COLLECTIVE INTELLIGENCE – LECTURE 10

# MATCHING MARKETS

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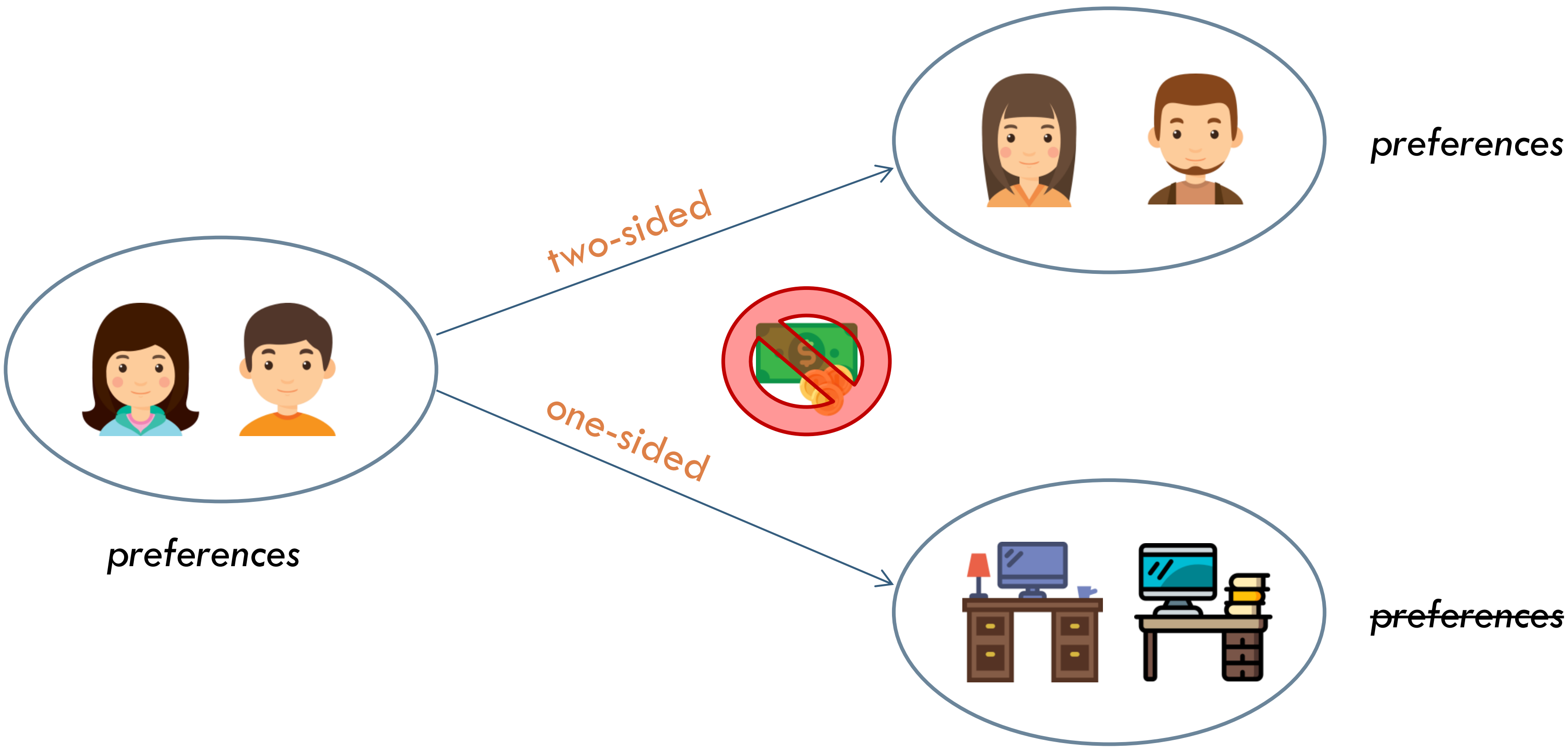
# Practicalities

- Contact info: arianna.novaro@irit.fr
- Last Week (23/11): *Social Choice and Voting*
  - Chapter 15 of EaC book
- Today (30/11): *Matching Markets*
  - Chapter 12 of EaC book

