GENERAL ASSEMBLY

INTRO TO DATA SCIENCE:
LOGISTIC REGRESSION

AGENDA

LECTURE:

I. LOGISTIC REGRESSION

II. OUTCOME VARIABLES

III. ERROR TERMS

IV. INTERPRETING RESULTS

LAB: IMPLEMENTING LOGISTIC REGRESSION IN PYTHON

FINAL PROJECT KICKOFF & “ELEVATOR PITCH” WORKING SESSION

I. LOGISTIC REGRESSION

	continuous	*categorical*
supervised	*???*	*???*
unsupervised	*???*	*???*

	continuous	***categorical***
supervised	*regression*	*classification*
unsupervised	*dimension reduction*	*clustering*

Q: What is **logistic regression***?*

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A: A generalization of the linear regression model to classification problems.

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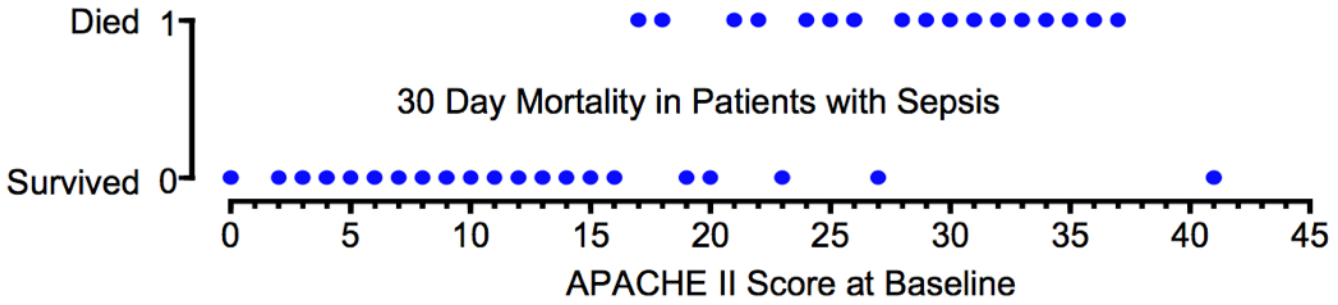
In logistic regression, we use a set of covariates to predict probabilities of (binary) class membership.

These probabilities are then mapped to class labels, thus solving the classification problem (categorical output).

A motivating problem:

The following figure shows 30 day mortality in a sample of septic patients as a function of their baseline APACHE II score. Patients are coded as 1 or 0 depending on whether they are dead or alive in 30 days, respectively.

How can we predict death from baseline APACHE II score in these patients?

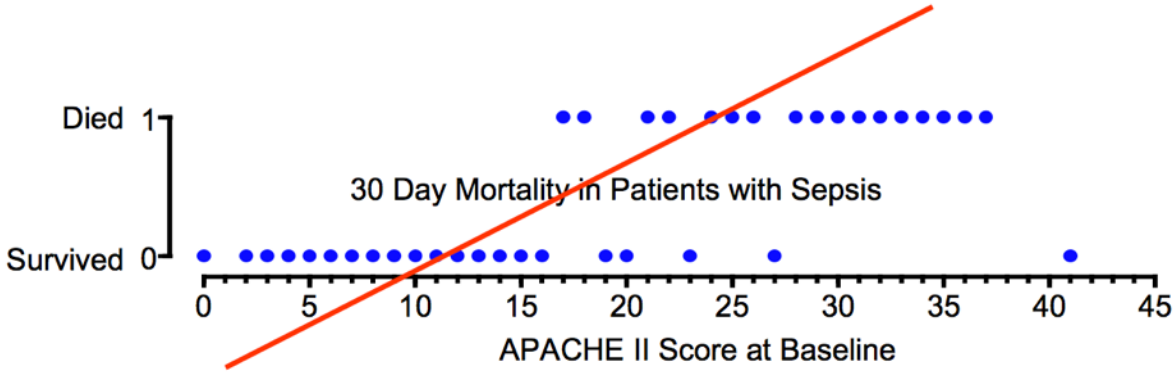


Source: http://www.mc.vanderbilt.edu/gcrc/workshop_files/2004-11-12.pdf

Q: How can we predict death from baseline APACHE II score in these patients?

