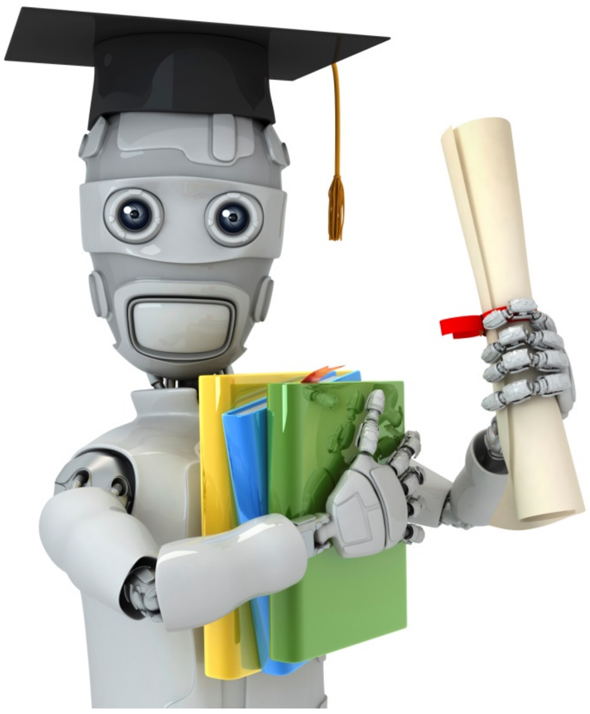

Machine Learning

Linear Algebra review (optional)

Matrices and vectors 矩阵&向量

Matrix: Rectangular array of numbers: 数字组成的矩形阵列

$$\begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

4 x2 matrix

$\boxed{\mathbb{R}^{4\times 2}}$ 4行2列

$$\begin{array}{cc} 2 & \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \\ & 3 \end{array}$$

2 x3 matrix

$\boxed{\mathbb{R}^{2\times 3}}$ 2行3列

Dimension of matrix: number of rows x number of columns

矩阵维数 行数 列数

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$\mathbb{R}^{m\times n}$ = m行n列的矩阵是集合$R^{m\times n}$的元素

Matrix Elements (entries of matrix) 矩阵元素/项

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

$A_{ij} =$ "i,j entry" in the i^{th} row, j^{th} column.

$A_{11} = 1402$

$A_{12} = 191$

$A_{32} = 1437$

$A_{41} = 147$

$\cancel{A_{43}}$ = undefined (error)

未定义(不存在此元素)

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Vector: An n x 1 matrix. 特殊的矩阵（只有一列）$n \times 1$

$\cancel{\mathbb{R}^{3\times 2}}$

n= 4

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

4-dimensional vector. $\mathbb{R}^4$ 4维向量

$y_i = i^{th}$ element

$y_1 = 460$

$y_2 = 232$

$y_3 = 315$

$\boxed{A, B, C, X}$ 矩阵

$\boxed{a, b, x, y}$
- 向量
- 数字
- 标量
- ...

1-indexed vs 0-indexed:

y[1]

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

1-indexed

下标从1开始（默认）

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

y[0]

0-indexed

下标从0开始

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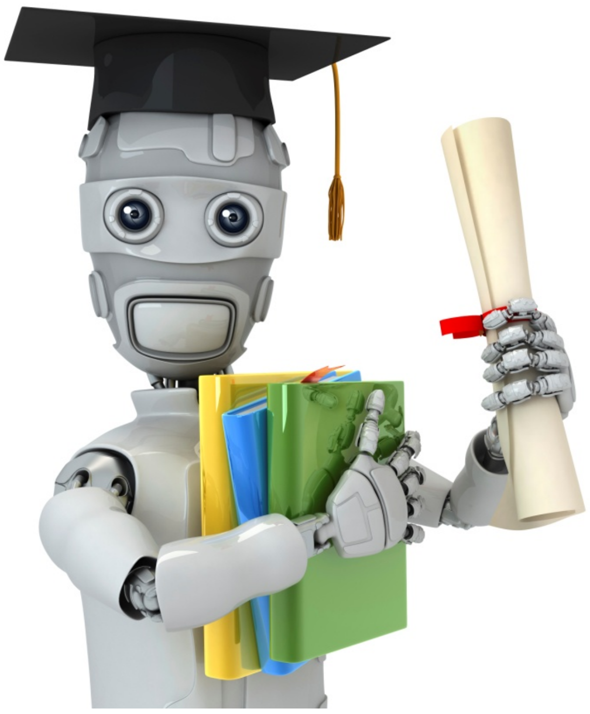


