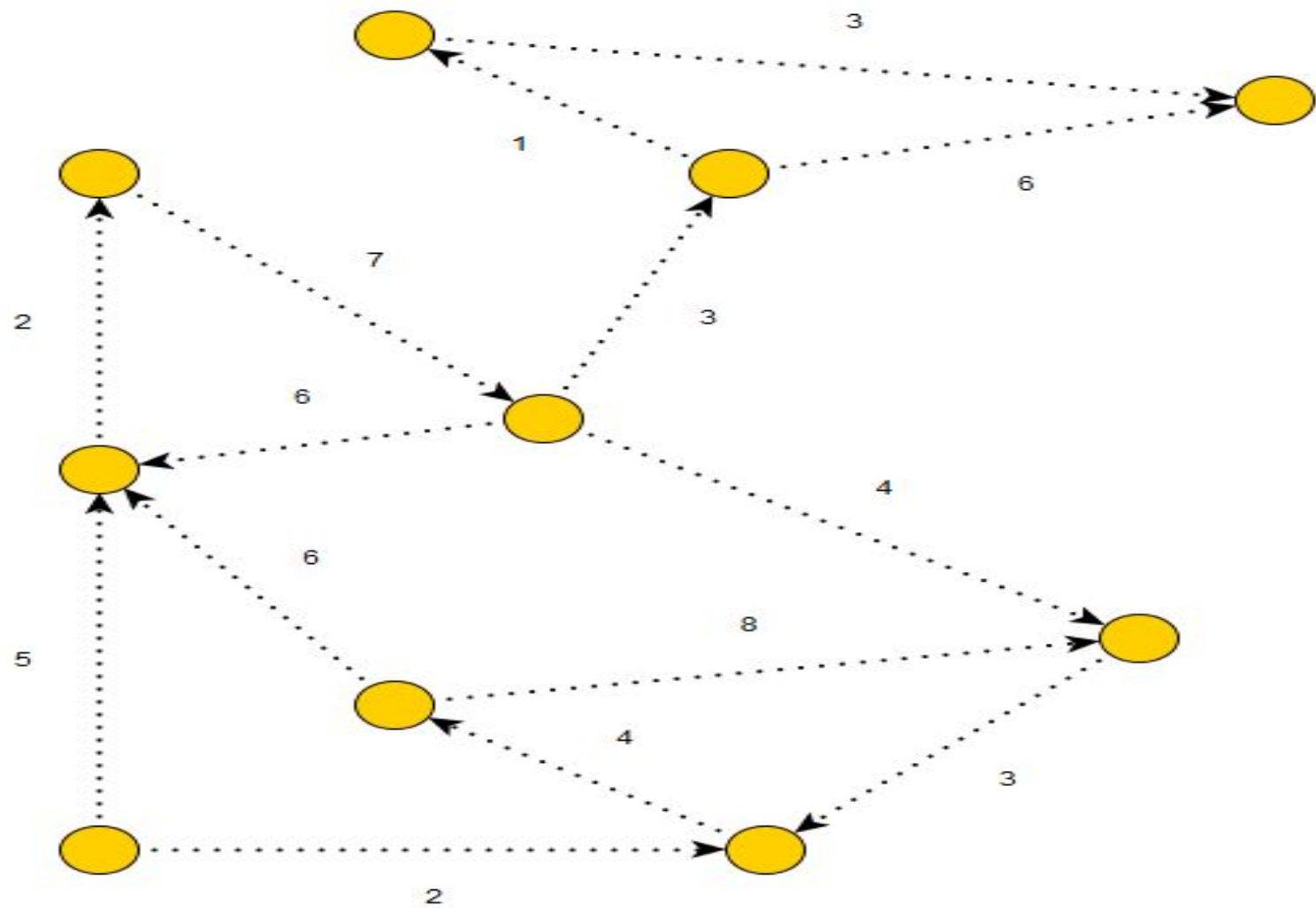


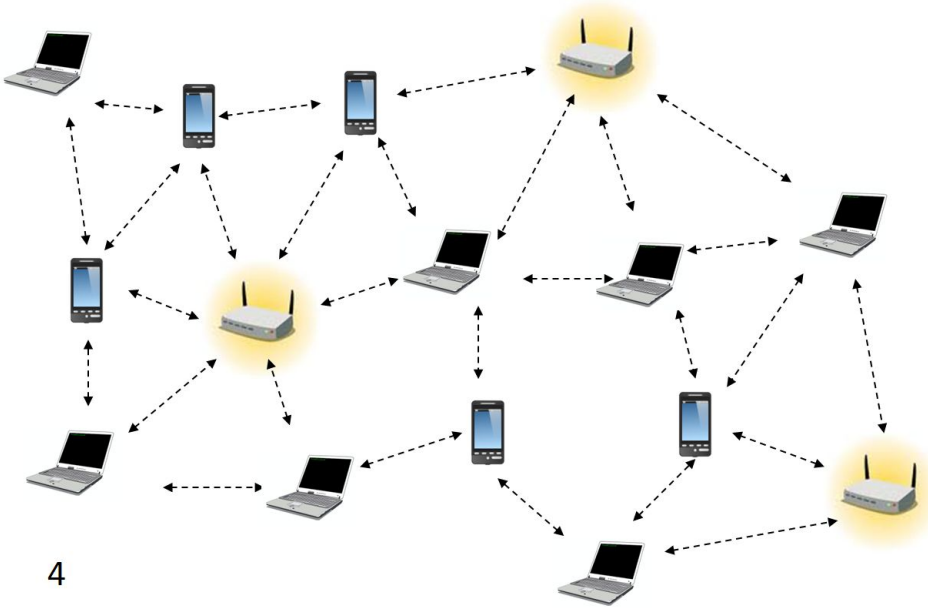
A New Approach to the Incremental All Pairs Shortest Paths Problem

Arie Slobbe

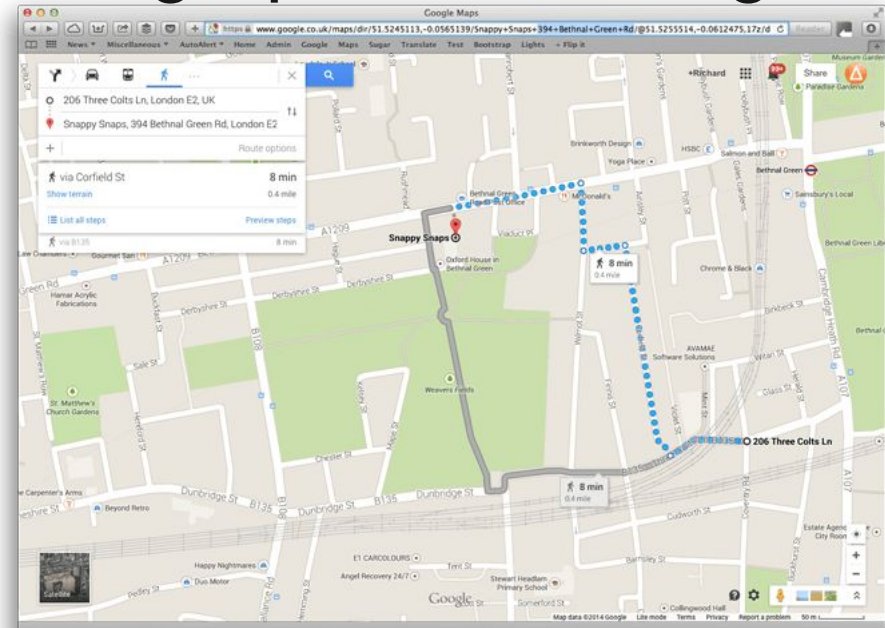


Examples and Applications

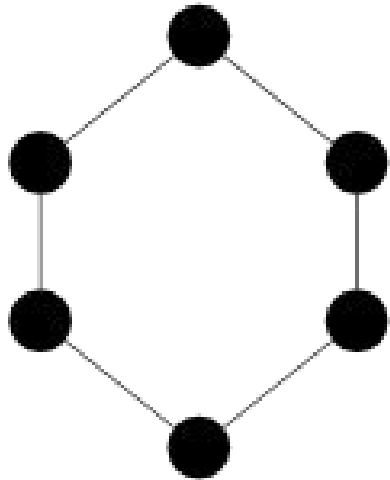
Internet Data Routing



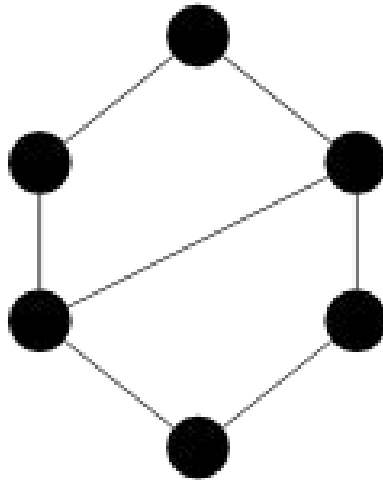
Geographical Routing



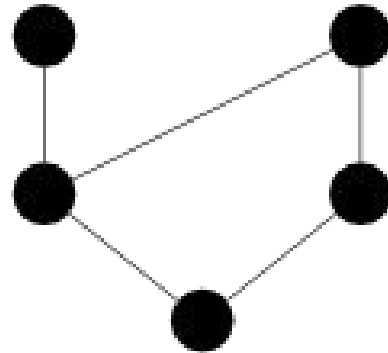
Dynamic Algorithms



Original graph



An edge added



A node removed



etc.

Dynamic algorithms

Goal: Avoid needless recomputation

Examples:

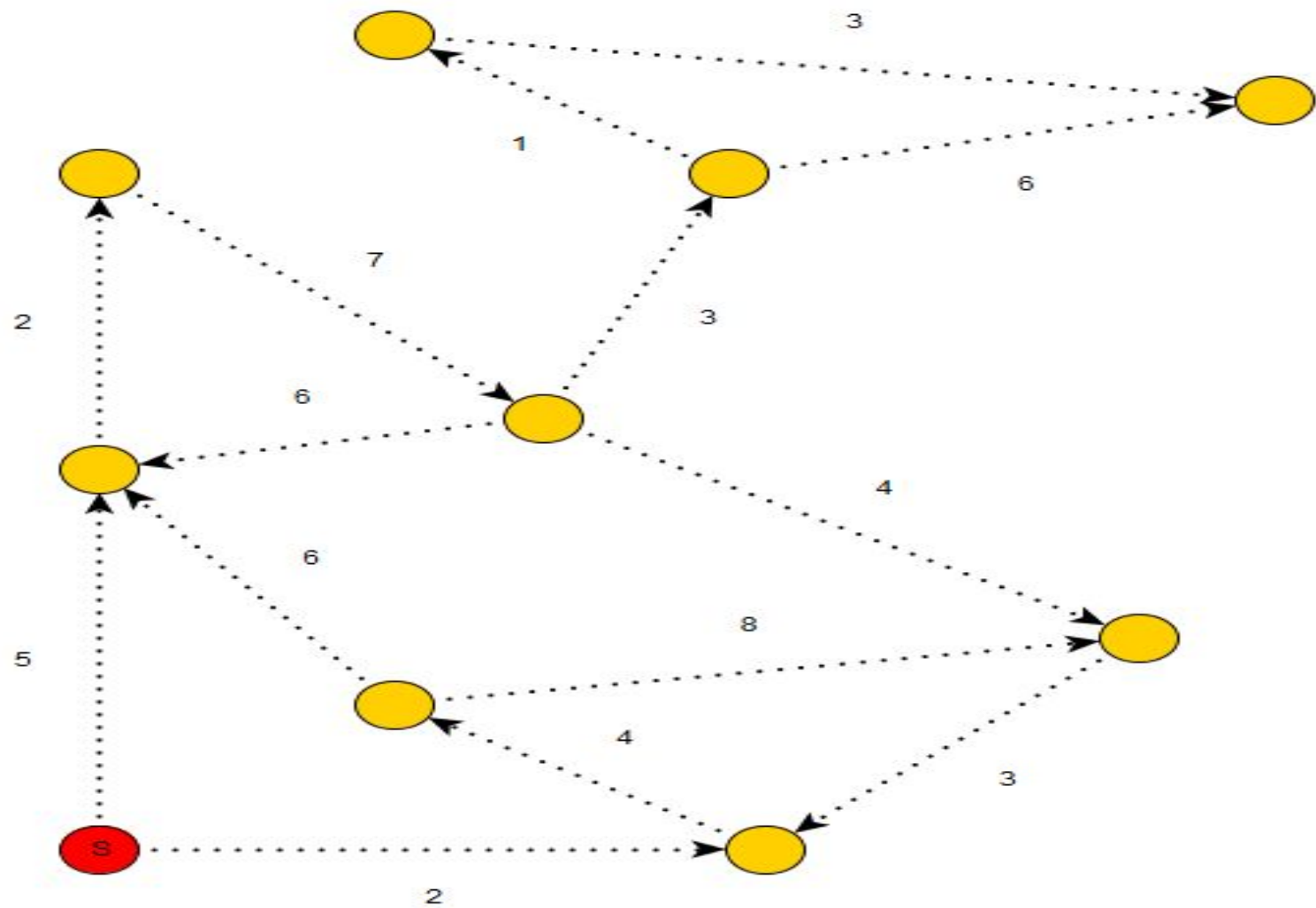
- Construction of a new road
- Phone connects to a wifi router
- New friendship added to a social network

