CÁLCULO IV

Lista de Exercício II

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1. a. $p = e^{10}$ e $\tilde{p} = 22.000$

$EA(\tilde{p}) = 22000 - e^{10}$

$\rightarrow EA(\tilde{p}) = 22000 - 22026,4657948$

$EA(\tilde{p}) = 26,4657948$

$\rightarrow ER(\tilde{p}) = EA(\tilde{p}) / |p| = 0,00120154522$

b. $p = 10^{\pi}$ e $\tilde{p} = 1400$

$EA(\tilde{p}) = 1400 - 10^{\pi}$

$\rightarrow EA(\tilde{p}) = 14,544268633$

$ER(\tilde{p}) = EA / |10^{\pi}|$

$\rightarrow ER(\tilde{p}) = 0,0104978227$

c. $p = 8!$ e $\tilde{p} = 39900$

$EA(\tilde{p}) = 39900 - 8! = 39900 - 40320$

$\rightarrow EA(\tilde{p}) = 420$

$ER(\tilde{p}) = EA(\tilde{p}) / |8!| = 420 / 40320$

$\rightarrow ER(\tilde{p}) = 0,0104166...$

d. $p = 9!$ e $\tilde{p} = \sqrt{18\pi}\,(9/e)^9$

$EA = \sqrt{18\pi}\,(9/e)^9 - 9!$

$EA = \sqrt{18\pi} \cdot 47811,4866467 - 9!$

$EA = 7,51988482389 \cdot$ "

$\rightarrow EA = 3343,12715805$

$ER = EA(\tilde{p}) / |p|$

$ER(\tilde{p}) = 0,00921276223$

2. a. π

$$\frac{|\pi - \tilde{p}|}{|\pi|} \leq 5 \times 10^{-4}$$

$$|\pi - \tilde{p}| \leq \pi \cdot 5 \cdot 10^{-4}$$
$$|\pi - \tilde{p}| \leq \pi \cdot 0{,}0005$$

a. $[-3{,}14002185726 \; ; \; 3{,}14316644992]$

b. $p = e$

$$\frac{|e - \tilde{p}|}{|e|} \leq 5 \cdot 10^{-4}$$

$$|e - \tilde{p}| \leq e \cdot 5 \times 10^{-4}$$

$$e + e \cdot 5 \cdot 10^{-4} \leq \tilde{p} \leq e + e \cdot 5 \cdot 10^{-4}$$

$$= \tilde{p} \in [2{,}71692268754 \; ; \; 2{,}71964096937]$$

c. $p = \sqrt{2}$

$$\frac{|\sqrt{2} - \tilde{p}|}{|\sqrt{2}|} \leq 5 \cdot 10^{-4}$$

$$p = \tilde{p} \in [1{,}41350645559 \; ; \; 1{,}41492066915]$$

d $\Rightarrow$ $p = \sqrt[3]{7}$

$$\frac{|\sqrt[3]{7} - \tilde{p}|}{|\sqrt[3]{7}|} \leq 5 \cdot 10^{-4}$$

~~$\tilde{p} \in [1,9138876483637772 ; 1,911974$~~

$\tilde{p} \in [1,911974717 ; 1,913887648]$

3. a. $f(x) = \ln x$; $a = 1,5$; $x_0 = 1$

(i) $P_1 = f(1) + f'(1)(a - 1)$

$P_1 = \ln(1) + \frac{1}{1} \cdot (1,5 - 1)$

$P_1 = 0 + 0,5$

$P_1 = 0,5$

b. $f(x) = \sqrt{x}$; $a = 9,5$; $x_0 = 9$

(i) $P_1 = f(9) + f'(9) \cdot (9,5 - 9)$

$P_1 = 3 + \frac{1}{2\sqrt{9}} \cdot (0,5)$

$P_1 = 3 + \frac{1}{6} \cdot \frac{1}{2}$

$P_1 = \frac{37}{12}$

Questão 3.

