

Machine Learning

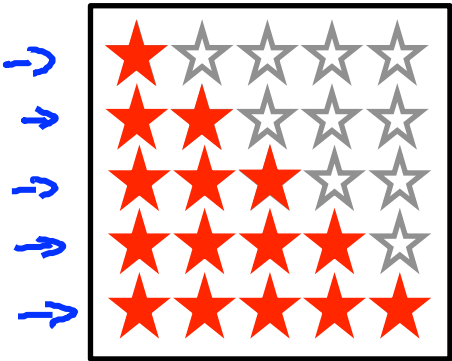
Recommender Systems

Problem formulation

问题规划

Example: Predicting movie ratings

→ User rates movies using ~~one~~ to five stars
zero



Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)
Love at last	5	5	0	0
Romance forever	5	?	?	0
Cute puppies of love	?	5	0	?
Nonstop car chases	0	0	5	4
Swords vs. karate	0	0	5	?

$n_u = 4$

$n_m = 5$

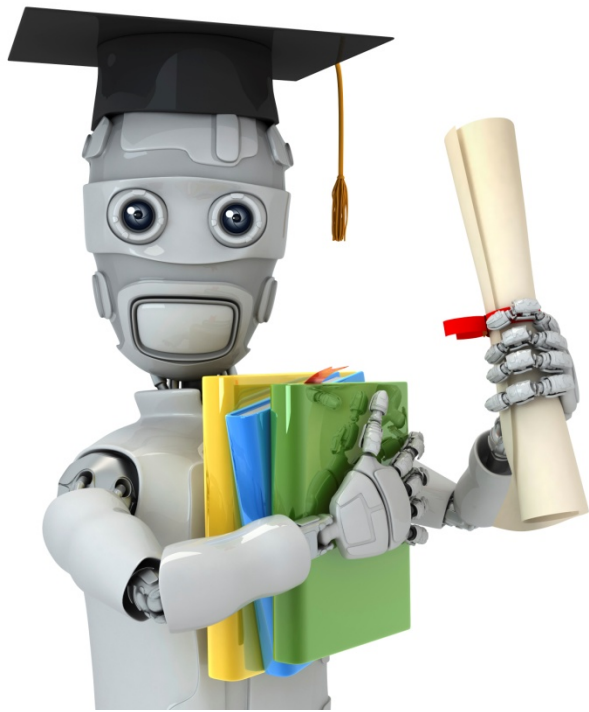
→ n_u = no. users

→ n_m = no. movies

→ $r(i, j) = 1$ if user j has rated movie i

→ $y^{(i,j)}$ = rating given by user j to movie i (defined only if $r(i, j) = 1$)

0, ..., 5



Machine Learning

Recommender Systems

Content-based recommendations

其内容相似性 (x → y)

Content-based recommender systems

$n_u = 4, n_m = 5$

$x_0 = 1$

$x^{(1)} = \begin{bmatrix} 1 \\ 0.9 \\ 0 \end{bmatrix}$

Movie	Alice (1) $\theta^{(1)}$	Bob (2) $\theta^{(2)}$	Carol (3) $\theta^{(3)}$	Dave (4) $\theta^{(4)}$	x_1 (romance)	x_2 (action)
Love at last 1	5	5	0	0	0.9	0
Romance forever 2	5	?	?	0	1.0	0.01
Cute puppies of love 3	?	4	?	?	0.99	0
Nonstop car chases 4	0	0	5	4	0.1	1.0
Swords vs. karate 5	0	0	5	?	0	0.9

$n=2$

For each user j , learn a parameter $\theta^{(j)} \in \mathbb{R}^3$. Predict user j as rating movie $(\theta^{(j)})^T x^{(i)}$ stars.

$x^{(3)} = \begin{bmatrix} 1 \\ 0.99 \\ 0 \end{bmatrix} \leftrightarrow \theta^{(1)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$

$(\theta^{(1)})^T x^{(3)} = 5 \times 0.99 = 4.95$

$h_{\theta}^{(1)} = \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots$

Alice: $h_{\theta}^{(1)}(x^{(1)}) = \theta_1 x_1^{(1)} + \theta_2 x_2^{(1)} + \theta_3 x_3^{(1)}$

$\theta^{(1)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$

$x^{(1)} = \begin{bmatrix} 1 \\ 0.9 \\ 0 \end{bmatrix}$

Bob: $h_{\theta}^{(2)}(x^{(2)}) = \theta_1 x_1^{(2)} + \theta_2 x_2^{(2)} + \theta_3 x_3^{(2)}$