# General Setting of Matching Markets



# Basic Notation

- $N = \{1, \ldots, n\}$ set of *agents*
- $o \in O$ an *outcome* (a *matching* or an *assignment*)
- $\succeq_i \in \boldsymbol{P}$ (weak) *preference of agent i* over outcomes
  - $o \succ_i o'$ i *strictly prefers* outcome o to o'
  - $o \sim_i o'$ i is indifferent between outcomes o and o'
- $\succeq = (\succeq_1, \ldots, \succeq_n) \in \boldsymbol{P}^n$ a *preference profile*

# Axiomatic Properties

- A matching mechanism $g$ is *strategy-proof* if it is dominant for all agents to report their true preferences.
- A mechanism $g$ is *dictatorial* if on all profiles it always selects the most preferred outcome of a given agent.
- A mechanism $g$ is *onto* if for every outcome o there is a profile such that $g$ gives o as the result on that profile.

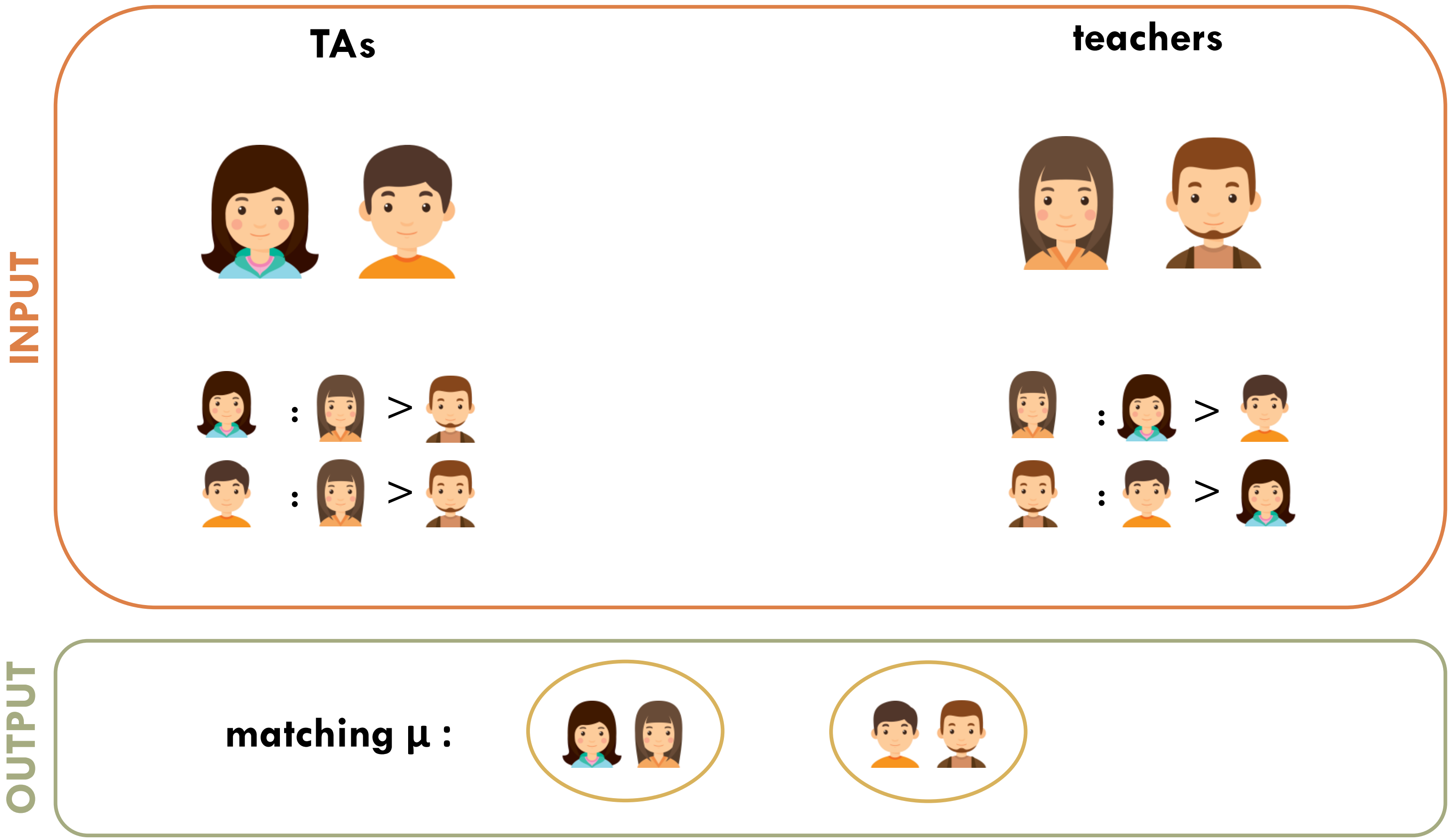
# An Impossibility Result

*(Gibbard-Satterthwaite)* In a domain with 3 or more outcomes and where all strict preference orders on outcomes are possible, any mechanism which is strategy-proof and onto is also dictatorial.

Assume special structure on the preferences in the domain:

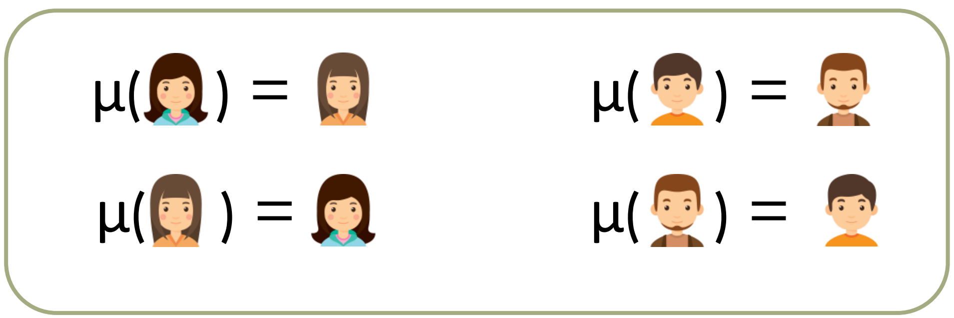
- agents are indifferent between two outcomes if *they* get the same thing in both
- agents prefer any outcome where they get a kidney to any where they don't

# Two-Sided Matching Markets



# Two-Sided Matching : Definition

- A *matching* $\mu : S \cup T \rightarrow S \cup T \cup \{\varphi\}$ is an assignment of each student in S to a teacher in T (or to $\varphi$ if unmatched) and of each teacher to a student (or to $\varphi$), such that $\mu(s) = t$ implies $\mu(t) = s$ and viceversa.



