hat{G'_{13}} & \hat{G'_{13}}^* \hat{G'_{23}} & \hat{G'_{13}}^* \hat{G'_{33}} \\ \hat{G'_{23}}^* \hat{G'_{13}} & \hat{G'_{23}}^* \hat{G'_{23}} & \hat{G'_{23}}^* \hat{G'_{33}} \\ \hat{G'_{33}}^* \hat{G'_{13}} & \hat{G'_{33}}^* \hat{G'_{23}} & \hat{G'_{33}}^* \hat{G'_{33}} \end{pmatrix} \tag{8.26}$$

If we define strike ϕ, dip δ and rake λ, we can define the following vectors:

$$e_1 = \begin{pmatrix} \sin\phi \\ \cos\phi \\ 0 \end{pmatrix},\ e_2 = \begin{pmatrix} \cos\phi \\ -\sin\phi \\ 0 \end{pmatrix},\ e_3 = \cos\delta e_2 - \sin\delta e_z = \begin{pmatrix} \cos\phi\cos\delta \\ -\sin\phi\cos\delta \\ -\sin\delta \end{pmatrix} \text{ and } e_4 = \sin\delta e_2 + \cos\delta e_z = \begin{pmatrix} \cos\phi\sin\delta \\ -\sin\phi\sin\delta \\ \cos\delta \end{pmatrix} \tag{8.27}$$

and the new coordinate system (u, v, w) with:

$$u = \cos\lambda e_1 - \sin\lambda e_3,\ v = -\sin\lambda e_1 - \cos\lambda e_3 \text{ and } w = e_4 \tag{8.28}$$

Thus we have:

$$u = \begin{pmatrix} \sin\phi\cos\lambda - \cos\phi\cos\delta\sin\lambda \\ \cos\phi\cos\lambda + \sin\phi\cos\delta\sin\lambda \\ \sin\delta\sin\lambda \end{pmatrix},\ v = \begin{pmatrix} -\sin\phi\sin\lambda - \cos\phi\cos\delta\cos\lambda \\ -\cos\phi\sin\lambda + \sin\phi\cos\delta\sin\lambda \\ -\sin\delta\cos\lambda \end{pmatrix} \text{ and } w = \begin{pmatrix} \cos\phi\sin\delta \\ -\sin\phi\sin\delta \\ \cos\delta \end{pmatrix} \tag{8.29}$$

We can choose:

$$N_1 = \begin{pmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{pmatrix} = \begin{pmatrix} \sin\phi\cos\lambda - \cos\phi\cos\delta\sin\lambda & -\sin\phi\sin\lambda - \cos\phi\cos\delta\cos\lambda & \cos\phi\sin\delta \\ \cos\phi\cos\lambda + \sin\phi\cos\delta\sin\lambda & -\cos\phi\sin\lambda + \sin\phi\cos\delta\sin\lambda & -\sin\phi\sin\delta \\ \sin\delta\sin\lambda & -\sin\delta\cos\lambda & \cos\delta \end{pmatrix} \tag{8.30}$$

to get Equation (19). To get Equations (21) and (23), we just have to permute the columns of N_1 to get N_2 and N_3. If we go back into the time domain, we have:

$$N_k^T \begin{pmatrix} u_1 \otimes u_1 & u_1 \otimes u_2 & u_1 \otimes u_3 \\ u_2 \otimes u_1 & u_2 \otimes u_2 & u_2 \otimes u_3 \\ u_3 \otimes u_1 & u_3 \otimes u_2 & u_3 \otimes u_3 \end{pmatrix} N_k = \begin{pmatrix} (F * G'_{1k}) \otimes (F * G'_{1k}) & (F * G'_{1k}) \otimes (F * G'_{2k}) & (F * G'_{1k}) \otimes (F * G'_{3k}) \\ (F * G'_{2k}) \otimes (F * G'_{1k}) & (F * G'_{2k}) \otimes (F * G'_{2k}) & (F * G'_{2k}) \otimes (F * G'_{3k}) \\ (F * G'_{3k}) \otimes (F * G'_{1k}) & (F * G'_{3k}) \otimes (F * G'_{2k}) & (F * G'_{3k}) \otimes (F * G'_{3k}) \end{pmatrix} \tag{8.31}$$

which can also be written as:

$$N_k^T \begin{pmatrix} u_1 \otimes u_1 & u_1 \otimes u_2 & u_1 \otimes u_3 \\ u_2 \otimes u_1 & u_2 \otimes u_2 & u_2 \otimes u_3 \\ u_3 \otimes u_1 & u_3 \otimes u_2 & u_3 \otimes u_3 \end{pmatrix} N_k = \begin{pmatrix} (F \otimes F) * (G'_{1k} \otimes G'_{1k}) & (F \otimes F) * (G'_{1k} \otimes G'_{2k}) & (F^S \otimes F^S) * (G'_{1k} \otimes G'_{3k}) \\ (F \otimes F) * (G'_{2k} \otimes G'_{1k}) & (F \otimes F) * (G'_{2k} \otimes G'_{2k}) & (F^S \otimes F^S) * (G'_{2k} \otimes G'_{3k}) \\ (F \otimes F) * (G'_{3k} \otimes G'_{1k}) & (F \otimes F) * (G'_{3k} \otimes G'_{2k}) & (F^S \otimes F^S) * (G'_{3k} \otimes G'_{3k}) \end{pmatrix} \tag{8.32}$$

Moment tensor We have:

$$M_{pq} = \int \int_\Sigma m_{pq} d\Sigma = \int \int_\Sigma \mu(\nu_p[u_q] + \nu_q[u_p]) d\Sigma = \mu A(\nu_p[u_q] + \nu_q[u_p]) \tag{8.33}$$

where ν is the normal to the fault surface and$[u]$ is the displacement discontinuity on the fault.
We define a new coordinates system with the unit vectors $n^{(1)}$, $n^{(2)}$ and $n^{(3)}$, and the matrix N by:

$$N = \begin{pmatrix} n_1^{(1)} & n_1^{(2)} & n_1^{(3)} \\ n_2^{(1)} & n_2^{(2)} & n_2^{(3)} \\ n_3^{(1)} & n_3^{(2)} & n_3^{(3)} \end{pmatrix} \tag{8.34}$$