Machine Learning

Linear Algebra review (optional)

Addition and scalar multiplication

矩阵+ & 矩阵×标量

Matrix Addition 只有相同维度的两个矩阵才可相加

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}}_{\substack{3\times 2 \\ \text{matrix}}} + \underbrace{\begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix}}_{3\times 2} = \underbrace{\begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}}_{3\times 2}$$

$$\cancel{\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}}_{3\times 2} + \underbrace{\begin{bmatrix} 4 & 0.5 \\ 2 & 5 \end{bmatrix}}_{2\times 2}} = \text{error}$$

标量

Scalar Multiplication

real number 实数

乘 $3 \times \underbrace{\begin{bmatrix}1 & 0\\ 2 & 5\\ 3 & 1\end{bmatrix}}_{3\times 2} = \underbrace{\boxed{\begin{bmatrix}3 & 0\\ 6 & 15\\ 9 & 3\end{bmatrix}}}_{3\times 2} = \begin{bmatrix}1 & 0\\ 2 & 5\\ 3 & 1\end{bmatrix} \times 3$

除 $\boxed{\begin{bmatrix}4 & 0\\ 6 & 3\end{bmatrix} / 4} = \frac{1}{4}\begin{bmatrix}4 & 0\\ 6 & 3\end{bmatrix} = \begin{bmatrix}1 & 0\\ \frac{3}{2} & \frac{3}{4}\end{bmatrix}$

Combination of Operands 运算的结合

$$3 \times \begin{bmatrix}1\\4\\2\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \begin{bmatrix}3\\0\\2\end{bmatrix} / 3$$

标量乘 Scalar multiplication

Scalar division 标量除

$$= \begin{bmatrix}3\\12\\6\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \begin{bmatrix}1\\0\\\frac{2}{3}\end{bmatrix}$$

matrix subtraction / vector subtraction $-$

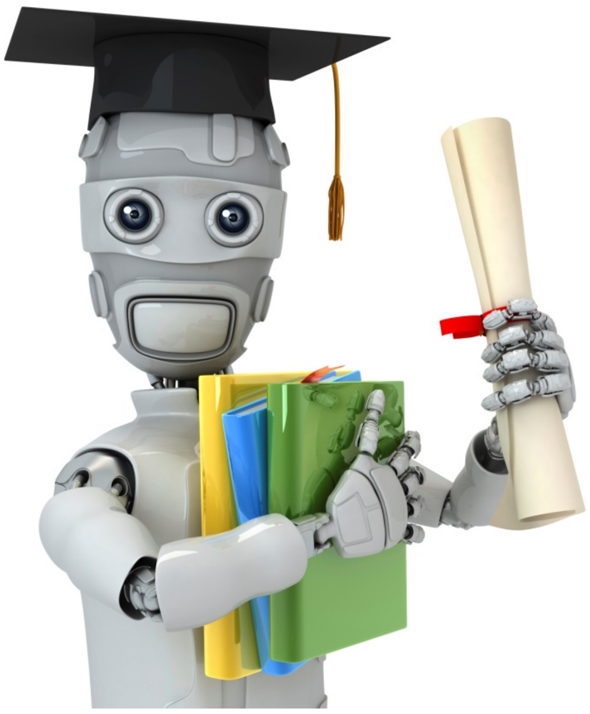
(矩阵以及向量)

matrix addition / vector addition $+$

$$= \begin{bmatrix}2\\12\\10\frac{1}{3}\end{bmatrix}$$

3x1 matrix

3-dimensional vector 3维



Machine Learning

Linear Algebra review (optional)

Matrix-vector multiplication

矩阵×向量

Example

$$\underbrace{\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} 1 \\ 5 \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}}_{3\times 1}$$

matrix

$$1\times 1 + 3\times 5 = 16$$

$$4\times 1 + 0\times 5 = 4$$

$$2\times 1 + 1\times 5 = 7$$

Details:

$$A \times x = y$$

m x n matrix
(m rows,
n columns)

n x 1 matrix
(n-dimensional
vector)

m-dimensional
vector

To get y_i , multiply A's i^{th} row with elements of vector x, and add them up.

