Home Work (1) Answers - Numerical Analysis

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1 Home Work

- in general, Taylor series need not be convergent at all. And in fact the set of functions with a convergent Taylor series is a meager set in the Fréchet space of smooth functions. And even if the Taylor series of a function f does converge, its limit need not in general be equal to the value of the function $f(x)$. Therefore, function $f(x) = e^{-\frac{1}{x^2}}$ where $x! = 0$ elsewhere, $f(x) = 0$. is infinitely differentiable at $x = 0$, and has all derivatives zero there. Consequently, the Taylor series of $f(x)$ about $x = 0$ is identically zero. However, $f(x)$ is not the zero function, so does not equal its Taylor series around the origin. Thus, $f(x)$ is an example of a non-analytic smooth function.

- Find n in Taylor series such that approximate the function $f(X) = \frac{sin(x)}{x}$ at point $x =$ with 6 digits after the dot. (Assume your computer using 64bit floating point representation).

  We use Lagrange's error for a generic n and upper bound it using

  $$\frac{\pi^n}{2*(n+1)!}\pi^{n+1} <$$

  We will look for n which satisfies the bound condition to be less then 10^{-6}. Using a simple loop, we get $n = 35$ to be the answer.

- def factorial(a: int) -¿ int:
  if a ¡ 1:
  raise Exception("Working only on positive numbers")
  if a == 1:
  return 1
  else:
  return a * f(a-1)

- def pow(a: int, b: int) -¿ int:
  if b ¡ 0:

```
raise Exception(”Working only on positive numbers”)
if b == 0:
return 1
else:
return a * f(a, b-1)
```

- ```
  def taylorexp(x: int, n: int) -¿ float:
  ans = 0
  for i in range(n):
  ans += pow(x, i) / factorial(i)
  return ans
  ```
  ```