In the new coordinates system, the Green's function is equal to $G' = N^T G N$, the moment tensor to $M' = N^T M N$, and the displacement to $u' = N^T u$. We choose N such that:

$$M' = \mu D A \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8.35}$$

Therefore, we have:

$$u'_i \hat{\otimes} u'_j = (2\pi)^{\frac{3}{2}} \mu D^* A [\frac{\partial \hat{G}'_{i1}}{\partial \xi'_1}^* - \frac{\partial \hat{G}'_{i2}}{\partial \xi'_2}^*] \mu D A [\frac{\partial \hat{G}'_{j1}}{\partial \xi'_1} - \frac{\partial \hat{G}'_{j2}}{\partial \xi'_1}] \tag{8.36}$$

If we come back in the time domain, we have:

$$u'_i \otimes u'_j = \mu^2 A^2 (D * [\frac{\partial G'_{i1}}{\partial \xi'_1} - \frac{\partial G'_{i2}}{\partial \xi'_2}]) \otimes (D * [\frac{\partial G'_{j1}}{\partial \xi'_1} - \frac{\partial G'_{j2}}{\partial \xi'_2}]) \tag{8.37}$$

which can also be written as:

$$u'_i \otimes u'_j = \mu^2 A^2 (D \otimes D) * ([\frac{\partial G'_{i1}}{\partial \xi'_1} - \frac{\partial G'_{i2}}{\partial \xi'_2}] \otimes [\frac{\partial G'_{j1}}{\partial \xi'_1} - \frac{\partial G'_{j2}}{\partial \xi'_2}]) \tag{8.38}$$

If we choose N such that:

$$M' = \mu D A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{8.39}$$

we would get:

$$u'_i \otimes u'_j = \mu^2 A^2 (D \otimes D) * ([\frac{\partial G'_{i1}}{\partial \xi'_1} - \frac{\partial G'_{i3}}{\partial \xi'_3}] \otimes [\frac{\partial G'_{j1}}{\partial \xi'_1} - \frac{\partial G'_{j3}}{\partial \xi'_3}]) \tag{8.40}$$

If we choose N such that:

$$M' = \mu D A \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{8.41}$$

we would get:

$$u'_i \otimes u'_j = \mu^2 A^2 (D \otimes D) * ([\frac{\partial G'_{i2}}{\partial \xi'_2} - \frac{\partial G'_{i3}}{\partial \xi'_3}] \otimes [\frac{\partial G'_{j2}}{\partial \xi'_2} - \frac{\partial G'_{j3}}{\partial \xi'_3}]) \tag{8.42}$$

How to compute the autocorrelation of the source term $F \otimes F$ or $D \otimes D$?

White noise At time t_i, we assume that the amplitude of the source function is A_i (random variable with expectancy $m = 0$ and standard deviation σ). We define a_i by:

$$P(A_i < p_0) = \int_{-\infty}^{p_0} a_i(p)dp \tag{8.43}$$

We have:

$$\int_{-\infty}^{\infty} a_i(p)dp = m \tag{8.44}$$

and:

$$\int_{-\infty}^{\infty} a_i^2(p)dp = m^2 + \sigma^2 \tag{8.45}$$

We suppose that the source function is a white noise, that is:

$$\int_{-\infty}^{\infty} (a_i(p) - m)(a_j(p) - m)dp = 0 \tag{8.46}$$

Thus, we have:

$$\int_{-\infty}^{\infty} a_i(p)a_j(p)dp = m^2 \tag{8.47}$$

We define the source time function $F(t_i) = A_i$ and we compute the autocorrelation. We have:

$$(F \otimes F)(t) = \int_{-\infty}^{\infty} F^*(\tau)F(t + \tau)d\tau \tag{8.48}$$

The expectancy of the term $F^*(\tau)F(t + \tau)$ is $m^2 + \sigma^2$ if $t = 0$ and m^2 if $t \neq 0$.

Stacking

8.1.3 Computation of amplitudes for P-, SV- and SH-waves

Changes of coordinates

We denote (x, y) the coordinates of the receiver array, (x_0, y_0) the coordinates of the tremor source, and d the depth of the tremor source.

In the $(\vec{e}_X, \vec{e}_Y, \vec{e}_Z)$ coordinate system, we have:

$$\vec{e}_R = \begin{pmatrix} \cos\beta \\ \sin\beta \\ 0 \end{pmatrix}, \vec{e}_T = \begin{pmatrix} -\sin\beta \\ \cos\beta \\ 0 \end{pmatrix} \text{ and } \vec{e}_Z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{8.49}$$

with $\beta = \text{atan2}(y - y_0, x - x_0)$

In the $(\vec{e}_R, \vec{e}_T, \vec{e}_Z)$ coordinate system, we have:

$$\vec{e}_X = \begin{pmatrix} \cos\beta \\ -\sin\beta \\ 0 \end{pmatrix}, \vec{e}_Y = \begin{pmatrix} \sin\beta \\ \cos\beta \\ 0 \end{pmatrix} \text{ and } \vec{e}_Z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{8.50}$$

