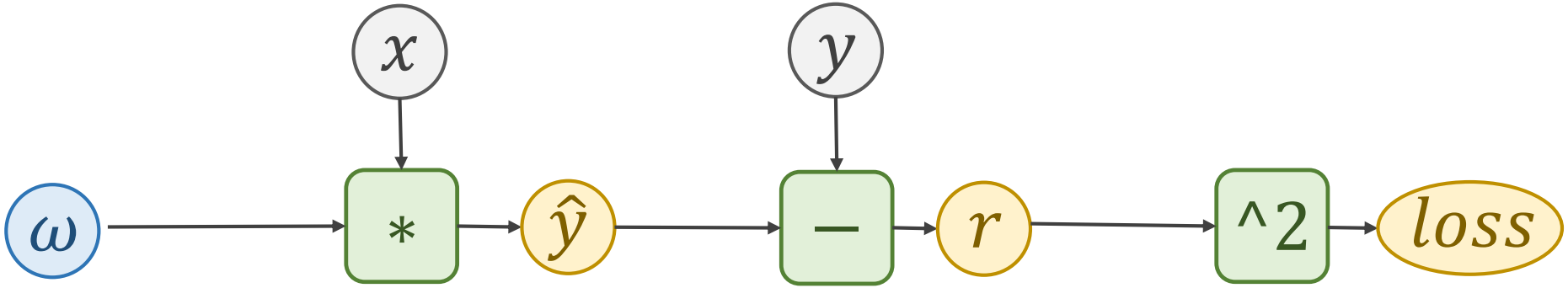
PyTorch Tutorial

05. Linear Regression with PyTorch

Revision

Linear Model	Loss Function
$\hat{y} = x * \omega$	$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$



Revision

```
print("predict (before training)",  4, forward(4).item())

for epoch in range(100):
    for x, y in zip(x_data, y_data):
        l = loss(x, y) forward
        l.backward() backward
        print('\tgrad:', x, y, w.grad.item())
        w.data = w.data - 0.01 * w.grad.data 更新

        w.grad.data.zero_() 手动清零

    print("progress:", epoch, l.item())

print("predict (after training)", 4, forward(4).item())
```

```
predict (before training) 4 4.0
        grad: 1.0 2.0 -2.0
        grad: 2.0 4.0 -7.840000152587891
        grad: 3.0 6.0 -16.228801727294922
progress: 0 7.315943717956543
        grad: 1.0 2.0 -1.478623867034912
        grad: 2.0 4.0 -5.796205520629883
        grad: 3.0 6.0 -11.998146057128906
progress: 1 3.9987640380859375
        grad: 1.0 2.0 -1.0931644439697266
        grad: 2.0 4.0 -4.285204887390137
        grad: 3.0 6.0 -8.870372772216797
progress: 2 2.1856532096862793
        grad: 1.0 2.0 -0.8081896305084229
        grad: 2.0 4.0 -3.1681032180786133
        grad: 3.0 6.0 -6.557973861694336
progress: 3 1.1946394443511963
        grad: 1.0 2.0 -0.5975041389465332
        grad: 2.0 4.0 -2.3422164916992188
        grad: 3.0 6.0 -4.848389625549316
progress: 4 0.6529689431190491
        grad: 1.0 2.0 -0.4417421817779541
        grad: 2.0 4.0 -1.7316293716430664
        grad: 3.0 6.0 -3.58447265625
progress: 5 0.3569012284278696
        grad: 1.0 2.0 -0.3265852928161621
        grad: 2.0 4.0 -1.2802143096923828
        grad: 3.0 6.0 -2.650045394897461
```

PyTorch Fashion

1 2
2 4
3 6

1 **Prepare dataset**

we shall talk about this later

2 **Design model using Class**

inherit from nn.Module

3 **Construct loss and optimizer**

using PyTorch API

4 **Training cycle**

forward, backward, update

Linear Regression – 1. Prepare dataset

In PyTorch, the computational graph is in mini-batch fashion, so X and Y are 3×1 Tensors.