$\theta^{(2)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x^{(2)} = \begin{bmatrix} 1 \\ 1.0 \\ 0.01 \end{bmatrix}$

Problem formulation

- $r(i, j) = 1$ if user j has rated movie i (0 otherwise) 所有样本都是一个0或1
- $y^{(i,j)}$ = rating by user j on movie i (if defined)
- $\theta^{(j)}$ = parameter vector for user j 每个用户样本一个 θ 向量 (与前面不同)
- $x^{(i)}$ = feature vector for movie i
- For user j , movie i , predicted rating: $(\theta^{(j)})^T (x^{(i)})$ $\theta^{(j)} \in \mathbb{R}^{n+1}$
- $m^{(j)}$ = no. of movies rated by user j 每个用户样本的评分数

To learn $\theta^{(j)}$.

$$\min_{\theta^{(j)}} \frac{1}{2} \sum_{i: r(i,j)=1} ((\theta^{(j)})^T (x^{(i)}) - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{k=1}^n (\theta_k^{(j)})^2$$

\downarrow 用户的个数 \downarrow 每个特征 $x^{(i)}$

Andrew Ng

针对所有用户 $\sum_{j=1}^m$

code to compute $J(\theta)$

$$J(\theta) = \left[-\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

$$h_{\theta}(x^{(i)}) = \theta^T x^{(i)}$$

$$\theta^T x^{(i)}$$

Optimization objective: 代价函数

To learn $\theta^{(j)}$ (parameter for user j): 每个用户的所有数据都取出来

$$\rightarrow \min_{\theta^{(j)}} \frac{1}{2} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{k=1}^n (\theta_k^{(j)})^2 \rightarrow \text{得最优的 } \theta^{(j)}$$

To learn $\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(n_u)}$: 多个用户的所有数据都取出来

$$\min_{\theta^{(1)}, \dots, \theta^{(n_u)}} \frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (\theta_k^{(j)})^2 \rightarrow \text{得最优的 } \theta^{(j)}$$

$\theta^{(1)}, \dots, \theta^{(n_u)}$

Optimization algorithm:

$$\min_{\theta^{(1)}, \dots, \theta^{(n_u)}} \underbrace{\frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (\theta_k^{(j)})^2}_{J(\theta^{(1)}, \dots, \theta^{(n_u)})}$$

Gradient descent update:

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right) x_k^{(i)} \quad (\text{for } k = 0)$$

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \left(\sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right) x_k^{(i)} + \lambda \theta_k^{(j)} \right) \quad (\text{for } k \neq 0)$$

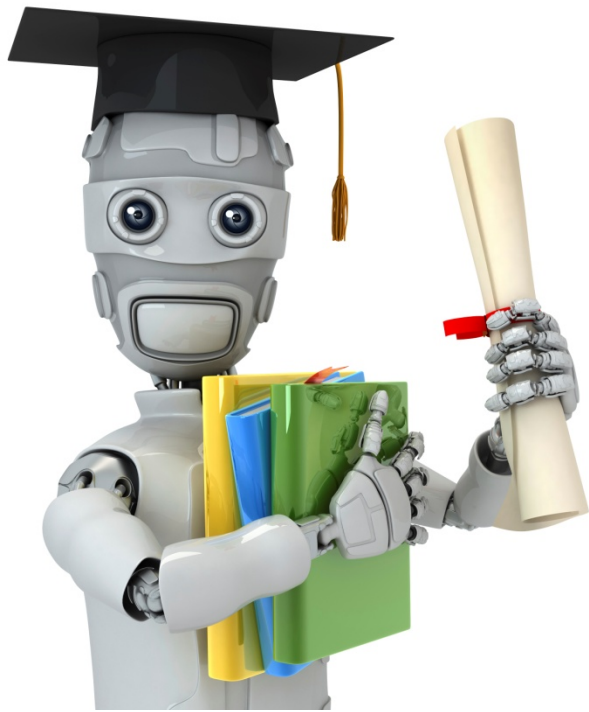
$$\frac{1}{m^{(j)}}$$

$$\frac{\partial}{\partial \theta_k^{(j)}} J(\theta^{(1)}, \dots, \theta^{(n_u)})$$

线性回归参数初始化

正则化参数初始化

$$\min_{\theta^{(j)}} \frac{1}{2} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{k=1}^n (\theta_k^{(j)})^2$$





Machine Learning

Recommender Systems

Collaborative filtering

共同兴趣、行为等所表现出的相似性 ($U \rightarrow X$)