For the direct wave, we have:

$$\vec{e}_P = \begin{pmatrix} \sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix}, \vec{e}_{SV} = \begin{pmatrix} \cos\alpha \\ 0 \\ -\sin\alpha \end{pmatrix} \text{ and } \vec{e}_{SH} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{8.51}$$

with $\alpha = \frac{\sqrt{(x-x_0)^2+(y-y_0)^2}}{d}$

For the reflected wave, we have:

$$\vec{e}_P = \begin{pmatrix} \sin\alpha \\ 0 \\ -\cos\alpha \end{pmatrix}, \vec{e}_{SV} = \begin{pmatrix} \cos\alpha \\ 0 \\ \sin\alpha \end{pmatrix} \text{ and } \vec{e}_{SH} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \tag{8.52}$$

where α is computed with the RayTracing code.
In the $(\vec{e}_P, \vec{e}_{SV}, \vec{e}_{SH})$ coordinate system, we have for the direct wave:

$$\vec{e}_R = \begin{pmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix}, \vec{e}_T = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ and } \vec{e}_Z = \begin{pmatrix} \cos\alpha \\ -\sin\alpha \\ 0 \end{pmatrix} \tag{8.53}$$

For the reflected wave, we have:

$$\vec{e}_R = \begin{pmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix}, \vec{e}_T = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \text{ and } \vec{e}_Z = \begin{pmatrix} -\cos\alpha \\ \sin\alpha \\ 0 \end{pmatrix} \tag{8.54}$$

We compute the seismic moment M in the $(\vec{e}_X, \vec{e}_Y, \vec{e}_Z)$ coordinate system. We have:

$$M_{ij} = u_i \nu_j + u_j \nu_i \tag{8.55}$$

with:

$$\vec{u} = \begin{pmatrix} -\cos\delta\cos\phi \\ \cos\delta\sin\phi \\ \sin\delta \end{pmatrix} \text{ and } \vec{\nu} = \begin{pmatrix} \sin\delta\cos\phi \\ -\sin\delta\sin\phi \\ \cos\delta \end{pmatrix} \tag{8.56}$$

where ϕ is the strike of the subducting plate, and δ is the dip of the subducting plate.
We then compute the value of M in the $(\vec{e}_P, \vec{e}_{SV}, \vec{e}_{SH})$ coordinate system. We have:

$$\begin{pmatrix} M_{RR} & M_{RT} & M_{RZ} \\ M_{TR} & M_{TT} & M_{TZ} \\ M_{ZR} & M_{ZT} & M_{ZZ} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} M_{XX} & M_{XY} & M_{XZ} \\ M_{YX} & M_{YY} & M_{YZ} \\ M_{ZX} & M_{ZY} & M_{ZZ} \end{pmatrix} \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8.57}$$

For the direct wave, we have:

$$\begin{pmatrix} M_{PP} & M_{PSV} & M_{PSH} \\ M_{SVP} & M_{SVSV} & M_{SVSH} \\ M_{SHP} & M_{SHSV} & M_{SHSH} \end{pmatrix} = \begin{pmatrix} \sin\alpha & 0 & \cos\alpha \\ \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} M_{RR} & M_{RT} & M_{RZ} \\ M_{TR} & M_{TT} & M_{TZ} \\ M_{ZR} & M_{ZT} & M_{ZZ} \end{pmatrix} \begin{pmatrix} \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \\ \cos\alpha & -\sin\alpha & 0 \end{pmatrix} \tag{8.58}$$

For the reflected wave, we have:

$$\begin{pmatrix} M_{PP} & M_{PSV} & M_{PSH} \\ M_{SVP} & M_{SVSV} & M_{SVSH} \\ M_{SHP} & M_{SHSV} & M_{SHSH} \end{pmatrix} = \begin{pmatrix} \sin\alpha & 0 & -\cos\alpha \\ \cos\alpha & 0 & \sin\alpha \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} M_{RR} & M_{RT} & M_{RZ} \\ M_{TR} & M_{TT} & M_{TZ} \\ M_{ZR} & M_{ZT} & M_{ZZ} \end{pmatrix} \begin{pmatrix} \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & -1 \\ -\cos\alpha & \sin\alpha & 0 \end{pmatrix} \tag{8.59}$$

In the $(\vec{e}_P, \vec{e}_{SV}, \vec{e}_{SH})$ coordinate system, we have:

$$\vec{\Gamma} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{8.60}$$

from Equation 9.13.1 of Pujol (2003), we have:

$$\begin{pmatrix} A_P \\ A_{SV} \\ A_{SH} \end{pmatrix} = \begin{pmatrix} M_{PP} \\ M_{SVP} \\ M_{SHP} \end{pmatrix} \tag{8.61}$$

Getting the reflection, conversion and transmission coefficients

We compute the reflection, conversion and transmission coefficients at the interface between two homogeneous media, following Aki and Richards (2002, ch. 5.2).

SH-wave

We have $u_x = 0$, $u_z = 0$ and $\frac{\partial}{\partial y} = 0$. Thus the wave equations become:

$$\begin{aligned}
\rho \frac{\partial^2 u_y}{\partial t^2} &= \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} \\
\sigma_{xy} &= \mu \frac{\partial u_y}{\partial x} \\
\sigma_{yz} &= \mu \frac{\partial u_y}{\partial z} \\
\sigma_{xx} &= \sigma_{yy} = \sigma_{zz} = \sigma_{xz} = 0
\end{aligned} \tag{8.62}$$

The incident SH-wave is of the form:

$$u_y(t) = \acute{S}_1 exp(i\omega(t - p_{\beta_1} x - q_{\beta_1} z)) \tag{8.63}$$

From Snell's law, we have $\frac{sinj_1}{\beta_1} = \frac{sinj_2}{\beta_2}$
At $z = 0$, we have $u_y(z^+) = u_y(z^-)$, and $\sigma_{yz}(z^+) = \sigma_{yz}(z^-)$ thus:

$$\begin{aligned}
&\acute{S}_1 + \grave{S}_1 = \acute{S}_2 \\
&-\mu_1 i\omega q_{\beta_1} \acute{S}_1 + \mu_1 i\omega q_{\beta_1} \grave{S}_1 = -\mu_2 i\omega q_{\beta_2} \acute{S}_2
\end{aligned} \tag{8.64}$$