第二章的线性模型，直接用到老师的数据

模型: $\hat{y} = wx + b$ $\begin{cases} x \in \mathbb{R} \\ \hat{y} \in \mathbb{R} \end{cases}$

原数据: $\begin{cases} (x_1, y_1) \\ (x_2, y_2) \\ (x_3, y_3) \end{cases}$

$$\begin{bmatrix} y_{pred}^{(1)} \\ y_{pred}^{(2)} \\ y_{pred}^{(3)} \end{bmatrix} = \omega \cdot \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} + b$$

```
import torch

x_data = torch.Tensor([[1.0], [2.0], [3.0]])
y_data = torch.Tensor([[2.0], [4.0], [6.0]])
```

1维向量

$\begin{bmatrix} [1.0] \\ [2.0] \\ [3.0] \end{bmatrix}$ $\begin{bmatrix} [2.0] \\ [4.0] \\ [6.0] \end{bmatrix}$

这里的乘，是矩阵元素对应位置相乘，不是矩阵相乘

计算所有 $\hat{y}$: $\begin{cases} \hat{y}_1 = wx_1 + b \\ \hat{y}_2 = wx_2 + b \\ \hat{y}_3 = wx_3 + b \end{cases}$

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = w \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b$$

$w \rightarrow \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}$ 自动广播为3×3的矩阵

$b \rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ 自动广播为3×1的矩阵

$\begin{bmatrix} 2.0 \\ 4.0 \\ 6.0 \end{bmatrix} = w \begin{bmatrix} 1.0 \\ 2.0 \\ 3.0 \end{bmatrix} + b$ → 矩阵

广播机制

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ 2 & 4 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

计算所有损失: $\begin{cases} loss_1 = (\hat{y}_1 - y_1)^2 \\ loss_2 = (\hat{y}_2 - y_2)^2 \\ loss_3 = (\hat{y}_3 - y_3)^3 \end{cases}$

$$\begin{bmatrix} loss_1 \\ loss_2 \\ loss_3 \end{bmatrix} = \left(\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right) ** 2$$

$loss = \frac{1}{N} \sum_{i}^{N} loss_i$ 最后值为标量

若为向量，则无法backward

Revision: Gradient Descent Algorithm

Derivative

$$\frac{\partial cost(\omega)}{\partial \omega} = \frac{\partial}{\partial \omega}\frac{1}{N}\sum_{n=1}^{N}(x_n \cdot \omega - y_n)^2$$

$$= \frac{1}{N}\sum_{n=1}^{N}\frac{\partial}{\partial \omega}(x_n \cdot \omega - y_n)^2$$

$$= \frac{1}{N}\sum_{n=1}^{N} 2 \cdot (x_n \cdot \omega - y_n)\frac{\partial (x_n \cdot \omega - y_n)}{\partial \omega}$$

$$= \frac{1}{N}\sum_{n=1}^{N} 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$

Gradient

$$\frac{\partial cost}{\partial \omega}$$

Update

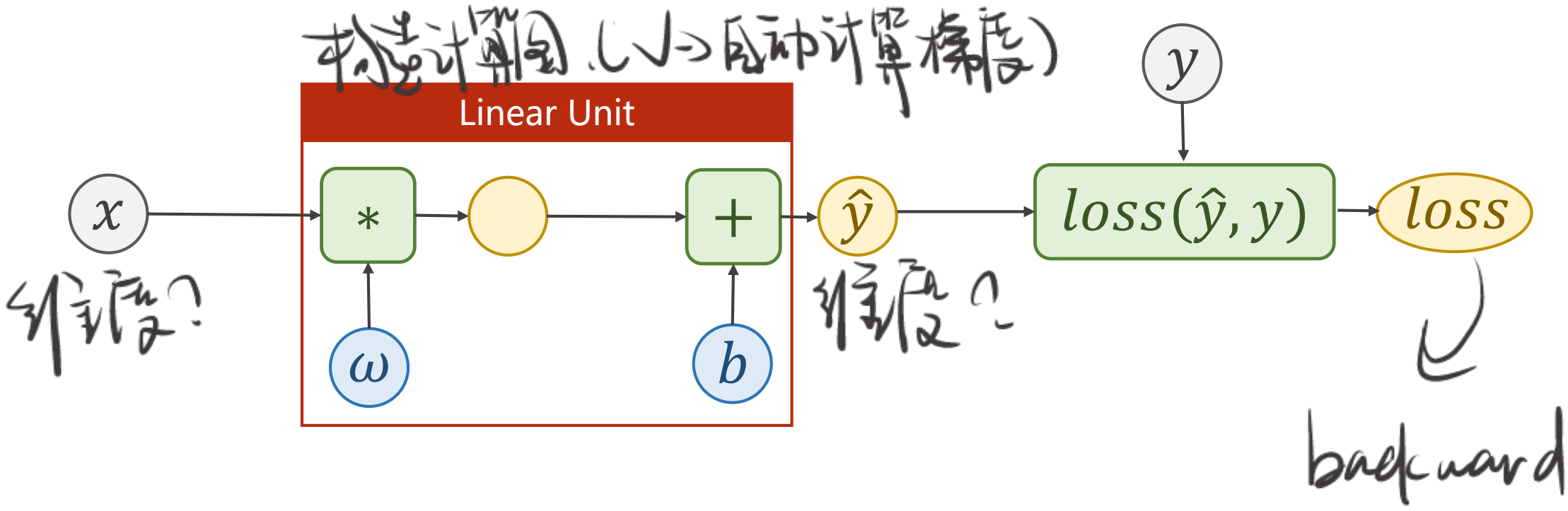
$$\omega = \omega - \alpha \frac{\partial cost}{\partial \omega}$$