Let p(x) be the probability that a patient with score x will die within 30 days.

Well, linear regression would not work well here, because it could produce probabilities less than zero or greater than one. Also, one new value could great change our model...



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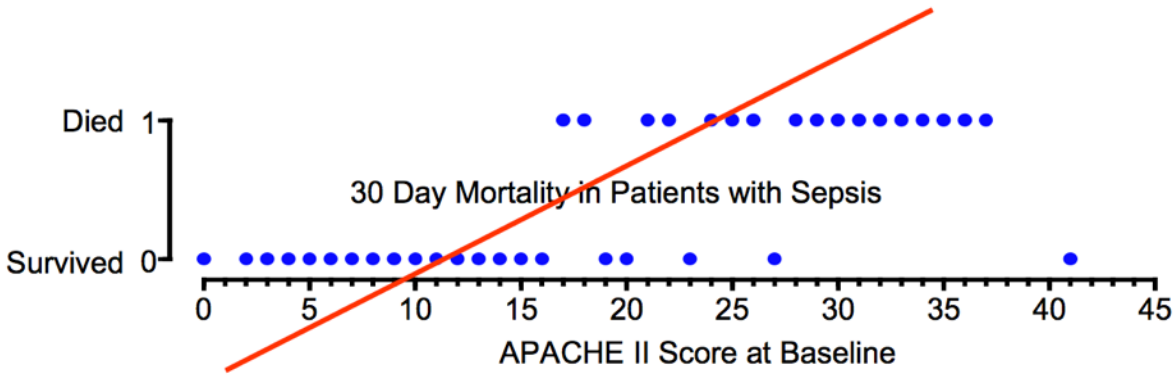
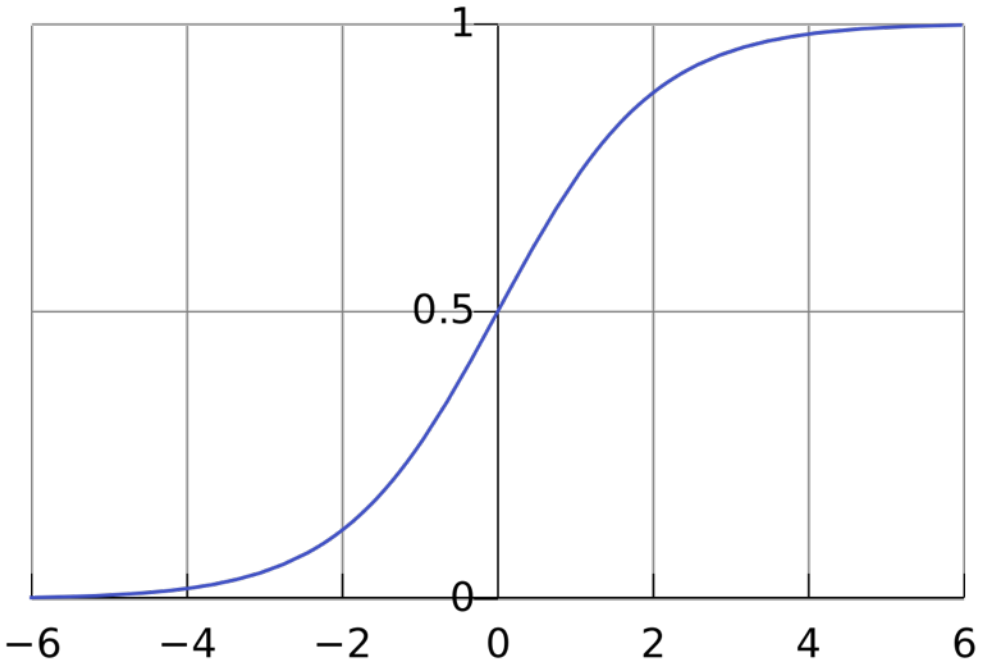
So, what can we do instead of linear regression?