Therefore, using $q_{\beta_1} = \frac{cosj_1}{\beta_1}$ and $q_{\beta_2} = \frac{cosj_2}{\beta_2}$, we find:

$$\begin{aligned}
\grave{S}_1 &= \acute{S}_1 \frac{\rho_1 \beta_1 cosj_1 - \rho_2 \beta_2 cosj_2}{\rho_1 \beta_1 cosj_1 + \rho_2 \beta_2 cosj_2} \\
\acute{S}_2 &= \acute{S}_1 \frac{2\rho_1 \beta_1 cosj_1}{\rho_1 \beta_1 cosj_1 + \rho_2 \beta_2 cosj_2}
\end{aligned} \tag{8.65}$$

8.1.4 P-wave

We have $u_y = 0$ and $\frac{\partial}{\partial y} = 0$. Thus the wave equations become:

$$\begin{aligned}
\rho \frac{\partial^2 u_x}{\partial t^2} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \\
\rho \frac{\partial^2 u_z}{\partial t^2} &= \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \\
\sigma_{xx} &= (\lambda + 2\mu) \frac{\partial u_x}{\partial x} + \lambda \frac{\partial u_z}{\partial z} \\
\sigma_{yy} &= \lambda (\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}) \\
\sigma_{zz} &= \lambda \frac{\partial u_x}{\partial x} + (\lambda + 2\mu) \frac{\partial u_z}{\partial z} \\
\sigma_{xz} &= \mu (\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}) \\
\sigma_{xy} &= \sigma_{yz} = 0
\end{aligned} \tag{8.66}$$

The incident P-wave is of the form:

$$\begin{aligned}
u_x(t) &= \acute{P}_1 sini_1 exp(i\omega(t - p_{\alpha_1} x - q_{\alpha_1} z)) \\
u_z(t) &= \acute{P}_1 cosi_1 exp(i\omega(t - p_{\alpha_1} x - q_{\alpha_1} z))
\end{aligned} \tag{8.67}$$

From Snell's law, we have $\frac{sini_1}{\alpha_1} = \frac{sini_2}{\alpha_2} = \frac{sinj_1}{\beta_1} = \frac{sinj_2}{\beta_2}$
At $z = 0$, we have $u_x(z^+) = u_x(z^-)$, $u_z(z^+) = u_z(z^-)$, $\sigma_{xz}(z^+) = \sigma_{xz}(z^-)$ and $\sigma_{zz}(z^+) = \sigma_{zz}(z^-)$ thus:

$$
\begin{aligned}
\acute{P}_1 sini_1 + \grave{P}_1 sini_1 + \grave{S}_1 cosj_1 = \acute{P}_2 sini_2 + \acute{S}_2 cosj_2 \\
-\acute{P}_1 cosi_1 + \grave{P}_1 cosi_1 - \grave{S}_1 sinj_1 = -\acute{P}_2 cosi_2 + \acute{S}_2 sinj_2 \\
\mu_1(-q\alpha_1 \acute{P}_1 sini_1 + q_{\alpha_1} \grave{P}_1 sini_1 + q_{\beta_1} \grave{S}_1 cosj_1) + \mu_1(-p_{\alpha_1} \acute{P}_1 cosi_1 + p_{\alpha_1} \grave{P}_1 cosi_1 - p_{\beta_1} \grave{S}_1 sinj_1) = \\
\mu_2(-q_{\alpha_2} \acute{P}_2 sini_2 - q_{\beta_2} \acute{S}_2 cosj_2) + \mu_2(-p_{\alpha_2} \acute{P}_2 cosi_2 + p_{\beta_2} \acute{S}_2 sinj_2) \\
\lambda_1(p_{\alpha_1} \acute{P}_1 sini_1 + p_{\alpha_1} \grave{P}_1 sini_1 + p_{\beta_1} \grave{S}_1 cosj_1) + (\lambda_1 + 2\mu_1)(q_{\alpha_1} \acute{P}_1 cosi_1 + q_{\alpha_1} \grave{P}_1 cosi_1 - q_{\beta_1} \grave{S}_1 sinj_1) = \\
\lambda_2(p_{\alpha_2} \acute{P}_2 sini_2 + p_{\beta_2} \acute{S}_2 cosj_2) + (\lambda_2 + 2\mu_2)(q_{\alpha_2} \acute{P}_2 cosi_2 - q_{\beta_2} \acute{S}_2 sinj_2)
\end{aligned}
\tag{8.68}
$$

that is:

$$
\begin{aligned}
-\acute{P}_2 sini_2 - \acute{S}_2 cosj_2 + \grave{P}_1 sini_1 + \grave{S}_1 cosj_1 = -\acute{P}_1 sini_1 \\
\acute{P}_2 cosi_2 - \acute{S}_2 sinj_2 + \grave{P}_1 cosi_1 - \grave{S}_1 sinj_1 = \acute{P}_1 cosi_1 \\
\mu_2 q_{\alpha_2} \acute{P}_2 sini_2 + \mu_2 p_{\alpha_2} \acute{P}_2 cosi_2 + \mu_2 q_{\beta_2} \acute{S}_2 cosj_2 - \mu_2 p_{\beta_2} \acute{S}_2 sinj_2 \\
+\mu_1 q\alpha_1 \grave{P}_1 sini_1 + \mu_1 p_{\alpha_1} \grave{P}_1 cosi_1 + \mu_1 q_{\beta_1} \grave{S}_1 cosj_1 - \mu_1 p_{\beta_1} \grave{S}_1 sinj_1 = \\
\mu_1 q_{\alpha_1} \acute{P}_1 sini_1 + \mu_1 p_{\alpha_1} \acute{P}_1 cosi_1 \\
-\lambda_2 p_{\alpha_2} \acute{P}_2 sini_2 - (\lambda_2 + 2\mu_2) q_{\alpha_2} \acute{P}_2 cosi_2 - \lambda_2 p_{\beta_2} \acute{S}_2 cosj_2 + (\lambda_2 + 2\mu_2) q_{\beta_2} \acute{S}_2 sinj_2 \\
+\lambda_1 p_{\alpha_1} \grave{P}_1 sini_1 + (\lambda_1 + 2\mu_1) q_{\alpha_1} \grave{P}_1 cosi_1 + \lambda_1 p_{\beta_1} \grave{S}_1 cosj_1 - (\lambda_1 + 2\mu_1) q_{\beta_1} \grave{S}_1 sinj_1 = \\
-\lambda_1 p_{\alpha_1} \acute{P}_1 sini_1 - (\lambda_1 + 2\mu_1) q_{\alpha_1} \acute{P}_1 cosi_1
\end{aligned}
\tag{8.69}
$$

