Belief and Opinion Dynamics and Aggregation in Multi-Agent Systems

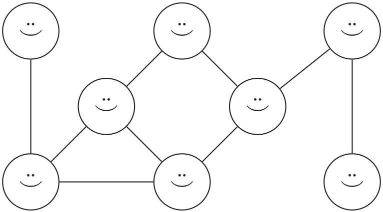
Part 2: Opinion Dynamics and Aggregation

Umberto Grandi · Emiliano Lorini · Arianna Novaro

IJCAI-2020 Tutorial (website)

Election day in Influenceville

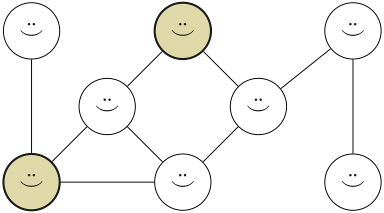
The inhabitants of *Influenceville* are notoriously open to new ideas.



Each day, if 50% (or more!) of an influenceviller's friends think a, the influenceviller will immediately change her opinion to a as well.

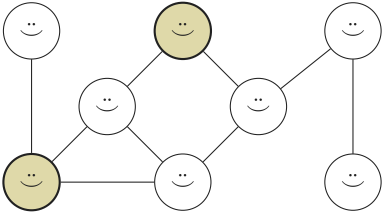
Election day in Influenceville

One day, two politically active influencevillers decide they will vote for the green candidate in the upcoming election.



Election day in Influenceville

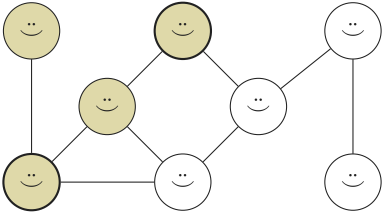
One day, two politically active influencevillers decide they will vote for the green candidate in the upcoming election.



*After how many days will **all** the influencevillers choose to vote for the green candidate (the one above is day zero)?*

Election day in Influenceville

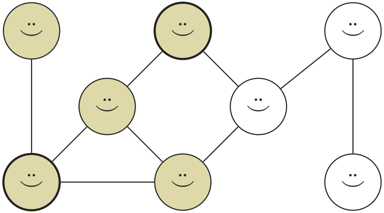
One day, two politically active influencevillers decide they will vote for the green candidate in the upcoming election.



Day 1

Election day in Influenceville

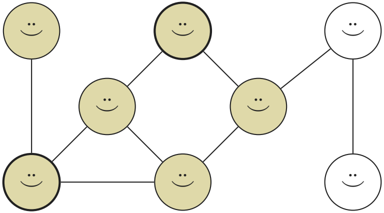
One day, two politically active influencevillers decide they will vote for the green candidate in the upcoming election.



Day 2

Election day in Influenceville

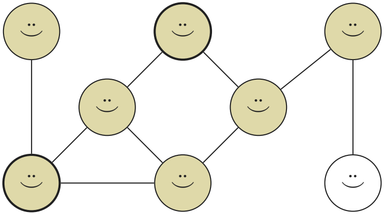
One day, two politically active influencevillers decide they will vote for the green candidate in the upcoming election.



Day 3

Election day in Influenceville

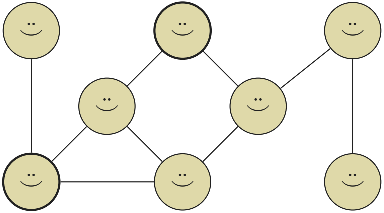
One day, two politically active influencevillers decide they will vote for the green candidate in the upcoming election.



Day 4

Election day in Influenceville

One day, two politically active influencevillers decide they will vote for the green candidate in the upcoming election.



