CÁLCULO IV

Lista de Exercício II

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1. a. $p = e^{10}$ e $\tilde{p} = 22.000$

$EA(\tilde{p}) = 22000 - e^{10}$

$\rightarrow EA(\tilde{p}) = 22000 - 22026,4657948$

$EA(\tilde{p}) = 26,4657948$

$\rightarrow ER(\tilde{p}) = EA(\tilde{p}) / |p| = 0,00120154522$

b. $p = 10^{\pi}$ e $\tilde{p} = 1400$

$EA(\tilde{p}) = 1400 - 10^{\pi}$

$\rightarrow EA(\tilde{p}) = 14,544268633$

$ER(\tilde{p}) = EA / |10^{\pi}|$

$\rightarrow ER(\tilde{p}) = 0,0104978227$

c. $p = 8!$ e $\tilde{p} = 39900$

$EA(\tilde{p}) = 39900 - 8! = 39900 - 40320$

$\rightarrow EA(\tilde{p}) = 420$

$ER(\tilde{p}) = EA(\tilde{p}) / |8!| = 420/40320$

$\rightarrow ER(\tilde{p}) = 0,0104166...$

d. $p = 9!$ e $\tilde{p} = \sqrt{18\pi}\,(9/e)^9$

$EA = \sqrt{18\pi}\,(9/e)^9 - 9!$

$EA = \sqrt{18\pi} \cdot 47811,4866467 - 9!$

$EA = 7,51988482389 \cdot$ "

$\rightarrow EA = 3343,12715805$

$ER = EA(\tilde{p}) / |p|$

$ER(\tilde{p}) = 0,00921276223$

2. a. π

$$\frac{|\pi - \tilde{p}|}{|\pi|} \leq 5 \times 10^{-4}$$

$$|\pi - \tilde{p}| \leq \pi \cdot 5 \cdot 10^{-4}$$
$$|\pi - \tilde{p}| \leq \pi \cdot 0{,}0005$$

a. $[-3{,}14002185726 \; ; \; 3{,}14316644992]$

b. $p = e$

$$\frac{|e - \tilde{p}|}{|e|} \leq 5 \cdot 10^{-4}$$

$$|e - \tilde{p}| \leq e \cdot 5 \times 10^{-4}$$

$$e + e \cdot 5 \cdot 10^{-4} \leq \tilde{p} \leq e + e \cdot 5 \cdot 10^{-4}$$

$= \tilde{p} \in [2{,}71692268754 \; ; \; 2{,}71964096937]$

c. $p = \sqrt{2}$

$$\frac{|\sqrt{2} - \tilde{p}|}{|\sqrt{2}|} \leq 5 \cdot 10^{-4}$$

$p = \tilde{p} \in [1{,}41350645559 \; ; \; 1{,}41492066915]$

d $\Rightarrow$ $p = \sqrt[3]{7}$

$$\frac{|\sqrt[3]{7} - \tilde{p}|}{|\sqrt[3]{7}|} \leq 5 \cdot 10^{-4}$$

~~$\tilde{p} \in [1{,}913887648363772 ; 1{,}911974]$~~

$\tilde{p} \in [1{,}911974717 ; 1{,}913887648]$

3. a. $f(x) = \ln x$; $a = 1{,}5$; $x_0 = 1$

(i) $P_1 = f(1) + f'(1)(a-1)$

$P_1 = \ln(1) + \frac{1}{1} \cdot (1{,}5 - 1)$

$P_1 = 0 + 0{,}5$

$P_1 = 0{,}5$

b. $f(x) = \sqrt{x}$; $a = 9{,}5$; $x_0 = 9$

(i) $P_1 = f(9) + f'(9) \cdot (9{,}5 - 9)$

$P_1 = 3 + \frac{1}{2\sqrt{9}} \cdot (0{,}5)$

$P_1 = 3 + \frac{1}{6} \cdot \frac{1}{2}$

$P_1 = \frac{37}{12}$

Questão 3.