that is:

$$
\begin{aligned}
-\acute{P}_2 sini_2 - \acute{S}_2 cosj_2 + \grave{P}_1 sini_1 + \grave{S}_1 cosj_1 = -\acute{P}_1 sini_1 \\
\acute{P}_2 cosi_2 - \acute{S}_2 sinj_2 + \grave{P}_1 cosi_1 - \grave{S}_1 sinj_1 = \acute{P}_1 cosi_1 \\
(\mu_2 q_{\alpha_2} sini_2 + \mu_2 p_{\alpha_2} cosi_2) \acute{P}_2 + (\mu_2 q_{\beta_2} cosj_2 - \mu_2 p_{\beta_2} sinj_2) \acute{S}_2 \\
+(\mu_1 q_{\alpha_1} sini_1 + \mu_1 p_{\alpha_1} cosi_1) \grave{P}_1 + (\mu_1 q_{\beta_1} cosj_1 - \mu_1 p_{\beta_1} sinj_1) \grave{S}_1 = \\
(\mu_1 q_{\alpha_1} sini_1 + \mu_1 p_{\alpha_1} cosi_1) \acute{P}_1 \\
-(\lambda_2 p_{\alpha_2} sini_2 + (\lambda_2 + 2\mu_2) q_{\alpha_2} cosi_2) \acute{P}_2 - (\lambda_2 p_{\beta_2} cosj_2 - (\lambda_2 + 2\mu_2) q_{\beta_2} sinj_2) \acute{S}_2 \\
+(\lambda_1 p_{\alpha_1} sini_1 + (\lambda_1 + 2\mu_1) q_{\alpha_1} cosi_1) \grave{P}_1 + (\lambda_1 p_{\beta_1} cosj_1 - (\lambda_1 + 2\mu_1) q_{\beta_1} sinj_1) \grave{S}_1 = \\
-(\lambda_1 p_{\alpha_1} sini_1 + (\lambda_1 + 2\mu_1) q_{\alpha_1} cosi_1) \acute{P}_1
\end{aligned}
\tag{8.70}
$$

that is:

$$
\begin{aligned}
-\alpha_2 p_{\alpha_2} \acute{P}_2 - cosj_2 \acute{S}_2 + \alpha_1 p_{\alpha_1} \grave{P}_1 + cosj_1 \grave{S}_1 = -\alpha_1 p_{\alpha_1} \acute{P}_1 \\
cosi_2 \acute{P}_2 - \beta_2 p_{\beta_2} \acute{S}_2 + cosi_1 \grave{P}_1 - \beta_1 p_{\beta_1} \grave{S}_1 = cosi_1 \acute{P}_1 \\
(2\rho_2 \beta_2^2 p_{\alpha_2} cosi_2) \acute{P}_2 + \rho_2 \beta_2 (1 - 2\beta_2^2 p_{\beta_2}^2) \acute{S}_2 \\
+(2\rho_1 \beta_1^2 p_{\alpha_1} cosi_1) \grave{P}_1 + \rho_1 \beta_1 (1 - 2\beta_1^2 p_{\beta_1}^2) \grave{S}_1 = \\
(2\rho_1 \beta_1^2 p_{\alpha_1} cosi_1) \acute{P}_1 \\
-\rho_2 \alpha_2 (1 - 2\beta_2^2 p_{\alpha_2}^2) \acute{P}_2 + 2\rho_2 \beta_2^2 p_{\beta_2} cosj_2 \acute{S}_2 \\
+\rho_1 \alpha_1 (1 - 2\beta_1^2 p_{\alpha_1}^2) \grave{P}_1 - 2\rho_1 \beta_1^2 p_{\beta_1} cosj_1 \grave{S}_1 = \\
-\rho_1 \alpha_1 (1 - 2\beta_1^2 p_{\alpha_1}^2) \acute{P}_1
\end{aligned}
\tag{8.71}
$$

that is:

$$
M \begin{pmatrix} \acute{P}_2 \\ \acute{S}_2 \\ \grave{P}_1 \\ \grave{S}_1 \end{pmatrix} = N \begin{pmatrix} 0 \\ 0 \\ \acute{P}_1 \\ 0 \end{pmatrix}
\tag{8.72}
$$

with:

$$M = \begin{pmatrix} -\alpha_2 p_{\alpha_2} & -cosj_2 & \alpha_1 p_{\alpha_1} & cosj_1 \\ cosi_2 & -\beta_2 p_{\beta_2} & cosi_1 & -\beta_1 p_{\beta_1} \\ 2\rho_2\beta_2^2 p_{\alpha_2} cosi_2 & \rho_2\beta_2(1-2\beta_2^2 p_{\beta_2}^2) & 2\rho_1\beta_1^2 p_{\alpha_1} cosi_1 & \rho_1\beta_1(1-2\beta_1^2 p_{\beta_1}^2) \\ -\rho_2\alpha_2(1-2\beta_2^2 p_{\alpha_2}^2) & 2\rho_2\beta_2^2 p_{\beta_2} cosj_2 & \rho_1\alpha_1(1-2\beta_1^2 p_{\alpha_1}^2) & -2\rho_1\beta_1^2 p_{\beta_1} cosj_1 \end{pmatrix} \tag{8.73}$$

and:

$$N = \begin{pmatrix} 0 & 0 & -\alpha_1 p_{\alpha_1} & 0 \\ 0 & 0 & cosi_1 & 0 \\ 0 & 0 & 2\rho_1\beta_1^2 p_{\alpha_1} cosi_1 & 0 \\ 0 & 0 & -\rho_1\alpha_1(1-2\beta_1^2 p_{\alpha_1}^2) & 0 \end{pmatrix} \tag{8.74}$$

