Belief and Opinion Dynamics and Aggregation in Multi-Agent Systems

Part I: Beliefs

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Belief vs. opinion

	Content	**Formation**
Belief	propositional (compositionality), higher-order (recursiveness)	reasoned (e.g., inference)
Opinion	atomic	effortless, automatic (e.g., influence, conformity)

$\Rightarrow$ Distinction between system 1 and system 2 (Kahneman, 2003)

Types of belief and opinion sociodynamics

- **Intentional influence** (communication-based):
  - Persuasion
  - Deception and manipulation
  - ...
- **Unintentional influence** (social learning):
  - Contagion
  - Imitation
  - ...

Types of belief and opinion aggregation

- Centralized [ADD FIGURE]
- Decentralized [ADD FIGURE]

Modeling tools

- Beliefs:
  - Epistemic logic (Fagin et al., 1995)
  - Type spaces (Harsanyi, 1967-1968)
  - Dynamic epistemic logic (van Ditmarsch et al., 2007)
  - Belief merging (Konieczny & Pino Pérez, 2002)
  - Belief revision games (Schwind et al., 2015)
  - Recursive reasoning models (Albrecht & Stone, 2018)
- Opinions:
  - Judgment aggregation (Grossi & Pigozzi, 2014)
  - Opinion and preference diffusion (Grandi et al., 2015; Brill et al., 2016)

AI applications

- Beliefs:
  - Epistemic planning
  - Theory of Mind (ToM) modelling for social robots and embodied conversational agents (ECAs)
  - Cryptographic protocols
  - Blockchain
- Opinions:
  - Analysis of opinion diffusion and polarization in social networks
  - E-democray (e.g., liquid democracy)

Epistemic logic (EL) and dynamic epistemic logic (DEL)

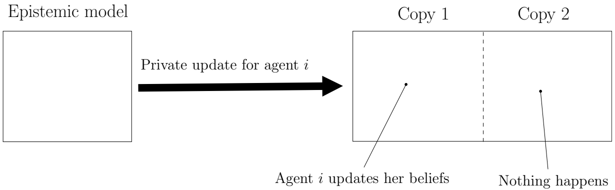
- Propositional and higher-order beliefs
- Multi-relational semantics: **Kripke models**
- Equivalent to type spaces (Galeazzi & Lorini, 2016)
- Large variety of multi-agent belief dynamics
  - Public announcement
  - Private announcement
  - Semi-private announcement
  - ...
- Distributed belief: simple notion of belief aggregation
  - Pooling together the agents' individual beliefs
  - Intersection of the individual epistemic accessibility relations

Belief merging

- Only propositional belief, no higher-order beliefs
- Syntactical approach to belief aggregation (**belief bases**)
- Compact and intuitive semantics (databases)
- Influence-based multi-agent belief dynamics: belief revision games

Private belief change in DEL

- Modelling private belief change in DEL requires world "duplication" (Baltag et al., 1998; Gerbrandy & Groeneveld, 1997)



- The epistemic model grows exponentially
- Using belief bases would radically simplify the approach

Focus of the tutorial

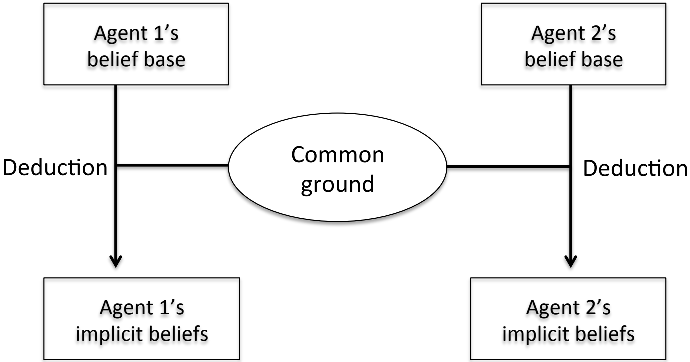
- Epistemic logic with a semantics exploiting belief bases (Lorini, 2018, 2019, 2020; Herzig et al., 2020; Lorini & Romero, 2019)
- $\Rightarrow$ Compact semantics for propositional and higher-order beliefs
- $\Rightarrow$ Rich variety of multi-belief dynamics:
  - Public announcement
  - Private belief change
- $\Rightarrow$ "Parsimonious" account of private belief change:
  - A private informative action modifies the belief base of a single agent
  - No need to "duplicate" models
- $\Rightarrow$ Connection between distributed belief and belief merging