Letra a.

i. $P_1(x) = f(1) + \frac{1}{1}(x-1)$

$P_1(x) = \ln(1) + \frac{(x-1)}{1}$

$P_1(x) = x - 1$

ii. $P_1(1,5) = 1,5 - 1$

$P_1(1,5) = 0,5$

$|\bar{E}R| = \frac{|0,5 - \ln(1,5)|}{|\ln(1,5)|} \approx 2,33 \times 10^{-1}$

iii. $|P_1(x) - f(x)| = \frac{1}{2} \cdot |f''(\xi) \cdot (x - x_0)^2|$

$= \frac{1}{2} \cdot \frac{1}{\xi^2} \cdot (x-1)^2$ para $1 \leq x \leq 2$

$\leq \frac{1}{2}$

Letna b.

i. $P_1(x) = f(9) + f'(9) \cdot (x-9)$

$P_1(x) = 3 + \frac{1}{6} \cdot (x-9)$

$P_1(x) = 3 + \frac{(x-9)}{6}$

$P_1(9.5) = 3{,}0833..$

ii. $|E_R| = \frac{|37/12 - \sqrt{9.5}|}{\sqrt{9.5}} \approx 0{,}000365$
$\approx 3{,}65 \times 10^{-4}$

iii. $|P_1(x) - f(x)| = \frac{1}{2} \cdot |f''(\xi) \cdot (x - x_0)^2|$

$= \frac{1}{2} \cdot \frac{1}{4 \cdot \xi^{3/2}} \cdot (x-9)^2 \quad , \quad 9 \leq x \leq 10$

$= \frac{1}{8} \cdot \underbrace{\frac{1}{\xi^{3/2}}}_{[3^3]} \cdot \underbrace{(x-9)^2}_{[0,1]}$

$\xi^{3/2} \to \sqrt[2]{\xi^3}$
$\sqrt{\xi^2 \cdot \xi^1}$
$\xi \sqrt{\xi}$
$\approx \xi^3$

$\leq \frac{1}{8} \cdot \frac{1}{3^3} \leq \frac{1}{216}$

Questão 4.

$$f(x) = (1-x)^{-1}, \quad x < 1 \quad e \quad x_0 < 1$$

a. $f(x) = (1-x)^{-1}$

$f'(x) = 1!\,(1-x)^{-2}$

$f'''(x) = 2!\,(1-x)^{-3}$

$f^n(x) = n!\,(1-x)^{-(n+1)}$

para $x_0 = 0$

$f^n(x) = n!$

$$P_n(x) = 0! + 1! \cdot x + \frac{2!}{2!} \cdot x^2 \ldots + \frac{n!}{n!} \cdot x^n$$

$$P_n(x) = 1 + x + x^2 + x^3 \ldots + x^n$$

$$P_n(x) = 1 \cdot \frac{(1-q^{n+1})}{1-q}, \text{ com } q = x$$

$$P_n(x) = \frac{1-x^{n+1}}{1-x}$$

b. $$|P_n(x) - f(x)| \leq \frac{1}{n!} \max_{0 \leq t \leq \frac{1}{2}} |t|^n \max_{0 \leq t \leq \frac{1}{2}} |f^n(t)|^n$$

$$\leq \frac{1}{n!} \cdot \frac{1}{2}^n \cdot \left(\frac{1}{2}\right)!^n \leq \frac{1}{2^n} \leq 10^{-6}$$

para $n \doteq 19.93156\ldots \rightarrow \boxed{n = 20}$

Questão 5.

$$\pm (d_0, d_1)_2 \times 2^e \qquad -1 \leq e \leq 1$$

Letra a.

$\pm(1,0) \times 2^{-1}$; $\pm(1,0) \times 2^{0}$; $\pm(1,0) \times 2^{+1}$

$\pm(0,1) \times 2^{-1}$; $\pm(0,0) \times 2^{-1}$; $\pm(1,1) \times 2^{-1}$

$\pm(1,1) \times 2^{0}$; $\pm(1,1) \times 2^{-1}$

Letra b.

i- $\left(\frac{1}{3}\right)_{10}$

$\frac{1}{3} \times 2 = \frac{2}{3} < 1$

$\frac{2}{3} \times 2 = \frac{4}{3} > 1$

$\frac{1}{3} \times 2 = \frac{2}{3} < 1$

$= (0,0101010101\overline{01}\ldots)_2$

$\cancel{= (1,010}$ $= (0,1010\overline{01}) \times 2^{-1}$

$= (0,1)_2 \times 2^{-1}$

Letra b.

ii. $2/3 \times 2 = \frac{4}{3} > 1$ $\rightarrow (2/3)_{10} \rightarrow (0,1010\overline{10})_2$