SV-wave

We have $u_y = 0$ and $\frac{\partial}{\partial y} = 0$. Thus the wave equations become:

$$\begin{aligned} \rho\frac{\partial^2 u_x}{\partial t^2} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \\ \rho\frac{\partial^2 u_z}{\partial t^2} &= \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \\ \sigma_{xx} &= (\lambda+2\mu)\frac{\partial u_x}{\partial x} + \lambda\frac{\partial u_z}{\partial z} \\ \sigma_{yy} &= \lambda(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}) \\ \sigma_{zz} &= \lambda\frac{\partial u_x}{\partial x} + (\lambda+2\mu)\frac{\partial u_z}{\partial z} \\ \sigma_{xz} &= \mu(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}) \\ \sigma_{xy} &= \sigma_{yz} = 0 \end{aligned} \tag{8.75}$$

The incident S-wave is of the form:

$$\begin{aligned} u_x(t) &= \acute{S}_1 cosj_1 exp(i\omega(t - p_{\beta_1}x - q_{\beta_1}z)) \\ u_z(t) &= -\acute{S}_1 sinj_1 exp(i\omega(t - p_{\beta_1}x - q_{\beta_1}z)) \end{aligned} \tag{8.76}$$

From Snell's law, we have $\frac{sini_1}{\alpha_1} = \frac{sini_2}{\alpha_2} = \frac{sinj_1}{\beta_1} = \frac{sinj_2}{\beta_2}$
At $z = 0$, we have $u_x(z^+) = u_x(z^-)$, $u_z(z^+) = u_z(z^-)$, $\sigma_{xz}(z^+) = \sigma_{xz}(z^-)$ and $\sigma_{zz}(z^+) = \sigma_{zz}(z^-)$ thus:

$$\begin{aligned} \acute{S}_1 cosj_1 + \grave{P}_1 sini_1 + \grave{S}_1 cosj_1 &= \acute{P}_2 sini_2 + \acute{S}_2 cosj_2 \\ \acute{S}_1 sinj_1 + \grave{P}_1 cosi_1 - \grave{S}_1 sinj_1 &= -\acute{P}_2 cosi_2 + \acute{S}_2 sinj_2 \\ \mu_1(-q_{\beta_1}\acute{S}_1 cosj_1 + q_{\alpha_1}\grave{P}_1 sini_1 + q_{\beta_1}\grave{S}_1 cosj_1) + \mu_1(p_{\beta_1}\acute{S}_1 sinj_1 + p_{\alpha_1}\grave{P}_1 cosi_1 - p_{\beta_1}\grave{S}_1 sinj_1) &= \\ \mu_2(-q_{\alpha_2}\acute{P}_2 sini_2 - q_{\beta_2}\acute{S}_2 cosj_2) + \mu_2(-p_{\alpha_2}\acute{P}_2 cosi_2 + p_{\beta_2}\acute{S}_2 sinj_2) & \\ \lambda_1(p_{\beta_1}\acute{S}_1 cosj_1 + p_{\alpha_1}\grave{P}_1 sini_1 + p_{\beta_1}\grave{S}_1 cosj_1) + (\lambda_1+2\mu_1)(-q_{\beta_1}\acute{S}_1 sinj_1 + q_{\alpha_1}\grave{P}_1 cosi_1 - q_{\beta_1}\grave{S}_1 sinj_1) &= \\ \lambda_2(p_{\alpha_2}\acute{P}_2 sini_2 + p_{\beta_2}\acute{S}_2 cosj_2) + (\lambda_2+2\mu_2)(q_{\alpha_2}\acute{P}_2 cosi_2 - q_{\beta_2}\acute{S}_2 sinj_2) & \end{aligned} \tag{8.77}$$

that is:

$$
\begin{aligned}
-\acute{P}_2 sini_2 - \acute{S}_2 cosj_2 + \grave{P}_1 sini_1 + \grave{S}_1 cosj_1 &= -\acute{S}_1 cosj_1 \\
\acute{P}_2 cosi_2 - \acute{S}_2 sinj_2 + \grave{P}_1 cosi_1 - \grave{S}_1 sinj_1 &= -\acute{S}_1 sinj_1 \\
\mu_2 q_{\alpha_2} \acute{P}_2 sini_2 + \mu_2 p_{\alpha_2} \acute{P}_2 cosi_2 + \mu_2 q_{\beta_2} \acute{S}_2 cosj_2 - \mu_2 p_{\beta_2} \acute{S}_2 sinj_2 & \\
+\mu_1 q\alpha_1 \grave{P}_1 sini_1 + \mu_1 p_{\alpha_1} \grave{P}_1 cosi_1 + \mu_1 q_{\beta_1} \grave{S}_1 cosj_1 - \mu_1 p_{\beta_1} \grave{S}_1 sinj_1 &= \\
\mu_1 q_{\beta_1} \acute{S}_1 cosj_1 - \mu_1 p_{\beta_1} \acute{S}_1 sinj_1 & \\
-\lambda_2 p_{\alpha_2} \acute{P}_2 sini_2 - (\lambda_2 + 2\mu_2) q_{\alpha_2} \acute{P}_2 cosi_2 - \lambda_2 p_{\beta_2} \acute{S}_2 cosj_2 + (\lambda_2 + 2\mu_2) q_{\beta_2} \acute{S}_2 sinj_2 & \\
+\lambda_1 p_{\alpha_1} \grave{P}_1 sini_1 + (\lambda_1 + 2\mu_1) q_{\alpha_1} \grave{P}_1 cosi_1 + \lambda_1 p_{\beta_1} \grave{S}_1 cosj_1 - (\lambda_1 + 2\mu_1) q_{\beta_1} \grave{S}_1 sinj_1 &= \\
-\lambda_1 p_{\beta_1} \acute{S}_1 cosj_1 + (\lambda_1 + 2\mu_1) q_{\beta_1} \acute{S}_1 sinj_1 &
\end{aligned}
\tag{8.78}
$$