Example

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{14} \\ \boxed{13} \\ \boxed{-7} \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

$\boxed{3} \times 4$ $\quad 4 \times 1 \quad 3 \times 1$

$$\left.\begin{array}{l} 1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 = 14 \\ 0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 = 13 \\ -1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 = -7 \end{array}\right]$$

House sizes:

- 2104
- 1416
- 1534
- 852

$$h_\theta(x) = -40 + 0.25x$$

$-40 \cdot 1 \quad 0.25 \cdot x$

$h_\theta(x)$

Matrix (4x2) × Vector (2x1) = 4x1 matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -40 \times 1 + 0.25 \times 2104 \\ -40 \times 1 + 0.25 \times 1416 \\ \\ \\ \end{bmatrix}$$

$h_\theta(2104)$

$h_\theta(1416)$

$\boxed{\text{prediction}} = \text{DataMatrix} \otimes \text{parameters}$

4x1

参数列加1路: 1, x

参数作为列向量

方法一

for i = 1:1000,

prediction (i) = ...

方法二

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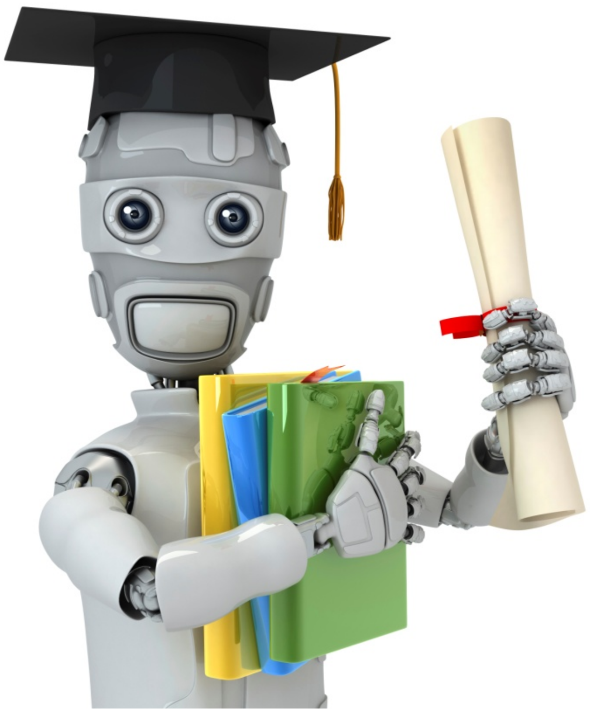


Machine Learning

Linear Algebra review (optional)

Matrix-matrix multiplication

矩阵×矩阵

Example

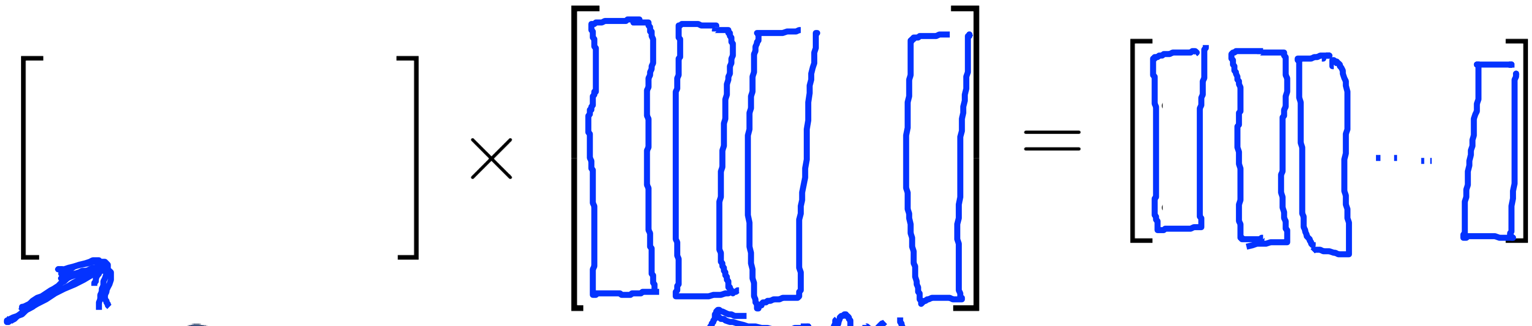
$$\underbrace{\begin{bmatrix}1 & 3 & 2\\4 & 0 & 1\end{bmatrix}}_{2\times 3}\underbrace{\begin{bmatrix}1 & 3\\0 & 1\\5 & 2\end{bmatrix}}_{3\times 2} = \underbrace{\begin{bmatrix}11 & 10\\9 & 14\end{bmatrix}}_{2\times 2}$$