$1/3 \times 2 = 2/3 < 1$

$\vdots$

$= (1,0101\bar{0}\bar{1})_2 \times 2^{-1}$

$\approx (1,1)_2 \times 2^{-1}$

iii. $0,9 \times 2 = 1,8 > 1$

$0,8 \times 2 = 1,6 > 1$

$0,6 \times 2 = 1,2 > 1$

$0,2 \times 2 = 0,4 < 1$

$0,4 \times 2 = 0,8 < 1$

$0,8 \times 2 = 1,6 > 1$

$\vdots$

$= (0,1\overline{1100})_2$

$= (1,1100)_2 \times 2^{-1}$

$= (1,1 + 0,1)_2 \times 2^{-1}$

$= (10,0)_2 \times 2^{-1}$

$= (1,0)_2 \times 2^{0}$

iv. Dado que $(1,1)_2 \times 2^1 = (3)_{10} < 9,6$

$fl(9,6) = +\infty$

Questão 6.

$a = fl(11,4) \quad b = fl(3,18) \quad c = fl(5,05)$

$(a \oplus b) \oplus c = 0,197 \times 10^2$

$a \oplus (b \oplus c) = 0,196 \times 10^2$

Questão 7.

Letra a.

$fl(1,33) = 1,33 \times 10^2$

$fl(0,921) = 9,21 \times 10^{-1}$

$= fl(1,33 \times 10^2 + 9,21 \times 10^{-1})$

$= fl(1,33921 \times 10^2) = 1,34 \times 10^2$

$EA = 1,3392 \times 10^2 - 1,34 \times 10^2$

$EA = 8 \times 10^{-2}$

$ER = \dfrac{EA}{1,3392 \times 10^2}$

$ER = 5,974 \times 10^{-4}$

Letra b.

$fl(1,33 \times 10^2 - 4,99 \times 10^{-1})$

$fl(1,32501 \times 10^2)$

$= 1,33 \times 10^2$

$EA = \cancel{1,32}\ 1,325 \times 10^2 - 1,33 \times 10^2$

$EA = 5 \times 10^{-1}$

$ER = 3,7736 \times 10^{-3}$

Letra c.

$fl(1,21 \times 10^2 - 3,27 \times 10^{-1}) - fl(1,19 \times 10^2)$

$fl(1,20673 \times 10^2) - fl(1,19 \times 10^2)$

$fl(0,01|6730 \times 10^2) = 0,02 \times 10^2$

$2,00 \times 10^0$

$EA = 1,6730 - 2 \times 10^0 = 0,327$

$ER = 1,9546 \times 10^{-1}$

Letra d.

$fl(1,21 \times 10^2 - 1,19 \times 10^2) - fl(3,27 \times 10^{-1})$

$fl(0,02 \times 10^2) - fl(3,27 \times 10^{-1})$

$\downarrow$

$fl(2 \times 10^0) - fl(3,27 \times 10^{-1})$

$fl(1,673 \times 10^0) = 1,678 \times 10^3$

$EA = 3 \times 10^{-3}$

$ER = 1,7932 \times 10^{-3}$

Questão 8.

$fl(1/3) = +(1,010101..01)_2 \times 2^{-2}$

$fl(2/3) = +(1,010101..01)_2 \times 2^{-1}$

$fl(1/3) + fl(2/3) = (1,11111111)_2 \times 2^{-1}$

mais proximo $= (1,011111) + 1 = (1,00000..) \times 2^0$

para o zero $= 2^{-1} \cdot \dfrac{(1 - 2^{-53})}{1 - 2^{-1}}$

$\approx 0,999999...$

Questão 8.

Letra b.

Como $2^n > \frac{b-a}{\varepsilon}$; $|p_n - \xi| \leq \frac{b-a}{2^n} < \varepsilon$

sendo $\varepsilon = 10^{-3}$ e para $n = 1000$

Seria necessário 1000 iterações para encontrar o valor exato $\xi = 1$ definindo o erro 10^{-3}

Questão 98. $x^3 - 2x + 1 = 0$

Letra a.