Day 5

Plan for this tutorial

- ☐ Review of opinion diffusion and social influence models
- ☐ Judgment and goal aggregation
- ☐ Propositional opinion diffusion
- ☐ Diffusion of constrained opinions
- ☐ Strategic opinion diffusion

Review of opinion diffusion and social influence models

Voter model

Well-known models, widely studied and used in simulations:

- At any point in time two voters meet and one of the two, again at random, takes the opinion of the other

Play with a simple voter model on `https://demonstrations.wolfram.com/VoterModel/`!

- R. A. Holley, T. M. Liggett. Ergodic Theorems for Weakly Interacting Infinite Systems and the Voter Model. *The Annals of Probability*. 1975.
- P. Clifford, A. Sudbury. A model for spatial conflict. *Biometrika*. 1973.

Weighted averages

Another well-known and studied model:

- Agents have opinions in $[0, 1]$, forming a vector o
- A stochastic matrix G represents the intensity of mutual influence among agents
- Multiply G with o to obtain the influenced opinions, as the weighted sums of the opinions of one's influencers

The linearity of influence and its clean mathematical formulation allows for the use of fixed-point theorems from linear algebra.

- M. H. DeGroot. Reaching a consensus. *Journal of the American Statistical Association*. 1974.
- K. Lehrer, C. Wagner. Applications of the Consensus Model. Rational Consensus in Science and Society. *Philosophical Studies Series in Philosophy*. 1981.

Independent cascades

A probabilistic model of influence mixing weights and probability:

- Agents are on a directed network and have binary $0/1$ opinions
- Each edge e has an influence probability p_e
- All agents with 0-opinion flip one coin for each edge that points to them from an agent with opinion 1, biased with the influence probability of the edge

The well-known paper below studies algorithms to find a set of seeds of fixed size which maximise the spread of the 1-opinion.

• D. Kempe, J. Kleinberg, E. Tardos. Maximizing the spread of influence through a social network. *ACM SIGKDD International Conference on Knowledge discovery and data mining*. 2003.

Our approach

We started from the less well-studied threshold models (Granovetter, Schelling, 1978) and from work on eliciting social influence from agents' choices (Grabisch and Rusinowska), and embarked on the study of diffusion of complex opinions:

- opinions as binary choices over interconnected issues, or preferences over alternatives
- social influence modelled through aggregation functions for each agent (decentralised)
- include elements of strategic reasoning into models of diffusion (intentional-unintentional)

Judgment and goal aggregation

Judgment aggregation

An elegant framework to represent binary collective decisions over a set of interrelated issues having an underlying integrity constraint.

	`valid`	`breached`	`liable`
Judge 1	✓	✓	✓
Judge 2	×	✓	×
Judge 3	✓	×	×
Majority	✓	✓	×

The doctrinal paradox: do you see why?

- L.A. Kornhauser, L.G. Sager. The One and the Many: Adjudication in Collegial Courts. *Calif. Law Review*. 1993.
- C. List, P. Pettit. Aggregating Sets of Judgments: An Impossibility Result. *Economics and Philosophy*. 2002.
- U. Endriss. Judgment Aggregation. In *Handbook of Computational Social Choice*. 2016.

Binary aggregation framework

- A set $\mathcal{N}$ of n agents deciding over a set $\mathcal{I}$ of m binary issues
- An integrity constraint Γ over the issues in $\mathcal{I}$
- The individual ballot $B_a \in \{0,1\}^m$ of agent $a \in \mathcal{N}$
- A profile $\boldsymbol{B} = (B_1, \ldots, B_n)$ with the ballots of the n agents
- An aggregation rule F associating ballot(s) to a profile

	p_1	p_2	$\ldots$	p_m
B_1	1	1	$\ldots$	0
B_2	0	1	$\ldots$	0
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
B_n	1	0	$\ldots$	0
$F(\boldsymbol{B})$	1	1	$\ldots$	0

- U. Grandi, U. Endriss. Binary Aggregation with Integrity Constraints. IJCAI-2011.