# Stable Matchings

- A *blocking pair* consists of a student and a teacher who prefer one another to the agents they are currently matched to.
- A matching is *stable* if there are no blocking pairs.

| Alice | Bob | Charlotte |
|---|---|---|

Alice : Eva > David > Fred
Bob : David > Fred > Eva
Charlotte : David > Eva > Fred

| David | Eva | Fred |
|---|---|---|

David : Alice > Charlotte > Bob
Eva : Charlotte > Alice > Bob
Fred : Alice > Charlotte > Bob

**Find a stable and an unstable matching.**

# Deferred Acceptance Procedure (S)

- Each student volunteers to her favourite teacher
- Each teacher tentatively accepts the best student among those that contacted her and rejects the others
- Each student who just got rejected volunteers to her favourite teacher (who has not rejected her already)
- Each teacher tentatively accepts the best student among those that contacted her (plus the one they accepted in a previous round) and rejects the others
- The process ends when no new proposals are made

# Deferred Acceptance Procedure IRL

We are now going to simulate the algorithm!

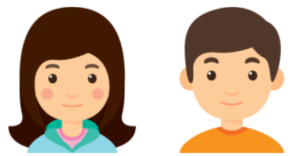
# Properties of the DA Procedure

- Terminates with a stable matching with respect to the reported preferences.
  - Why?
- Gives as outcome the student-optimal (best achievable teacher) stable matching with respect to the reported preferences.
  - Why? What about truthfulness?
- It can be computed in $O(mn)$ time.
  - Why?