Problem motivation

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)	 x_1	 x_2
					(romance)	(action)
Love at last	5	5	0	0	0.9	0
Romance forever	5	?	?	0	1.0	0.01
Cute puppies of love	?	4	0	?	0.99	0
Nonstop car chases	0	0	5	4	0.1	1.0
Swords vs. karate	0	0	5	?	0	0.9

Problem motivation

Movie	Alice (1) $\theta^{(1)}$	Bob (2) $\theta^{(2)}$	Carol (3) $\theta^{(3)}$	Dave (4) $\theta^{(4)}$	$x_0 = 1$	
					x_1 (romance)	x_2 (action)
$x^{(1)}$ Love at last	5	5	0	0	1.0	0.0
Romance forever	5	?	?	0	?	?
Cute puppies of love	?	4	0	?	?	?
Nonstop car chases	0	0	5	4	?	?
Swords vs. karate	0	0	5	?	?	?

$\theta^{(1)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \theta^{(2)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \theta^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}, \theta^{(4)} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$

每个用户对电影的爱爱程度

$\theta^{(j)}$
 $(\theta^{(1)})^T x^{(1)} \approx 5$
 $(\theta^{(2)})^T x^{(1)} \approx 5$
 $(\theta^{(3)})^T x^{(1)} \approx 0$
 $(\theta^{(4)})^T x^{(1)} \approx 0$

IPU对电影的评价
 $\theta^{(j)} x^{(i)}$

Optimization algorithm

Given $\theta^{(1)}, \dots, \theta^{(n_u)}$ to learn $x^{(i)}$: 全部数据, 针对所有用户数据求和

$$\min_{x^{(i)}} \frac{1}{2} \sum_{j: r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{k=1}^n (x_k^{(i)})^2$$

← 得到最优 $x^{(i)}$

Given $\theta^{(1)}, \dots, \theta^{(n_u)}$ to learn $x^{(1)}, \dots, x^{(n_m)}$: 所有数据, 针对所有用户数据求和

$$\min_{x^{(1)}, \dots, x^{(n_m)}} \frac{1}{2} \sum_{i=1}^{n_m} \sum_{j: r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2$$

← 得到最优 $x^{(i)}$

Andrew Ng

Optimization objective: 优化目标

To learn $\theta^{(j)}$ (parameter for user j): 单个用户, 针对所有数据求和

$$\min_{\theta^{(j)}} \frac{1}{2} \sum_{i: r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{k=1}^n (\theta_k^{(j)})^2$$

→ 得到最优 $\theta^{(j)}$

To learn $\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(n_u)}$: 多个用户, 针对所有数据求和

$$\min_{\theta^{(1)}, \dots, \theta^{(n_u)}} \frac{1}{2} \sum_{j=1}^{n_u} \sum_{i: r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (\theta_k^{(j)})^2$$

$\theta^{(1)}, \dots, \theta^{(n_u)}$

Collaborative filtering

已知电影特征 $x^{(i)}$ \rightarrow $\Sigma \theta^{(j)}$

$\sigma(i,j)$
 $y^{(i,j)}$

相似性

Given $x^{(1)}, \dots, x^{(n_m)}$ (and movie ratings),
can estimate $\theta^{(1)}, \dots, \theta^{(n_u)}$

\nwarrow

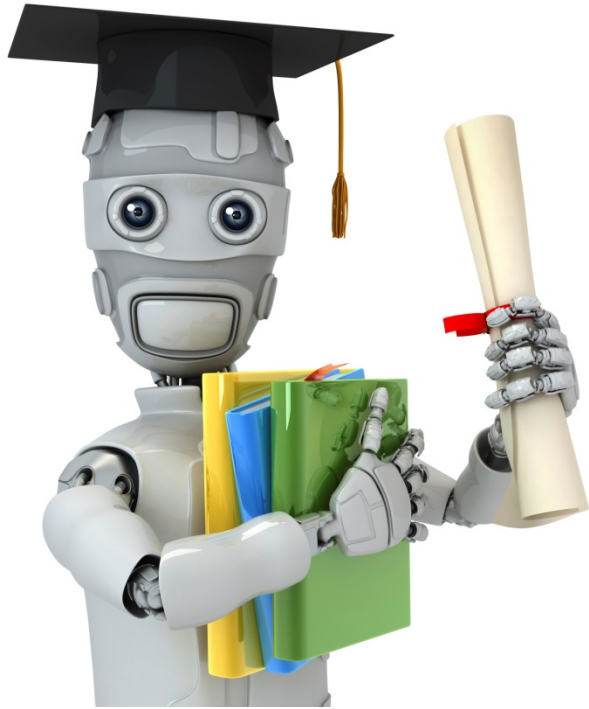
Given $\theta^{(1)}, \dots, \theta^{(n_u)}$,
can estimate $x^{(1)}, \dots, x^{(n_m)}$