Binary aggregation rules

- Majority: accept an issue if $(> / \geq)$ 50% of agents accept it
- Unanimity: accept/reject an issue if all agents accept/reject it

	p_1	p_2	p_3
B_1	1	1	0
B_2	0	1	0
B_3	1	1	0
Majority($\boldsymbol{B}$)	1	1	0
Unanimity($\boldsymbol{B}$)	0	1	0

If the majority outcome is not consistent with the integrity constraint Γ, more sophisticated rules needed (e.g., *Kemeny* rule).

- J. Lang, M. Slavkovik. Judgment aggregation rules and voting rules. ADT-2013.

Axiomatic analysis of rules

How can we analyse aggregation rules, to choose one to use?

$\Rightarrow$ By checking whether they satisfy some desirable properties (called *axioms* in the literature), expressed formally.

How to capture formally our intuition for a "good" property?

Axiomatic analysis of rules

How can we analyse aggregation rules, to choose one to use?

⇒ By checking whether they satisfy some desirable properties (called *axioms* in the literature), expressed formally.

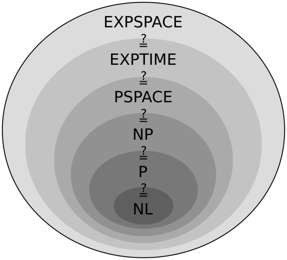
How to capture formally our intuition for a "good" property?

- Anonymity: it should not matter which agent submitted a certain ballot to compute the outcome of the rule
- Unanimity: if every agent agrees on an issue (or on a whole ballot), the outcome of the rule should agree with them
- Monotonicity: if an issue is currently accepted in the outcome, it should still be accepted if more support is given to it

Computational analysis of rules

How can we analyse aggregation rules, to choose one to use?

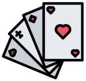
⇒ By studying the computational complexity of calculating their outcome (i.e., the *winner determination problem*)



Goal-based voting

M: Movie night?

T: Take-away?



C: Playing cards?

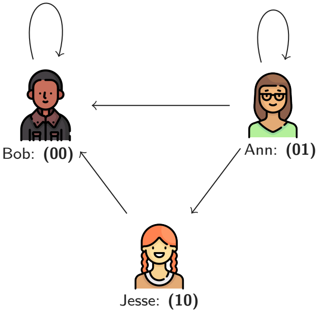
- Aggregation of goals instead of judgments ($\Rightarrow$ collective plan)
- Balance expressivity, compactness and complexity:
  - Propositional formula γ_i instead of ballot B_i for $i \in \mathcal{N}$
  - Restrictions on the language of goals to improve results
  - Need to generalize known voting rules to the new input

• A. Novaro, U. Grandi, D. Longin, E. Lorini. Goal-Based Collective Decisions: Axiom. & Complexity. IJCAI-2018.

Propositional opinion diffusion

Opinion diffusion on a network

Agents are connected to each other over a network and they can influence one another on their opinions over some binary issues.

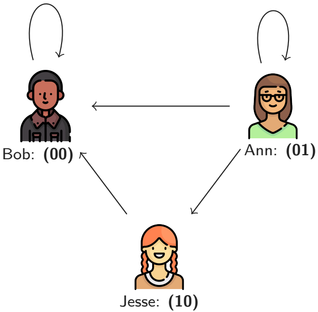


- U. Grandi, E. Lorini, L. Perrussel. Propositional Opinion Diffusion. AAMAS-2015.