Update

$$\omega = \omega - \alpha \frac{1}{N}\sum_{n=1}^{N} 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$

Linear Regression – 2. Design Model

Affine Model	Loss Function
$\hat{y} = x * \omega + b$	$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$



Linear Regression – 2. Design Model

```
class LinearModel(torch.nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = self.linear(x)
        return y_pred

model = LinearModel()
```

Our model class should be inherit from *nn.Module*, which is Base class for all neural network modules.

Linear Regression – 2. Design Model

```
class LinearModel(torch.nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = self.linear(x)
        return y_pred

model = LinearModel()
```

Member methods *__init__()* and *forward()* have to be implemented.

Linear Regression – 2. Design Model

```
class LinearModel(torch.nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = self.linear(x)
        return y_pred

model = LinearModel()
```

Just do it. :)

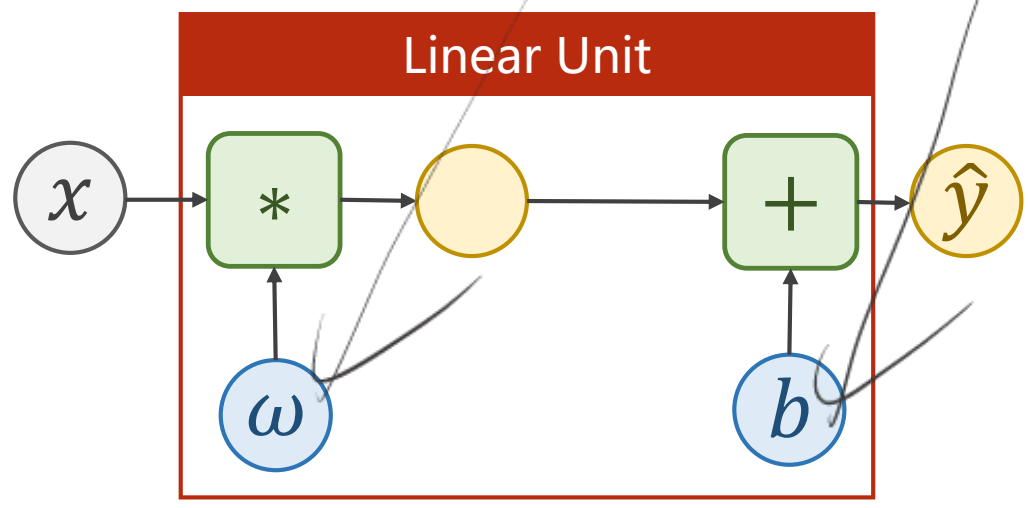
Linear Regression – 2. Design Model

```python
class LinearModel(torch.nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = self.linear(x)
        return y_pred

model = LinearModel()
```

Class *nn.Linear* contain two member **Tensors**: **weight** and **bias**.



Linear Regression – 2. Design Model

class **torch.nn.Linear**(*in_features*, *out_features*, *bias=True*) [source]

Applies a linear transformation to the incoming data: $y = Ax + b$

Parameters:

- **in_features** – size of each input sample
- **out_features** – size of each output sample
- **bias** – If set to False, the layer will not learn an additive bias. Default: `True`

Shape:

- Input: $(N, *, in\_features)$ where $*$ means any number of additional dimensions
- Output: $(N, *, out\_features)$ where all but the last dimension are the same shape as the input.

Variables:

- **weight** – the learnable weights of the module of shape *(out_features x in_features)*
- **bias** – the learnable bias of the module of shape *(out_features)*

Linear Regression – 2. Design Model

class **torch.nn.Linear**(*in_features, out_features, bias=True*) [source]

Applies a linear transformation to the incoming data: $y = Ax + b$

Parameters:

$$\begin{bmatrix} y_{pred}^{(1)} \\ y_{pred}^{(2)} \\ y_{pred}^{(3)} \end{bmatrix} = \omega \cdot \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} + b$$

Output

Input

每个输入/输出样本~特征

tive bias. Default: `True`

Shape:

- Input: $(N, *, in\_features)$ where $*$ means any number of additional dimensions
- Output: $(N, *, out\_features)$ where all but the last dimension are the same shape as the input.

