COLLECTIVE INTELLIGENCE – LECTURE 9

SOCIAL CHOICE AND VOTING

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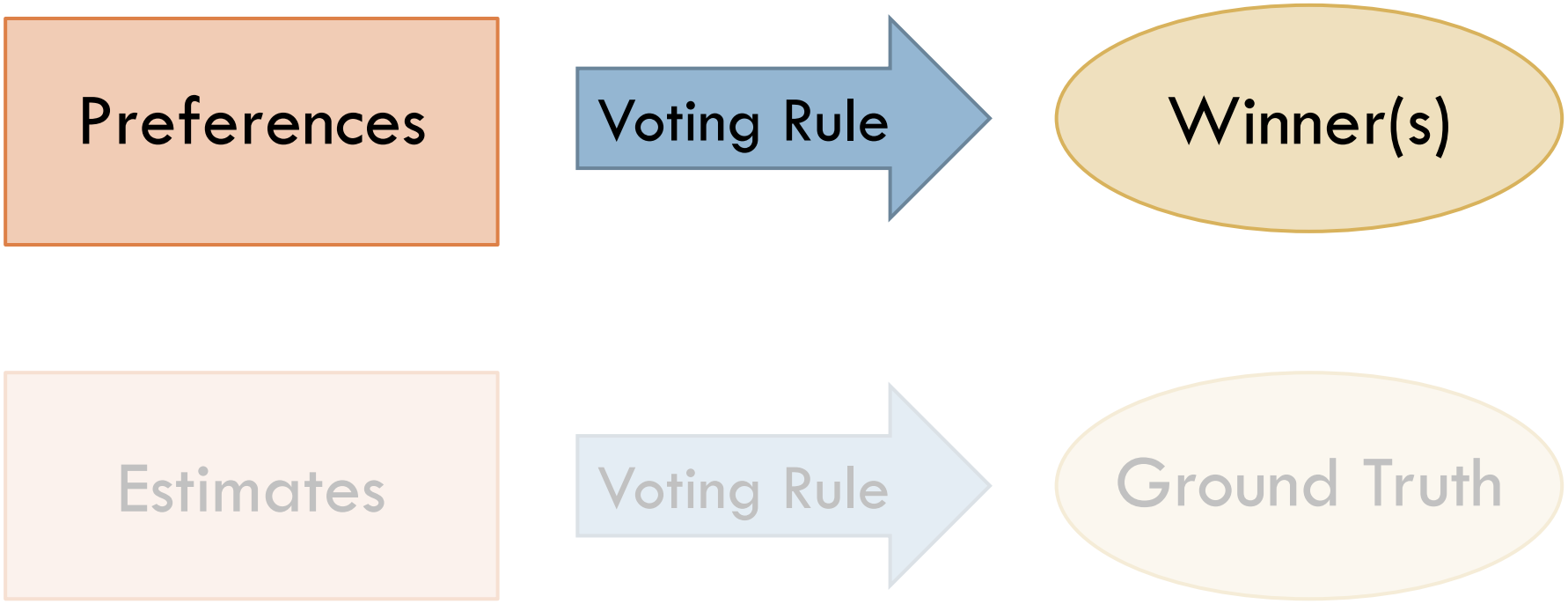
Practicalities

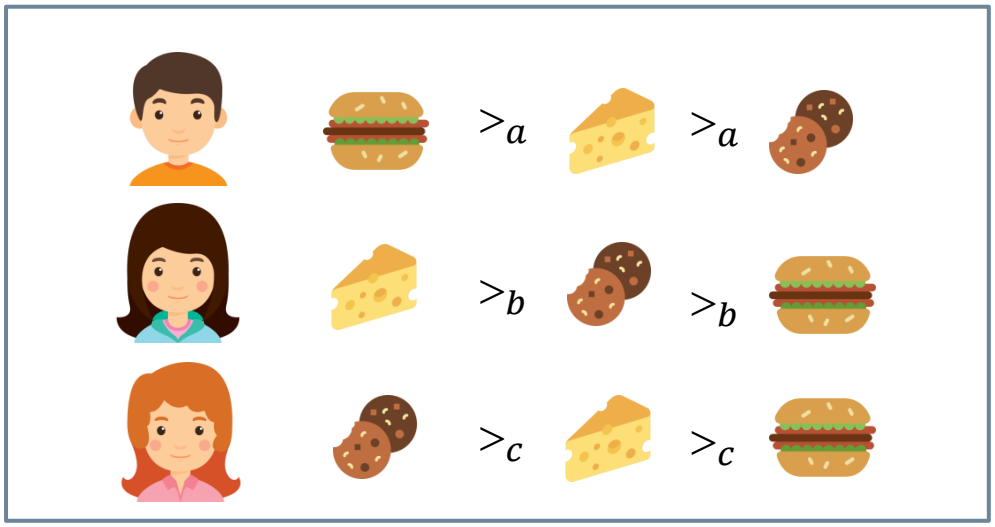
- Contact info: arianna.novaro@irit.fr
- Today (23/11): *Social Choice and Voting*
  - Chapter 15 of EaC book
- Next week (30/11): *Matching Markets*
  - Chapter 12 of EaC book

What is (the goal of) Social Choice?