Outline

Outline

1. Beliefs: static picture
2. Belief dynamics
3. Belief merging
4. Bonus track I: recovering introspection
5. Bonus track II: graded belief

Conceptual framework



Common ground can be described as the "...presumed background information shared by participants in a conversation..." (Stalnaker, 2002, p. 701).

Logic of Doxastic Attitudes (LDA)

- Two primitive operators:
  - Explicit belief $\triangle_i$
  - Implicit belief $\Box_i$
- New semantics for epistemic logic: epistemic accessibility relations are computed from belief bases

Language

Language of Logic of Doxastic Attitudes (LDA):

$$\mathcal{L}_0 : \quad \alpha \quad ::= \quad p \mid \neg\alpha \mid \alpha_1 \wedge \alpha_2 \mid \triangle_i \alpha$$

$$\mathcal{L}_{\mathsf{LDA}} : \quad \varphi \quad ::= \quad \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box_i \varphi$$

p ranges over infinite countable set Atm and i ranges over finite set Agt

$\triangle_i \alpha$: agent i explicitly (actually) believes α

: α is in agent i's belief base

$\Box_i \varphi$: agent i implicitly (potentially) believes that φ

Language of Epistemic Logic (EL):

$$\mathcal{L}_{\mathsf{EL}} : \quad \varphi \quad ::= \quad p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box_i \varphi$$

$$\mathcal{L}_{\mathsf{EL}} \subset \mathcal{L}_{\mathsf{LDA}}$$

States

Definition (State)

A state is a tuple $B = (B_1, \dots, B_n, V)$ where:

- $B_i \subseteq \mathcal{L}_0$ is agent i's belief base,
- $V \subseteq Atm$ is the actual environment.

The set of states is noted $\mathbf{S}$.

Definition (Satisfaction relation)

Let $B = (B_1, \dots, B_n, V) \in \mathbf{S}$. Then:

$$
\begin{aligned}
B \models p \quad &\iff \quad p \in V \\
B \models \neg\alpha \quad &\iff \quad B \not\models \alpha \\
B \models \alpha_1 \wedge \alpha_2 \quad &\iff \quad B \models \alpha_1 \text{ and } B \models \alpha_2 \\
B \models \triangle_i \alpha \quad &\iff \quad \alpha \in B_i
\end{aligned}
$$

Epistemic accessibility relation

Definition (Epistemic alternatives)

Let $B = (B_1, \ldots, B_n, V), B' = (B'_1, \ldots, B'_n, V') \in \mathbf{S}$. Then,

$$B\mathcal{R}_i B' \text{ if and only if } \forall \alpha \in B_i : B' \models \alpha.$$

Models

Definition (Multi-agent belief model)

A multi-agent belief model (or simply model) is a pair (B, Cxt), where:

- $B \in \mathbf{S}$, and
- $Cxt \subseteq \mathbf{S}$ is the agents' common ground (or context).

The class of multi-agent belief models is denoted by $\mathbf{M}$.

Definition (Satisfaction relation (cont.))