已知用户偏好 $\Sigma \theta^{(j)}$ \rightarrow $\Sigma x^{(i)}$

Guess

$\Theta \rightarrow x \rightarrow \Theta \rightarrow x \rightarrow \Theta \rightarrow x \rightarrow \dots$

随机取 θ



Machine Learning

Recommender Systems

Collaborative
filtering algorithm

共同研究

Collaborative filtering optimization objective

$(i,j) : r(i,j)=1$ 即 x_0

迭代计算 $\theta, x \rightarrow x \rightarrow \theta \rightarrow x \dots$

→ Given $x^{(1)}, \dots, x^{(n_m)}$, estimate $\theta^{(1)}, \dots, \theta^{(n_u)}$:

$$\min_{\theta^{(1)}, \dots, \theta^{(n_u)}} \left[\frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (\theta_k^{(j)})^2 \right]$$

$x \in \mathbb{R}^n$
 $\theta \in \mathbb{R}^n$
 $x_1=1$

→ Given $\theta^{(1)}, \dots, \theta^{(n_u)}$, estimate $x^{(1)}, \dots, x^{(n_m)}$:

$$\min_{x^{(1)}, \dots, x^{(n_m)}} \left[\frac{1}{2} \sum_{i=1}^{n_m} \sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2 \right]$$

同时计算出 θ, x

Minimizing $x^{(1)}, \dots, x^{(n_m)}$ and $\theta^{(1)}, \dots, \theta^{(n_u)}$ simultaneously:

$$J(x^{(1)}, \dots, x^{(n_m)}, \theta^{(1)}, \dots, \theta^{(n_u)}) = \frac{1}{2} \sum_{(i,j):r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (\theta_k^{(j)})^2$$

$$\min_{\substack{x^{(1)}, \dots, x^{(n_m)} \\ \theta^{(1)}, \dots, \theta^{(n_u)}}} J(x^{(1)}, \dots, x^{(n_m)}, \theta^{(1)}, \dots, \theta^{(n_u)})$$

$\theta \rightarrow x \rightarrow \theta \rightarrow x \rightarrow \dots$

Andrew Ng

如何初始化: 现在是对所有特征 (θ, x) , 设初始偏置 $x_0=1$

并给可学习到的参数定个初值 ($x_1=1$)

Collaborative filtering algorithm

- 1. Initialize $x^{(1)}, \dots, x^{(n_m)}, \theta^{(1)}, \dots, \theta^{(n_u)}$ to small random values.
- 2. Minimize $J(x^{(1)}, \dots, x^{(n_m)}, \theta^{(1)}, \dots, \theta^{(n_u)})$ using gradient descent (or an advanced optimization algorithm). E.g. for every $j = 1, \dots, n_u, i = 1, \dots, n_m$:

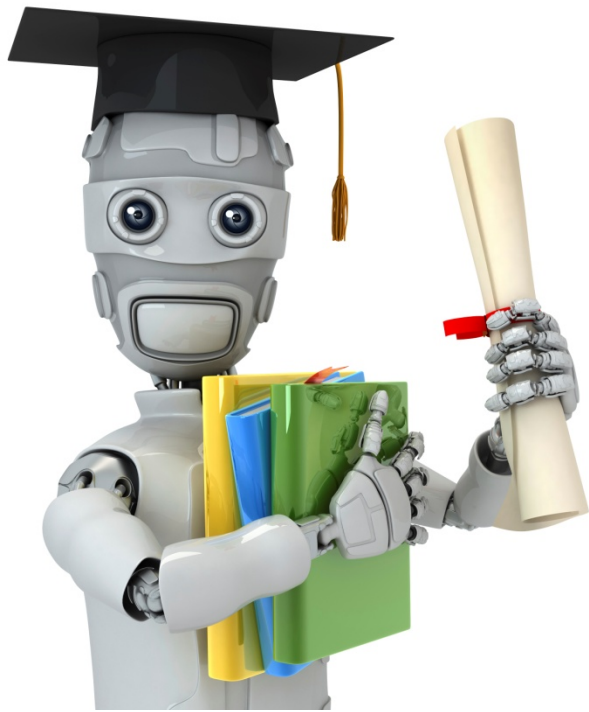
$$x_k^{(i)} := x_k^{(i)} - \alpha \left(\sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) \theta_k^{(j)} + \lambda x_k^{(i)} \right)$$