i. $x^3 - 2x + 1 = 0$

$$x^2 + 1 = 2x$$

$$x = \frac{1}{2}(x^3 + 1)$$

ii. $x^3 - 2x + 1 = 0$

$$\frac{x^3}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2} = 0$$

$$x = + \frac{2}{x} - \frac{1}{x^2}$$

iii. $x^3 - 2x + 1 = 0$

$$\frac{x^3}{x} - \frac{2x}{x} + \frac{1}{x} = 0$$

$$x^2 = 2 - \frac{1}{x}$$

$$x = \sqrt{2 - \frac{1}{x}}$$

iv. $x^3 - 2x + 1 = 0$

$$-x^3 = -2x + 1$$

$$-x = \sqrt[3]{-2x + 1}$$

$$x = -\sqrt[3]{-2x + 1}$$

Letra b. $p_0 = \frac{1}{2}$

Como sabemos que $\sqrt{z}$:

$f'(x) = 3x^2 - 2 + 1 \quad > 0 \quad \forall \ |x| \geq 1$

$f(x) = x^3 - 2x + 1 \quad \geq 0 \quad \forall \ x > 0$

i. $g(x) = \frac{1}{2}(x^3 + 1)$

g é contínua em $[0, 1]$; $g(0) = \frac{1}{2}$ e $g(1) = 1$

$g'(x) = \frac{1}{2}(3x^2) > 0 \quad \forall x \in [0, 1]$

Logo g possui ao menos um ponto fixo em $[0, 1]$

$\therefore p_0 = 1/2$

$p_1 = g(1/2) = 0,5625$

$p_2 = g(0,5625) = 0,588989257$

$p_3 = g(p_2) = 0,602162...$

$p_4 = g(p_3) = 0,609172$

ii. $g(x) = \frac{2}{x} - \frac{1}{x^2}$, contínua em $[0, 1]$

$g'(x) = -\frac{2(x-1)}{x^3} > 0 \quad \forall (x) > 0$

$g(0)$ vai tender a 0 e $g(1) = 1$.

ii. $p_0 = 1/2$

$p_1 = g(p_0) = 0$

$p_2 = g(0) =$ Sem solução real

iii. $g(x) = \sqrt{2 - \frac{1}{x}}$

$p_0 = \frac{1}{2}$

$p_1 = g(p_0) = 0$

$p_2 = g(0) =$ Sem solução real

iv. $g(x) = -\sqrt[3]{1-2x}$

$p_0 = 1/2$

$p_1 = g(1/2) = -\sqrt[3]{0} = 0$

$p_2 = g(0) =$ ~~sem solução real~~ -1

$p_3 = g(1) = -1,44225$

$p_4 = g(p_3) = -1,57197$

Letra c. Com base nisso a função $g(x)$ que parece convergir a um ponto fixo são:

i. $1/2\,(x^3+1)$

iv. $-\sqrt[3]{1-2x}$

Questão 10. $x\cos x - x^2 - 8x - 1$ em $[-1, 0]$

$\rightarrow$ $g(x) = \dfrac{x\cos x - x^2 - 1}{8}$

~~g(0)~~ $g(-1)$ e $g(0) \in [-1, 0]$

$$g'(x) = -\frac{x\,\text{sen}(x) - \cos(x) + 2x}{8}$$

$g'(x) > 0 \quad \forall x \in [-1, 0]$

~~Logo~~

ponto-fixo no $g(x) = \dfrac{x\cos x - x^2 - 1}{8}$

Questão 11. $x^2 - 3x + 2 = 0$; 1 e 2

$$g(x) = \frac{1}{w}(x^2 - (3-w)x + 2$$

$$g'(x) = \frac{1}{w}(2x - 3 + w)$$

$$\left|\frac{1}{w}(2x - 3 + w)\right| < 1 \qquad \forall\ x < 2$$

quando

$$|2x - 3 + w| < w$$

$$|2x| < w + w + 3$$

$$|x| < \frac{2w+3}{2} < 2$$

$$2w < 1$$

$$w < \frac{1}{w}$$

Questão 11.