Let $(B, Cxt) \in \mathbf{M}$. Then:

$$
\begin{aligned}
(B, Cxt) \models \alpha &\iff B \models \alpha \\
(B, Cxt) \models \neg\varphi &\iff (B, Cxt) \not\models \varphi \\
(B, Cxt) \models \varphi \wedge \psi &\iff (B, Cxt) \models \varphi \text{ and } (B, Cxt) \models \psi \\
(B, Cxt) \models \Box_i\varphi &\iff \forall B' \in Cxt : \text{if } B\mathcal{R}_i B' \text{ then } (B', Cxt) \models \varphi
\end{aligned}
$$

Integrity constraint

Definition (Integrity constraint)

Let $\alpha \in \mathcal{L}_0$. Then,

$$Cxt_\alpha = \{B \in \mathbf{S} : B \models \alpha\}$$

is the context induced by the integrity constraint α.

Notice $Cxt_\top = \mathbf{S}$

Validity and satisfiability

Let $\varphi \in \mathcal{L}_{\mathsf{LDA}}$:

- φ is valid, noted $\models_{\mathbf{M}} \varphi$, if and only if $(B, Cxt) \models \varphi$ for every $(B, Cxt) \in \mathbf{M}$
- φ is satisfiable if and only if $\neg\varphi$ is not valid

Axiomatics

Logic LDA:

Axioms of CPL	**(CPL)**
$(\Box_i\varphi \wedge \Box_i(\varphi \to \psi)) \to \Box_i\psi$	$(\mathbf{K}_{\Box_i})$
$\triangle_i\alpha \to \Box_i\alpha$	$(\mathbf{Int}_{\triangle_i,\Box_i})$
$\dfrac{\varphi, \varphi \to \psi}{\psi}$	**(MP)**
$\dfrac{\varphi}{\Box_i\varphi}$	$(\mathbf{Nec}_{\Box_i})$

Axiomatics (cont.)

Theorem

The logic LDA is sound and complete for the class $\mathbf{M}$.

Complexity

- Polynomial embedding of LDA into the logic of *general* awareness (Fagin & Halpern, 1987), whose satisfiability problem is known to be PSPACE-complete
  - Explicit belief $\rightsquigarrow$ (Implicit belief + Awareness)
  - Implicit belief $\rightsquigarrow$ Implicit belief

Theorem (Complexity)

Checking satisfiability of formulas in $\mathcal{L}_{LDA}$ is a PSPACE-complete problem.

Recent result: polysize reduction of sat. problem for logic of *propositional* awareness to LDA-sat. problem (Lorini & Song, DALí 2020)

Model checking problem

Recall: $Cxt_\alpha = \{B \in \mathbf{S} : B \models \alpha\}$

α-context model checking
Given: $\varphi \in \mathcal{L}_{\mathsf{LDA}}$, $\alpha \in \mathcal{L}_0$ and finite $B \in \mathbf{S}_\alpha$.
Question: Do we have $(B, Cxt_\alpha) \models \varphi$?

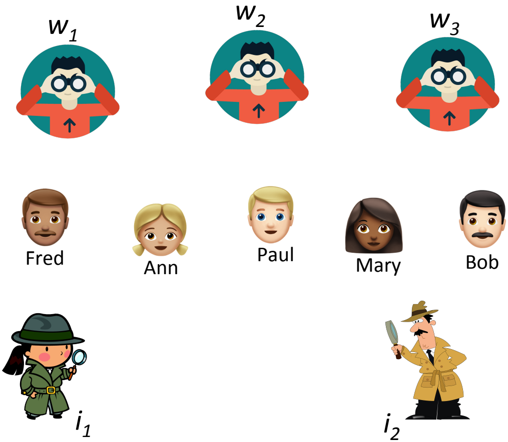
Theorem

The α-context model checking problem is PSPACE-complete.

Consequence: α-context model checking has the same complexity as satisfiability checking

Example: detective story

- Five suspects: Ann, Bob, Fred, Mary and Paul
- Three witnesses: w_1, w_2 and w_3
- Two police investigators: i_1 and i_2



Example: detective story (cont.)