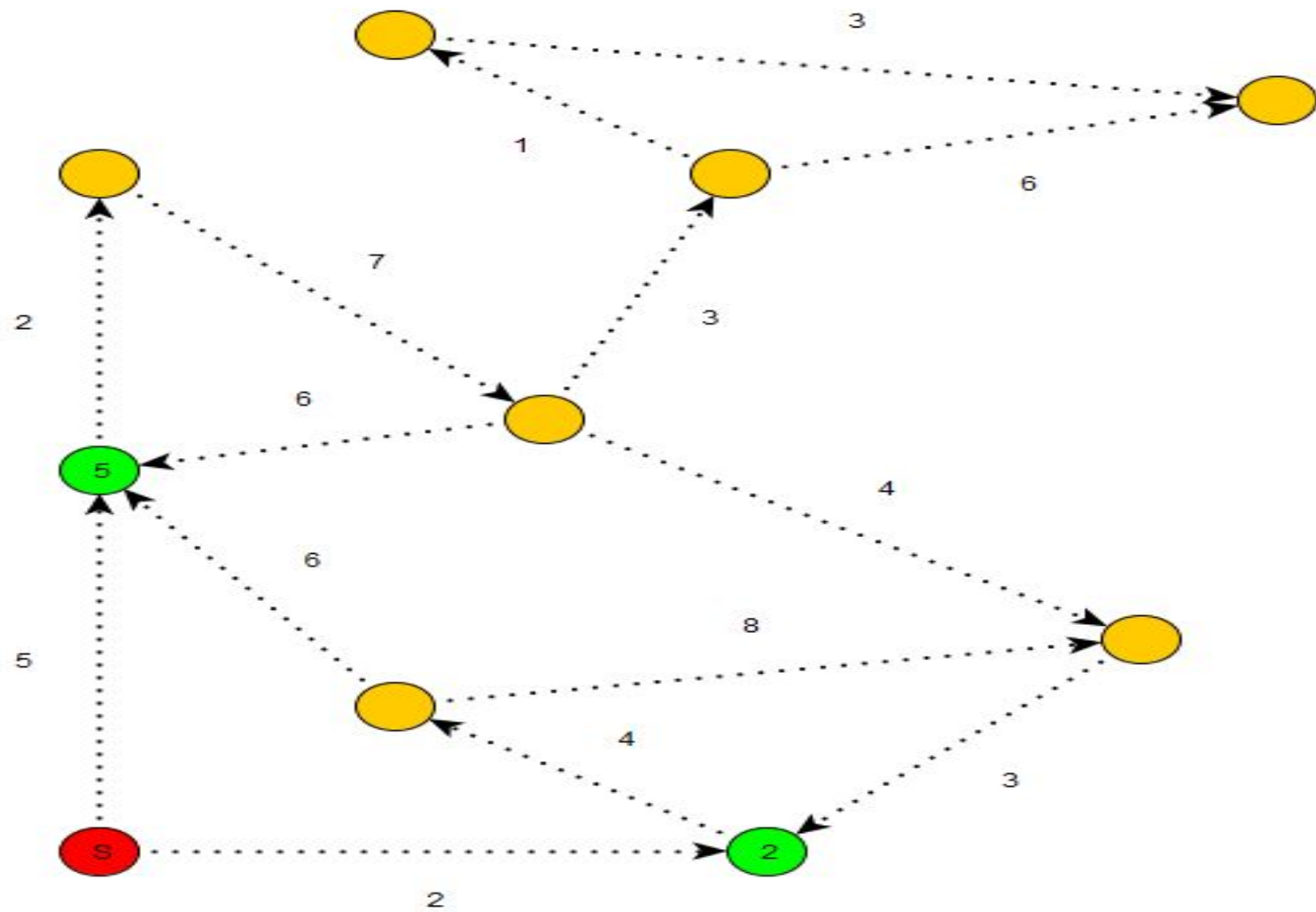
Contributions

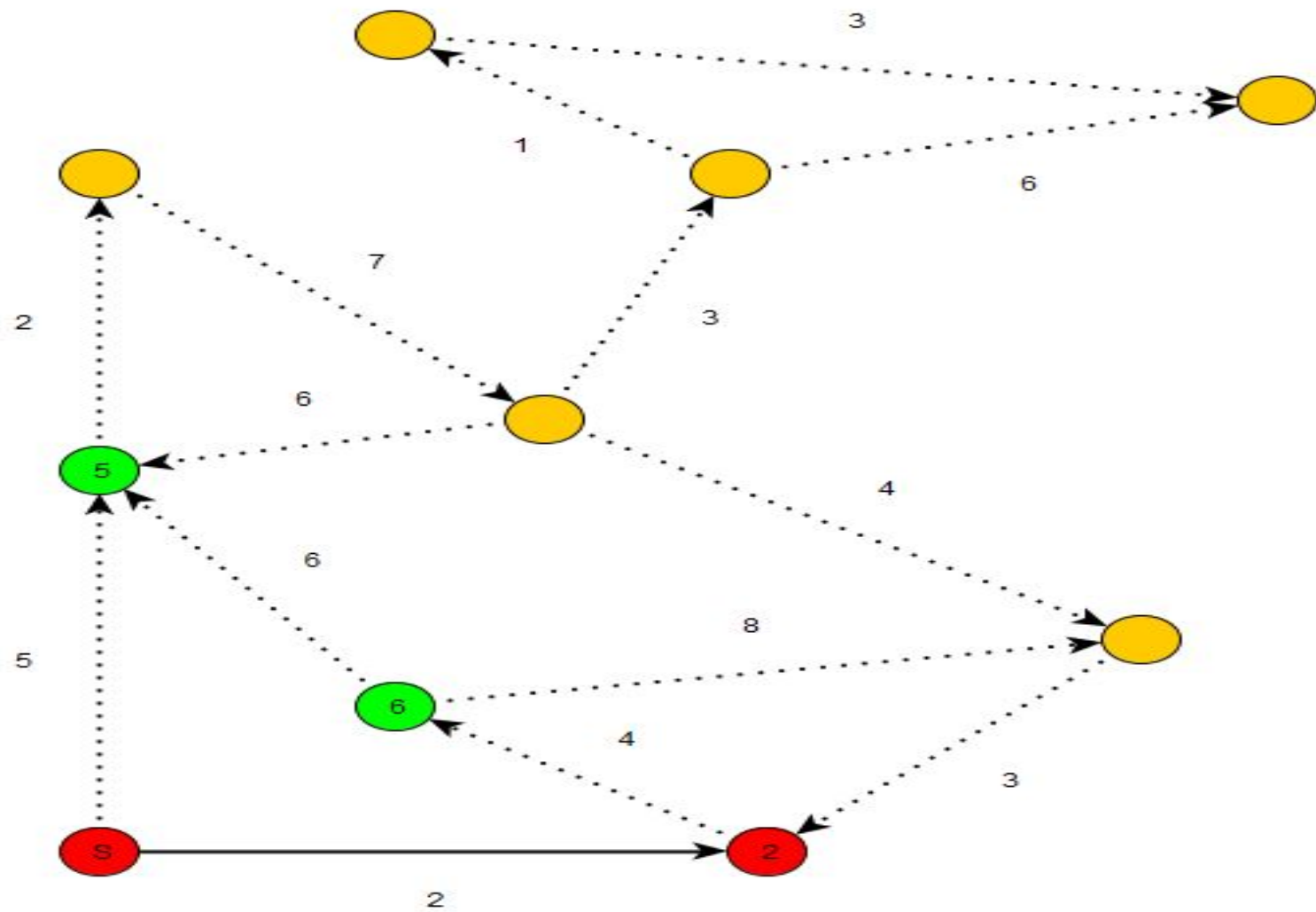
- A new algorithm for updating shortest paths after a graph undergoes changes
- Optimal worst-case running time $O(n^2)$
- Performs well in practice

