Lec 1

1. w.l.o.g == 不失一般性

2. 递归数列求通项式

1. 将F_n设为X_2, 再第一次的设为X , -2项设为常数项，之后列方程解出来特征根ϕ 和 ψ
2. 将特征根ϕ 和 ψ 以及 F_0, F_1 带入公式：$F_n = a\phi^n + b\psi^n$ 得到**a, b** 。
3. 将**a, b** 带回公式，得到F_n 的通项公式。

3. Logical connectives:

- Logical connectives:
  - *Negation*
  - *Conjunction*
  - *Disjunction*
  - *Exclusive or*
  - *Implication*
  - *Biconditional*

1. exclusive or 异或

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

2. implication

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \to q$:

- if p then q
- p implies q
- p is sufficient for q
- q is necessary for p
- q follows from p
- q unless $\neg p$
- p only if q

$\neg q \to \neg p$ is *equivalent* to $p \to q$

3. Biconditional

- p is necessary and sufficient for q
- if p then q, and conversely
- p iff q

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Lec 2

名词:

- tautology 永真式： always True
- contradiction 永假式：always False
- contingency 偶然式:

P	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

偶然 永真 永假

1. Equivalent Propositions 等价命题

1. 对比真值表

2. Lows

- *Identity laws*
  - $p \wedge T \equiv p$
  - $p \vee F \equiv p$

- *Domination laws*
  - $p \vee T \equiv T$
  - $p \wedge F \equiv F$

- *Idempotent laws*
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$

- *Double negation laws*
  - $\neg(\neg p) \equiv p$

- *Commutative laws*
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$

- *Associative laws*
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

- Distributive laws
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- De Morgan's laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Others
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$

    Absorption laws
  - $p \vee \neg p \equiv T$
  - $p \wedge \neg p \equiv F$

    Negation laws
  - $p \rightarrow q \equiv \neg p \vee q$

3. 证明一个命题的方法：

1. 公式迭代证明
2. 真值表证明
3. 反证法

4. 命题逻辑的局限性：

5. propositional logic 命题逻辑 || Predicate Logic 谓词逻辑

1. Predicate Logic 谓词逻辑 : P(x) depend on x！！ P(x)不是命题但是在确定x的情况下是命题。
2. truth set

6. quantifiers

1. depends on domain
2. $\forall x\ P(x) \vee Q(x)$ means $(\forall x\ P(x)) \vee Q(x)$ rather than $\forall x\ (P(x) \vee Q(x))$
3.

Lec 3

1. review:

Important Logical Equivalences

- *Identity laws*
  - $p \wedge T \equiv p$
  - $p \vee F \equiv p$
- *Domination laws*
  - $p \vee T \equiv T$
  - $p \wedge F \equiv F$
- *Idempotent laws*
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- *Double negation laws*
  - $\neg(\neg p) \equiv p$
- *Commutative laws*
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- *Associative laws*
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- *Distributive laws*
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- *De Morgan's laws*
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- *Others*
  - $p \vee (p \wedge q) \equiv p$ — *Absorption laws*
  - $p \wedge (p \vee q) \equiv p$
  - $p \vee \neg p \equiv T$ — *Negation laws*
  - $p \wedge \neg p \equiv F$
  - $p \to q \equiv \neg p \vee q$ — *Useful*

1. Order of Quantifiers

1. **Example**: $\forall x \exists y\ L(x,y) \not\equiv \exists y \forall x\ L(x,y)$

   ◇ $L(x,y)$ denotes "x loves y"

   ◇ $\forall x \exists y\ L(x,y)$: Everybody loves somebody.

   ◇ $\exists y \forall x\ L(x,y)$: There is someone who is loved by everyone.

2. There is **exactly** one person whom everybody loves.
   $\exists y(\forall x\ L(x,\ y) \land \forall z(\ L(x,\ z) \to z = y))$
3. Rules of Inference for Propositional Logic

modus ponens (*law of detachment*) 肯定前件式

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

corresponding tautology:
$(p \land (p \to q)) \to q$

- **modus tollens** 否定后件式

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

corresponding tautology:
$(\neg q \land (p \to q)) \to \neg p$

- **hypothetical syllogism** 假言三段论

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

corresponding tautology:
$((p \to q) \land (q \to r)) \to (p \to r)$

- **disjunctive syllogism** 选言三段论

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

corresponding tautology:
$(\neg p \wedge (p \vee q)) \rightarrow q$

- **Addition**

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}$$

corresponding tautology:
$p \rightarrow (p \vee q)$

- **Simplication**

$$\begin{array}{c} p \wedge q \\ \hline \therefore q \end{array}$$

corresponding tautology:
$(p \wedge q) \rightarrow p$

- **Conjunction**

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

corresponding tautology:
$((p) \wedge (q)) \rightarrow (p \wedge q)$

- **Resolution**

$$\begin{array}{c} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

corresponding tautology:
$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

- **Universal Instantiation** (UI)

$$\frac{\forall x P(x)}{\therefore P(c)}$$

- **Universal Generalization** (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

- **Existential Instantiation** (EI)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

- **Existential Generalization** (EG)

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$

Lec 4

1. Sets

- Natural numbers:
  - $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$
- Integers:
  - $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
- Positive integers:
  - $\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$
- Rational numbers:
  - $\mathbf{Q} = \{\frac{p}{q} \mid p \in \mathbf{Z}, q \in \mathbf{Z}, q \neq 0\}$
- Real numbers:
  - $\mathbf{R}$
- Complex numbers:
  - $\mathbf{C}$

2. Cardinality (基数；集的势)

$A = \{1, 2, 3, \ldots, 20\}$ $(|A| = 20)$

1. $B = \{1, 2, 3, \ldots\}$ (infinite)

   $|\emptyset| = 0$

3. Power Set

Given a set S, the power set of S is the set of all subsets of the set S, denoted by P(S).

Examples:

0 ◇ $\emptyset$ $\{\emptyset\}$ $1 = 2^0$

1 ◇ $\{1\}$ $\{\emptyset, \{1\}\}$ $2 = 2^1$

2 ◇ $\{1, 2\}$ $\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ $4 = 2^2$

3 ◇ $\{1, 2, 3\}$ $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ $8 = 2^3$

4. Cartesian Product : 笛卡尔积

1. Let A and B be sets. The *Cartesian product of A and B*, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

   Example:

$$A = \{1, 2\}, \ B = \{a, b, c\}$$

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

2. ■ $A \times B \neq B \times A$

   ■ $|A \times B| = |A| \times |B|$

3. $A \cap B = \emptyset$

   $\rightarrow A - b = A$

   $\rightarrow A \cap \overline{B} = A$

5. Function

1. injective --- one-to-one --- 单射 --- image只被指向一次

   $g(x) = g(y) \quad iff \quad x = y$

   $x \neq y \rightarrow f(x) \neq f(y)$

   surjective --- Onto --- 满射 --- image 全部用到

Bijective --- both one-to-one and onto --- 一一对应

Suppose that $f : A \to B$.

To show that f is *injective*	Show that if $f(x) = f(y)$ for all $x, y \in A$, then $x = y$
To show that f is not *injective*	Find specific element $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$
To show that f is *surjective*	Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$
To show that f is not *surjective*	Find a specific element $y \in B$ such that $f(x) \neq y$ for all $x \in A$

2. inverse Function 必须是bijection， 也就是必须是onto和one-to-one。