APPENDIX B2

GRID DIAGNOSTIC EQUATIONS

The following are mathematical expressions of the GEMPAK functions involving differential operators. The terms involving the map scale factors (m_x , m_y) can be derived by expanding the differential operators, applying the partial derivatives to the unit basis vectors as well as the components of the vectors. The partial derivatives of the basis vectors are obtained by evaluating the Christoffel symbols defined in tensor analysis. The deformation terms are derived by expanding the velocity in a vector Taylor series from which the quadratic and higher order terms are dropped. The resulting differential operators are expanded as described above allowing the result to be expressed in terms of divergence, vorticity, and the deformation components.

The following two-dimensional vectors are defined:

$$\underset{\sim}{V} = u\hat{i} + v\hat{j}$$

$$\underset{\sim}{V}_n = u_n\hat{i} + v_n\hat{j}$$

where

$$n = 1 \text{ or } 2$$

Advection of a scalar:

$$ADV(s, \underset{\sim}{V}) = -\underset{\sim}{V} \bullet \nabla s = -um_x\left(\frac{\partial s}{\partial x}\right) - vm_y\left(\frac{\partial s}{\partial y}\right)$$

Absolute vorticity:

$$AVOR(\underset{\sim}{V}) = \zeta + f$$

where

$$\zeta = VOR(\underset{\sim}{V})$$

Vector cross product:

$$CROS(\underset{\sim}{V}_1, \underset{\sim}{V}_2) = \left|\underset{\sim}{V}_1 \times \underset{\sim}{V}_2\right| = u_1 v_2 - u_2 v_1$$

Derivative with respect to x:

$$DDX(s) = m_x\left(\frac{\partial s}{\partial x}\right)$$

Derivative with respect to y:

$$DDY(s) = m_y\left(\frac{\partial s}{\partial y}\right)$$

Deformation:

$$DEF(\underset{\sim}{V}) = \sqrt{(T^2 + H^2)}$$

where

$$H = shearing\ deformation = SHR(\underset{\sim}{V})$$
$$T = stretching\ deformation = STR(\underset{\sim}{V})$$

Divergence:

$$DIV(\underset{\sim}{V}) = \nabla \bullet \underset{\sim}{V} = m_x\left(\frac{\partial u}{\partial x}\right) + m_y\left(\frac{\partial v}{\partial y}\right) - u\frac{m_x}{m_y}\left(\frac{\partial}{\partial x}m_y\right) - v\frac{m_y}{m_x}\left(\frac{\partial}{\partial y}m_x\right)$$

Dot product:

$$DOT(\underset{\sim}{V}_1, \underset{\sim}{V}_2) = \underset{\sim}{V}_1 \bullet \underset{\sim}{V}_2 = u_1u_2 + v_1v_2$$

Partial x derivative of a vector:

$$DVDX(\underset{\sim}{V}) = \left(m_x\frac{\partial u}{\partial x} - v\frac{m_y}{m_x}\left(\frac{\partial}{\partial y}m_x\right)\right)\hat{i} + \left(m_x\frac{\partial v}{\partial x} + u\frac{m_y}{m_x}\left(\frac{\partial}{\partial y}m_x\right)\right)\hat{j}$$

Partial y derivative of a vector:

$$DVDY(\underset{\sim}{V}) = \left(m_y\frac{\partial u}{\partial y} + v\frac{m_x}{m_y}\left(\frac{\partial}{\partial x}m_y\right)\right)\hat{i} + \left(m_y\frac{\partial v}{\partial y} - u\frac{m_x}{m_y}\left(\frac{\partial}{\partial x}m_y\right)\right)\hat{j}$$

Surface frontogensis:

$$FRNT(\theta, \underset{\sim}{V}) = \frac{1}{2} c |\nabla\theta| [E\cos(2\beta) - D]$$

where

$$\beta = \sin^{-1}\left(\frac{-\cos(\delta) m_x\left(\frac{\partial\theta}{\partial x}\right) - \sin(\delta) m_y\left(\frac{\partial\theta}{\partial y}\right)}{|\nabla\theta|}\right)$$

$$\delta = \frac{1}{2}\tan^{-1}\left(\frac{H}{T}\right)$$

$$D = DIV(\underset{\sim}{V})$$

δ = angle of local axis of dilatation
c = 1.08 x 10^9, a unit conversion
E = total deformation
H = shearing deformation
T = stretching deformation

Gradient:

$$GRAD(s) = \nabla s = m_x\left(\frac{\partial s}{\partial x}\right)\hat{i} + m_y\left(\frac{\partial s}{\partial y}\right)\hat{j}$$

Inertial Advection:

$$INAD(\underset{\sim}{V}_1, \underset{\sim}{V}_2) = (\underset{\sim}{V}_1 \bullet \nabla)\underset{\sim}{V}_2 =$$

$$\left[\underset{\sim}{V}_1 \bullet \nabla u_2 + v_1 v_2 \frac{m_x}{m_y}\left(\frac{\partial}{\partial x} m_y\right) - u_1 v_2 \frac{m_y}{m_x}\left(\frac{\partial}{\partial y} m_x\right)\right]\hat{i} +$$
$$\left[\underset{\sim}{V}_1 \bullet \nabla v_2 + u_1 u_2 \frac{m_y}{m_x}\left(\frac{\partial}{\partial y} m_x\right) - v_1 u_2 \frac{m_x}{m_y}\left(\frac{\partial}{\partial x} m_y\right)\right]\hat{j}$$