Alternatives, Agents and Preferences

- A = {a, b, c, …, m} set of *alternatives* (m > 1)
- N = {1, 2, …, n} set of *agents* (or voters)
- $>_i$ in $P_>$ the *preference of agent i*
- $(>_1, \ldots, >_n)$ in $P_>^n$ is a *preference profile*



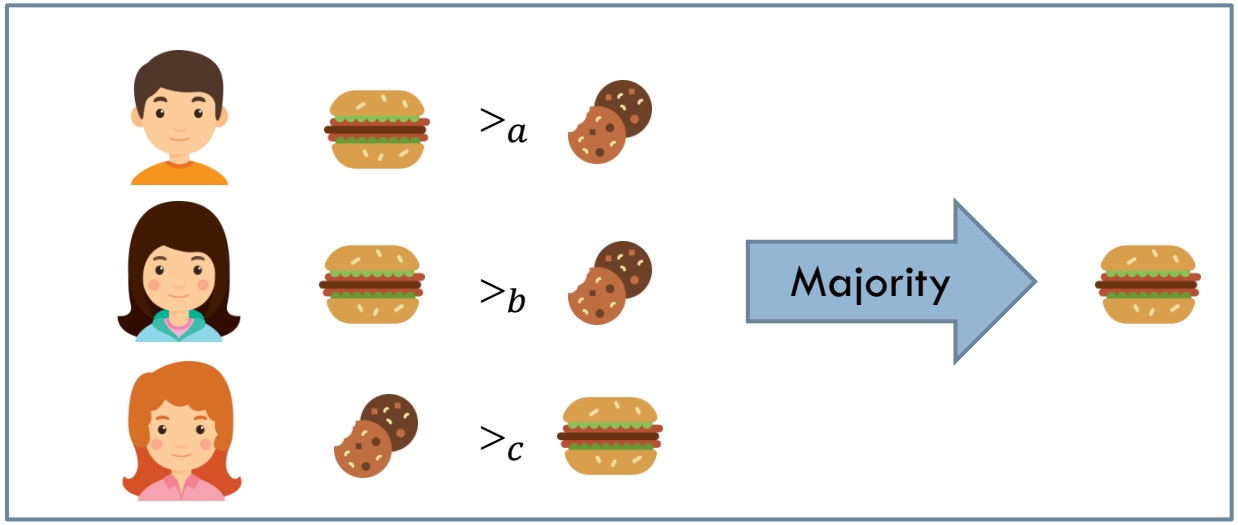
Social Choice/Ranking Rules

- A *social choice rule* f selects an alternative from a profile
  - $f : P_{>}^{n} \rightarrow A$

- A *social ranking rule* R selects a rank of alternatives from a profile
  - $R : P_{>}^{n} \rightarrow P_{>}$

The Simple Majority Rule

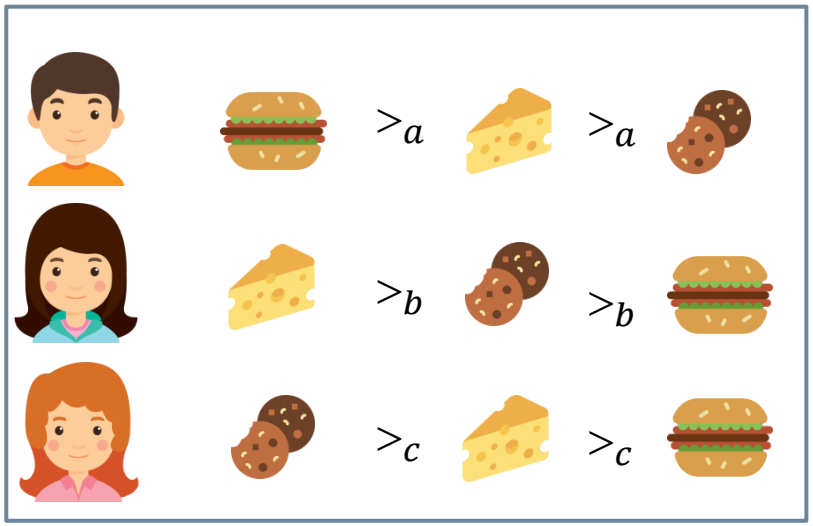
- (m = 2) Select the alternative that is *ranked first by most agents*
  - with some tie-breaking mechanism