Letra a.

i. $P_1(x) = f(1) + \frac{1}{1}(x-1)$

$P_1(x) = \ln(1) + \frac{(x-1)}{1}$

$P_1(x) = x - 1$

ii. $P_1(1,5) = 1,5 - 1$

$P_1(1,5) = 0,5$

$|ER| = \frac{|0,5 - \ln(1,5)|}{|\ln(1,5)|} \approx 2,33 \times 10^{-1}$

iii. $|P_1(x) - f(x)| = \frac{1}{2} \cdot |f''(\xi) \cdot (x - x_0)^2|$

$= \frac{1}{2} \cdot \frac{1}{\xi^2} \cdot (x-1)^2$ para $1 \leq x \leq 2$

$\leq \frac{1}{2}$

Letna b.

i. $P_1(x) = f(9) + f'(9) \cdot (x - 9)$

$P_1(x) = 3 + \frac{1}{6} \cdot (x - 9)$

$P_1(x) = 3 + \frac{(x-9)}{6}$

$P_1(9.5) = 3{,}0833\ldots$

ii. $|ER| = \frac{|37/12 - \sqrt{9{,}5}|}{\sqrt{9{,}5}} \approx 0{,}000365 \approx 3{,}65 \times 10^{-4}$

iii. $|P_1(x) - f(x)| = \frac{1}{2} \cdot |f''(\xi) \cdot (x - x_0)^2|$

$= \frac{1}{2} \cdot \frac{1}{4 \cdot \xi^{3/2}} \cdot (x-9)^2 \quad , \quad 9 \leq x \leq 10$

$= \frac{1}{8} \cdot \underbrace{\frac{1}{\xi^{3/2}}}_{[3^3]} \cdot \underbrace{(x-9)^2}_{[0,1]}$

$\xi^{3/2} \to \sqrt[2]{\xi^3}$

$\sqrt{\xi^2 \cdot \xi^1}$

$\xi\sqrt{\xi}$

$\approx \xi^3$

$\leq \frac{1}{8} \cdot \frac{1}{3^3} \leq \frac{1}{216}$

Questão 4.

$$f(x) = (1-x)^{-1} \quad , \quad x < 1 \quad e \quad x_0 < 1$$

a. $f(x) = (1-x)^{-1}$

$f'(x) = 1!\,(1-x)^{-2}$

$f'''(x) = 2!\,(1-x)^{-3}$

$f^n(x) = n!\,(1-x)^{-(n+1)}$

para $x_0 = 0$

$f^n(x) = n!$

$$P_n(x) = 0! + 1! \cdot x + \frac{2!}{2!} \cdot x^2 \dots + \frac{n!}{n!} \cdot x^n$$

$$P_n(x) = 1 + x + x^2 + x^3 \dots + x^n$$

$$P_n(x) = 1 \cdot \frac{(1-q^{n+1})}{1-q}, \text{ com } q = x$$

$$P_n(x) = \frac{1-x^{n+1}}{1-x}$$

b. $$|P_n(x) - f(x)| \leq \frac{1}{n!} \max_{0 \leq t \leq \frac{1}{2}} |t|^n \max_{0 \leq t \leq \frac{1}{2}} |f^n(t)|^n$$

$$\leq \frac{1}{n!} \cdot \frac{1}{2}^n \cdot \left(\frac{1}{2}\right)!^n \leq \frac{1}{2^n} \leq 10^{-6}$$

para $n \geq 19{,}93156\ldots \rightarrow \boxed{n = 20}$

Questão 5.

$$\pm (d_0, d_1)_2 \times 2^e \quad -1 \leq e \leq 1$$

Letra a.