Variables:

- **weight** – the learnable weights of the module of shape *(out_features x in_features)*
- **bias** – the learnable bias of the module of shape *(out_features)*

Linear Regression – 2. Design Model

```
class LinearModel(torch.nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = self.linear(x)
        return y_pred

model = LinearModel()
```

Class *nn.Linear* has implemented the magic method ***__call__()***, which enable the instance of the class can be called just like a function. Normally the ***forward()*** will be called.

Pythonic!!!

Linear Regression – 2. Design Model

```
class LinearModel(torch.nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = self.linear(x)
        return y_pred

model = LinearModel()
```

Create a instance of class **LinearModel**.

callable

$\hat{y} = model(x)$

Linear Regression – 3. Construct Loss and Optimizer

```
criterion = torch.nn.MSELoss(size_average=False)

optimizer = torch.optim.SGD(model.parameters(), lr=0.01)
```

class **torch.nn.MSELoss**(*size_average=True, reduce=True*) [source]

Creates a criterion that measures the mean squared error betwee
target *y*.

The loss can be described as:

$$\ell(x, y) = L = \{l_1, \dots, l_N\}^\top, \quad l_n = (x_n - y_n)^2,$$

where N is the batch size.

Also inherit from **nn.Module**.

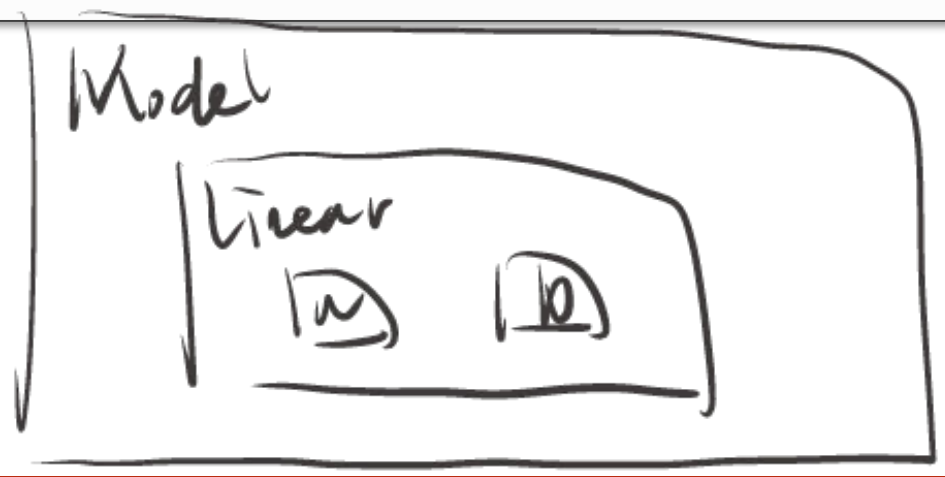
Linear Regression – 3. Construct Loss and Optimizer

```
criterion = torch.nn.MSELoss(size_average=False)

optimizer = torch.optim.SGD(model.parameters(), lr=0.01)
```

class **torch.optim.SGD**(*params, lr=<object object>, momentum=0, dampening=0, weight_decay=0, nesterov=False*) [source]

Implements stochastic gradient descent (optionally with momentum).



Linear Regression – 3. Construct Loss and Optimizer

```
criterion = torch.nn.MSELoss(size_average=False)

optimizer = torch.optim.SGD(model.parameters(), lr=0.01)
```

Parameters:

- **params** (*iterable*) – iterable of parameters to optimize or dicts defining parameter groups
- **lr** (*float*) – learning rate

Linear Regression – 4. Training Cycle

训练过程

```
for epoch in range(100):
    y_pred = model(x_data)
    loss = criterion(y_pred, y_data)
    print(epoch, loss)

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

前馈：用模型model(x)计算个

Forward: Predict

Linear Regression – 4. Training Cycle

```
for epoch in range(100):
    y_pred = model(x_data)
    loss = criterion(y_pred, y_data)
    print(epoch, loss)

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

Forward: Loss

用损失函数criterion计算损失loss

Linear Regression – 4. Training Cycle

```python
for epoch in range(100):
    y_pred = model(x_data)
    loss = criterion(y_pred, y_data)
    print(epoch, loss)

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

NOTICE:

The grad computed by *.backward()* will be **accumulated**.

So before backward, remember set the grad to **ZERO**!!!

Linear Regression – 4. Training Cycle

```
for epoch in range(100):
    y_pred = model(x_data)
    loss = criterion(y_pred, y_data)
    print(epoch, loss)