LOGISTIC REGRESSION

probability of belonging to class



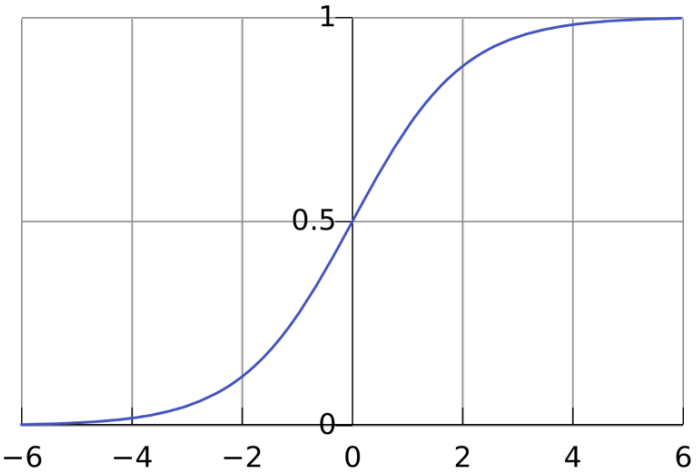
value of independent variable

NOTE

Probability predictions look like this.

This function fits our problem much better:

$$0 \leq h_\theta(x) \leq 1$$



In other words, our classifier will output values between 0 and 1. It asymptotically approaches 0 and 1.

This is called the Sigmoid Function, or the Logistic Function (synonymous)

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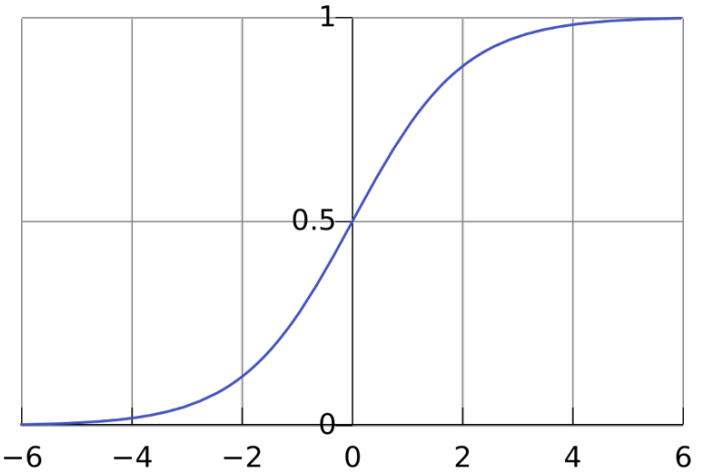
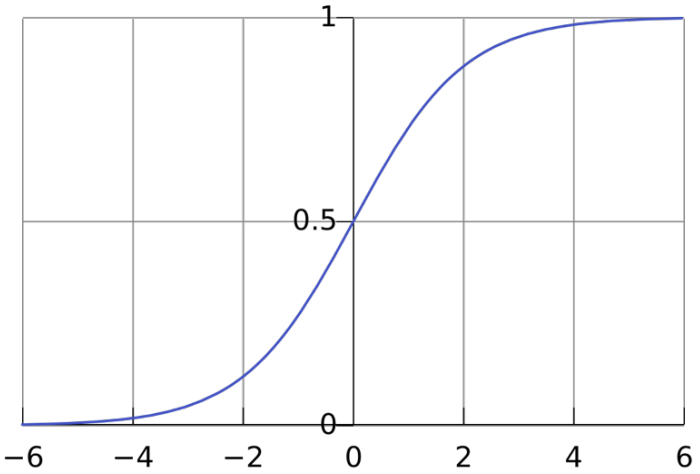


NOTE

This function gives Logistic Regression its name!