Vocabulary:

- c_x with $x \in Suspect = \{Ann, Bob, Fred, Mary, Paul\}$: "$x$ is the culprit"
- $Clue = \{e, t, f\}$
  - t: "the culprit is tall"
  - e: "the crime was committed after 8 pm"
  - f: "the culprit speaks French"

Example: detective story (cont.)

Common ground:

$$\alpha_1 \stackrel{\text{def}}{=} \bigvee_{x \in Suspect} c_x$$

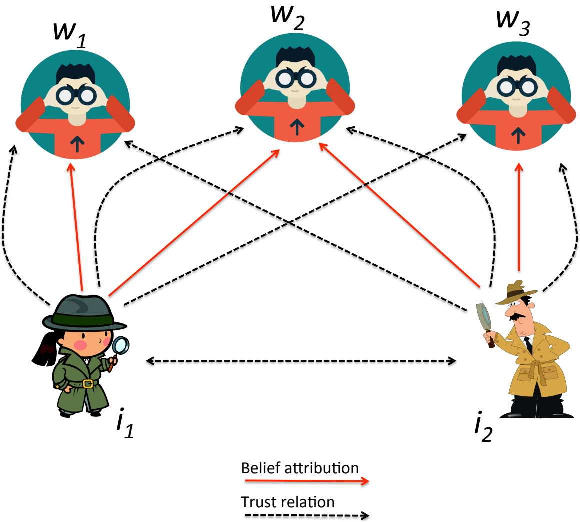
$$\alpha_2 \stackrel{\text{def}}{=} \bigvee_{x,y \in Suspect : x \neq y} (c_x \rightarrow \neg c_y)$$

$$\alpha_3 \stackrel{\text{def}}{=} t \rightarrow (c_{Ann} \vee c_{Fred} \vee c_{Bob} \vee c_{Paul})$$

$$\alpha_4 \stackrel{\text{def}}{=} e \rightarrow (c_{Ann} \vee c_{Mary} \vee c_{Fred} \vee c_{Bob})$$

$$\alpha_5 \stackrel{\text{def}}{=} f \rightarrow (c_{Mary} \vee c_{Fred} \vee c_{Bob} \vee c_{Paul})$$

Example: detective story (cont.)



Example: detective story (cont.)

Actual state $B = (B_{w_1}, B_{w_2}, B_{w_3}, B_{i_1}, B_{i_2}, V)$ with:

$$
\begin{aligned}
B_{w_1} &= \{t\},\\
B_{w_2} &= \{e\},\\
B_{w_3} &= \{f\},\\
B_{i_1} &= \{ \bigwedge_{k \in \{1,2,3\}, p \in Clue} (\triangle_{w_k} p \to p),\\
&\quad \bigwedge_{k \in \{w_1, w_2, w_3\}, p \in Clue} (\triangle_{i_2} \triangle_{w_k} p \to \triangle_{w_k} p), \triangle_{w_1} t, \triangle_{w_2} e\},\\
B_{i_2} &= \{ \bigwedge_{k \in \{1,2,3\}, p \in Clue} (\triangle_{w_k} p \to p),\\
&\quad \bigwedge_{k \in \{w_1, w_2, w_3\}, p \in Clue} (\triangle_{i_1} \triangle_{w_k} p \to \triangle_{w_k} p), \triangle_{w_2} e, \triangle_{w_3} f\},\\
V &= \{c_{Bob}, t, e, f\}.
\end{aligned}
$$

Example: detective story (cont.)

We have:

$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models \Box_{i_1}(c_{Ann} \vee c_{Fred} \vee c_{Bob}) \wedge \Box_{i_2}(c_{Mary} \vee c_{Fred} \vee c_{Bob})$$
$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models \Box_{i_1}(\neg c_{Mary} \wedge \neg c_{Paul}) \wedge \Box_{i_2}(\neg c_{Ann} \wedge \neg c_{Paul})$$

Outline

Outline

Multi-agent belief dynamics

- Public information $\Rightarrow$ common ground change
- Private information $\Rightarrow$ belief base change:
  - Belief base expansion
  - Forgetting
  - Belief base revision
  - ....