$\pm(1,0) \times 2^{-1}$; $\pm(1,0) \times 2^{0}$; $\pm(1,0) \times 2^{+1}$
$\pm(0,1) \times 2^{-1}$; $\pm(0,0) \times 2^{-1}$; $\pm(1,1) \times 2^{-1}$
$\pm(1,1) \times 2^{0}$; $\pm(1,1) \times 2^{-1}$

Letra b.

i- $\left(\frac{1}{3}\right)_{10}$

$\frac{1}{3} \times 2 = \frac{2}{3} < 1$

$\frac{2}{3} \times 2 = \frac{4}{3} > 1$

$\frac{1}{3} \times 2 = \frac{2}{3} < 1$

$= (0,0101010\overline{10}1\ldots)_2$
$\cancel{= (1,010}$ $= (0,1010\overline{1}) \times 2^{-1}$
$= (0,1)_2 \times 2^{-1}$

Letra b.

ii. $2/3 \times 2 = \frac{4}{3} > 1$ $\quad \rightarrow (2/3)_{10} \rightarrow (0,1010\overline{10})_2$
$1/3 \times 2 = 2/3 < 1$
$\vdots$

$= (1,0101\overline{01})_2 \times 2^{-1}$

$\approx (1,1)_2 \times 2^{-1}$

iii. $0,9 \times 2 = 1,8 > 1$

$0,8 \times 2 = 1,6 > 1$

$0,6 \times 2 = 1,2 > 1$

$0,2 \times 2 = 0,4 < 1$

$0,4 \times 2 = 0,8 < 1$

$0,8 \times 2 = 1,6 > 1$

$\vdots$

$= (0,1\overline{1100})_2$

$= (1,1100)_2 \times 2^{-1}$

$= (1,1 + 0,1)_2 \times 2^{-1}$

$= (10,0)_2 \times 2^{-1}$

$= (1,0)_2 \times 2^{0}$

iv. Dado que $(1,1)_2 \times 2^1 = (3)_{10} < 9,6$

$fl(9,6) = +\infty$

Questão 6.

$a = fl(11,4) \quad b = fl(3,18) \quad c = fl(5,05)$

$(a \oplus b) \oplus c = 0,197 \times 10^2$

$a \oplus (b \oplus c) = 0,196 \times 10^2$

Questão 7.

Letra a.

$fl(1,33) = 1,33 \times 10^2$

$fl(0,921) = 9,21 \times 10^{-1}$

$= fl(1,33 \times 10^2 + 9,21 \times 10^{-1})$

$= fl(1,33921 \times 10^2) = 1,34 \times 10^2$

$EA = 1,3392 \times 10^2 - 1,34 \times 10^2$

$EA = 8 \times 10^{-2}$

$ER = \frac{EA}{1,3392 \times 10^2}$

$ER = 5,974 \times 10^{-4}$

Letra b.

$fl(1,33 \times 10^2 - 4,99 \times 10^{-1})$

$fl(1,32501 \times 10^2)$

$= 1,33 \times 10^2$

$EA = \cancel{1,32}\ 1,325 \times 10^2 - 1,33 \times 10^2$

$EA = 5 \times 10^{-1}$

$ER = 3,7736 \times 10^{-3}$

Letra c.

$fl(1,21 \times 10^2 - 3,27 \times 10^{-1}) - fl(1,19 \times 10^2)$

$fl(1,20673 \times 10^2) - fl(1,19 \times 10^2)$

$fl(0,01|6730 \times 10^2) = 0,02 \times 10^2$

$2,00 \times 10^0$

$EA = 1,6730 - 2 \times 10^0 = 0,327$

$ER = 1,9546 \times 10^{-1}$

Letra d.

$fl(1,21 \times 10^2 - 1,19 \times 10^2) - fl(3,27 \times 10^{-1})$

$fl(0,02 \times 10^2) - fl(3,27 \times 10^{-1})$

$\downarrow$

$fl(2 \times 10^0) - fl(3,27 \times 10^{-1})$

$fl(1,673 \times 10^0) = 1,678 \times 10^3$

$EA = 3 \times 10^{-3}$

$ER = 1,7932 \times 10^{-3}$

Questão 8.

$fl(1/3) = +(1,010101..01)_2 \times 2^{-2}$

$fl(2/3) = +(1,010101..01)_2 \times 2^{-1}$

$fl(1/3) + fl(2/3) = (1,11111111)_2 \times 2^{-1}$

mais próximo $= (1,011111) \to +1 = (1,00000..) \times 2^0$

para o zero $= 2^{-1} \cdot \dfrac{(1 - 2^{-53})}{1 - 2^{-1}}$

$\approx 0,999999...$