← 更新 $x_k^{(i)}$ 的值

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \left(\sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) x_k^{(i)} + \lambda \theta_k^{(j)} \right)$$

← $\frac{\partial J}{\partial \theta_k^{(j)}}$

3. For a user with parameters θ and a movie with (learned) features x , predict a star rating of $\theta^T x$.
- $(\theta^{(i)})^T (x^{(i)})$ ← 预测评分



Machine Learning

Recommender Systems

Vectorization:
Low rank matrix
factorization

低秩矩阵分解

Collaborative filtering

$n_m = 5$
 $n_u = 4$

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)
Love at last	5	5	0	0
Romance forever	5	?	?	0
Cute puppies of love	?	4	0	?
Nonstop car chases	0	0	5	4
Swords vs. karate	0	0	5	?

$Y = \begin{bmatrix} 5 & 5 & 0 & 0 \\ 5 & ? & ? & 0 \\ ? & 4 & 0 & ? \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix}$

$y^{(i,j)}$

Y matrix

Collaborative filtering

$$X \Theta^T \leftarrow$$

$$(\Theta^{(j)})^T (x^{(i)})$$

$$(i,j) \rightarrow$$

Predicted ratings:

$$Y = \begin{bmatrix} 5 & 5 & 0 & 0 \\ 5 & ? & ? & 0 \\ ? & 4 & 0 & ? \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} (\theta^{(1)})^T (x^{(1)}) & (\theta^{(2)})^T (x^{(1)}) & \dots & (\theta^{(n_u)})^T (x^{(1)}) \\ (\theta^{(1)})^T (x^{(2)}) & (\theta^{(2)})^T (x^{(2)}) & \dots & (\theta^{(n_u)})^T (x^{(2)}) \\ \vdots & \vdots & \vdots & \vdots \\ (\theta^{(1)})^T (x^{(n_m)}) & (\theta^{(2)})^T (x^{(n_m)}) & \dots & (\theta^{(n_u)})^T (x^{(n_m)}) \end{bmatrix}$$

$$X = \begin{bmatrix} -(x^{(1)})^T \\ -(x^{(2)})^T \\ \vdots \\ -(x^{(n_m)})^T \end{bmatrix}$$

$$\Theta = \begin{bmatrix} -(\theta^{(1)})^T \\ -(\theta^{(2)})^T \\ \vdots \\ -(\theta^{(n_u)})^T \end{bmatrix}$$

→ Low rank matrix factorization

低秩矩阵分解

Andrew Ng

$$x^{(i)} = \begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \\ \vdots \\ x_n^{(i)} \end{bmatrix}$$

$$(x^{(i)})^T = [x_1^{(i)} \ x_2^{(i)} \ \dots \ x_n^{(i)}]$$

$$\theta^{(j)} =$$

$$X = \begin{bmatrix} (x^{(1)})^T \\ (x^{(2)})^T \\ \vdots \\ (x^{(n_m)})^T \end{bmatrix} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \dots & x_n^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{(n_m)} & x_2^{(n_m)} & \dots & x_n^{(n_m)} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} (\theta^{(1)})^T \\ (\theta^{(2)})^T \\ \vdots \\ (\theta^{(n_u)})^T \end{bmatrix}$$

Finding related movies 找到与电影i相关的电影

For each product i , we learn a feature vector $x^{(i)} \in \mathbb{R}^n$.

→ $x_1 = \text{romance}$, $x_2 = \text{action}$, $x_3 = \text{comedy}$, $x_4 = \dots$