Propositional Opinion Diffusion (POD)

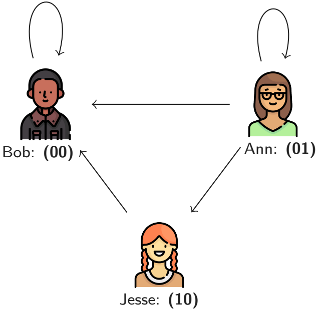
Extension of the binary aggregation framework with addition of:

- an influence network: $E \subseteq \mathcal{N} \times \mathcal{N}$
- the influencers for each agent i: $\mathit{Inf}(i) = \{j \mid (j, i) \in E\}$



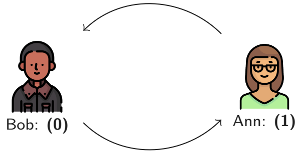
Synchronous iterative diffusion

$$B_i^t = \begin{cases} B_i^{t-1} & \text{if } Inf(i) = \emptyset \\ F_i(\boldsymbol{B}^{t-1}_{Inf(i)}) & \text{otherwise} \end{cases}$$



POD termination

POD terminates on a class of graphs $\mathcal{E}$ if there does not exist an infinite sequence of effective updates starting from any initial opinion profile on any graph $G \in \mathcal{E}$.



Simplest non-terminating graph

- If F_i satisfies ballot-monotonicity for all $i \in \mathcal{N}$, then POD terminates on the class of directed acyclic graphs with loops after at most $diam(E) + 1$ number of steps.

Sufficient and necessary conditions

Related work found necessary and sufficient conditions for POD-termination:

- F_i are independent, monotonic, and responsive, the graph G is serial, and
- *[intricate conditions in terms of winning and losing coalitions of F_i interlocking on G]*

- Z. Christoff, D. Grossi. Stability in Binary Opinion Diffusion. LORI-2017.

Diffusion of constrained opinions

Example

Four individuals need to decide on whether to build a skyscraper (S), a new road (R), or a hospital (H). Law says that if S and H are built then R should also be built.

(Hosp and SkyS) implies Road

Voter 1: Y N N

Voter 2: N N Y

Voter 3: Y Y Y

Voter 4: N N N

What happens if voter 4 is influenced on **all three issues***?*

Example

Four individuals need to decide on whether to build a skyscraper (S), a new road (R), or a hospital (H). Law says that if S and H are built then R should also be built.

(Hosp and SkyS) implies Road

Voter 1: Y N N

Voter 2: N N Y

Voter 3: Y Y Y

Voter 4: N N N

What happens if voter 4 is influenced on **all three issues***?*

What happens if voter 4 is influenced on **one issue at the time***?*

POD vs. proposition-wise updates

Individuals update using aggregation functions F on all issues:

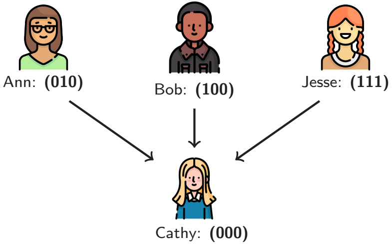
- POD_F is a transformation function that updates the opinion of a subset of individuals on all issues at the same time (provided the update is consistent with the constraint Γ).

Or on subsets of issues of limited size:

- PWOD_F^k is a transformation function that updates the opinion of a subset of individuals on a subset of issues of size at most k (provided the update is consistent with Γ).

Example (continued)

An influence network between four agents, with $\Gamma = (S \wedge H \rightarrow R)$:

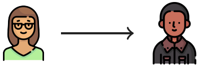


If F_{Cathy} is the strict majority rule and $\boldsymbol{B}$ the above profile, then:

- $\mathrm{POD}_F(\boldsymbol{B}) = \{\boldsymbol{B}\}$, $\boldsymbol{B}$ is a termination profile
- $\mathrm{PWOD}^1_F(\boldsymbol{B}) = \{(010, 100, 111, \mathbf{010}), (010, 100, 111, \mathbf{100}), \boldsymbol{B}\}$.
- $\mathrm{PWOD}^2_F(\boldsymbol{B}) = \mathrm{PWOD}^1_F(\boldsymbol{B})$

Problematic example

Let there be two issues and $\Gamma = p \,\mathrm{XOR}\, q = \{01, 10\}$



Ann: **(01)** Bob: **(10)**

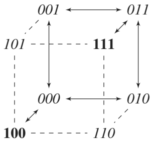
Whatever the unanimous F:

- $\mathrm{POD}_F(\boldsymbol{B}) = \{\boldsymbol{B}, \boldsymbol{B}'\}$ where $B'_1 = B'_2 = (0, 1)$
- $\mathrm{PWOD}^1_F(\boldsymbol{B}) = \{\boldsymbol{B}\}$

Question: Can we characterise the set of integrity constraints on which PWOD^k_F-reachability corresponds to POD_F-reachability?

