Neural temporal point processes for earthquake catalogs

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What is a point process?

A point process is a random collection of points falling in some space.

For a temporal point process, the space is a portion of the real line.

$\rightarrow$ Ex: A collection of timings of earthquake events.

For a marked point process, there is also some information associated with the timing of the events.

$\rightarrow$ Ex: A collection of timings and magnitudes of earthquake events.

Conditional intensity function (CIF)

$N_\delta(t)$: Number of events between t and $t+\delta$.

$\mathcal{H}_t = \{((t_i, m_i)\,\forall i : t_i < t\}$: Event history up to time t.

Conditional intensity function ($\sim$ event rate):

$\lambda(t \mid \mathcal{H}_t) = \lim_{\delta \to 0} Pr\{N_\delta(t) > 0 \mid \mathcal{H}_t\}$

Our objective: Computing the value of $\lambda(t)$ knowing the event sequence $\{(t_1, m_1), \cdots, (t_n, m_n)\}$ between T_1 and T_2.

Epidemic-Type Aftershock Sequence (ETAS) model

- Magnitude-frequency distribution law of Gutenberg and Richter: $Pr\left(m_i > m\right) = e^{-\beta(m - m_C)}$ for $m > m_C$.
- Omori-Utsu law of aftershock decay: Number of aftershocks decays as $1/t^p$.
- Each event, irrespective of whether it is a small or a big event, can trigger its own offspring.

$$\lambda\left(t \mid \mathcal{H}_t\right) = \mu + \sum_{i:t_i<t} A e^{\alpha(m_i - m_C)} \left(1 + \frac{t - t_i}{c}\right)^{-p}$$

Fitting an ETAS model

Find the parameters (μ, A, α, c, p) that maximizes the likelihood:

$$
\begin{aligned}
L = & Pr\,\{\text{The first event occurs between } t_1 \text{ and } t_1 + dt \mid \\
& \mathcal{H}_{T_1} \text{ and the first event does not occur between } T_1 \text{ and } t_1\} * \\
& Pr\,\{\text{The second event occurs between } t_2 \text{ and } t_2 + dt \mid \\
& \mathcal{H}_{t_1} \text{ and the second event does not occur between } t_1 \text{ and } t_2\} * \\
& \dots \\
& Pr\,\{\text{The last event occurs between } t_n \text{ and } t_n + dt \mid \\
& \mathcal{H}_{t_{n-1}} \text{ and the last event does not occur between } t_{n-1} \text{ and } t_n\}
\end{aligned}
$$

Fitting an ETAS model

Find the parameters (μ, A, α, c, p) that maximizes the log-likelihood:

$$l = \sum_{i:T_1 \leq t_i \leq T_2} \log \lambda \left(t \mid \mathcal{H}_{t_i} \right) - \int_{T_1}^{T_2} \lambda \left(t \mid \mathcal{H}_t \right) dt$$

Goodness of fit of ETAS model

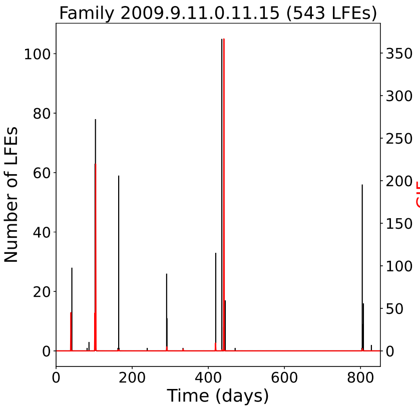
Residual process:

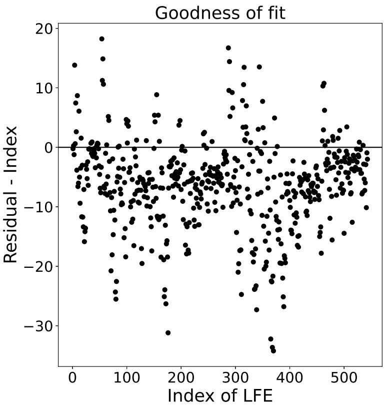
Transform times $\tau_i = \int_{T_1}^{t_i} \hat{\lambda}\,(t \mid \mathcal{H}_t)\,dt$ where $\hat{\lambda}$ is the fitted conditional intensity function.

Plot τ_i as a function of i. There is a good fit if the points follow the straight line $y = x$.

Plot $\tau_i - i$ as a function of i. There is a good fit if the points are horizontally aligned.

Example on an LFE family using R package PtProcess





LFE catalog from Chestler and Creager (2017).

First step: Encoding the timings of the events

Timing t_j of earthquake event is transformed into a vector y_j:

- Inter-event times: $y_j = t_j - t_{j-1}$ or $y_j = \log(t_j - t_{j-1})$.
- Decomposition into a sum of M cosine and sine functions: y_j is the vector of the M weights associated.

Second step: Encode the history of the process

The hidden state h_i contains all the history of the process until time t_i.

- Update at each time step $h_{i+1} = f\left(h_i, y_i\right)$.
- Encode the history of all events together: $H = f\left(y_1, y_2, \cdots, y_n\right)$

Third step: Use the hidden states to get the conditional intensity function

- Parametric density function: $\lambda(\tau_i) = f(\tau_i, \theta_i)$ and $\theta_i = \sigma(Wh_i + b)$.
- Non parametric: $\lambda(\tau_i) = f(Wh_i + b)$.

Compute the loss function $l = -\sum_{i=1}^{n} \log \lambda(t_i) + \int_0^T \lambda(t)\, dt$ and minimize it using a gradient method.

More about ETAS and NTPP

- CORSSA website: `http://www.corssa.org/en/home/`
- Technical session at SSA meeting: New Developments in Physics- and Statistics-based Earthquake Forecasting oral session (Friday 10am to 11:15 am)
- Oleksandr Shchur blog: `https://shchur.github.io/blog/`

Questions?