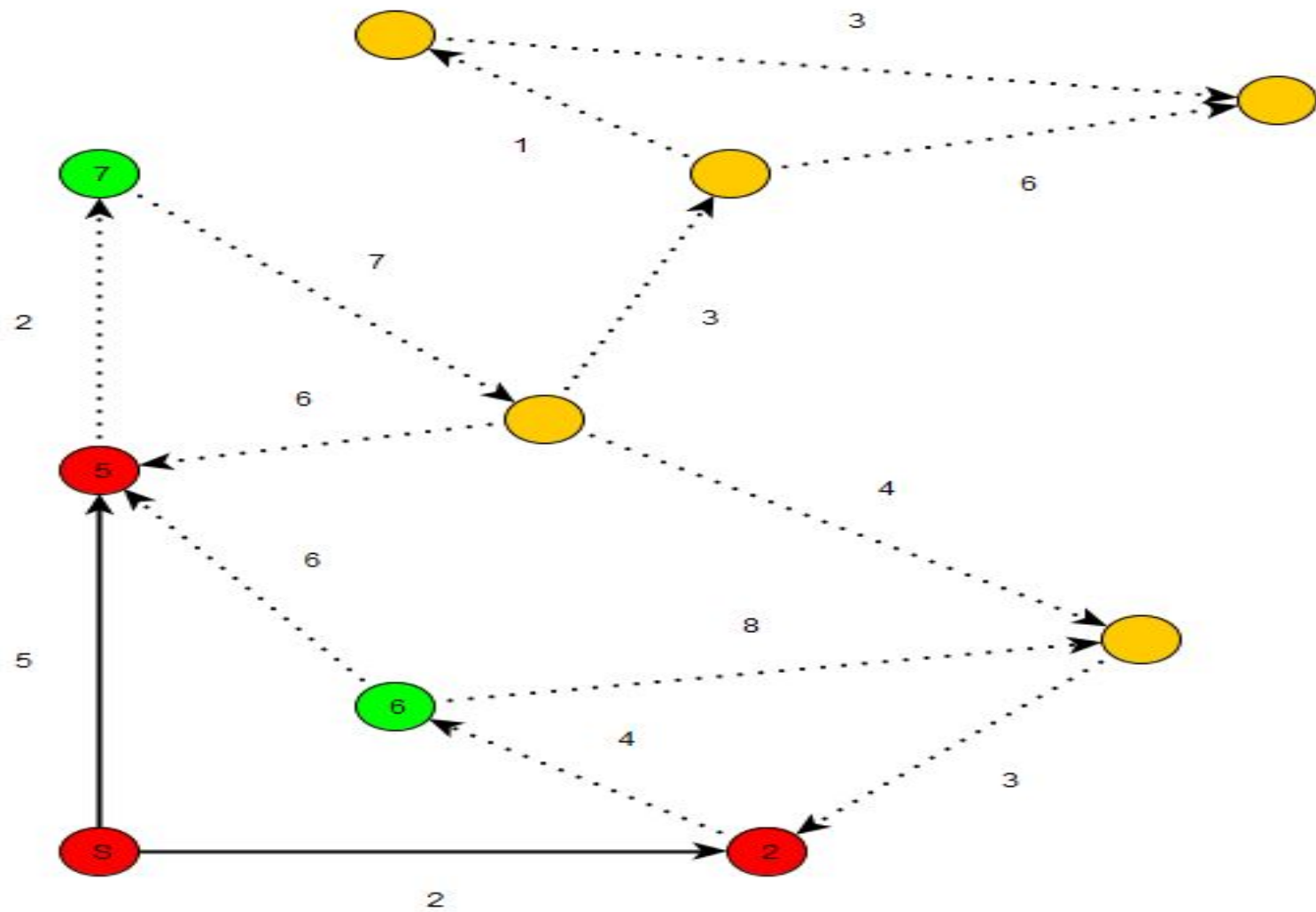
Dijkstra's algorithm

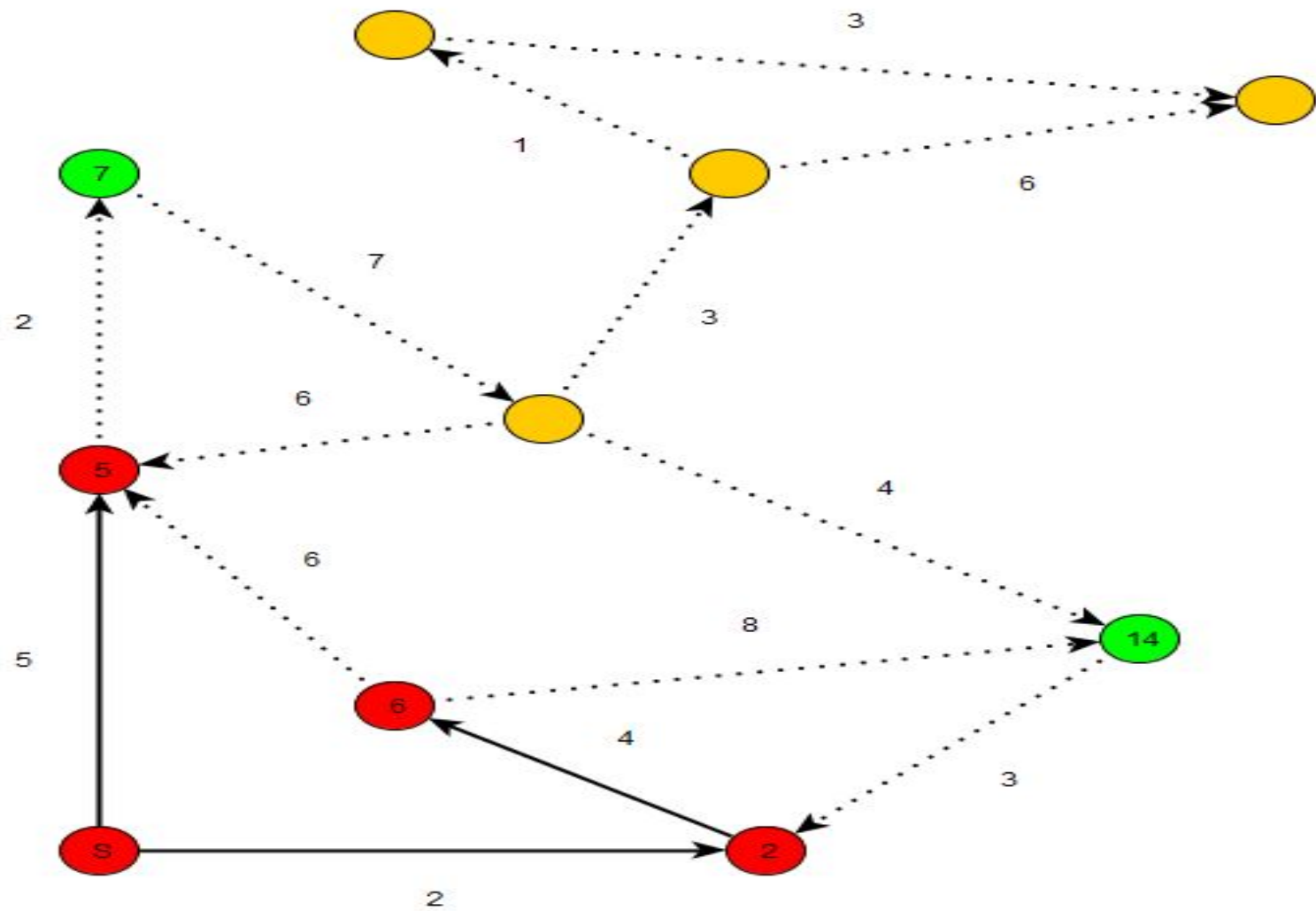
- Conceived by computer scientist Edsger W. Dijkstra in 1956 and published three years later.
- For a given source node in the graph, the algorithm finds the shortest path between that node and every other.