Reachability result

An integrity constraint Γ is k-geodetic iff for all models B and B' of Γ, at least one of the shortest paths connecting them is composed of nodes that are all models of Γ. A non-example:



- Let Γ be an integrity constraint. Any profile $\boldsymbol{B}'$ that is POD_F-reachable from a Γ-consistent initial profile $\boldsymbol{B}$ is also PWOD_F^k-reachable from $\boldsymbol{B}$ if and only if Γ is k-geodetic.

- S. Botan, U. Grandi and L. Perrussel. Multi-Issue Opinion Diffusion under Constraints. *AAMAS-2019*.

Examples of 1-geodetic constraints

Preferences. Let $a > b$ be a set of binary questions for candidates $a, b, c, \dots$. The constraints are that of transitivity, completeness and anti-symmetry.

- Set of constraints is 1-geodetic: two distinct linear orders always differ on at least one adjacent pair.

• Brill, Elkind, Endriss, Grandi. Pairwise Diffusion of Preference Rankings in Social Networks. IJCAI-2016.

Budget constraints. Enumerate all combinations of items that exceed a given budget. They are *negative formulas*, i.e., one DNF representation only has negative literals: a sufficient condition for 1-geodeticity.

• O. Ekin, P. L. Hammer, A. Kogan. On Connected Boolean Functions. *Discrete Mathematics*, 1999.

Cost of constraints and termination

- Cost of constraints: Can we quantify the gain in terms of influence that is given by allowing updates on k issues?

Answer: the influence gap is the sum of the distances between each individual's opinion and the aggregated one of its influencers. We show that this figure for POD_F is larger than for PWOD_F^k and give precise bounds.

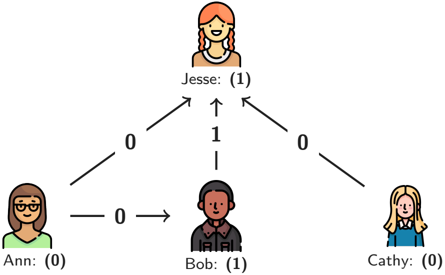
- Termination: Can we find conditions on the graph and aggregator that guarantee termination?

Answer: preliminary findings on complete graphs and DAGs. For arbitrary graphs we have to assume consistent aggregation of influencers' opinions. Ongoing: general termination results.

Strategic opinion diffusion

Exerting influence over opinions

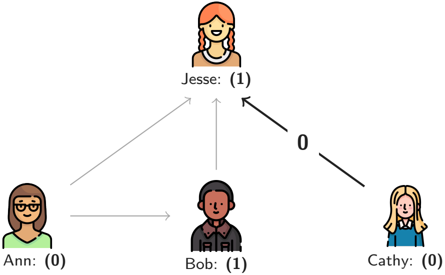
- Extension of POD where agents have individual goals and they choose whether to exert their influence on the issues.



- U. Grandi, E. Lorini, A. Novaro, L. Perrussel. Games of Influence. *Journal of Logic and Computation*. 2021.

Exerting influence over opinions

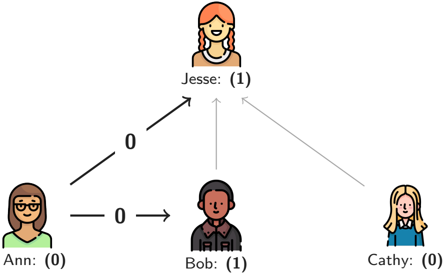
- Extension of POD where agents have individual goals and they choose whether to exert their influence on the issues.