Isallobaric wind:

$$ISAL(s) = \frac{1}{f}\hat{k} \times \frac{\partial}{\partial t}\underset{\sim}{V}_g$$

Jacobian operator:

$$JCBN(s_1, s_2) = m_x\left(\frac{\partial}{\partial x} s_1\right) m_y\left(\frac{\partial}{\partial y} s_2\right) - m_y\left(\frac{\partial}{\partial y} s_1\right) m_x\left(\frac{\partial}{\partial x} s_2\right)$$

Cross product with vertical unit vector:

$$KCRS(\underset{\sim}{V}) = \hat{k} \times \underset{\sim}{V} = -v\hat{i} + u\hat{j}$$

Laplacian operator:

$$LAP(s) = \nabla^2 s = \nabla \bullet (\nabla s)$$

Solve Poisson equation of a forcing function with the given boundary values:

$$POIS(s_1, s_2) \text{ defined as } \nabla^2 \phi = s_1$$

where

s_1 = forcing function
s_2 = boundary values
ϕ = function for which to solve

Isentropic potential vorticity (IPV) of a layer:

$$PVOR(\theta, \underset{\sim}{V}) = -g(\bar{\zeta}_\theta + f)\frac{\partial\theta}{\partial p}$$

where

$\bar{\zeta}_\theta$ = isentropic vorticity; the overbar denotes layer average

In isobaric coordinates, potential vorticity of a layer is:

$$PVOR(\theta, \underset{\sim}{V}) = -g\left(\bar{\zeta}_p + f + \left(\hat{k} \times \frac{\partial}{\partial\theta}\overline{\underset{\sim}{V}}\right)\right)\frac{\partial\theta}{\partial p}$$

where the overbar denotes layer average

Q-vector at a level (map factors are not shown):

$$QVEC(s, \underset{\sim}{V}) = -\left(\frac{\partial \underset{\sim}{V}}{\partial x} \bullet \nabla s\right)\hat{i} - \left(\frac{\partial \underset{\sim}{V}}{\partial y} \bullet \nabla s\right)\hat{j}$$

Q-vector of a layer (map factors are not shown):

$$QVCL(s, \underset{\sim}{V}) = \left(\frac{1}{\left(\frac{\partial s}{\partial P}\right)}\right)\left(\left(\frac{\partial \overline{\underset{\sim}{V}}}{\partial x} \bullet \nabla\bar{s}\right)\hat{i} + \left(\frac{\partial \overline{\underset{\sim}{V}}}{\partial y} \bullet \nabla\bar{s}\right)\hat{j}\right)$$

where the overbar denotes layer average

Richardson number:

$$RICH(\underset{\sim}{V}) = \frac{g\partial\theta}{\theta\partial z}\left(\left|\frac{\partial}{\partial z}\underset{\sim}{V}\right|\right)^{-2}$$

Rossby number:

$$ROSS(\underset{\sim}{V}_1, \underset{\sim}{V}_2) = \frac{\left|(\underset{\sim}{V}_1 \bullet \nabla)\underset{\sim}{V}_2\right|}{f\left|\underset{\sim}{V}_1\right|}$$

Flux divergence:

$$SDIV(s, \underset{\sim}{V}) = \nabla \bullet s\underset{\sim}{V} = s(\nabla \bullet \underset{\sim}{V}) + \underset{\sim}{V} \bullet \nabla s$$

Shearing deformation:

$$SHR(\underset{\sim}{V}) = m_x\frac{\partial v}{\partial x} + m_y\frac{\partial u}{\partial y} + v\frac{m_x}{m_y}\left(\frac{\partial}{\partial x}m_y\right) + u\frac{m_y}{m_x}\left(\frac{\partial}{\partial y}m_x\right)$$

Stretching deformation:

$$STR(\underset{\sim}{V}) = m_x\frac{\partial u}{\partial x} - m_y\frac{\partial v}{\partial y} + u\frac{m_x}{m_y}\left(\frac{\partial}{\partial x}m_y\right) - v\frac{m_y}{m_x}\left(\frac{\partial}{\partial y}m_x\right)$$

Vector component of V1 along V2:

$$VASV(\underset{\sim}{V}_1, \underset{\sim}{V}_2) = \left(\frac{\underset{\sim}{V}_1 \bullet \underset{\sim}{V}_2}{\left|\underset{\sim}{V}_2\right|^2}\right)\underset{\sim}{V}_2$$

Relative vorticity:

$$VOR(\underset{\sim}{V}) = \hat{k} \bullet \nabla \times \underset{\sim}{V} = m_x\left(\frac{\partial v}{\partial x}\right) - m_y\left(\frac{\partial u}{\partial y}\right) - v\frac{m_x}{m_y}\left(\frac{\partial}{\partial x}m_y\right) + u\frac{m_y}{m_x}\left(\frac{\partial}{\partial y}m_x\right)$$