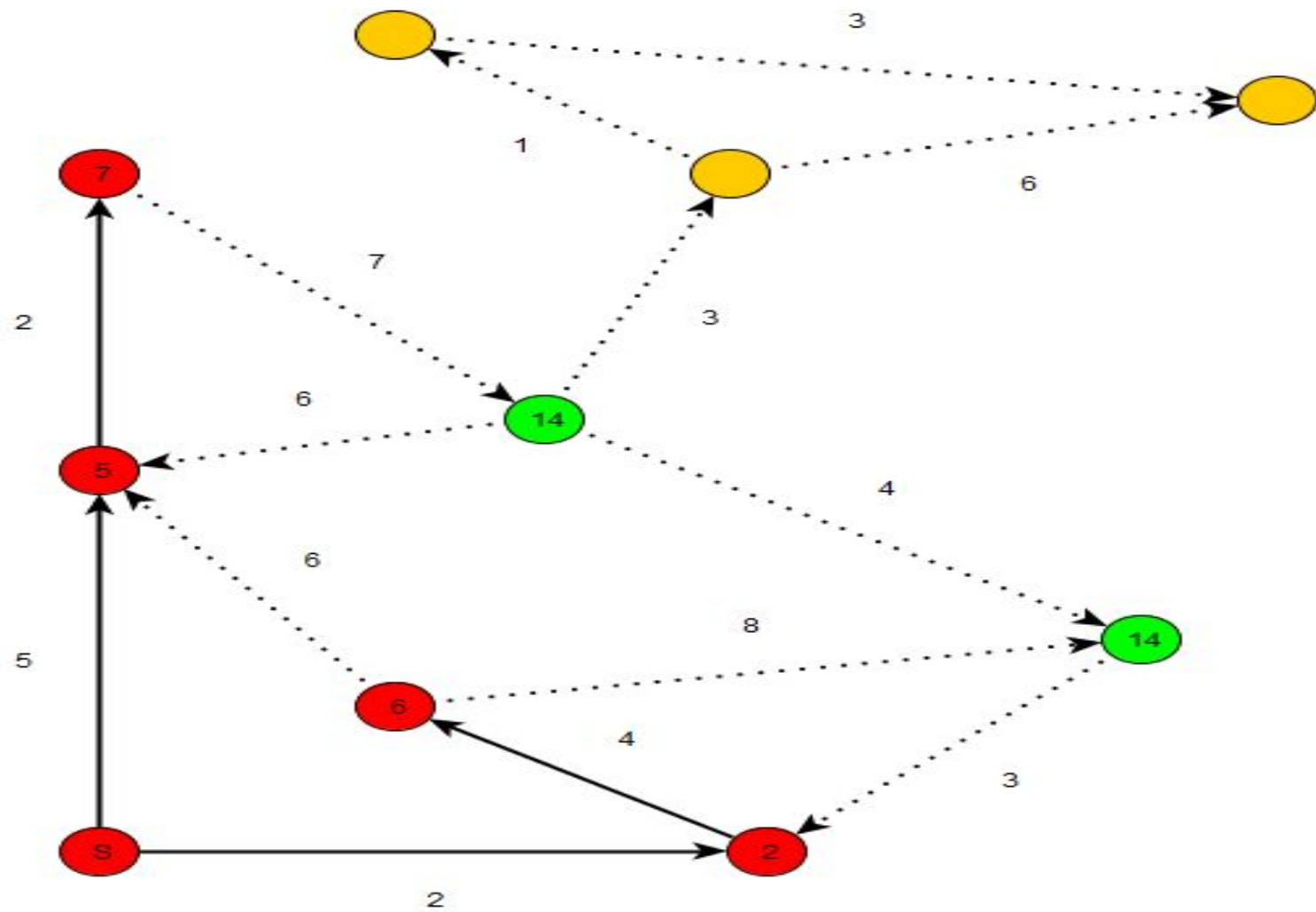


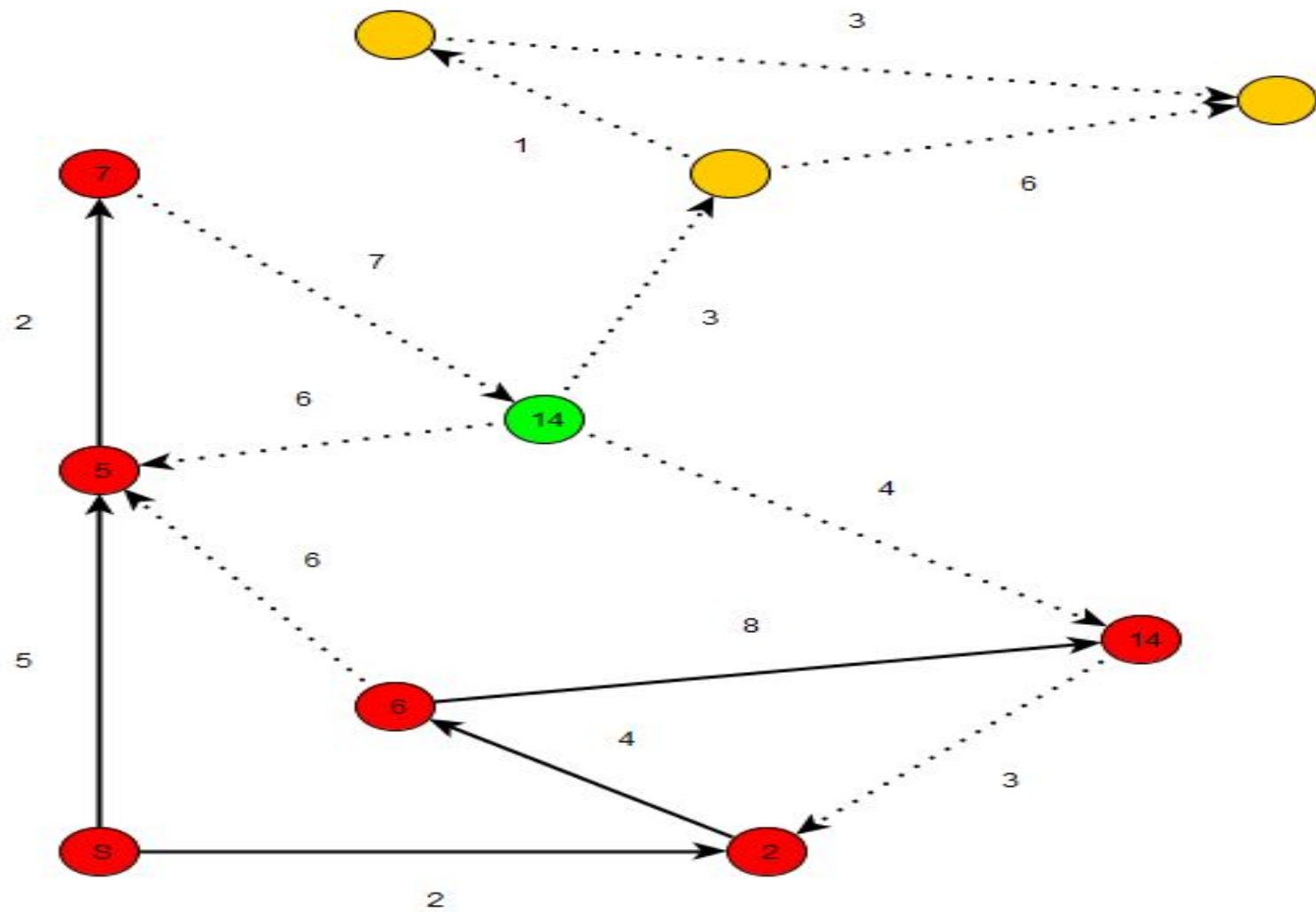


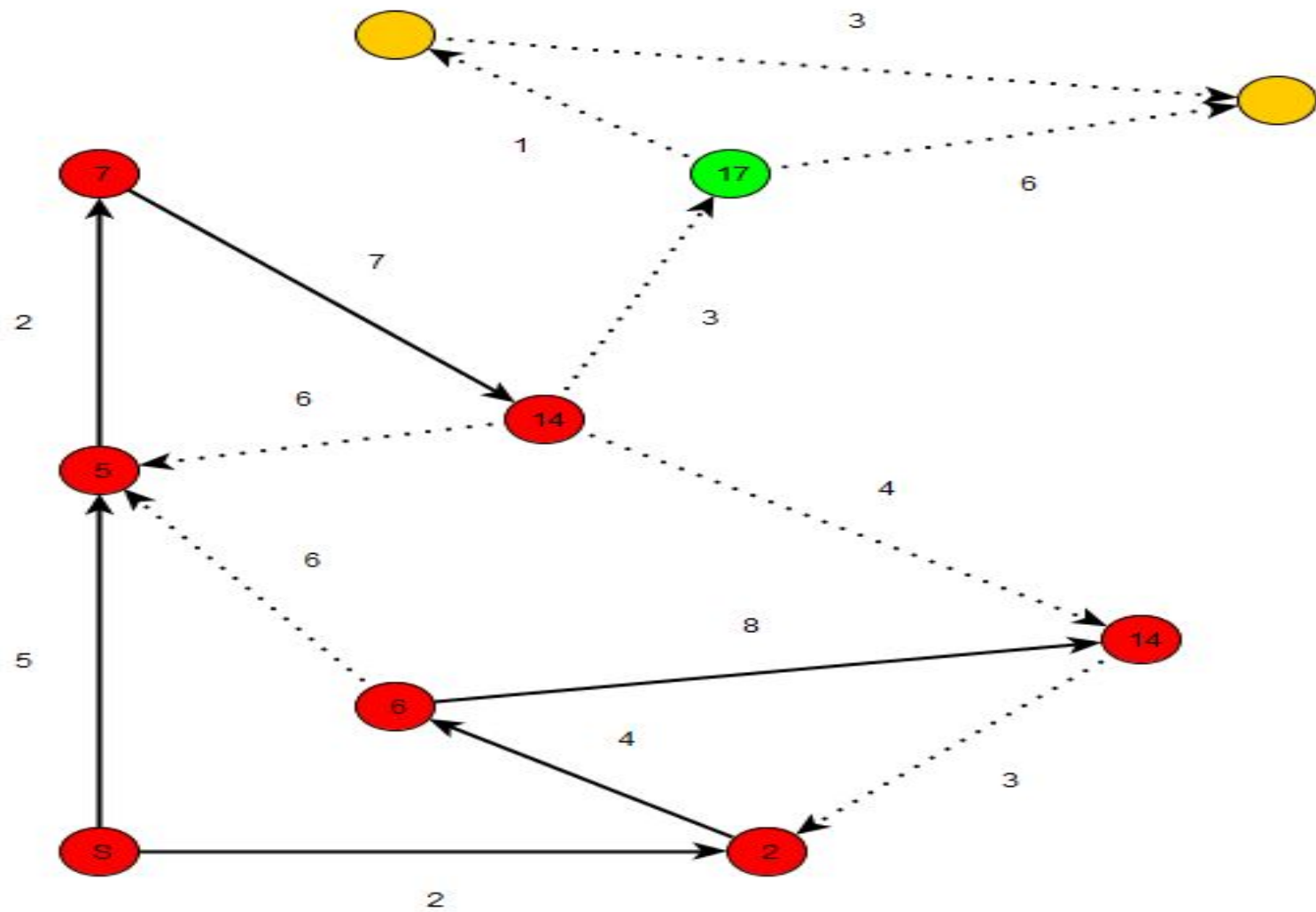


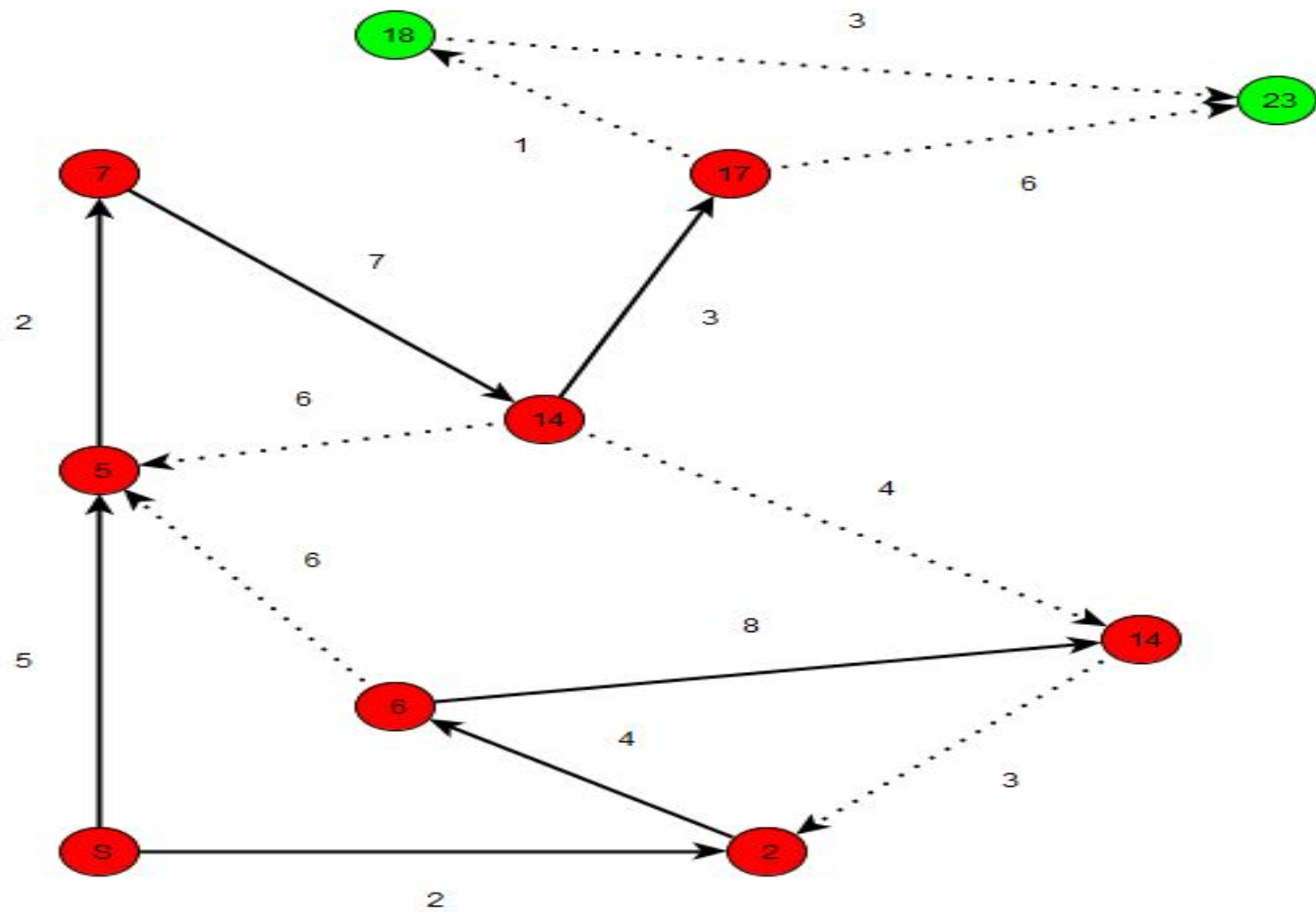


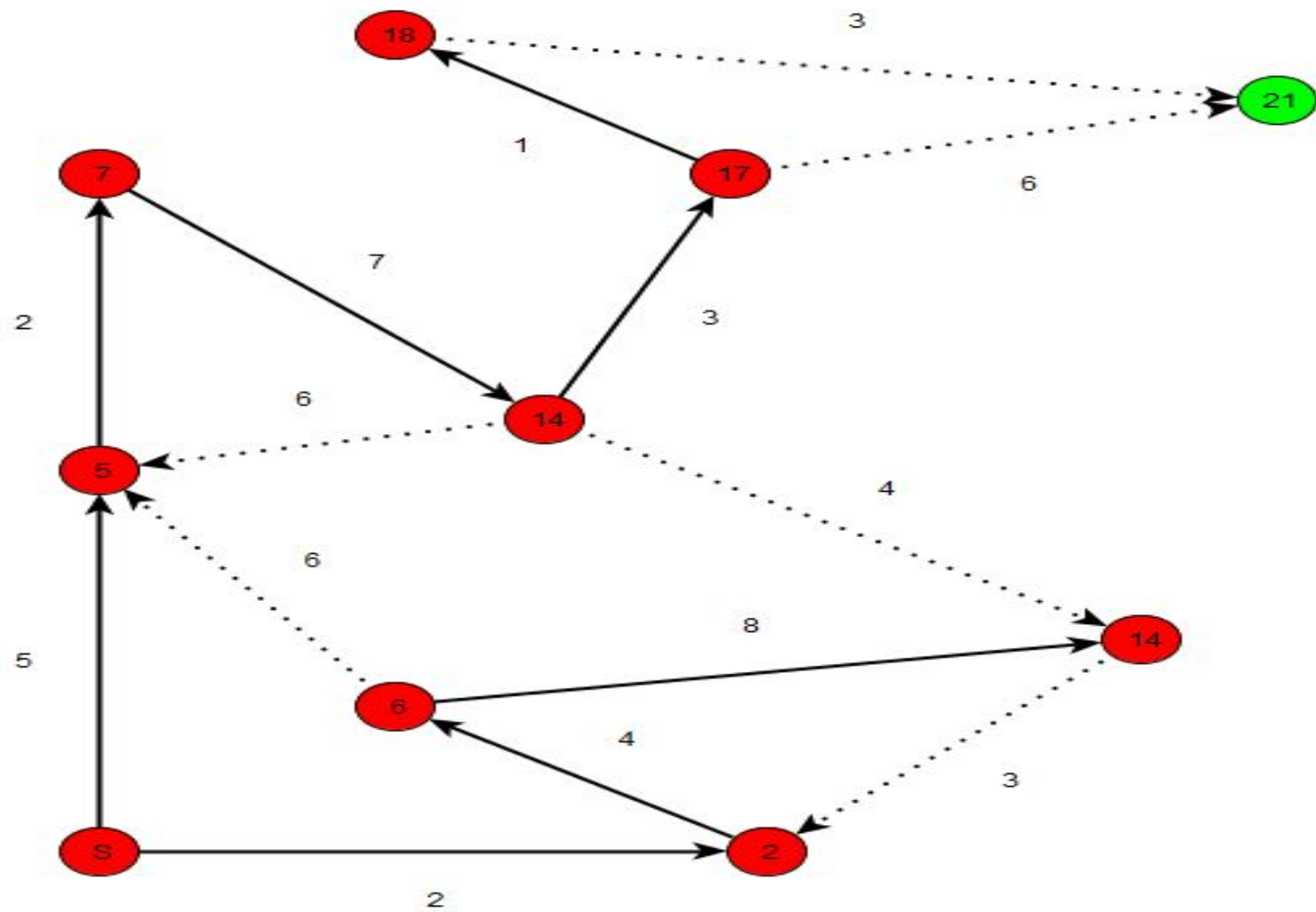


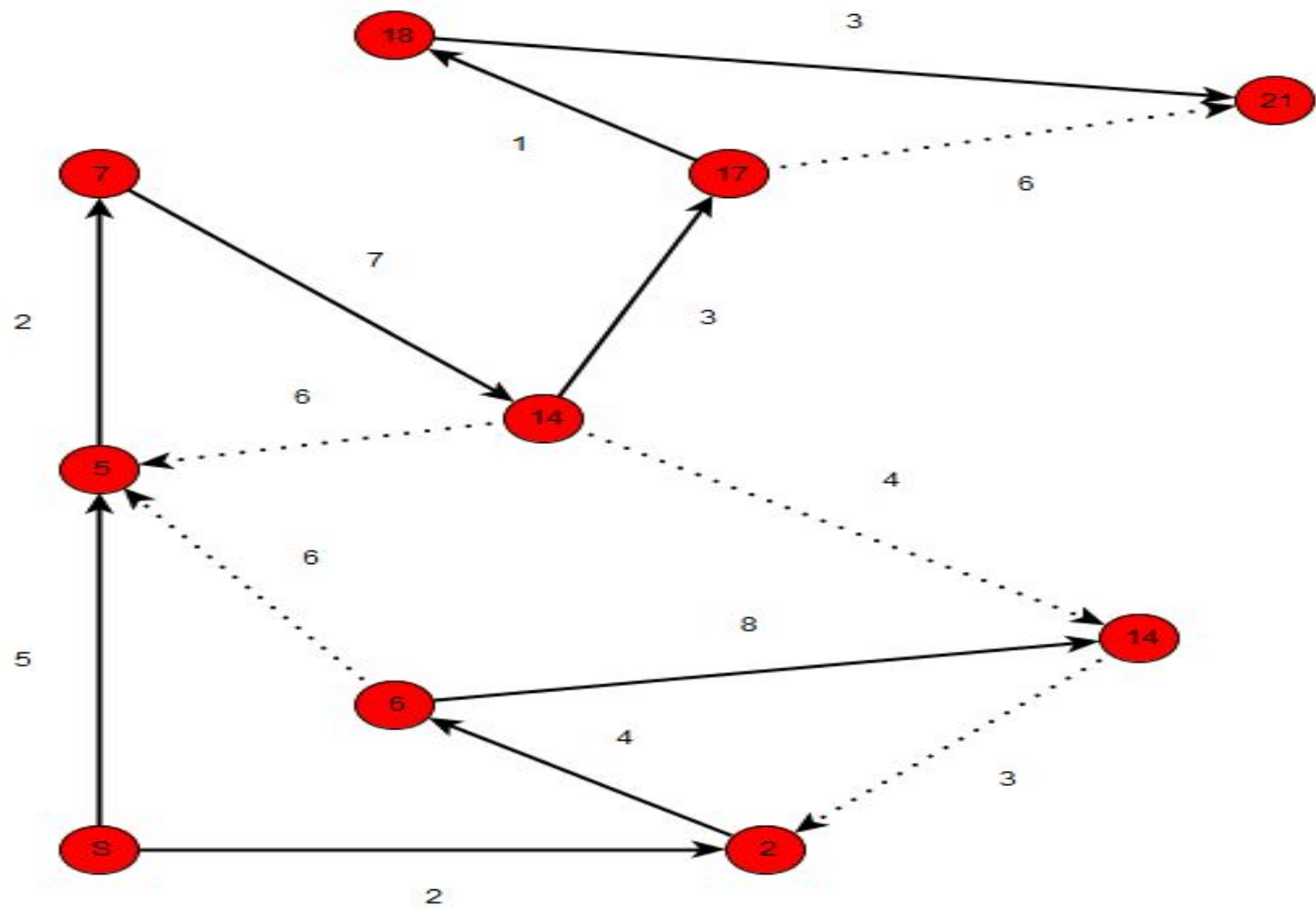