$$\begin{bmatrix}1 & 3 & 2\\4 & 0 & 1\end{bmatrix} \times \underbrace{\begin{bmatrix}1\\0\\5\end{bmatrix}}_{\text{第一列}} = \begin{bmatrix}11\\9\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & 2\\4 & 0 & 1\end{bmatrix} \times \underbrace{\begin{bmatrix}3\\1\\2\end{bmatrix}}_{\text{第二列}} = \begin{bmatrix}10\\14\end{bmatrix}$$

Details:

$$A \times B = C$$



m x n matrix
(m rows,
n columns)

$n \times 1$

n x o matrix
(n rows,
o columns)

m x o
matrix

The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B. (for i = 1,2,…,o)

Example

$$\underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times 0 + 3\times 3 \\ 2\times 0 + 5\times 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 3\times 2 \\ 2\times 1 + 5\times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

House sizes:

$$\begin{Bmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{Bmatrix}$$

Have 3 competing hypotheses:

1. $h_\theta(x) = -40 + 0.25x$ 第一列参数
2. $h_\theta(x) = 200 + 0.1x$ 第二列参数
3. $h_\theta(x) = -150 + 0.4x$ 第三列参数

Matrix X

Matrix

$$\underbrace{\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix}}_{\text{Data Matrix}} \times \underbrace{\begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix}}_{\text{Parameters}} = \begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

1 2 3

1 2 3

Prediction of first hθ

Predictions of 2nd hθ

Predictions

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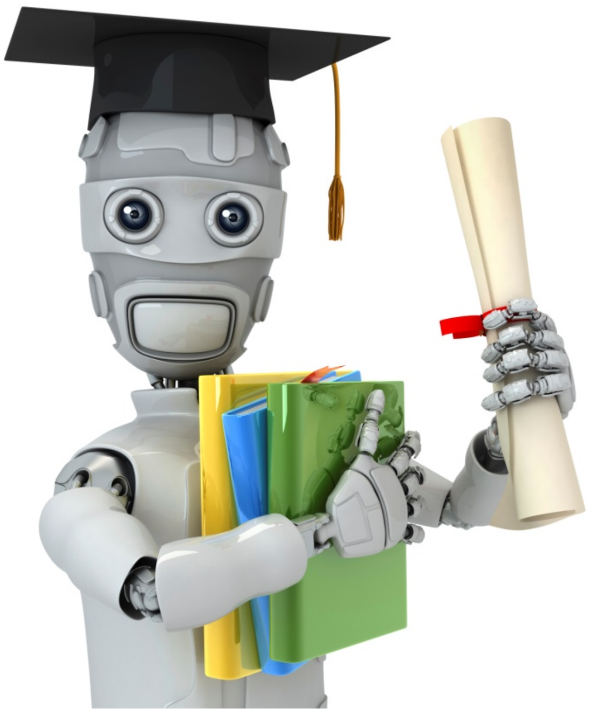


Machine Learning

Linear Algebra review (optional)

Matrix multiplication properties

矩阵乘法的特性

$3 \times 5 = 5 \times 3$ "Commutative" 交换律

Let A and B be matrices. Then in general,

$A \times B \neq B \times A$. (not commutative.)