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

Backward: Autograd

Linear Regression – 4. Training Cycle

```
for epoch in range(100):
    y_pred = model(x_data)
    loss = criterion(y_pred, y_data)
    print(epoch, loss)

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

重新打重

```
for x, y in zip(x_data, y_data):
    ……
    w.data = w.data - 0.01 * w.grad.data
```

Update

步骤：

1. 准备数据集 torch.Tensor([[1.0],[2.0],[3.0]]) → 元组

2. 构建模型：

面向 class LinearModel(torch.nn.Module){ __init__(self) { super torch.nn.Linear(1,1) }

forward(self, x): self.linear(x)

model = LinearModel()

3. 创建损失对象：

损失函数 criterion = torch.nn.MSELoss(size_average=False)

4. 构造优化器用于更新权重

优化器 optimizer = torch.optim.SGD(model.parameters(), lr=0.01)

5. 训练

前馈 $\hat{y}$ = model(x)

loss = criterion($\hat{y}$, y)

梯度清零，pytorch会动态计算图 optimizer.zero_grad()

反馈：调用backward 计算梯度grad：loss.backward()

更新权重：optimizer.step()

Linear Regression – Test Model

```
# Output weight and bias
print('w = ', model.linear.weight.item())
print('b = ', model.linear.bias.item())

# Test Model
x_test = torch.Tensor([[4.0]])
y_test = model(x_test)
print('y_pred = ', y_test.data)
```

```
86 0.3036523759365082
87 0.2992883026599884
88 0.29498720169067383
89 0.2907477021217346
90 0.28656935691833496
91 0.28245046734809875
92 0.27839142084121704
93 0.27439042925834656
94 0.2704470157623291
95 0.2665606141090393
96 0.262729674577713
97 0.25895369052886963
98 0.2552322745323181
99 0.2515641450881958
w = 1.666100263595581
b = 0.7590328454971313
y_pred = tensor([[ 7.4234]])
```

100 Iterations

```
986 3.594939812501252e-07
987 3.5411068211033125e-07
988 3.4917979974125046e-07
989 3.4428359185767476e-07
990 3.392528924450744e-07
991 3.3442694302721065e-07
992 3.294019847999152e-07
993 3.247135396122758e-07
994 3.199925231456291e-07
995 3.1540417921860353e-07
996 3.1097857799977646e-07
997 3.0668098816022393e-07
998 3.020934400410624e-07
999 2.977626536448952e-07
w = 1.9996366500854492
b = 0.0008257834706455469
y_pred = tensor([[ 7.9994]])
```

1000 Iterations

$$\begin{cases} \hat{y}_1 = w x_1 + b \\ \hat{y}_2 = w x_2 + b \\ \hat{y}_3 = w x_3 + b \end{cases}$$

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = w \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b$$

$$\begin{bmatrix} 2.0 \\ 4.0 \\ 6.0 \end{bmatrix} = w \begin{bmatrix} 1.0 \\ 2.0 \\ 3.0 \end{bmatrix} + b$$

广播机制

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

Linear Regression

```python
import torch

x_data = torch.Tensor([[1.0], [2.0], [3.0]])
y_data = torch.Tensor([[2.0], [4.0], [6.0]])

class LinearModel(torch.nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = self.linear(x)
        return y_pred
model = LinearModel()

criterion = torch.nn.MSELoss(size_average=False)
optimizer = torch.optim.SGD(model.parameters(), lr=0.01)

for epoch in range(1000):
    y_pred = model(x_data)
    loss = criterion(y_pred, y_data)
    print(epoch, loss.item())

    optimizer.zero_grad()
    loss.backward()
    optimizer.step()

print('w = ', model.linear.weight.item())
print('b = ', model.linear.bias.item())

x_test = torch.Tensor([[4.0]])
y_test = model(x_test)
print('y_pred = ', y_test.data)
```

1. **Prepare dataset**
   we shall talk about this later
2. **Design model using Class**
   inherit from nn.Module
3. **Construct loss and optimizer**
   using PyTorch API
4. **Training cycle**
   forward, backward, update

Exercise 5-1: Try Different Optimizer in Linear Regression

- torch.optim.Adagrad
- torch.optim.Adam
- torch.optim.Adamax
- torch.optim.ASGD
- torch.optim.LBFGS
- torch.optim.RMSprop
- torch.optim.Rprop
- torch.optim.SGD

Exercise 5-2: Read more example from official tutorial

Table of Contents

https://pytorch.org/tutorials/beginner/pytorch_with_examples.html

PyTorch Tutorial

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