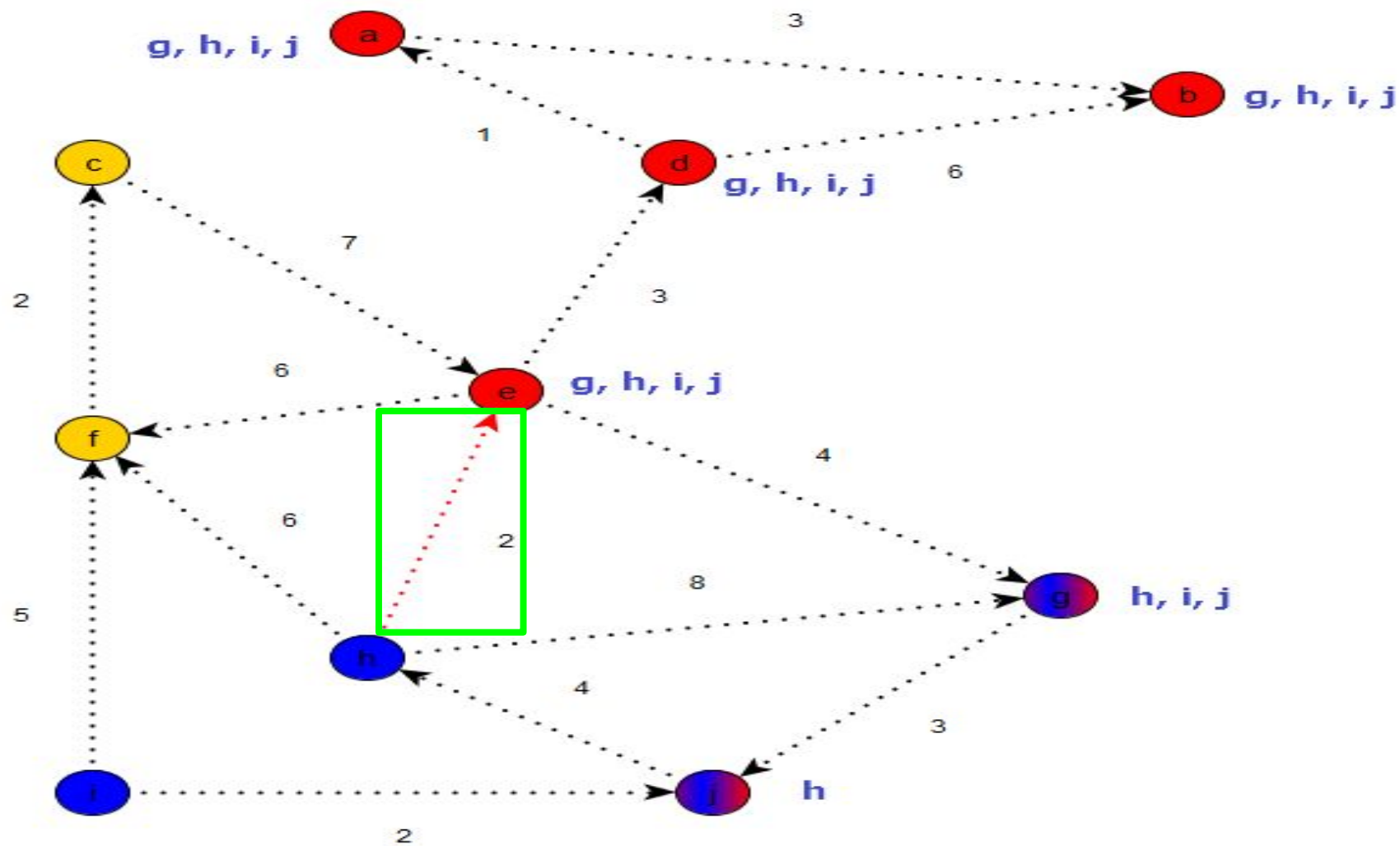






All Pairs Shortest Paths

- Find shortest path from A to B for all A and B.
- From scratch running time $O(m \cdot n + n^2 \log n)$
- In the incremental version, we introduce a new edge and efficiently recompute shortest paths in $O(n^2)$.