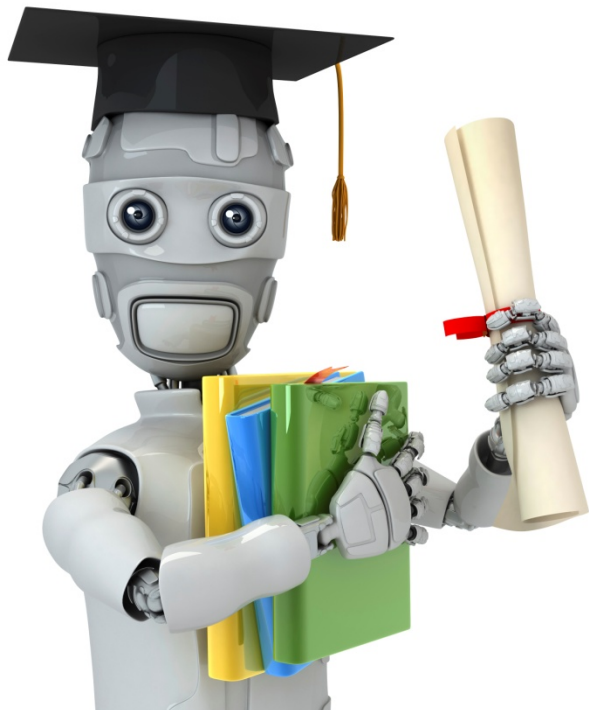
How to find movies j related to movie i ? 找到与电影i相关的电影j

Small $\|x^{(i)} - x^{(j)}\| \rightarrow$ movie j and i are "similar"

电影相似度 = 电影特征向量之差

5 most similar movies to movie i :

Find the 5 movies j with the smallest $\|x^{(i)} - x^{(j)}\|$.



Machine Learning

Recommender Systems

Implementational
detail: Mean
normalization

均值归一化

Users who have not rated any movies

某用户没有评分任何一部电影

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)	Eve (5)
Love at last	5	5	0	0	?
Romance forever	5	?	?	0	?
Cute puppies of love	?	4	0	?	?
Nonstop car chases	0	0	5	4	?
Swords vs. karate	0	0	5	?	?

$Y = \begin{bmatrix} 5 & 5 & 0 & 0 \\ 5 & ? & ? & 0 \\ ? & 4 & 0 & ? \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix}$

对于第5个用户，评分 $r(i,5)$ 未知 $\Rightarrow \sum_{i,j: r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2$ 无和项

更新 $\theta^{(5)}$ 的唯一项

$$\min_{x^{(1)}, \dots, x^{(n_m)}, \theta^{(1)}, \dots, \theta^{(n_u)}} \frac{1}{2} \sum_{(i,j): r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (\theta_k^{(j)})^2$$

特征 $n=2$

$\theta^{(5)} \in \mathbb{R}^2$

$\theta^{(5)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

不含 $\theta^{(5)}$ 项

$\frac{\lambda}{2} [\theta_1^{(5)^2} + \theta_2^{(5)^2}] \leftarrow$

$(\theta^{(5)})^T x^{(i)} = 0$

求min, 该项越小越好

无意义

Mean Normalization:

$$Y = \begin{bmatrix} 5 & 5 & 0 & 0 & ? \\ 5 & ? & ? & 0 & ? \\ ? & 4 & 0 & ? & ? \\ 0 & 0 & 5 & 4 & ? \\ 0 & 0 & 5 & 0 & ? \end{bmatrix}$$

Handwritten notes: Blue circles around 5s and 0s. Blue arrows point to the right column of values (2.5, 2.5, 2, ..., 1.25). A blue box highlights the bottom row.

$$\mu = \begin{bmatrix} 2.5 \\ 2.5 \\ 2 \\ 2.25 \\ 1.25 \end{bmatrix}$$

Handwritten note: A red arrow points from the first element of μ to the first element of the resulting matrix Y .

$$Y = \begin{bmatrix} 2.5 & 2.5 & -2.5 & -2.5 & ? \\ 2.5 & ? & ? & -2.5 & ? \\ ? & 2 & -2 & ? & ? \\ -2.25 & -2.25 & 2.75 & 1.75 & ? \\ -1.25 & -1.25 & 3.75 & -1.25 & ? \end{bmatrix}$$

Handwritten notes: Blue circles around 2.5s and -2.5s. A blue box highlights the bottom row. A red arrow points from the first element of μ to the first element of the resulting matrix Y .

For user j , on movie i predict:

$$\hat{y}_{ji} \rightarrow (\theta^{(i)})^T (x^{(j)}) + \mu_i$$

Handwritten note: A red box around the μ_i term.

Handwritten note: A blue arrow points from the matrix Y to the text "learn $\theta^{(i)}, x^{(j)}$ ".

User 5 (Eve):

$$\theta^{(5)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Handwritten note: A blue underline under the vector $\theta^{(5)}$.

$$(\theta^{(5)})^T (x^{(i)}) + \mu_i = \mu_i \neq 0$$

Handwritten note: A red box around the μ_i term.

Handwritten note: 这部电影的平均评分 + 偏差

Handwritten note: 这部电影的平均评分 \rightarrow 偏差