The logistic function:

$$F(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-t}}$$



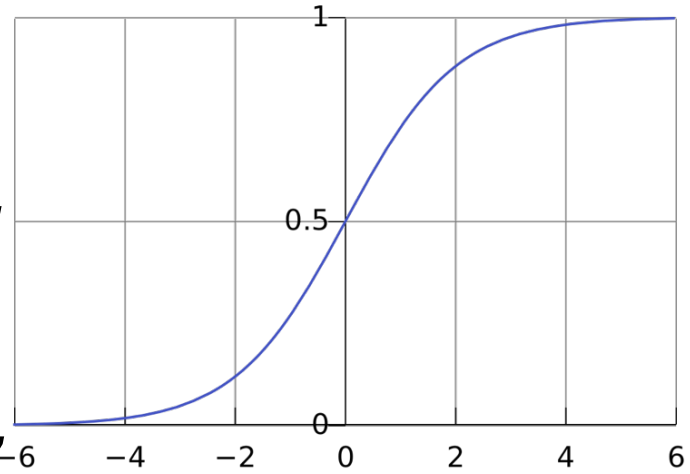
Notice that f(t) = 0.5 when t = 0

f(t) >= 0.5 when t > 0

f(t) <= 0.5 when t < 0

Suppose we predict class 1 when f(t) >= 0.5 and class 0 when f(t) < 0.5

$$F(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-t}}$$



So, if the t in the logistic function is a linear function of an explanatory variable x, or a linear combination of explanatory variables, the logistic function becomes:

$$F(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

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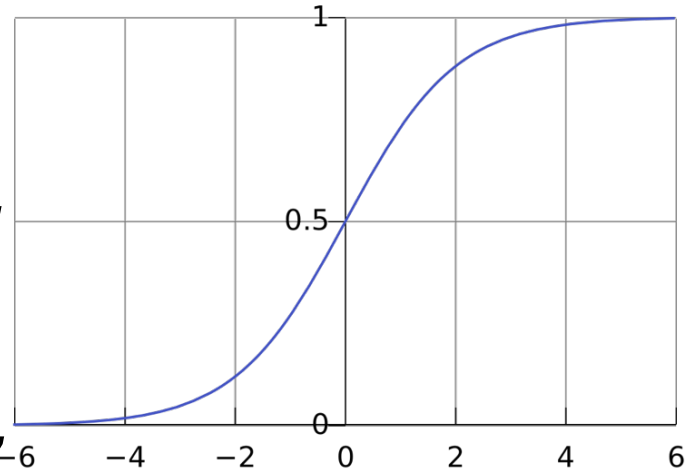

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Does that exponent look familiar…?

