Example 1: Computation of an LU decomposition

Find the LU decomposition of

$$A = \begin{pmatrix} 9 & 3 & 6 \\ 4 & 6 & 1 \\ 1 & 1 & 7 \end{pmatrix}$$

Solution:

We use Gaussian elimination on A, so that the resulting upper triangular matrix will be U and the lower triangular matrix which is formed from the opposite numbers of the coefficients used will be L.

$$\begin{pmatrix} 9 & 3 & 6 \\ 4 & 6 & 1 \\ 1 & 1 & 7 \end{pmatrix} \sim \begin{pmatrix} 9 & 3 & 6 \\ 0 & 14/3 & -5/3 \\ 0 & 2/3 & 19/3 \end{pmatrix} \sim \begin{pmatrix} 9 & 3 & 6 \\ 0 & 14/3 & -5/3 \\ 0 & 0 & 46/7 \end{pmatrix}$$

-Add the 1st row multiplied by -4/9 to the 2nd row =>the 1st entry on the 2nd row of L is -(-4/9) = 4/9 -Add the 1st row multiplied by -1/9 to the 3rd row =>the 1st entry on the 3rd row of L is -(-1/9)=1/9	-Add the 2nd row multiplied by -1/7 to the 3rd row =>the 2nd entry on the 3rd row of L is -(-1/7) = 1/7	-We have the upper triangular matrix U

Moreover,

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 4/9 & 1 & 0 \\ 1/9 & 1/7 & 1 \end{pmatrix}$$

so that the LU decomposition is

$$A = \begin{pmatrix} 9 & 3 & 6 \\ 4 & 6 & 1 \\ 1 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 4/9 & 1 & 0 \\ 1/9 & 1/7 & 1 \end{pmatrix} \begin{pmatrix} 9 & 3 & 6 \\ 0 & 14/3 & -5/3 \\ 0 & 0 & 46/7 \end{pmatrix} = LU$$

The result can be checked by multiplying L and U.

Go back to theory