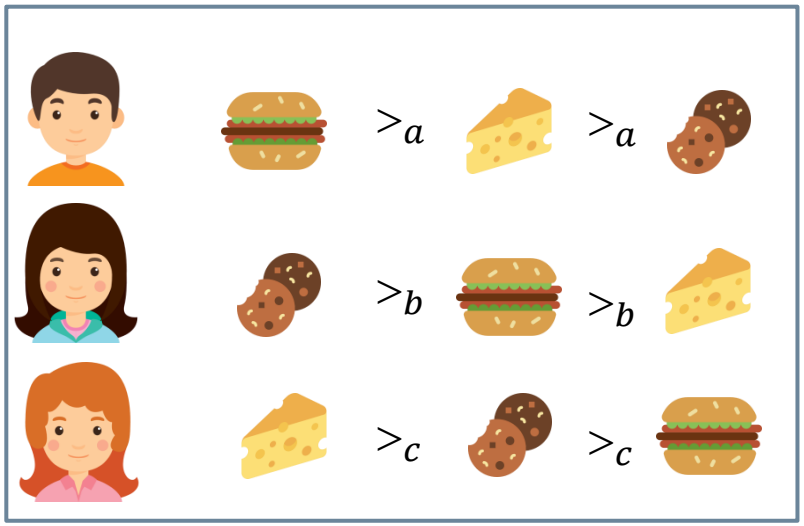
… but what to do when the setting is more complex?

Condorcet Winner

- (m > 2) Select the alternative that *defeats* every *other alternative* by simple majority



Condorcet Winner is 



Condorcet Winner is … nobody!

Voting Rules on Whale

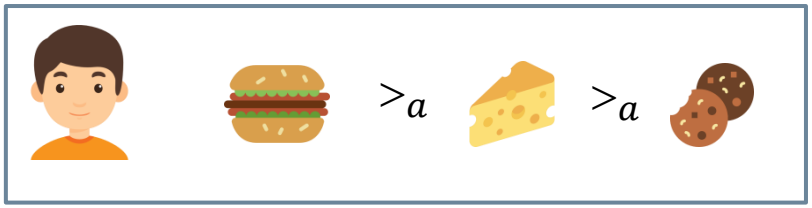
We are going to see other rules by using the Platform *Whale* (developed in Grenoble):

To the (Virtual) Ballot Box!

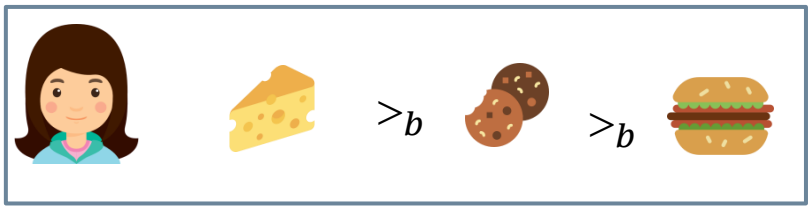
- *Borda* and *Alternative Vote*:
  - http://whale.imag.fr/polls/vote/091e1f23-a80a-4e4a-9961-fba8f1e2a740
- *Approval* and *Range Voting*:
  - http://whale.imag.fr/polls/vote/015e2d4f-cfe4-4a6e-9000-c1e1f5825c3c

Positional Scoring Rules

- *Positional Scoring Rule*: For each vote, a score is assigned to each candidate depending on its position in the ranking. The candidate with maximum total score (summed across all votes) wins.



Borda:



Plurality:

Condorcet Criterion

- A social choice rule satisfies the *Condorcet criterion* if when there is one candidate that beats every other candidate in pairwise duels, then it wins.

- The rule is said to be *Condorcet consistent*.

P-S Rules VS. Condorcet Criterion

- For $m > 2$, no positional scoring rule is Condorcet consistent.

  - *Proof idea.*
    Show that for any vector of scores (i.e., a rule) we can construct a profile where there is a Condorcet winner, but it is not selected as the winner by the positional scoring rule.

Axiomatic Analysis

We want to check if the rules that we are studying satisfy some good *mathematical properties* (*axioms*).

- *Unanimity*: if every agent thinks that A is better than B, the result of the rule should rank A before B
- *Independence of Irrelevant Alternatives* (IIA): if in two profiles the relative position of A and B is unchanged, their relative position in the outcomes should also be unchanged
- *Non-dictatorial*: there is no agent such that on every profile the rule returns her ranking as the output

Arrow's Theorem (1950)

- For $m > 2$, any social ranking rule satisfying unanimity and IIA is also dictatorial.

Characterization of Positional Scoring Rules

- A social choice rule satisfies neutrality, anonymity, weak monotonicity, non-constancy, consistency and continuity if and only if it is a positional-scoring rule (e.g., plurality and Borda).

Strategy-Proofness

- A social-choice rule is *strategy-proof* if an agent is always better off (*dominant strategy*) by reporting her truthful preference.
  - Is *Approval* strategy-proof? Why?
  - Is *Borda* strategy-proof? Why?

A rule *may be* manipulable, but it may be *hard in general* to find a way to manipulate (cf. Chapter 15.6.2).

Summary

- Social choice/ranking rules
  - (Majority, Borda, Alternative Vote, Range Voting, …)
- Condorcet winner and Condorcet criterion
- Positional Scoring rules
  - (Plurality, Borda, …)
- Axiomatic Analysis
  - Arrow's Theorem
  - Characterization of Positional Scoring Rules
- Strategy-Proofness