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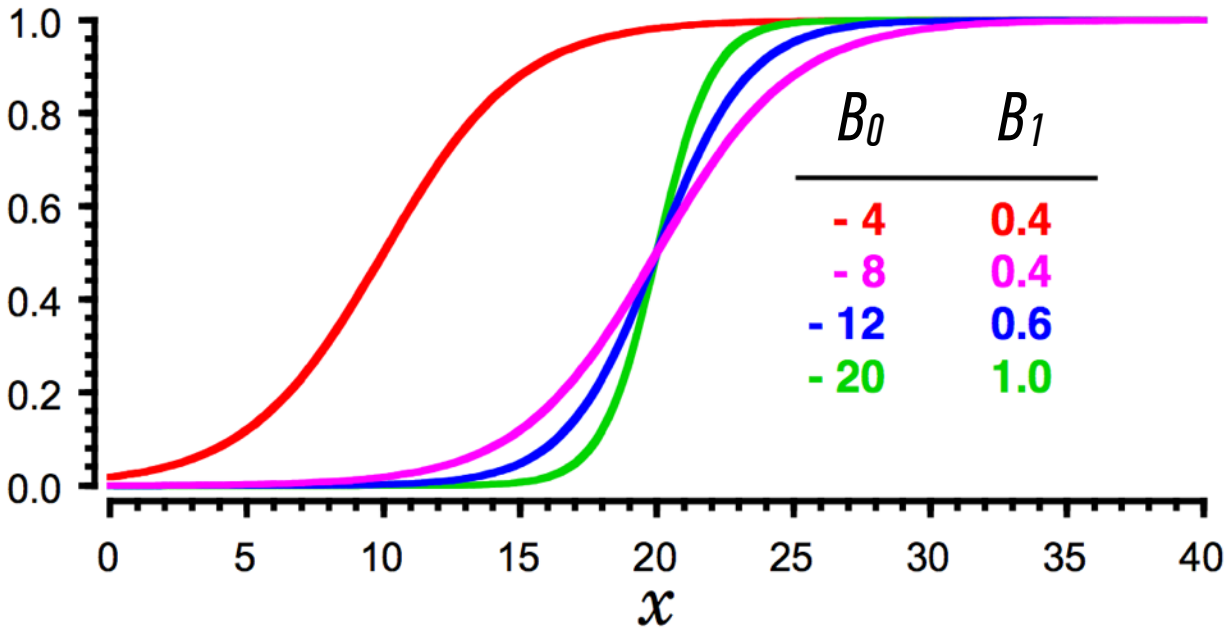
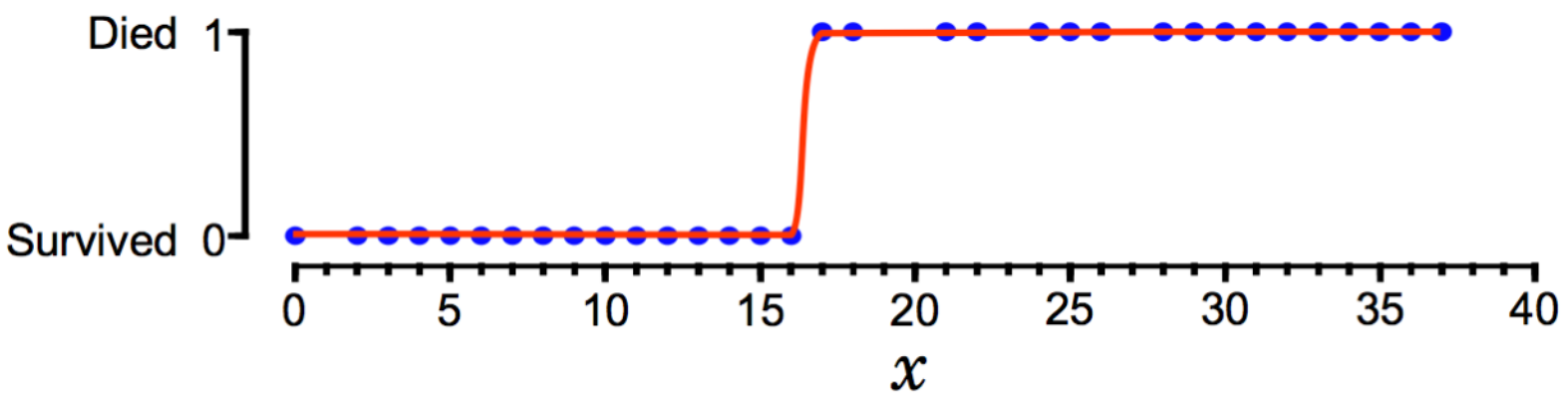
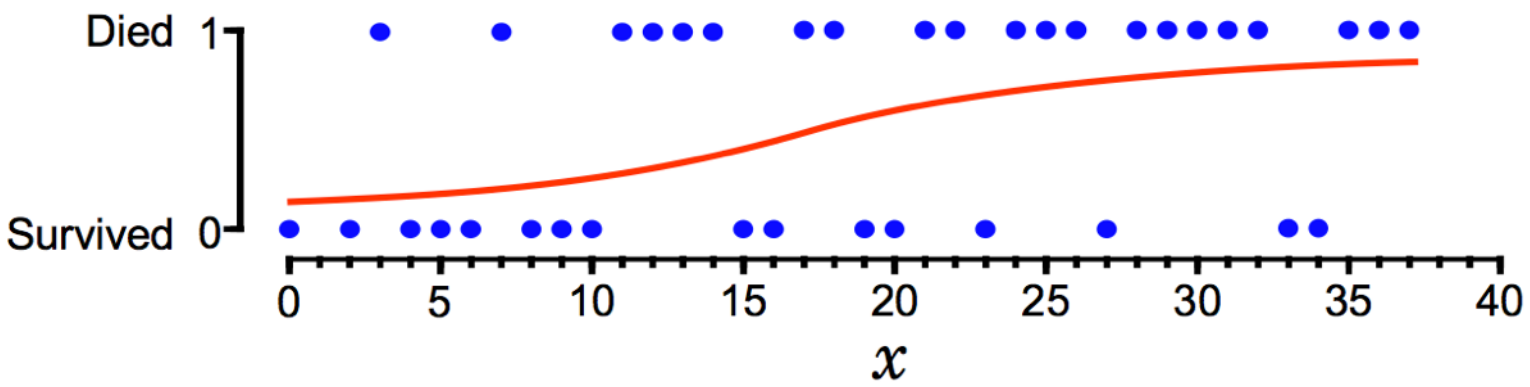