# Assignment Problems

INPUT

**Agents**



**Items**

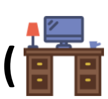
OUTPUT

**assignment** $x$ **:**

# Assignment Problems: Definitions

- $G = \{1, \ldots, n\}$ is the set of *items* to allocate
- $N = \{1, \ldots, n\}$ is the set of *agents*
- $x = (x_1, \ldots, x_n)$ is a *feasible assignment*
  - $x_i$ is the item assigned to agent i
- $>_i$ is the (strict) *preference of agent i* over items in G

$N = \{$  ,  $\}$

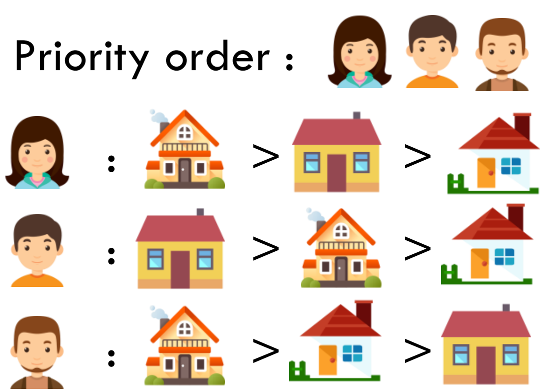
$x = ($  ,  $)$

# House Allocation Problem

- Allocate an house to each agent.
- An assignment is *Pareto optimal* if there is no other assignment that makes all agents as happy, and at least 1 agent strictly happier.

# Serial Dictatorship

- There is a given *priority order* of the agents
- Each agent reports their strict preference over items
- At step $k$, the agent at position $k$ in the order is given their preferred item among those not already assigned




**What is the result of the SD?**

# Serial Dictatorship: Properties and Variations

- The SD mechanism is strategy-proof and Pareto-optimal for the house allocation problem.
  - Why?

- In the *Random Serial Dictatorship* the priority order is randomly chosen, and the Pareto-optimality property can be adapted to obtain an analogous result.

# House Markets Problem

- Agents already possess one house each at the beginning, and start swapping houses to improve their happiness (they are *individually rational*).
- A set of agents form a *blocking coalition* if they would get a better outcome (weakly for all, strictly for at least one) by trading among themselves.
- An assignment is in the core if there is no blocking coalition (and it is also Pareto-optimal).

# Top-Trading Cycles

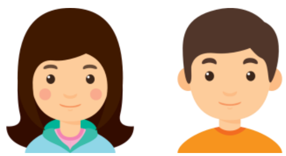
- Each agent reports their preferences on houses
- Form a graph: agents as vertices, one edges to the owner of their favourite house (self-loops possible)
- Swap houses on cycles, then remove agents who traded (and their houses)
- Repeat the two previous steps with leftover agents and houses until there are no more people

- Outcome of TTC is in the core, strategy-proof and can be determined in $O(n^2)$.

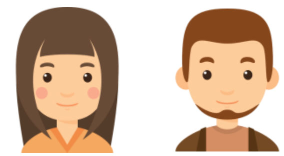
# Top-Trading Cycles IRL

We are now going to simulate the algorithm!

# Kidney-Paired Donation



Alice Bob



Charlotte David

Bob and Charlotte need a kidney transplant.
Alice wants to donate a kidney to Bob. David wants to donate a kidney to Charlotte.
However, Alice and Bob are incompatible, as well as Charlotte and David.
Turns out that Alice is compatible with Charlotte, and David with Bob.
Hence, the four agree to swap donors.

# Kidney Donations Set-Up

- A graph $G = (V, E)$ where each vertex is a pair of patient and donor, and an edge from $a$ to $b$ means that $a$ is compatible with $b$.
- A *matching* is a set of vertex-disjoint cycles on G.

Patients don't have *preferences*, but compatibilities.

They don't have an *ordering*: just compatible or not.

Cycles *have to be limited* for logistical reasons.

# Summary

- Matching and Axiomatic Properties
  - Impossibility Result
- Two-Sided Matchings
  - Blocking Pairs and Stability
  - Deferred Acceptance Procedure
- Assignment Problems
  - House Allocation Problem
    - Serial Dictatorship
  - House Markets Problem
    - Top-Trading Cycles
- Kidney-Paired Donations

Icons by Flaticon