- U. Grandi, E. Lorini, A. Novaro, L. Perrussel. Games of Influence. *Journal of Logic and Computation*. 2021.

Exerting influence over opinions

- Extension of POD where agents have individual goals and they choose whether to exert their influence on the issues.

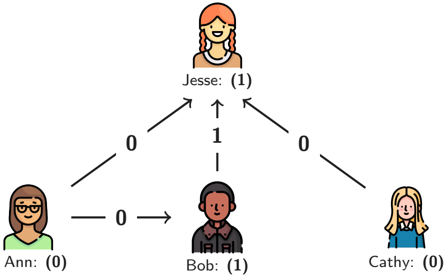


- U. Grandi, E. Lorini, A. Novaro, L. Perrussel. Games of Influence. *Journal of Logic and Computation*. 2021.

Unanimous aggregation of opinions

- Agents need to aggregate the opinions of their influencers

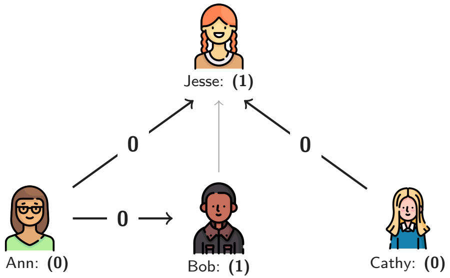
$\Rightarrow$ Unanimity of those influencers who exert their opinion



Unanimous aggregation of opinions

- Agents need to aggregate the opinions of their influencers

$\Rightarrow$ Unanimity of those influencers who exert their opinion



Influence games

Agents first decide whether to exert their influence on some issues, then they update their opinions as per those of their influencers.

- An influence game is a tuple $(\mathcal{N}, \mathcal{I}, E, S_0, F_i, \gamma_i)$
- Each γ_i is a formula in a language of Linear Temporal Logic

  - *Example of issue-wise consensus goal*:

$$\Diamond\Box \wedge_{p\in J} \left((\wedge_{i\in C}\mathsf{op}(i,\{p\})) \vee (\wedge_{i\in C}\neg\mathsf{op}(i,\{p\}))\right)$$

for $C \subseteq N$ and $J \subseteq \mathcal{I}$.

Game-theoretic and computational complexity results for influence games

- If E is a graph such that $Inf(i) \neq \emptyset$ for all $i \in \mathcal{N}$ and γ_i is the issue-wise consesus goal for all $i \in \mathcal{N}$ and $J \subseteq \mathcal{I}$, then there is a Nash equilibrium for any initial state S_0.
- Checking if a profile is a Nash equilibrium (Nash-membership) for influence games with unanimous aggregation rule is in PSPACE for memory-less strategies.

Conclusions

Summary

- Election day in Influenceville: *hard-threshold cascade model*
- A partial review of opinion diffusion and social influence models
- Judgment aggregation: *doctrinal paradox*, *binary aggregation with constraints*, *aggregation rules* (majority, unanimity), *axioms* (anonimity, unanimity, monotonicity); Goal-based voting
- Propositional opinion diffusion (POD): *iterative diffusion process*, *convergence results*
- Diffusion of constrained opinions: *reachability result* and k*-geodetic constraints*
- Influence games: *exerting one's influence*, *unanimous aggregation*, *game-theory and complexity results*

Future work

- Investigate the effects of opinion diffusion on voting:
  - Preliminary results in simulation show that the frequency of Condorcet winners increases after preference diffusion is run
  - Recent work explored control by an external player of the result of an election via opinion diffusion
- Go further in the analysis of constrained opinion diffusion obtaining general termination results
- Develop a full-fledged model of strategic opinion diffusion, in which agents can lie about their opinions towards the satisfaction of their goals