that is:

$$
\begin{aligned}
-\acute{P}_2 sini_2 - \acute{S}_2 cosj_2 + \grave{P}_1 sini_1 + \grave{S}_1 cosj_1 &= -\acute{S}_1 cosj_1 \\
\acute{P}_2 cosi_2 - \acute{S}_2 sinj_2 + \grave{P}_1 cosi_1 - \grave{S}_1 sinj_1 &= -\acute{S}_1 sinj_1 \\
(\mu_2 q_{\alpha_2} sini_2 + \mu_2 p_{\alpha_2} cosi_2) \acute{P}_2 + (\mu_2 q_{\beta_2} cosj_2 - \mu_2 p_{\beta_2} sinj_2) \acute{S}_2 & \\
+(\mu_1 q_{\alpha_1} sini_1 + \mu_1 p_{\alpha_1} cosi_1) \grave{P}_1 + (\mu_1 q_{\beta_1} cosj_1 - \mu_1 p_{\beta_1} sinj_1) \grave{S}_1 &= \\
(\mu_1 q_{\beta_1} cosj_1 - \mu_1 p_{\beta_1} sinj_1) \acute{S}_1 & \\
-(\lambda_2 p_{\alpha_2} sini_2 + (\lambda_2 + 2\mu_2) q_{\alpha_2} cosi_2) \acute{P}_2 - (\lambda_2 p_{\beta_2} cosj_2 - (\lambda_2 + 2\mu_2) q_{\beta_2} sinj_2) \acute{S}_2 & \\
+(\lambda_1 p_{\alpha_1} sini_1 + (\lambda_1 + 2\mu_1) q_{\alpha_1} cosi_1) \grave{P}_1 + (\lambda_1 p_{\beta_1} cosj_1 - (\lambda_1 + 2\mu_1) q_{\beta_1} sinj_1) \grave{S}_1 &= \\
(-\lambda_1 p_{\beta_1} cosj_1 + (\lambda_1 + 2\mu_1) q_{\beta_1} sinj_1) \acute{S}_1 &
\end{aligned}
\tag{8.79}
$$

that is:

$$
\begin{aligned}
-\alpha_2 p_{\alpha_2} \acute{P}_2 - cosj_2 \acute{S}_2 + \alpha_1 p_{\alpha_1} \grave{P}_1 + cosj_1 \grave{S}_1 &= -cosj_1 \acute{S}_1 \\
cosi_2 \acute{P}_2 - \beta_2 p_{\beta_2} \acute{S}_2 + cosi_1 \grave{P}_1 - \beta_1 p_{\beta_1} \grave{S}_1 &= -\beta_1 p_{\beta_1} \acute{S}_1 \\
(2\rho_2 \beta_2^2 p_{\alpha_2} cosi_2) \acute{P}_2 + \rho_2 \beta_2 (1 - 2\beta_2^2 p_{\beta_2}^2) \acute{S}_2 & \\
+(2\rho_1 \beta_1^2 p_{\alpha_1} cosi_1) \grave{P}_1 + \rho_1 \beta_1 (1 - 2\beta_1^2 p_{\beta_1}^2) \grave{S}_1 &= \\
\rho_1 \beta_1 (1 - 2\beta_1^2 p_{\beta_1}^2) \acute{S}_1 & \\
-\rho_2 \alpha_2 (1 - 2\beta_2^2 p_{\alpha_2}^2) \acute{P}_2 + 2\rho_2 \beta_2^2 p_{\beta_2} cosj_2 \acute{S}_2 & \\
+\rho_1 \alpha_1 (1 - 2\beta_1^2 p_{\alpha_1}^2) \grave{P}_1 - 2\rho_1 \beta_1^2 p_{\beta_1} cosj_1 \grave{S}_1 &= \\
2\rho_1 \beta_1^2 p_{\beta_1} cosj_1 \acute{S}_1 &
\end{aligned}
\tag{8.80}
$$

that is:

$$
M \begin{pmatrix} \acute{P}_2 \\ \acute{S}_2 \\ \grave{P}_1 \\ \grave{S}_1 \end{pmatrix} = N \begin{pmatrix} 0 \\ 0 \\ 0 \\ \acute{S}_1 \end{pmatrix}
\tag{8.81}
$$

with:

$$
M = \begin{pmatrix}
-\alpha_2 p_{\alpha_2} & -cosj_2 & \alpha_1 p_{\alpha_1} & cosj_1 \\
cosi_2 & -\beta_2 p_{\beta_2} & cosi_1 & -\beta_1 p_{\beta_1} \\
2\rho_2 \beta_2^2 p_{\alpha_2} cosi_2 & \rho_2 \beta_2 (1 - 2\beta_2^2 p_{\beta_2}^2) & 2\rho_1 \beta_1^2 p_{\alpha_1} cosi_1 & \rho_1 \beta_1 (1 - 2\beta_1^2 p_{\beta_1}^2) \\
-\rho_2 \alpha_2 (1 - 2\beta_2^2 p_{\alpha_2}^2) & 2\rho_2 \beta_2^2 p_{\beta_2} cosj_2 & \rho_1 \alpha_1 (1 - 2\beta_1^2 p_{\alpha_1}^2) & -2\rho_1 \beta_1^2 p_{\beta_1} cosj_1
\end{pmatrix}
\tag{8.82}
$$

and:

$$
N = \begin{pmatrix}
0 & 0 & 0 & -cosj_1 \\
0 & 0 & 0 & -\beta_1 p_{\beta_1} \\
0 & 0 & 0 & \rho_1 \beta_1 (1 - 2\beta_1^2 p_{\beta_1}^2) \\
0 & 0 & 0 & 2\rho_1 \beta_1^2 p_{\beta_1} cosj_1
\end{pmatrix}
\tag{8.83}
$$

8.1.5 Ray tracing

We solve the eikonal and the transport equations following Čeverný (2001, ch. 3.1).