Extension by public announcements

$[\varphi!]\psi$: ψ holds after the public announcement of φ

$$(B, Cxt) \models [\varphi!]\psi \iff \text{if } (B, Cxt) \models \varphi \text{ then } (B, Cxt^{\varphi!}) \models \psi$$

where:

$$Cxt^{\varphi!} = \{B' \in Cxt : (B', Cxt) \models \varphi\}$$

Reduction axioms

$$\models_{\mathbf{M}} [\varphi!]p \leftrightarrow (\varphi \to p)$$
$$\models_{\mathbf{M}} [\varphi!]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi!]\psi)$$
$$\models_{\mathbf{M}} [\varphi!](\psi_1 \wedge \psi_2) \leftrightarrow ([\varphi!]\psi_1 \wedge [\varphi!]\psi_2)$$
$$\models_{\mathbf{M}} [\varphi!]\triangle_i\alpha \leftrightarrow \triangle_i\alpha$$
$$\models_{\mathbf{M}} [\varphi!]\Box_i\varphi \leftrightarrow (\varphi \to \Box_i[\varphi!]\varphi)$$

Example: detective story (cont.)

Public announcement by the police department:

"Fred was with his family after 8 pm!"

$(e \rightarrow \neg c_{Fred})!$

We have:

$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models [(e \rightarrow \neg c_{Fred})!]\big(\Box_{i_1}(c_{Ann} \vee c_{Bob}) \wedge \Box_{i_2}(c_{Mary} \vee c_{Bob})\big)$$

$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models [(e \rightarrow \neg c_{Fred})!]\big(\Box_{i_1}(\neg c_{Mary} \wedge \neg c_{Paul} \wedge \neg c_{Fred}) \wedge \Box_{i_2}(\neg c_{Ann} \wedge \neg c_{Paul} \wedge \neg c_{Fred})\big)$$

Extension by private belief base expansion

$[+_i\alpha]\varphi$: φ holds after agent $i \in Agt$ has privately expanded her belief base by α

$$(B, Cxt) \models [+_i\alpha]\varphi \iff (B^{+_i\alpha}, Cxt) \models \varphi$$

with $B^{+_i\alpha} = (B_1^{+_i\alpha}, \ldots, B_n^{+_i\alpha}, V)$ and:

$$B_i^{+_i\alpha} = B_i \cup \{\alpha\}$$
$$B_j^{+_i\alpha} = B_j \text{ if } i \neq j$$

Reduction axioms and complexity

$$\models_{\mathbf{M}} [+_i\alpha]p \leftrightarrow p$$
$$\models_{\mathbf{M}} [+_i\alpha]\neg\varphi \leftrightarrow \neg[+_i\alpha]\varphi$$
$$\models_{\mathbf{M}} [+_i\alpha](\varphi_1 \wedge \varphi_2) \leftrightarrow \big([+_i\alpha]\varphi_1 \wedge [+_i\alpha]\varphi_2\big)$$
$$\models_{\mathbf{M}} [+_i\alpha]\triangle_i\alpha \leftrightarrow \top$$
$$\models_{\mathbf{M}} [+_i\alpha]\triangle_j\beta \leftrightarrow \triangle_j\beta \text{ if } i \neq j \text{ or } \alpha \neq \beta$$
$$\models_{\mathbf{M}} [+_i\alpha]\Box_i\varphi \leftrightarrow \Box_i(\alpha \rightarrow \varphi)$$
$$\models_{\mathbf{M}} [+_i\alpha]\Box_j\varphi \leftrightarrow \Box_j\varphi \text{ if } i \neq j$$

Theorem

The satisfiability problem and α-context model checking problem of LDA extended by private belief base expansion operators are PSPACE-complete.

Example: detective story (cont.)