E.g. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ $\neq$

$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$

$\underset{m \times n}{A} \times \underset{n \times m}{B}$

$A \times B$ is $m \times m$

$B \times A$ is $n \times n$

> 会改变矩阵维度

$3 \times 5 \times 2$

$3 \times 10 = 30 = 15 \times 2$

$3 \times (5 \times 2) = (3 \times 5) \times 2$

$\boxed{\text{“Associative”}}$ 結合律

$A \times (B \times C)$

$(A \times B) \times C$

$A \times B \times C.$

Let $D = B \times C$. Compute $A \times D$.

Let $E = A \times B$. Compute $E \times C$.

$A \times (B \times C)$

$(A \times B) \times C$

Same answer.

Identity Matrix 单位矩阵 1 is identity

$\boxed{1 \times z = z \times 1 = z}$ for any z

Denoted I (or $\boxed{I_{n \times n}}$). 方阵

Examples of identity matrices: 对角线=1

Informally:

$[1]$ 1×1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
2 x 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
3 x 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
4 x 4

$$\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & \ddots \;1 \end{bmatrix}$$

For any matrix A,

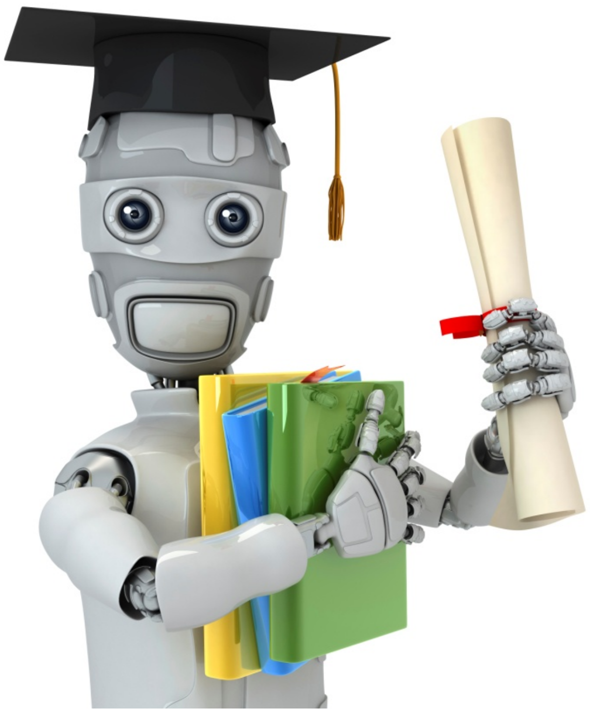
$$A \cdot \boxed{I} = \boxed{I} \cdot A = A$$ 交换律 $I_{n\times n}$

$m\times n$ $n\times n$ $m\times m$ $m\times n$ $m\times n$ 维度隐含

Note:

$AB \neq BA$ in general

$AI = \cancel{}\; IA$ ✓



Machine Learning

Linear Algebra review (optional)

Inverse and transpose

逆&转置

1 = "identity" $3\,\boxed{(3^{-1})} = 1$ $12 \times (12^{-1}) = 1$

$\frac{1}{3}$ $\frac{1}{12}$

Not all numbers have an inverse. 0无倒数   0 (0^{-1}) undefined

Matrix inverse: 方阵 square matrix (#rows = #columns) A^{-1}

If A is an m x m matrix, and if it has an inverse,

$$A(A^{-1}) = A^{-1}A = I.$$

不存在逆矩阵 $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 可想成近似为0

e.g. $\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_{\substack{2\times 2 \\ A}} * \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$

$A^{-1}A$

Octave: pinv(A)

Matrices that don't have an inverse are "singular" or "degenerate"

奇异矩阵   退化矩阵

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Matrix Transpose

Example: $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$ 行→列 $B = A^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$

2×3 3×2

Let A be an m x n matrix, and let $B = A^T$.
纵行
Then B is an n x m matrix, and

元素 $B_{ij} = A_{ji}$.

$B_{12} = A_{21} = 2$

$B_{32} = 9 \qquad A_{23} = 9.$