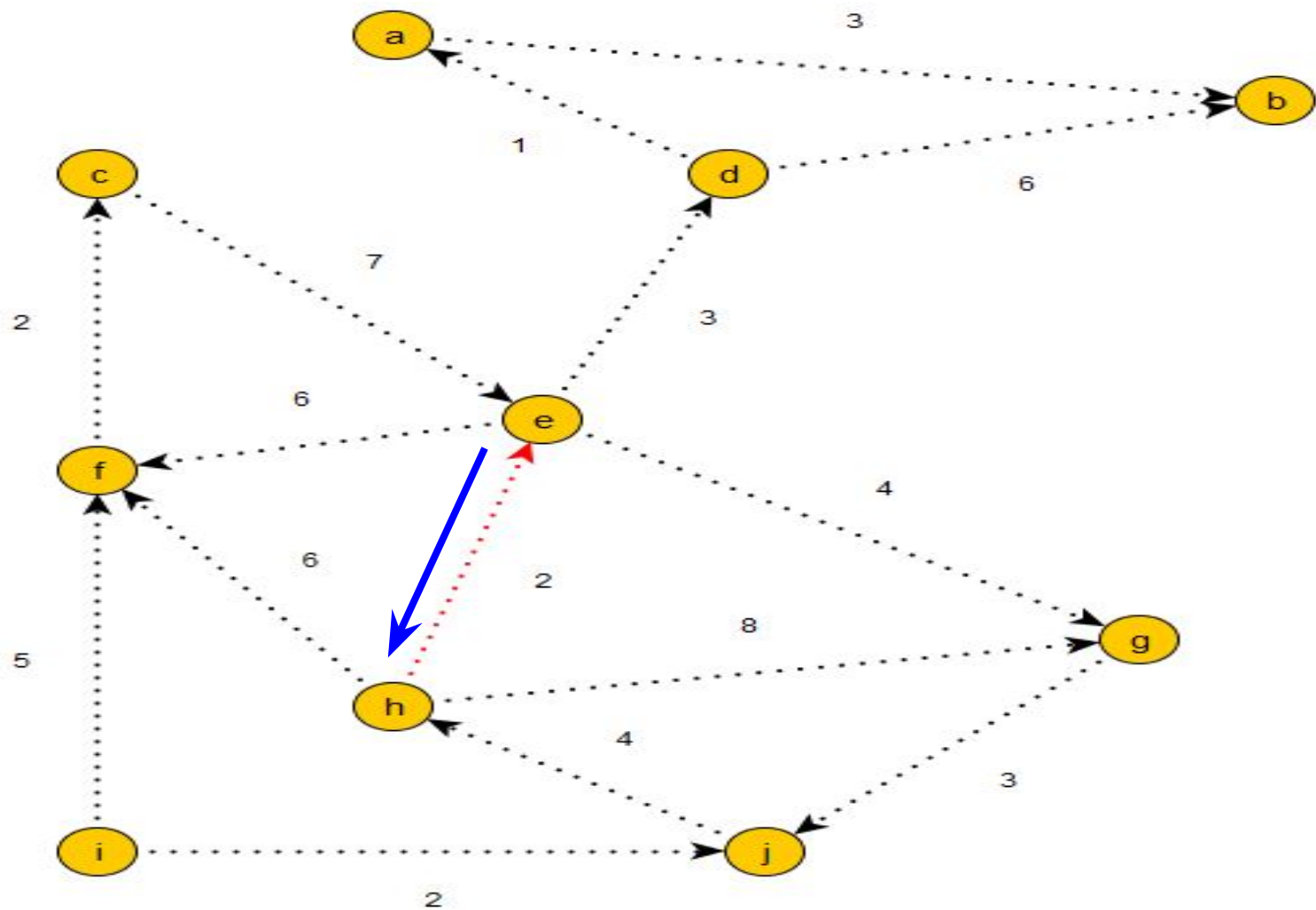


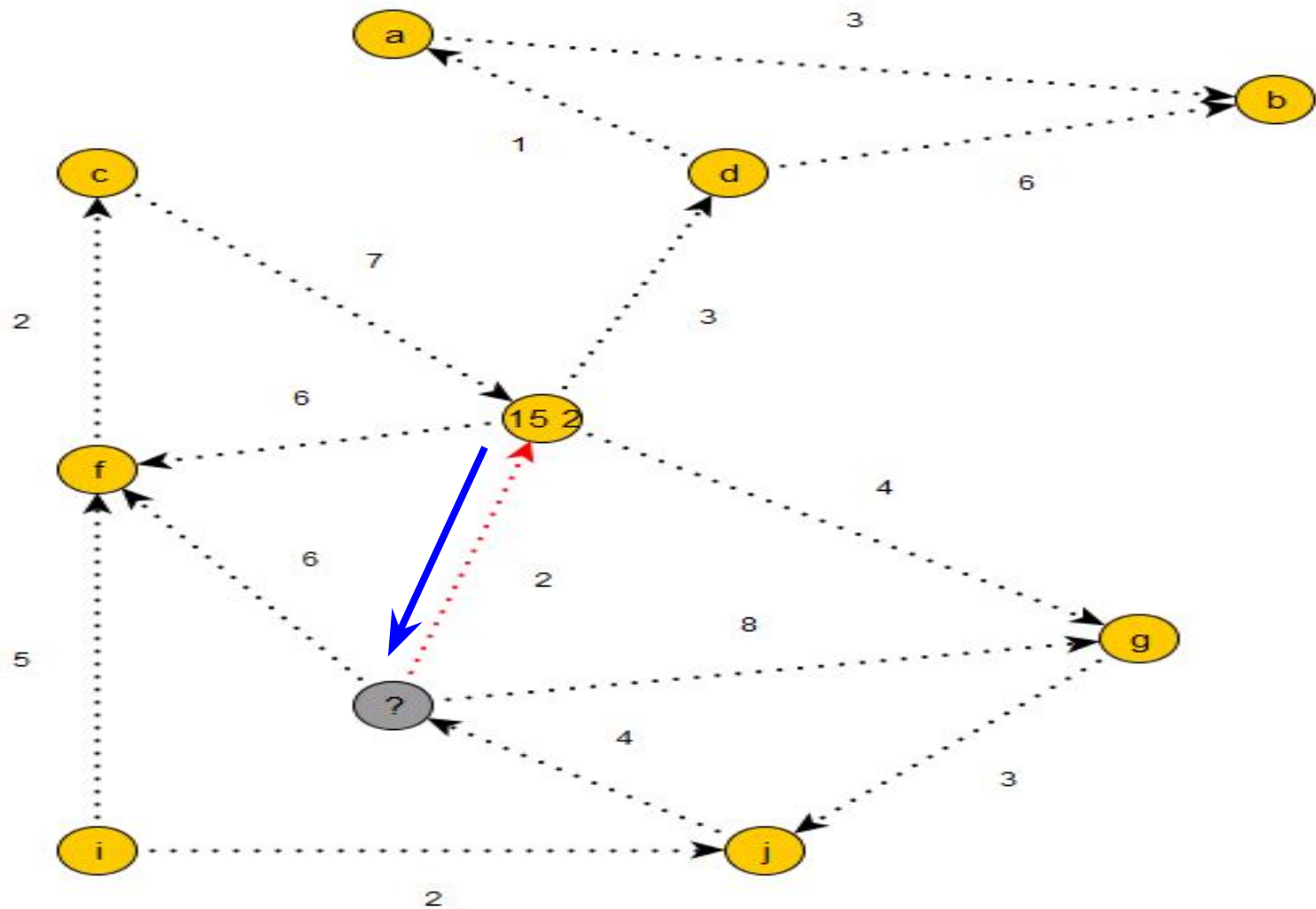
All Pairs Shortest Paths

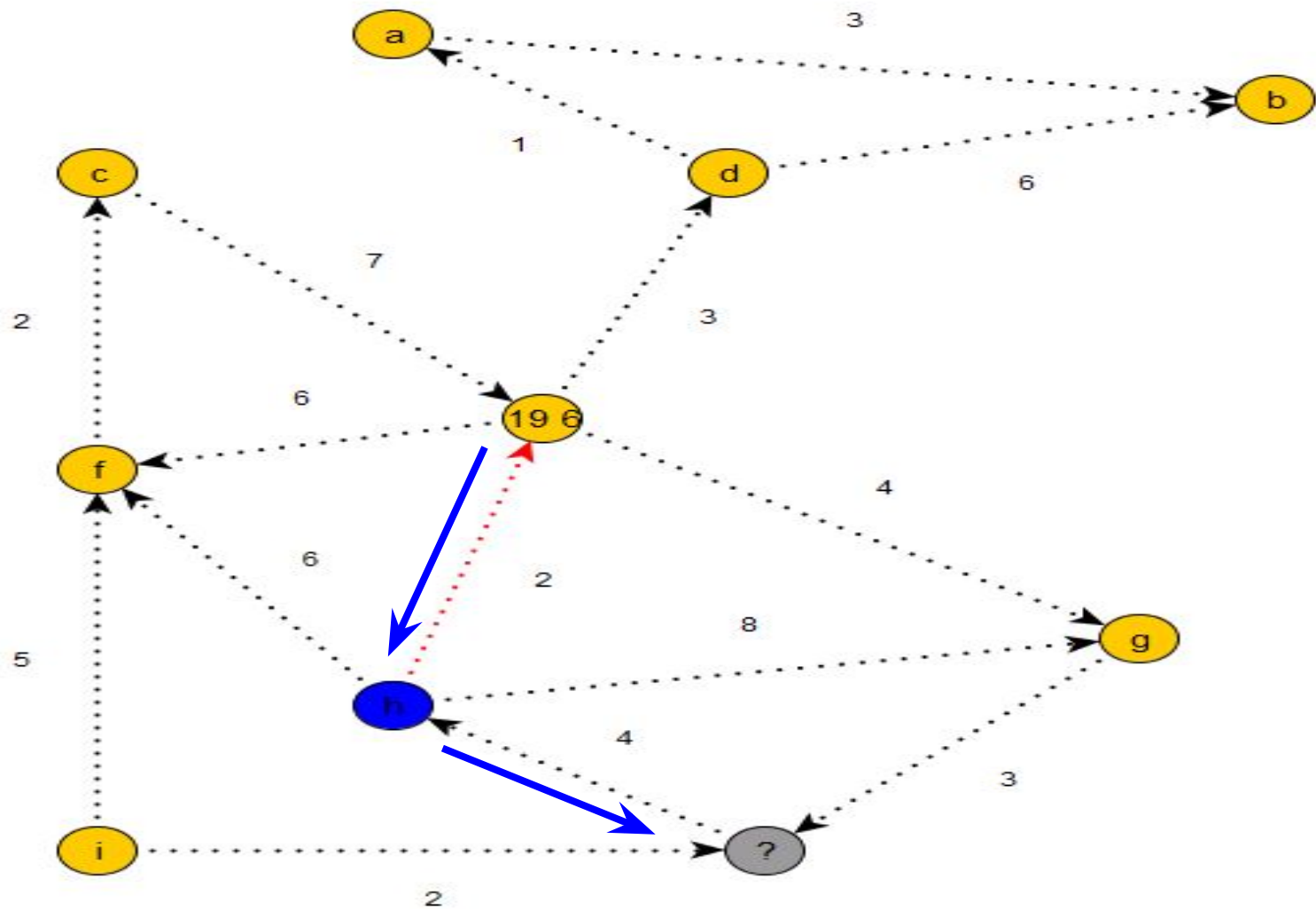
- Phase 1: Find the blue nodes

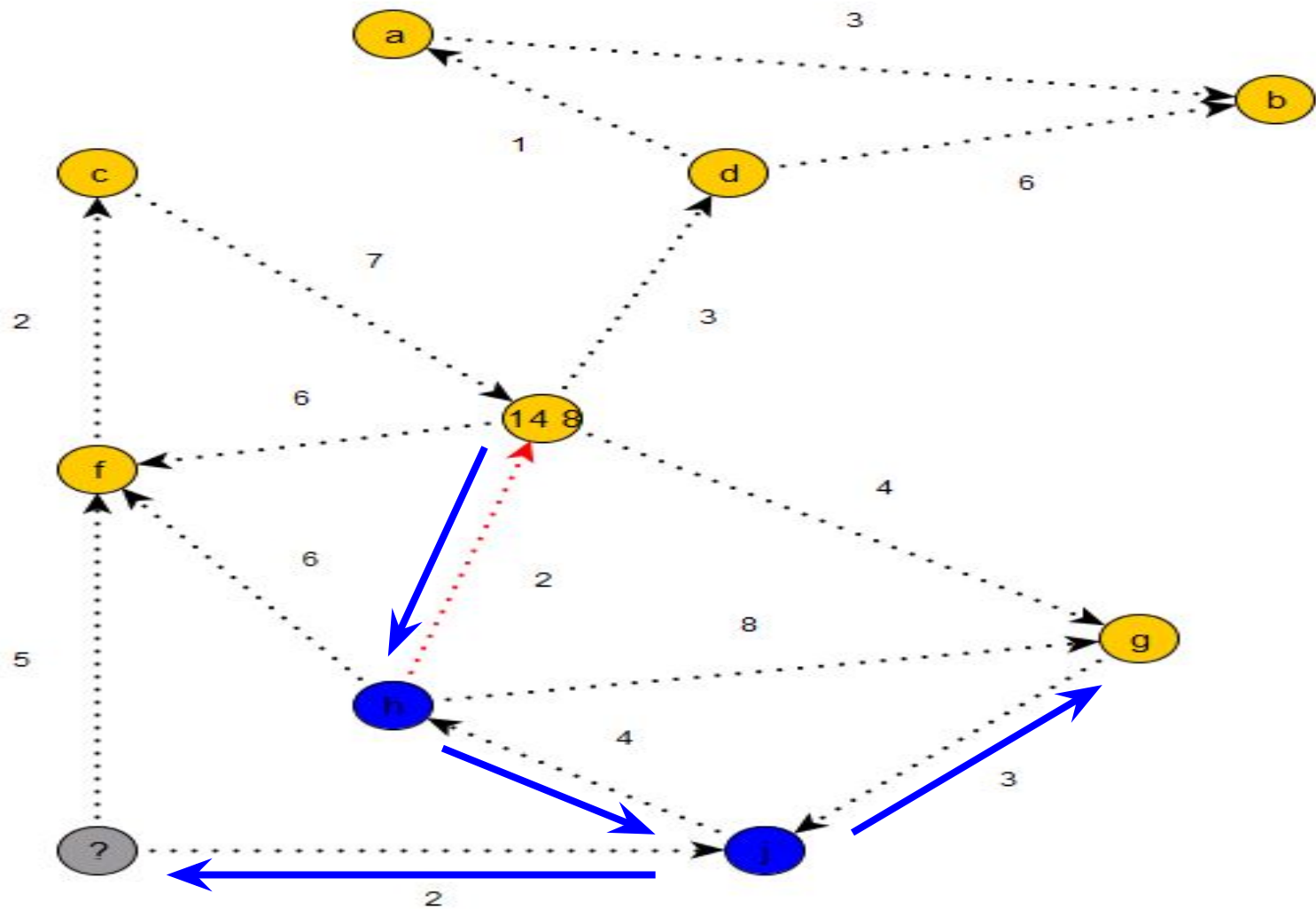
All Pairs Shortest Paths

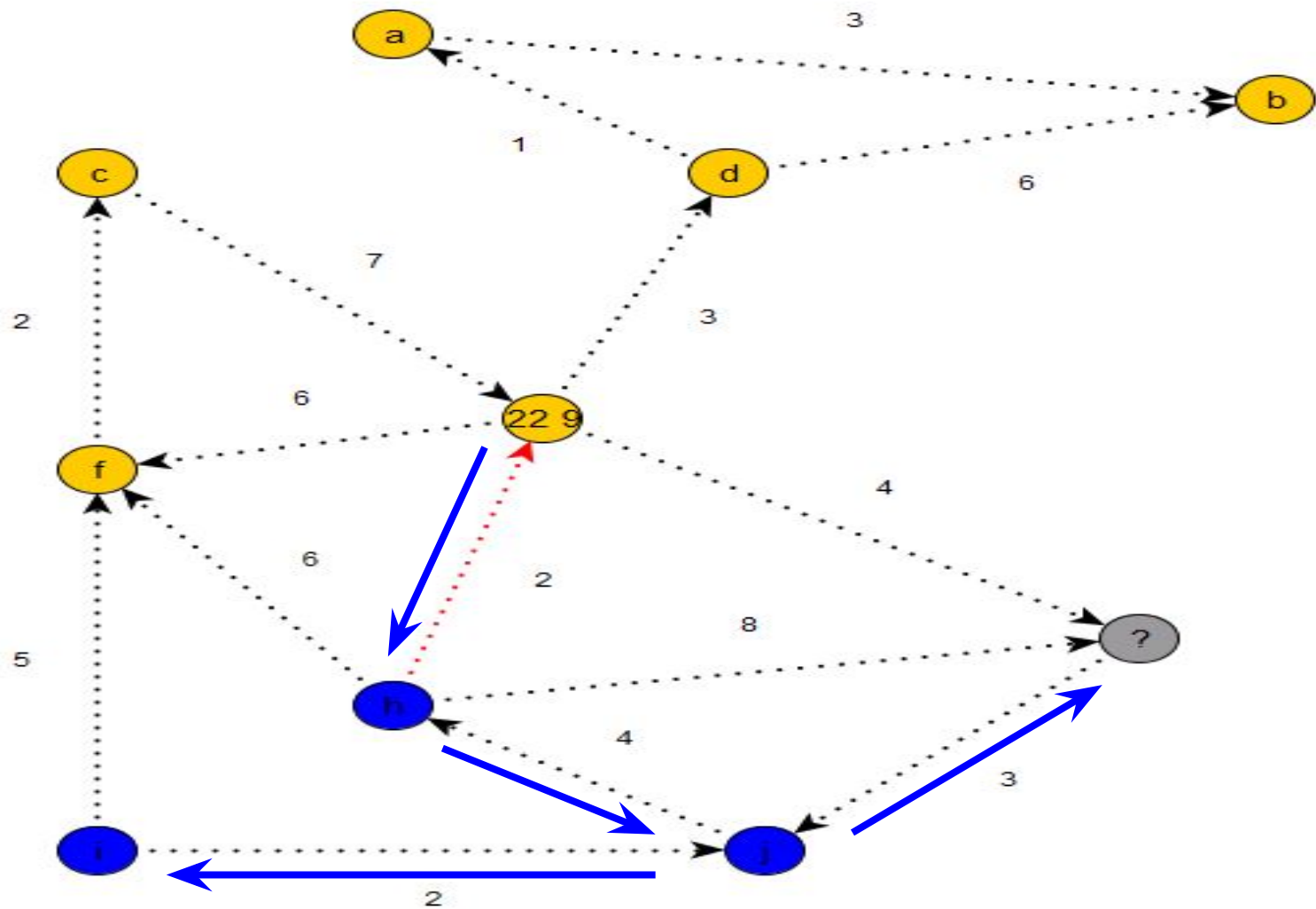
- Phase 1: Find the blue nodes
- Phase 2: Find the red nodes