When $B_0 + B_1x = 0$, then $F(x) = 0.5$, which is the inflection point on all these curves.

Going back to our example of patient survival given a sepsis test score: Data that has a sharp cut off point between the two classes (living / dying) should have a large value of B_1.

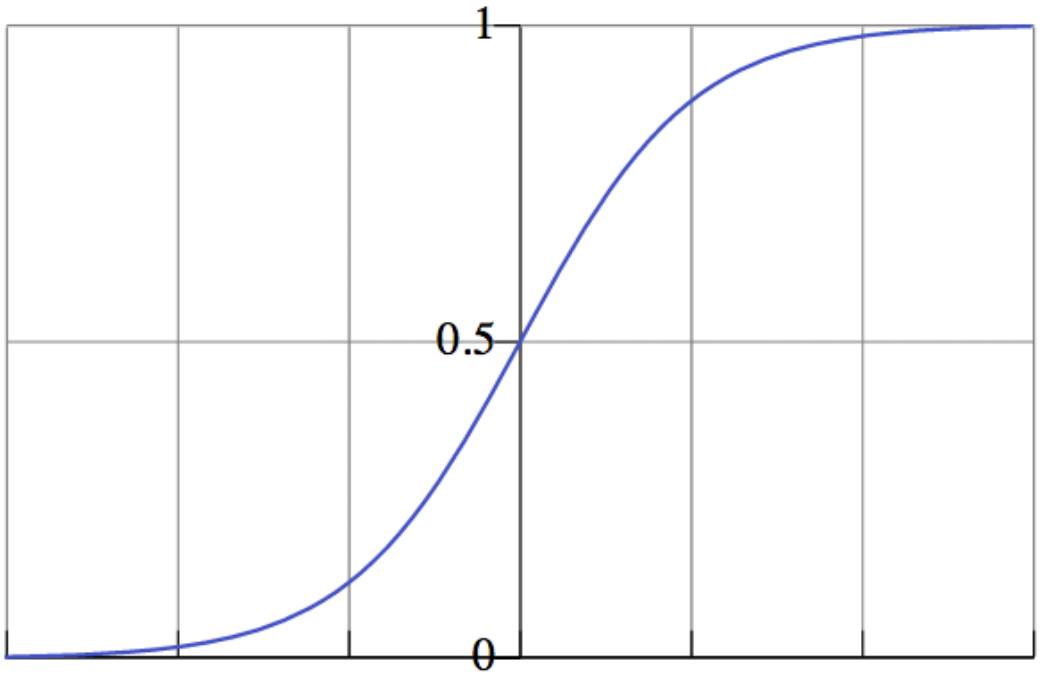


Going back to our example of patient survival given a sepsis test score: Data that has a lengthy transition between the two classes (living / dying) should have a small value of B_1.