The investigators exchange their private information about witnesses' beliefs

$$\text{inform}(i,j,\alpha) \stackrel{\texttt{def}}{=} +_j \triangle_i \alpha$$

We have:

$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models [\text{inform}(i_1,i_2,\triangle_{w_1} t)][\text{inform}(i_2,i_1,\triangle_{w_3} f)]\big(\Box_{i_1}(c_{Fred} \vee c_{Bob}) \wedge \Box_{i_2}(c_{Fred} \vee c_{Bob})\big)$$

$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models [\text{inform}(i_1,i_2,\triangle_{w_1} t)][\text{inform}(i_2,i_1,\triangle_{w_3} f)]\big(\Box_{i_1}(\neg c_{Ann} \wedge \neg c_{Mary} \wedge \neg c_{Paul}) \wedge \Box_{i_2}(\neg c_{Ann} \wedge \neg c_{Mary} \wedge \neg c_{Paul})\big)$$

Example: detective story (cont.)

Public announcement by the police departement followed by private information exchange between the investigators:

$$
\begin{aligned}
(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models & [(e \rightarrow \neg c_{Fred})!][\mathrm{inform}(i_1, i_2, \triangle_{w_1} t)] \\
& [\mathrm{inform}(i_2, i_1, \triangle_{w_3} f)]\big(\Box_{i_1} c_{Bob} \wedge \Box_{i_2} c_{Bob}\big)
\end{aligned}
$$

Extension by private forgetting

$[-_i\alpha]\varphi$: φ holds after agent $i \in Agt$ has privately forgotten α

$$(B, Cxt) \models [-_i\alpha]\varphi \iff (B^{-_i\alpha}, Cxt) \models \varphi$$

with $B^{-_i\alpha} = (B_1^{-_i\alpha}, \ldots, B_n^{-_i\alpha}, V)$ and:

$$B_i^{-_i\alpha} = B_i \setminus \{\alpha\}$$
$$B_j^{-_i\alpha} = B_j \text{ if } i \neq j$$

Outline

Outline

Distributed belief language

$$\varphi \quad ::= \quad \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box_G \varphi$$

where α ranges over $\mathcal{L}_0(Atm, Agt)$ and G ranges over $2^{Agt*} = 2^{Agt} \setminus \{\emptyset\}$

$\Box_G \varphi$: coalition G implicitly (potentially) believes that φ

Semantics

Definition (Pooling)

Let $B = (B_1, \ldots, B_n, V) \in \mathbf{S}$ and let $G \in 2^{Agt*}$. Then

$$Pool_G(B) = \bigcup_{i \in G} B_i.$$

Definition (Collective epistemic alternatives)

Let $G \in 2^{Agt*}$ and $B = (B_1, \ldots, B_n, V), B' = (B'_1, \ldots, B'_n, V') \in \mathbf{S}$.
Then,

$$B \mathcal{R}_G B' \text{ if and only if } \forall \alpha \in Pool_G(B) : B' \models \alpha.$$

Semantics (cont.)

Definition (Satisfaction relation (cont.))

Let (B, Cxt) be a MAB. Then:

$$(B, Cxt) \models \Box_G \varphi \iff \forall B' \in Cxt : \text{if } B \mathcal{R}_G B' \text{ then } (B', Cxt) \models \varphi$$

Axiomatics

Axioms of CPL	(**CPL**)
$(\Box_i\varphi \wedge \Box_i(\varphi \to \psi)) \to \Box_i\psi$	($\mathbf{K}_{\Box_i}$)
$\triangle_i\alpha \to \Box_i\alpha$	($\mathbf{Int}_{\triangle_i,\Box_i}$)
$\Box_G\varphi \to \Box_{G'}\varphi$ if $G \subseteq G'$	($\mathbf{Mon}_{\Box_G}$)
$\dfrac{\varphi, \varphi \to \psi}{\psi}$	(**MP**)
$\dfrac{\varphi}{\Box_i\varphi}$	($\mathbf{Nec}_{\Box_i}$)

Model checking: complexity

Theorem

The α-context model checking problem for the extension of $\mathcal{L}_{LDA}$ by implicit distributed belief ($\Box_G$) is PSPACE-complete.