Eikonal equation

Following Čeverný (2001, ch. 2.4), the eikonal equation is:

$$\nabla T.\nabla T = \frac{1}{V^2} \tag{8.84}$$

with $V = \alpha$ or $V = \beta$. Using the Hamiltonian, it can also be written as:

$$\mathcal{H}(x_i, p_i) = \frac{1}{2}(p_i^2 - \frac{1}{V^2}) = 0 \tag{8.85}$$

where $p_i = \frac{\partial T}{\partial x_i}$.
We define the auxiliary variable σ by:

$$\frac{dx_i}{d\sigma} = \frac{\partial \mathcal{H}}{\partial p_i} \text{ and } \frac{dp_i}{d\sigma} = -\frac{\partial \mathcal{H}}{\partial x_i} \tag{8.86}$$

We get:

$$\frac{dT}{d\sigma} = \frac{\partial T}{\partial x_i}\frac{\partial x_i}{\partial \sigma} = p_i \frac{\partial \mathcal{H}}{\partial p_i} = \frac{1}{V^2} \tag{8.87}$$

thus we have:

$$T = T_0 + \frac{1}{V^2}\sigma \text{ and } \sigma = V^2(T - T_0) \tag{8.88}$$

Constant velocity We have:

$$\frac{dp_i}{d\sigma} = -\frac{\partial \mathcal{H}}{\partial x_i} = \frac{1}{2}\frac{\partial}{\partial x_i}(\frac{1}{V^2}) = -\frac{1}{V^3}\frac{\partial V}{\partial x_i} = 0 \tag{8.89}$$

thus:

$$\begin{aligned} p_1 &= p_{10} \\ p_2 &= p_{20} \\ p_3 &= p_{30} \end{aligned} \tag{8.90}$$

We have:

$$x_i = x_{i0} + \frac{\partial \mathcal{H}}{\partial p_i}\sigma = x_{i0} + p_i\sigma \tag{8.91}$$

thus:

$$\begin{aligned} x_1 &= x_{10} + p_{10}V^2(T - T_0) \\ x_2 &= x_{20} + p_{20}V^2(T - T_0) \\ x_3 &= x_{30} + p_{30}V^2(T - T_0) \end{aligned} \tag{8.92}$$

Constant gradient of velocity We write the velocity as $V = az + b$.
We have:

$$\frac{dp_1}{d\sigma} = 0,\ \frac{dp_2}{d\sigma} = 0 \text{ and } \frac{dp_3}{d\sigma} = -\frac{1}{V^3}\frac{\partial V}{\partial z} = -\frac{a}{(az+b)^3} \tag{8.93}$$

thus:

$$\begin{aligned} p_1 &= p_{10} \\ p_2 &= p_{20} \\ p_3 &= p_{30} - \frac{a}{(az+b)^3}\sigma = p_{30} - \frac{a}{az+b}(T - T_0) \end{aligned} \tag{8.94}$$

We have:

$$
\begin{aligned}
x_1 &= x_{10} + p_{10}\sigma \\
x_2 &= x_{20} + p_{20}\sigma \\
x_3 &= x_{30} + p_{30}\sigma - \frac{1}{2}\frac{a}{(az+b)^3}\sigma^2
\end{aligned}
\tag{8.95}
$$

thus:

$$
\begin{aligned}
x_1 &= x_{10} + p_{10}(az+b)^2(T-T_0) \\
x_2 &= x_{20} + p_{20}(az+b)^2(T-T_0) \\
x_3 &= x_{30} + p_{30}(az+b)^2(T-T_0) - \frac{1}{2}a(az+b)(T-T_0)^2
\end{aligned}
\tag{8.96}
$$

Transport equation

Following Čeverný (2001, ch. 2.4), the transport equation is:

$$2\nabla T.\nabla(\sqrt{\rho V^2}A) + \sqrt{\rho V^2}A\nabla^2 T = 0 \tag{8.97}$$

with $V = \alpha$ or $V = \beta$ and A is the amplitude of the P-wave or one of the two components of the S-wave.

Constant velocity We have $(\nabla T)_i = p_i = p_{i0}$ thus $\nabla^2 T = 0$ and the wave equation becomes:

$$2\nabla T.\nabla(\sqrt{\rho V^2}A) = 0 \tag{8.98}$$

As ρ and V are constant, we get:

$$\nabla T.\nabla A = p_i\frac{\partial A}{\partial x_i} = 0 \tag{8.99}$$

However, we have:

$$\frac{\partial A}{\partial \sigma} = \frac{\partial A}{\partial x_i}\frac{\partial x_i}{\partial \sigma} = \frac{\partial A}{\partial x_i}\frac{\partial \mathcal{H}}{\partial p_i} = \frac{\partial A}{\partial x_i}p_i \tag{8.100}$$

Thus:

$$\frac{\partial A}{\partial \sigma} = 0 \text{ that is } A = A_0 \tag{8.101}$$

Constant gradient of velocity We have:

$$\nabla^2 T = \frac{a^2}{(az+b)^2}(T-T_0) \tag{8.102}$$

If we assume constant density, we get the transport equation:

$$2A\nabla T.\nabla V + 2V\nabla T.\nabla A + A\frac{a^2}{az+b}(T-T_0) = 0 \tag{8.103}$$

8.2 LFE catalog

8.3 Slow slip

Some bla bla about DWT and MODWT.