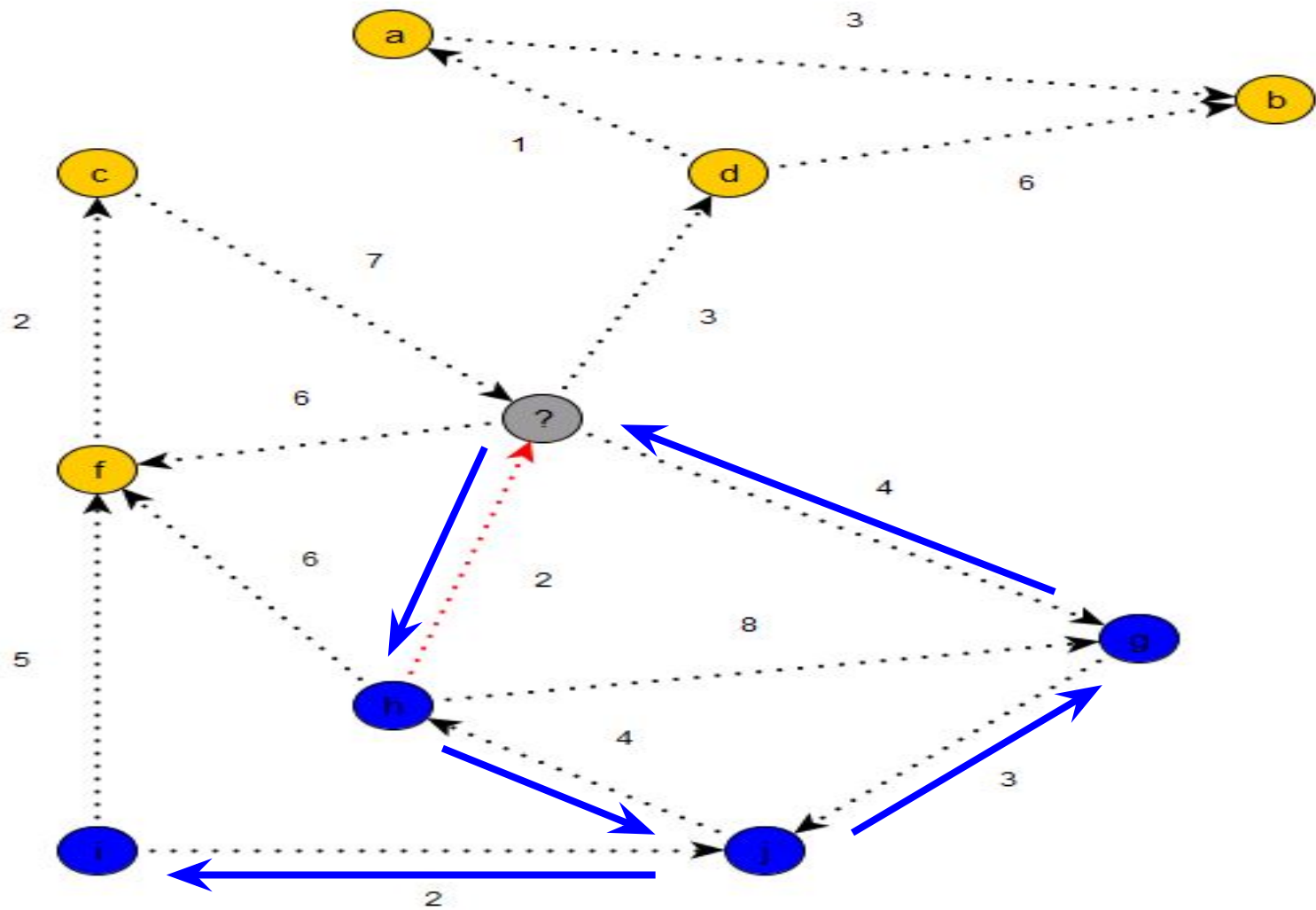


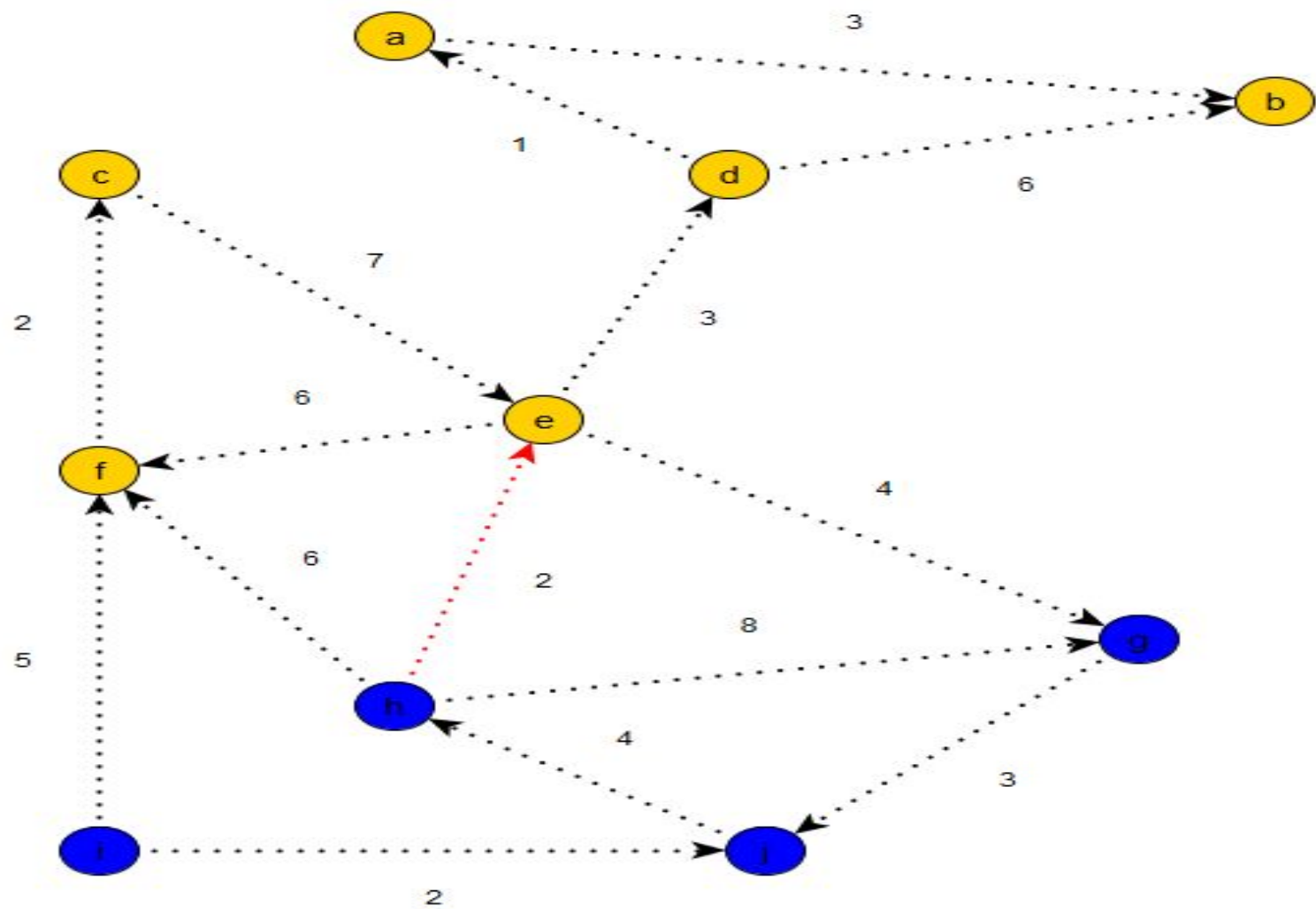


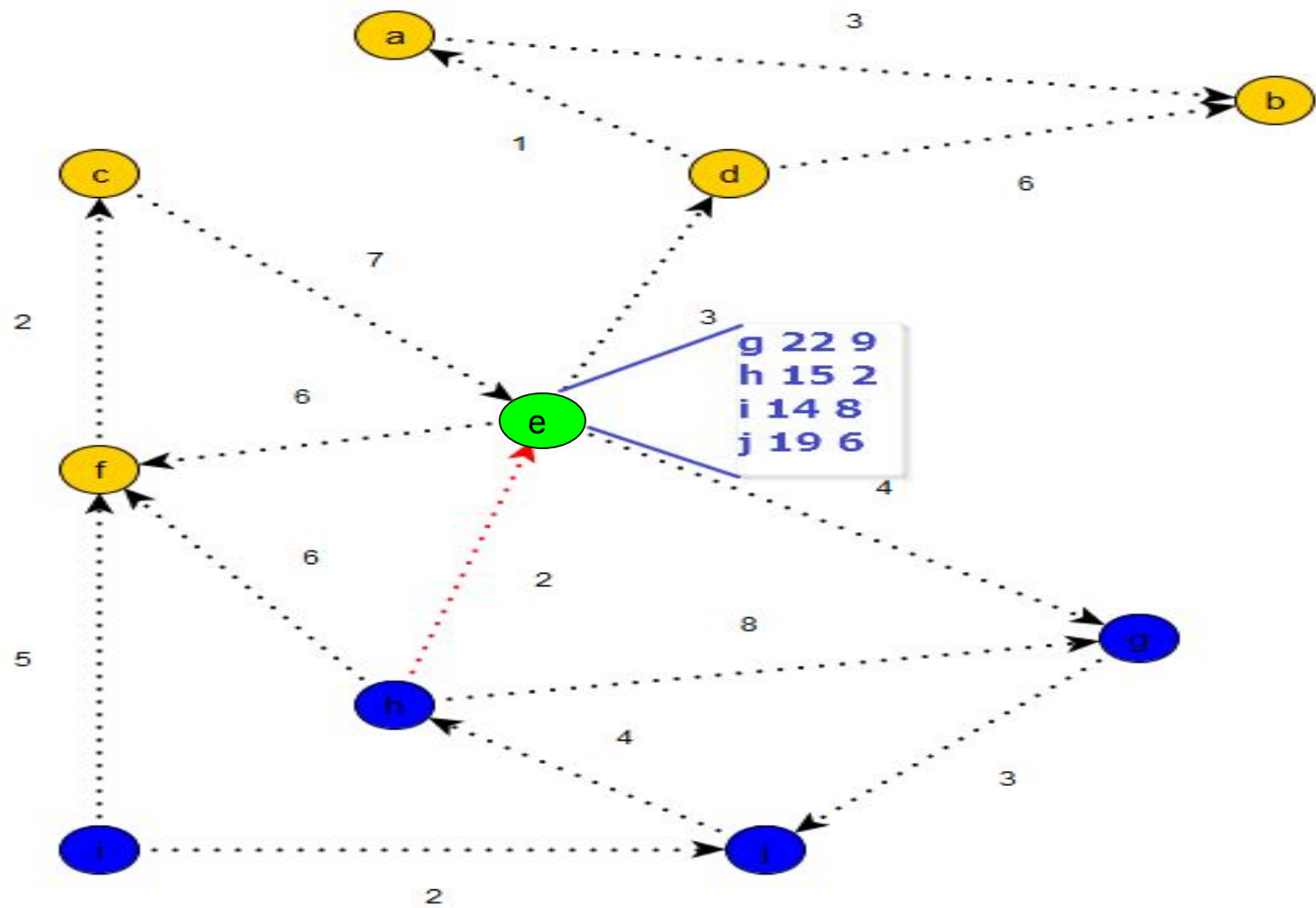


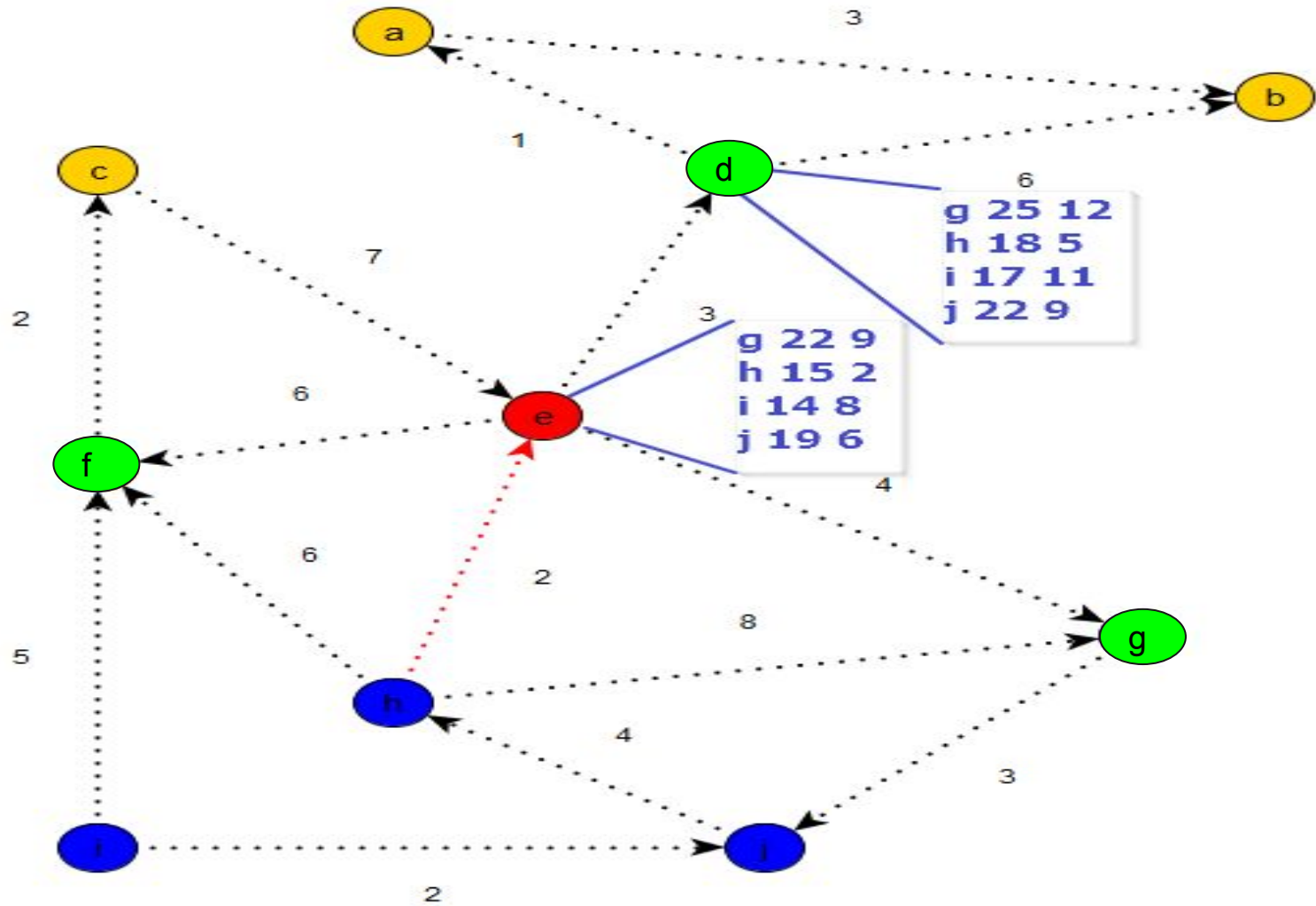


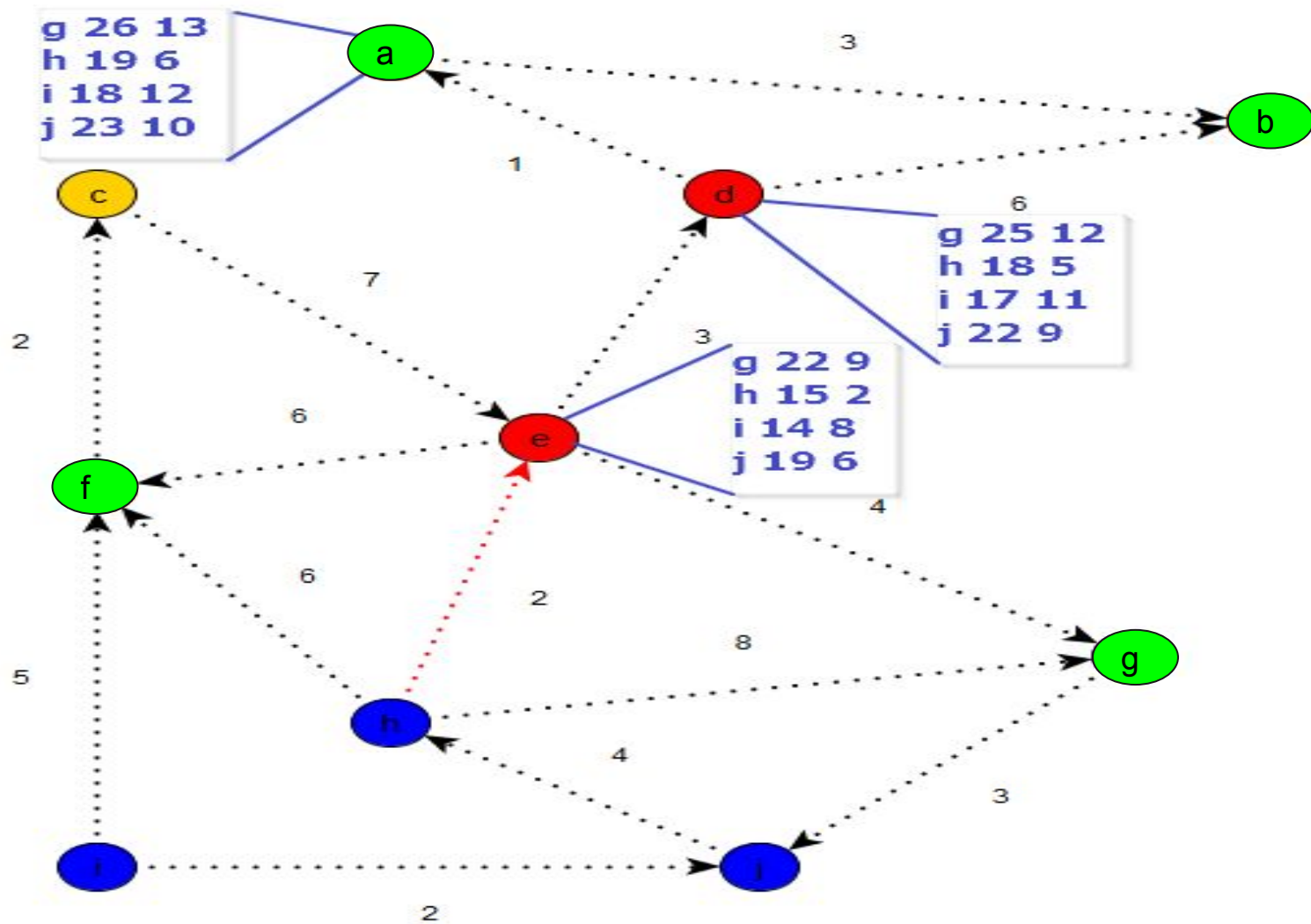


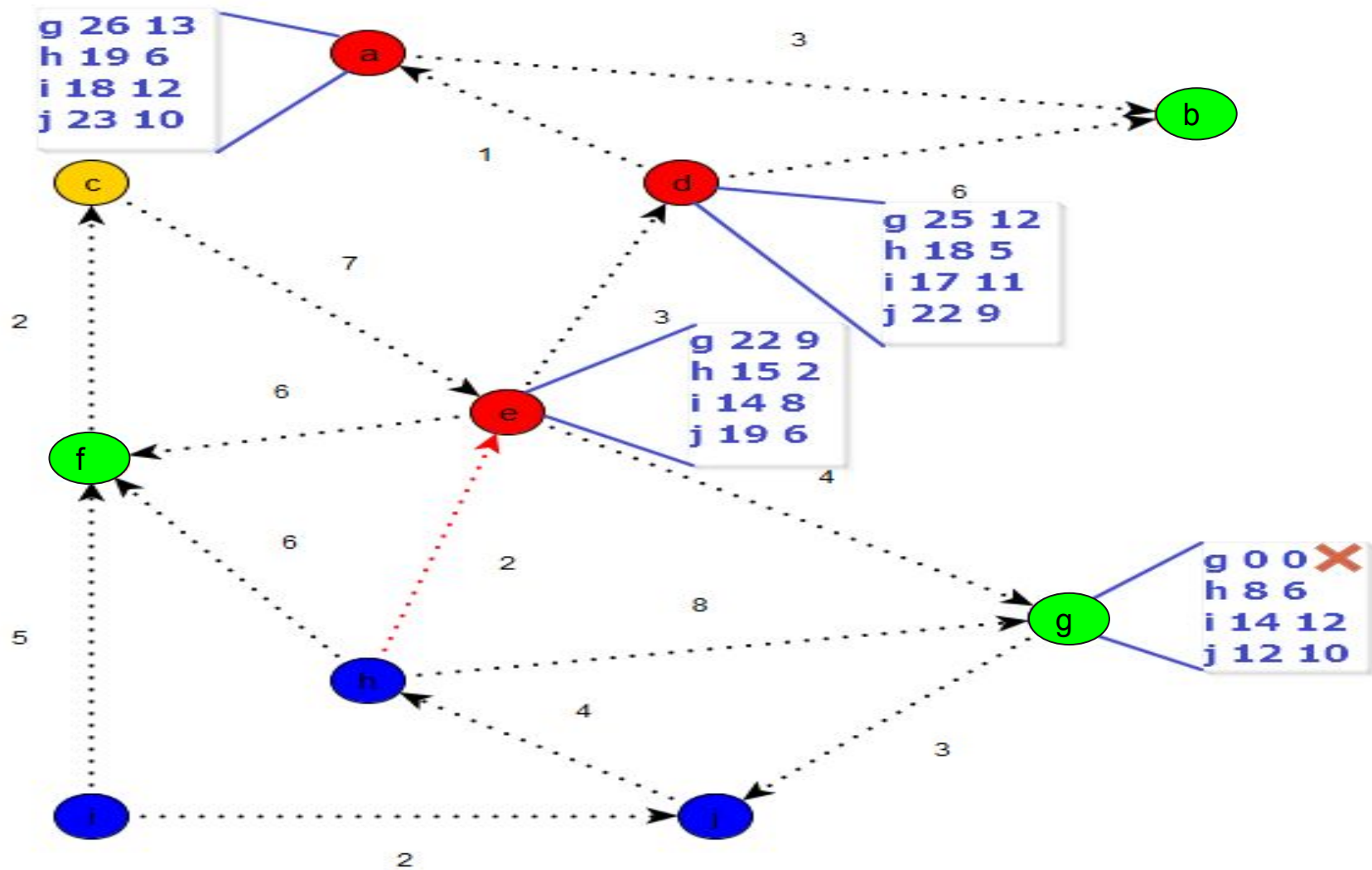


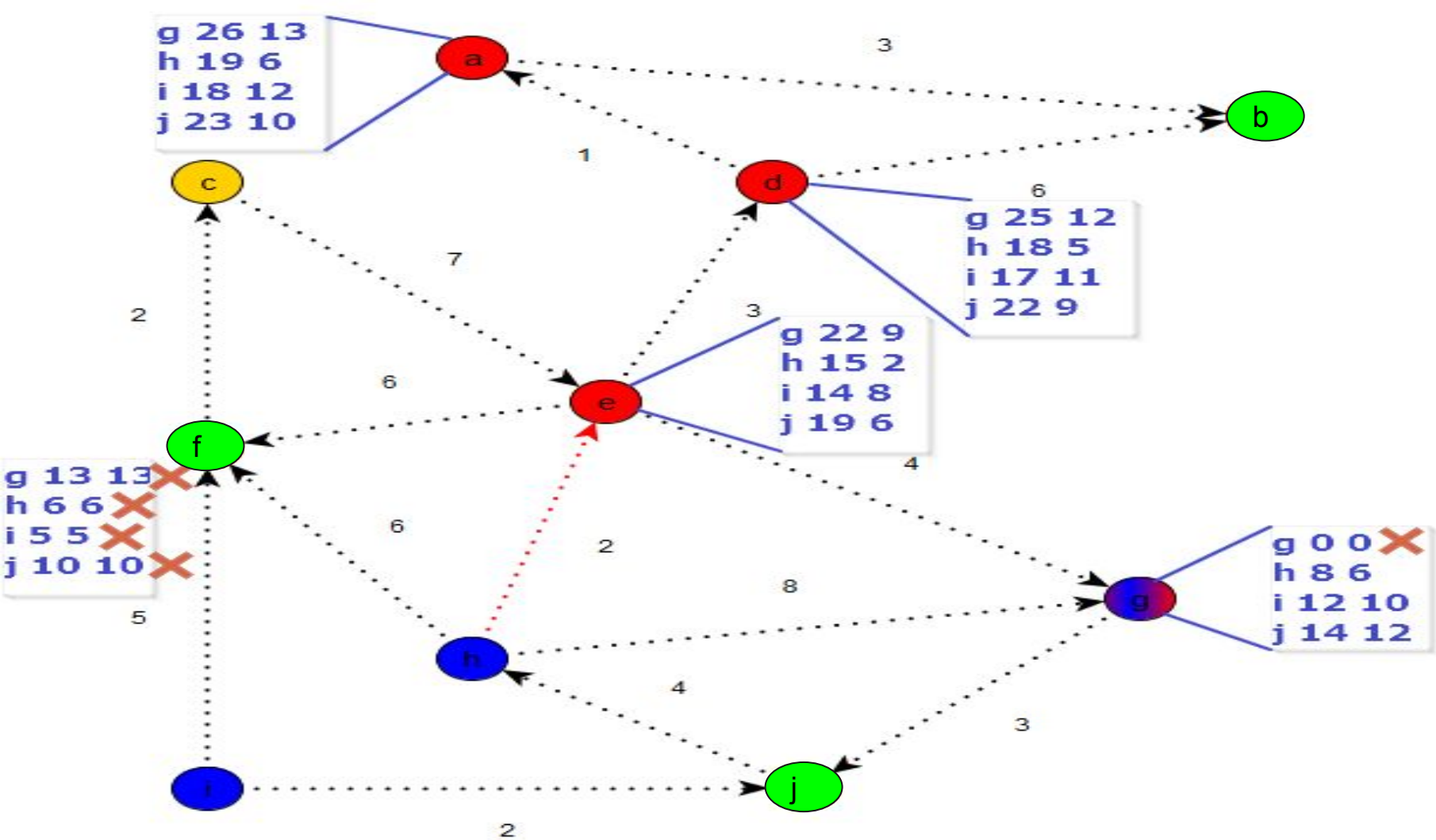


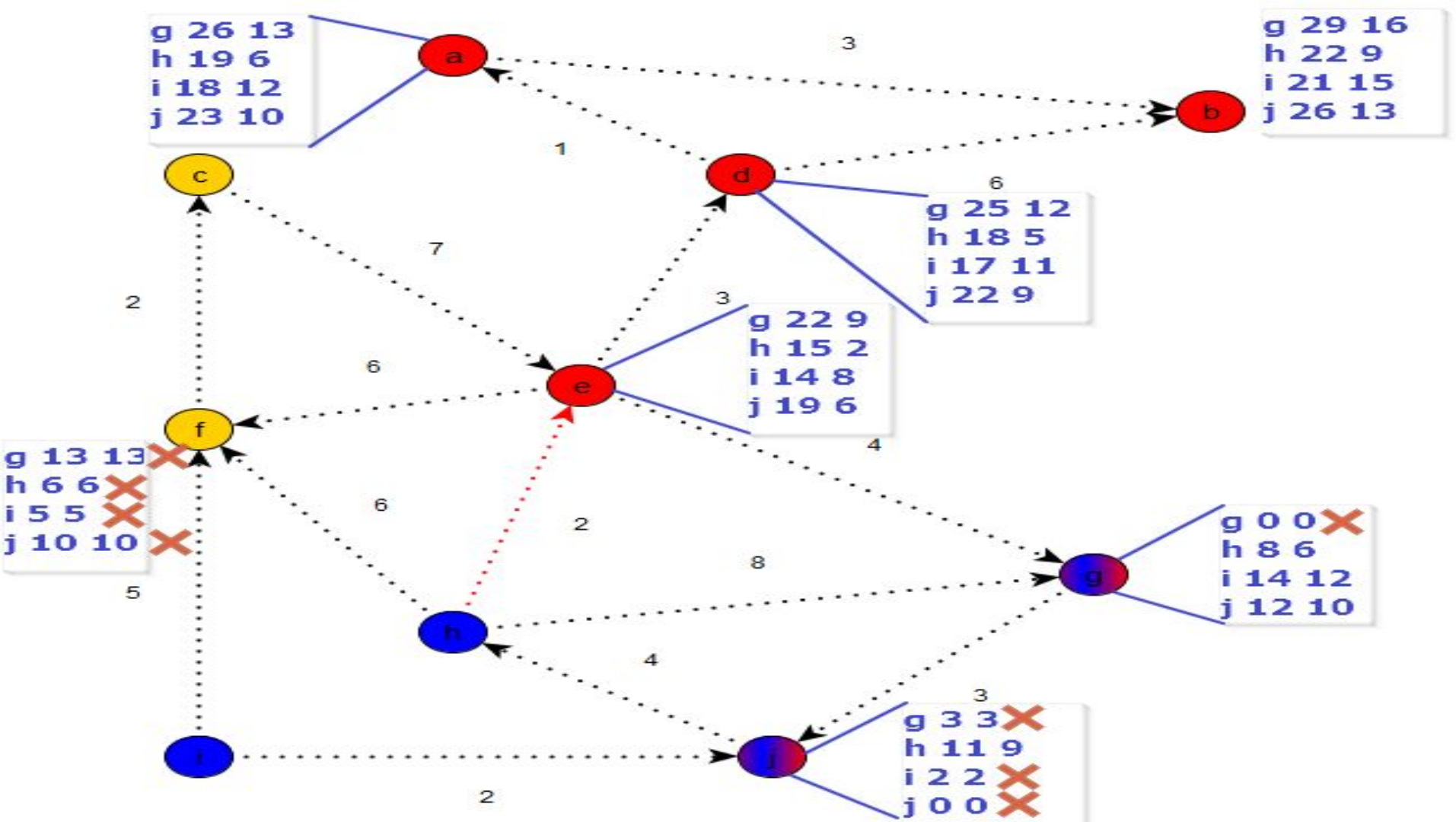












Experimental Results

- Max. distance insertion: connect two nodes that are very far apart
- Random insertion: connect two random nodes that were previously unconnected.
- Khopkar et al. is optimal theoretically
- Ramalingam and Reps (R & R) is best in practice

Max. distance insertion

Graph	Nodes	Edges	Static	Khopkar	R & R	New approach
PGPgiantcompo	10680	24316	32.19	2.3275	0.1625	0.138
subelj_cora_cora	23166	89157	184.78	10.8915	0.627	0.349
Caida_20071105	26475	53381	196.5	14.2305	0.362	0.0875
cfinder_google	15763	148585	62.65	5.1475	0.057	0.0225
p2p-Gnutella25	22687	54705	157.31	10.464	0.2075	0.082

Random insertion

Graph	Nodes	Edges	Static	Khopkar	R & R	New approach
PGPgiantcompo	10680	24316	32.19	2.327	0.1495	0.114
subelj_cora_cora	23166	89157	184.78	10.893	0.425	0.335
Caida_20071105	26475	53381	196.5	14.2295	0.124	0.042
cfinder_google	15763	148585	62.65	5.135	0.0155	0.0104
p2p-Gnutella25	22687	54705	157.31	10.4515	0.169	0.08

Speedups

	Max. distance insertion			Random edge insertion		
Graph	Khopkar	R & R	New approach	Khopkar	R & R	New approach
PGPgiantcompo	13.83	198.09	233.26	13.83	215.32	282.37
subelj_cora_cora	16.97	294.70	529.46	16.96	434.78	551.58
Caida_20071105	13.81	542.82	2245.71	13.81	1584.68	4678.57
cfinder_google	12.17	1099.12	2784.44	12.20	4041.94	6024.04
p2p-Gnutella25	15.03	758.12	1918.41	15.05	930.83	1966.38

Thank you