Example: detective story (cont.)

We have:

$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models \Box_{\{i_1, i_2\}}(c_{Fred} \vee c_{Bob})$$
$$(B, Cxt_{\alpha_1 \wedge \ldots \wedge \alpha_5}) \models \Box_{\{i_1, i_2\}}(\neg c_{Ann} \wedge \neg c_{Mary} \wedge \neg c_{Paul})$$

Recovering consistency

Definition (Maximally consistent subsets of collective belief base)

Let $G \in 2^{Agt*}$ and $B \in \mathbf{S}$. Then, $X \in MCS_G(B)$ if and only if:

- $X \subseteq Pool_G(B)$,
- $||X||_{\mathbf{S}} \neq \emptyset$, and
- there is no $X' \subseteq Pool_G(B)$ such that $X \subset X'$ and $||X'||_{\mathbf{S}} \neq \emptyset$,

where, for every $Y \subseteq \mathcal{L}_0$, $||Y||_{\mathbf{S}} = \{B' \in \mathbf{S} : \forall \alpha \in Y, B' \models \alpha\}$.

Recovering consistency (cont.)

Definition (Combining)

Let $B = (B_1, \ldots, B_n, V) \in \mathbf{S}$. Then,

$$\mathit{Comb}_G(B) = \bigcap_{X \in \mathit{MCS}_G(B)} X.$$

Definition (Doxastic alternatives for doxastically consistent (dc) coalitions)

Let $G \in 2^{Agt*}$ and $B = (B_1, \ldots, B_n, V), B' = (B'_1, \ldots, B'_n, V') \in \mathbf{S}$. Then,

$$B \mathcal{R}^{dc}_G B' \text{ if and only if } \forall \alpha \in \mathit{Comb}_G(B) : B' \models \alpha.$$

Recovering consistency (cont.)

$$(B, Cxt) \models \Box^{dc}_G \varphi \iff \forall B' \in Cxt : \text{if } B\mathcal{R}^{dc}_G B' \text{ then } (B', Cxt) \models \varphi$$

Proposition

Let $G \in 2^{Agt*}$ and $B \in \mathbf{S}$. Then,

$$(B, \mathbf{S}) \models \Diamond^{dc}_G \top,$$

where $\Diamond^{dc}_G \varphi \overset{\texttt{def}}{=} \neg\Box^{dc}_G \neg\varphi$.

Theorem

The α-context model checking problem for the extension of $\mathcal{L}_{LDA}$ by doxastically consistent implicit distributed belief ($\Box^{dc}_G$) is PSPACE-complete.

Outline

Recovering introspection

Private belief base expansion does not necessarily preserve positive introspection closure on implicit beliefs

Proposition

There exists $(B, Cxt) \in \mathbf{M}$ *such that:*

- $\forall \varphi \in \mathcal{L}_{LDA}, \forall i \in Agt$:

$$(B, Cxt) \models \Box_i \varphi \rightarrow \Box_i \Box_i \varphi,$$

- $\exists i \in Agt, \exists \alpha \in \mathcal{L}_0, \exists \varphi \in \mathcal{L}_{LDA}$:

$$(B^{+_i \alpha}, Cxt) \models \Box_i \varphi \text{ and } (B^{+_i \alpha}, Cxt) \models \neg \Box_i \Box_i \varphi.$$

The same for negative introspection

Recovering introspection (cont.)

