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1.1. Simplify $\frac{x^{32}}{x^9 x^2} \cdot \frac{x^7}{x^2}$

$= \frac{x^{39}}{x^{13}} = \boxed{x^{26}}$

1.2. Solve for x : $8^2 \cdot 4^x \cdot 2^x = 8^4$

$8^2 \cdot 4^x \cdot 2^x \cdot 8^{-2} = 8^4 \cdot 8^{-2}$

$4^x \cdot 2^x = 8^2$

$(4 \cdot 2)^x = 8^2$

$8^x = 8^2$

$\boxed{x = 2}$

1.3. If $\frac{x}{y} = 3$ then $x^{-4} y^4 = \ldots$

$x^{-4} y^4 = \frac{y^4}{x^4} = \left[\frac{x}{y}\right]^{-4} = 3^{-4} = \boxed{0.012345679 01}$

1.4. Calculate $\frac{\sqrt{4^{15}}}{\sqrt{16^7}}$

$\frac{\sqrt{4^{15}}}{\sqrt{16^7}} = \frac{2^{15}}{4^7} = \frac{2^{15}}{(2^2)^7} = \frac{2^{15}}{2^{14}} = \boxed{2}$

1.5. a. TRUE

b. TRUE

c. FALSE

d. FALSE

1.6. $\ln(x) \geq e$

$$e^{\ln(x)} \geq e^{e}$$

$$\boxed{x \geq e^{e}}$$

2.1. Let $y =$ temperature in F

$x =$ temperature in C

$y = mx + b$

① $32 = 0m + b$

$32 = b$

② $212 = 100m + b$

$212 = 100m + 32$

$212 - 32 = 100m$

$180 = 100m$

$m = 1.8$

If $x = y$:

$x = \cancel{1.8m}\ 1.8x + 32$

$x - 1.8x = 32$

$-0.8x = 32$

$x = -40$

$\therefore \boxed{-40^\circ C = -40^\circ F}$

2.2. $f(x) = 3x - 12$

Find y if $f(y) = 0$

$f(y) = 3y - 12 = 0$

$3y = 12$

$\boxed{y = 4}$

2.3. $9^{x^2 - 6x + 2} = 81$

$9^{x^2 - 6x + 2} = 9^2$

$x^2 - 6x + 2 = 2$

$x^2 - 6x + 2 - 2 = 0$

$x^2 - 6x = 0$

$x(x - 6) = 0$

$\Rightarrow \boxed{x = 0}, \boxed{x = 6}$

2.4. $g = 3\%$

$$(1.03)^x = 3$$

$$x \ln(1.03) = \ln 3$$

$$x = \frac{\ln 3}{\ln(1.03)}$$

$$x = 37.167$$

$$\approx 37 \text{ years}$$

2.5. $\log_\pi \left(\frac{1}{\pi^5}\right) = x$

$$\pi^x = \frac{1}{\pi^5}$$

$$\pi^x = \pi^{-5}$$

$$x = \boxed{-5}$$

3.1. $\sum_{i=0}^{\infty} \left(\frac{1}{5^i} + 0.3^i \right)$

$$= \sum_{i=0}^{\infty} \frac{1}{5^i} + \sum_{i=0}^{\infty} 0.3^i$$

$$= \sum_{i=0}^{\infty} \left(\frac{1}{5}\right)^i + \sum_{i=0}^{\infty} 0.3^i$$

$$= \frac{1}{1-\frac{1}{5}} + \frac{1}{1-0.3}$$

$$= 1.25 + \frac{1}{0.7}$$

$$= \boxed{2.67857}$$

3.2. $\lim_{x \to 5} \dfrac{x^2 - 25}{x-5}$

$= \lim_{x \to 5} \dfrac{(x+5)(x-5)}{x-5}$

$= \lim_{x \to 5} x+5$

$= 5+5 = \boxed{10}$

3.3. Find the slope at $(-2, -12)$

$f(x) = x^3 - 4$

$f'(x) = 3x^2$

$f'(-2) = 3(-2)^2$

$= 3(4) = \boxed{12}$

3.4. $f(x) = \dfrac{x^5 + 3}{x^2 - 1}$

$f'(x) = \dfrac{(x^2-1)(5x^4) - (x^5+3)(2x)}{(x^2-1)^2}$

$= \dfrac{5x^6 - 5x^4 - 2x^6 - 6x}{(x^2-1)^2}$

$= \boxed{\dfrac{x(3x^5 - 5x^3 - 6)}{(x^2-1)^2}}$

3.5. $f(x) = x^9 + 3$

$f'(x) = 9x^8$

$f''(x) = 9(8)x^7$

$= \boxed{72x^7}$

3.6 Is $f(x) = \frac{1}{x}$ continuous at 0?

No. $\lim_{x \to 0} \frac{1}{x} = \infty$

3.7. $f(x) = 4x^3 - 12x$

$f'(x) = 12x^2 - 12$

$= 12(x^2 - 1)$

$= 12(x-1)(x+1)$

$x = 1$ $\quad\quad$ $x = -1$

$f''(x) = 24x$

at $x = 1$:

$f(1) = 4 - 12 = -8$

$f''(1) = 24$ $\quad\quad$ min at $(1, -8)$

at $x = -1$:

$f(-1) = -4 + 12 = 8$

$f''(-1) = -24$ $\quad\quad$ max at $(-1, 8)$

3.8. $f(x, y) = x^3 - y^2$

$f(2, 3) = 2^3 - 3^2 = 8 - 9 = \boxed{-1}$

3.9. $f(x, y) = \ln(x - 3y)$

$\ln(x - 3y)$ is only defined for $(x - 3y) > 0$

$x - 3y > 0$

$\boxed{x > 3y}$

3.10 $\frac{d}{dx}\left(x^5 y^7 + \frac{x^2}{y^3}\right)$

$$= \boxed{5x^4 y^7 + \frac{2x}{y^3}}$$

3.11

3.12. max x^2y^2 s.t. $2x + y = 9$

$$\mathcal{L} = f(x,y) - \lambda g(x,y)$$
$$= x^2y^2 - \lambda(2x + y - 9)$$

$$\frac{d\mathcal{L}}{dx} = 2xy^2 - 2\lambda = 0 \Rightarrow 2xy^2 = 2\lambda$$
$$xy^2 = \lambda$$

$$xy^2 = 2x^2y$$
$$y\cancel{x} = 2x$$

$$\frac{d\mathcal{L}}{dy} = 2x^2y - \lambda = 0 \Rightarrow 2x^2y = \lambda$$

$$\frac{d\mathcal{L}}{d\lambda} = 2x + y - 9 = 0$$

$$\Rightarrow 2x + y - 9 = 0$$
$$y + y - 9 = 0$$
$$2y = 9$$
$$y = \frac{9}{2}$$

$$2x = y$$
$$2x = \frac{9}{2} \Rightarrow x = \frac{9}{4}$$

$$\boxed{\left(\frac{9}{4}, \frac{9}{2}\right)}$$

4.1. $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}$

$B \cdot A$:

$$\begin{array}{c|cc} & 2 & 5 \\ & 2 & 1 \\ & 7 & 6 \\ \hline 1\,0\,1 & 9 & 11 \\ 9\,1\,5 & 55 & 76 \end{array}$$

$BA = \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}$

4.2. $A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

AB :

$$\begin{array}{c|ccc} & 8 & 4 & 0 \\ & 2 & 1 & 2 \\ \hline 5\;3 & 46 & 23 & 6 \\ 0\;1 & 2 & 1 & 2 \\ 1\;2 & 12 & 6 & 4 \end{array}$$

$AB = \begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{bmatrix}$

4.3

$$\begin{bmatrix} e & 93 & 4.7 \\ 2 & 6.1 & 4.22 \\ 4 & \pi & 0 \end{bmatrix}^T = \begin{bmatrix} e & 2 & 4 \\ 93 & 6.1 & \pi \\ 4.7 & 4.22 & 0 \end{bmatrix}$$

4.4. $\det\left(\begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix} \right) = 2(8) - 2(6) = 4$

5.1. $\Omega = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

5.2. Let U = event that someone is a drug user

P = event that the test comes out positive

$P(U) = 0.1\%$

$P(P \mid U) = 98\%$

$P(P^c \mid U^c) = 99.7\%$

$$P(P) = P(P|U)P(U) + P(P|U^c)P(U^c)$$
$$= 0.98(0.001) + (1-0.997)(1-0.001)$$
$$= 0.003977$$

$$P(U|P) = \frac{P(P|U)\cdot P(U)}{P(P)}$$
$$= \frac{(0.98)(0.001)}{0.003977} = \boxed{0.2464}$$

5.3. $n = 20$ tosses

p = Probability of getting a 5 in 1 toss $= \frac{1}{6}$

Expected value for binomial random variable $= np$

$$= 20\left(\frac{1}{6}\right) = \boxed{\frac{20}{6} = 3.3\overline{3}}$$