PROBABILITIES

probability of belonging to class

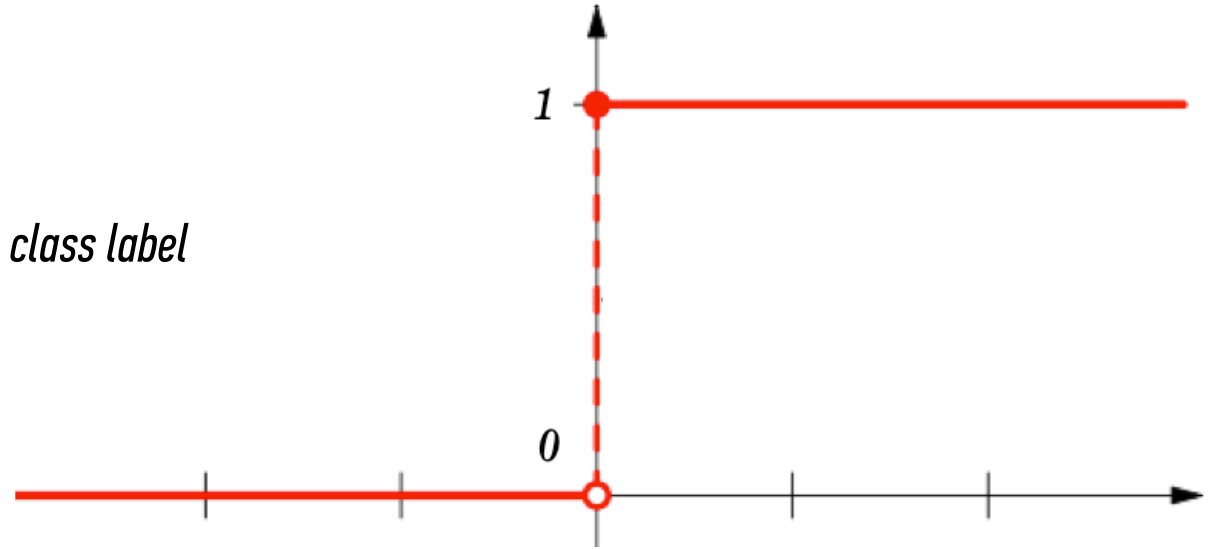


value of independent variable

NOTE

Probability predictions look like this.

CLASS LABELS



NOTE

Probabilities are "snapped" to class labels (e.g. by thresholding at 50%).

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The first difference is in the outcome variable.

The second difference is in the error term.

II. OUTCOME VARIABLES

OUTCOME VARIABLES

The key variable in any regression problem is the **conditional mean** *of the outcome variable y given the value of the covariate x:*

$$E(y|x)$$

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$$E(y|x)$$

In linear regression, we assume that this conditional mean is a linear function taking values in (-∞, +∞):

$$E(y|x) = \alpha + \beta x$$

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Q: How do we do this?

A: By using a transformation called the **logistic function***:*

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

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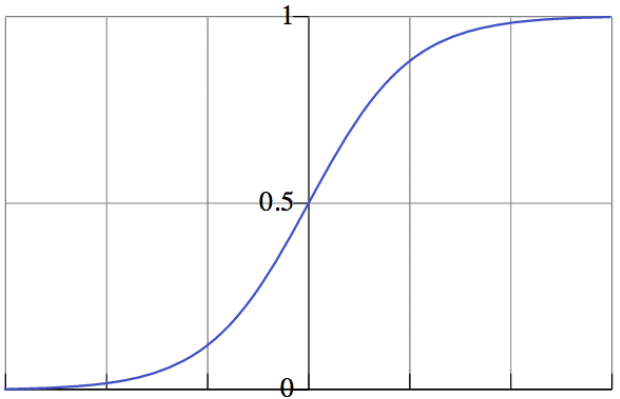