Definition (Epistemic alternatives for introspectively discerning agents)

Let $B = (B_1, \ldots, B_n, V), B' = (B'_1, \ldots, B'_n, V') \in \mathbf{S}$. Then,

$$B\mathcal{R}_i^{ID}B' \text{ if and only if}$$
$$(i) \forall \alpha \in B_i : B' \models \alpha, \textit{and}$$
$$(ii) B_i = B'_i.$$

Proposition

Let $i \in Agt$. Then,

- $\mathcal{R}_i^{ID} \subseteq \mathcal{R}_i$, *and*
- $\mathcal{R}_i^{ID}$ *is transitive and Euclidean.*

$$(B, Cxt) \models \blacksquare_i \varphi \iff \quad \forall B' \in Cxt : \text{if } B\mathcal{R}_i^{ID}B' \text{ then } (B', Cxt) \models \varphi$$

Recovering introspection (cont.)

$$\models_{\mathbf{M}} (\blacksquare_i\varphi \wedge \blacksquare_i(\varphi \to \psi)) \to \blacksquare_i\psi$$

If $\models_{\mathbf{M}} \varphi$ then $\models_{\mathbf{M}} \blacksquare_i\varphi$

$$\models_{\mathbf{M}} \triangle_i\alpha \to \blacksquare_i\alpha$$

$$\models_{\mathbf{M}} \triangle_i\alpha \to \blacksquare_i\triangle_i\alpha$$

$$\models_{\mathbf{M}} \neg\triangle_i\alpha \to \blacksquare_i\neg\triangle_i\alpha$$

$$\models_{\mathbf{M}} \blacksquare_i\varphi \to \blacksquare_i\blacksquare_i\varphi$$

$$\models_{\mathbf{M}} \neg\blacksquare_i\varphi \to \blacksquare_i\neg\blacksquare_i\varphi$$

$$\models_{\mathbf{M}_{BC}} \blacksquare_i\varphi \to \varphi$$

Recovering introspection (cont.)

$$\models_{\mathbf{M}} [+_i\alpha](\blacksquare_i\varphi \rightarrow \blacksquare_i\blacksquare_i\varphi)$$

$$\models_{\mathbf{M}} [+_i\alpha](\neg\blacksquare_i\varphi \rightarrow \blacksquare_i\neg\blacksquare_i\varphi)$$

Outline

Graded belief

$$\varphi \quad ::= \quad \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box_i^k \varphi$$

where α ranges over $\mathcal{L}_0$, i ranges over Agt and k ranges over $\mathbb{N}$

$\Box_i^k \varphi$: agent i would believe that φ,
if she removed k pieces of information from her belief base
: agent i implicitly believes that φ with strength k

Graded belief (cont.)

Definition (Graded doxastic alternatives)

Let $i \in Agt$, $k \in \mathbb{N}$ and $B = (B_1, \ldots, B_n, V), B' = (B'_1, \ldots, B'_n, V') \in \mathbf{S}$:

$$B\mathcal{R}_i^k B' \text{ if and only if } |Sat(B', B_i)| \geq \big(|B_i| - k\big),$$

where $Sat(B', B_i) = \{\alpha \in B_i : B' \models \alpha\}$.

$$(B, Cxt) \models \Box_i^k \varphi \iff \forall B' \in Cxt : \text{if } B\mathcal{R}_i^k B' \text{ then } (B', Cxt) \models \varphi$$

Axiomatics

Axioms and rules of inference of LDA *plus*:

$$\Box_i^k \varphi \to \Box_i k' \varphi \text{ if } k' \leq k$$

$$\Big(\bigwedge_{\alpha \in X} \triangle_i \alpha \Big) \to \Big(\Box_i^k \Big(\bigvee_{X' \subseteq X : |X'| \geq |X| - k} \bigwedge_{\beta \in X'} \beta \Big) \Big) \text{ if } |X| > k$$

Expressivity and model checking

Theorem

The extension of the language $\mathcal{L}_{LDA}$ by graded belief operators $\Box_i^k$ is strictly more expressive than $\mathcal{L}_{LDA}$.

Theorem

The α-context model checking problem for the extension of $\mathcal{L}_{LDA}$ by graded belief ($\Box_i^k$) is PSPACE-complete.

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