Letra a.

para $g'(1) = \frac{1}{w}(2-3+w) = \frac{w-1}{w}$

$$\left|\frac{w-1}{w}\right| < 1 \quad \text{para } x > \frac{1}{2}$$

$\xi = 1 > \frac{1}{2}$ ✗

Letra b.

para $g'(2) = \frac{1}{w}(4-3+w)$

$$\left|\frac{1+w}{w}\right| < 1 \rightarrow x < -\frac{1}{2}$$

$\xi = 2 > -\frac{1}{2}$, não converge

Questão 12. $g \in C'[a,b]$, $p \in [a,b]$

$g(p) = p$ e $\underbrace{|g'(p)| > 1}_{\text{Divergência}}$

Se temos $0 < |p_0 - p| < \delta$

$|p_0 - p| < \delta$ para g contínua

$|g'(p_0) - g'(p)| < \varepsilon$

$g'(p_0) - \varepsilon < g'(p_0) < g'(p_0) + \varepsilon$

Sendo $|g'(p)| > 1$

$-1 > g'(p) > 1$
$g'(p) - \varepsilon = 1$
$g'(p) + \varepsilon = -1$

assim $\varepsilon = |g'(p)| - 1 > 0 \rightsquigarrow g'(p) > 1$

Como g' é contínua, Existe p_0 tal que

$p_0 \in [p-\delta, p+\delta]$ e $g'(p_0) > 1$

$|p_0 - p| = |g(p_0) - g(p)|$

Pelo teorema do valor médio, como $x_0 \in [a, b]$ existe um c entre p_0 e p, tal que

$$|g'(c)| \cdot |(p_0 - p)| = |g(p_0) - g(p)|$$

sendo que $|g'(c)| > 1$

e como $p_1 = g(p_0)$, então

$$|p_1 - p| = |g'(c)| \cdot |g(p_0) - g(p)|$$

$$|p_1 - p| > |p_0 - p|$$

Logo, $|p_0 - p| < |p_1 - p|$

Questão 13.

Letra a. $e^x - 2^{-x} + 2\cos x - 6 = 0$; $1 \leq x \leq 2$

$$f(x) = e^x - 2^{-x} + 2\cos x - 6$$
$$f'(x) = e^x + \frac{\ln(2)}{2^x} - 2\operatorname{sen} x$$

	p_0	p	$\lvert p - p_0\rvert$
1.	1.5	1.95649	$\geq 10^{-2}$
2.	1.95649	1.84153	$> 10^{-2}$
3.	1.84153	1.82951	$> 10^{-2}$
4.	1.82951	$\boxed{1.82938}$	$< 10^{-2}$

Letra b.

$$p_{n+1} = p_n - \frac{(p_n - 2)^2 - \ln(p_n)}{2p_n - 4 - \frac{1}{p_n}}$$

	p_0	p	$\lvert p - p_0 \rvert$
1.	1.5	1.40672	$> 10^{-2}$
2.	1.40672	$\boxed{1.41237}$	$< 10^{-2}$

Questão 14. $x_0 = 1 \quad x_1 = 2 \quad x_2 = 1 \quad x_3 = 2 \dots$

$P(x) = a_2 x^2 + a_1 x + a_0$

$\downarrow$ Derivando

$P'(x) = 2 \cdot a_2 \cdot x + a_1$

$P_0 = 1$
$P_1 = 2$

$$\cancel{P_1 = P_0 - \frac{a_2 x^2 + a_1 x + a_0}{2a_2 x + a_1}}$$

$$\cancel{2 = 1 - \frac{ax^2 + a_1 x + a_0}{2a_2 x + a_1}} = -1$$

~~Como~~ Questão 14.

$$2 = 1 - \frac{a_2 + a_1 + a_0}{2a_2 + a_1}$$

$$\frac{a_2 + a_1 + a_0}{2a_2 + a_1} = -1$$

$$a_2 + a_1 + a_0 = -2a_2 - a_1$$
$$3a_2 + 2a_1 + a_0 = 0$$

~~~ Para $P_2 = 1$ e $P_1 = 2$

$$1 = 2 - \frac{2^2 a_2 + 2a_1 + a_0}{2a_2 2 + a_1}$$

$$\frac{4a_2 + 2a_1 + a_0}{4a_2 + a_1} = 1$$

$$4a_2 + 2a_1 + a_0 = 4a_2 + a_1$$
$$a_1 + a_0 = 0$$
$$a_1 = -a_0$$

$$3a_2 + 2(-a_0) + a_0 = 0$$
$$3a_2 - a_0 = 0$$
$$a_2 = \frac{a_0}{3}$$
~~~

$$P(x) = \frac{a_0}{3}x^2 + (-a_0)x + a_0$$

se $a_0 = 1$

$$P(x) = \frac{1}{3}x^2 - 1x + 1$$

se $a_0 = 3$

$$\boxed{P(x) = x^2 - 3x + 3}$$