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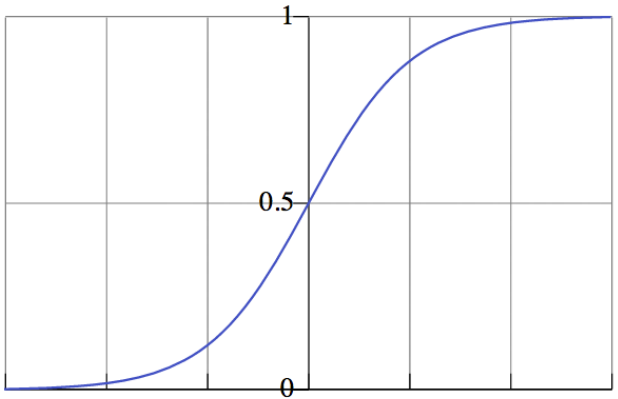

NOTE

For any value of x, y is in the interval [0, 1]

This is a nonlinear transformation!

The **logit function** *is an important transformation of the logistic function. Notice that it returns the linear model!*

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THE LOGISTIC FUNCTION

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NOTE

This name hints at its usefulness in interpreting our results.

We will see why shortly.

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III. ERROR TERMS

The second difference between linear regression and the logistic regression model is in the error term.

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One of the key assumptions of linear regression is that the error terms follow independent Gaussian distributions with zero mean and constant variance:

$$\epsilon \sim N(0, \sigma^2)$$

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It's easy to show from this that instead of following a Gaussian distribution, the error term in logistic regression follows a Bernoulli distribution:

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NOTE

This is the same distribution followed by a coin toss.

Think about why this makes sense!

These two key differences define the logistic regression model, and they also lead us to a kind of unification of regression techniques called **generalized linear models**.

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Briefly, GLMs generalize the distribution of the error term, and allow the conditional mean of the response variable to be related to the linear model by a **link function**.

In the present case, the error term follows a Bernoulli distribution, and the logit is the link function that connects us to the linear predictor.

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Since the Bernoulli distribution and the logit function share a common parameter π*, we say that the logit is the* **canonical link function** *for the Bernoulli distribution.*

AN ASIDE: GLM

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NOTE

This terminology is just FYI!

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IV. INTERPRETING RESULTS

In linear regression, the parameter β represents the change in the response variable for a unit change in the covariate.

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In logistic regression, β represents the change in the logit function for a unit change in the covariate.

Interpreting this change in the logit function requires another definition first.

The **odds** *of an event are given by the ratio of the probability of the event by its complement:*

$$O(x = 1) = \frac{\pi(1)}{(1-\pi(1))}$$

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The **odds ratio** *of a binary event is given by the odds of the event divided by the odds of its complement:*

$$OR = \frac{O(x=1)}{O(x=0)} = \frac{\pi(1)/[1-\pi(1)]}{\pi(0)/[1-\pi(0)]}$$

Substituting the definition of $\pi(x)$ into this equation yields (after some algebra),

$$OR = e^{\beta}$$

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This simple relationship between the odds ratio and the parameter β is what makes logistic regression such a powerful tool.

Q: So how do we interpret this?

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A: The odds ratio of a binary event gives the increase in likelihood of an outcome if the event occurs.

INTERPRETING RESULTS – AN EXAMPLE

Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote a mobile OS (for example, iOS).

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Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote a mobile OS (for example, iOS).

In this case, an odds ratio of 2 (eg, $\beta = \log(2)$) indicates that a purchase is twice as likely for an iOS user as for a non-iOS user.

LAB: LOGISTIC REGRESSION