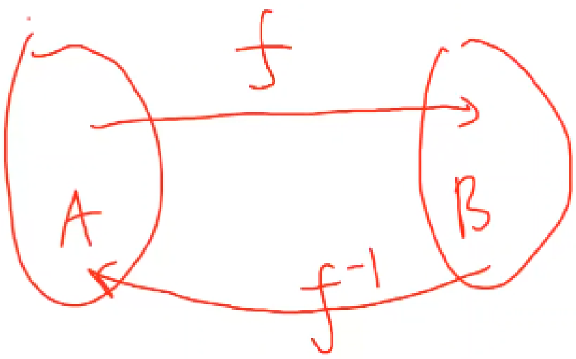
Lec 5

1. Composition of function

- Suppose that f is a bijection from A to B. Then $f \circ f^{-1} = I_B$ and $f^{-1} \circ f = I_A$. Since

$$(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$$
$$(f \circ f^{-1})(b) = f(f^{-1}(b)) = f(a) = b,$$

where I_A, I_B denote the *identity functions* on the sets A and B, respectively.



$\underline{f \circ f^{-1}}(b) = f(f^{-1}(b))$
$= f(a) = b$ $\quad 1_B$

$f^{-1} \circ f$

Sequence

1. Words:

   arithmetic progression 等差数列

   $S = \sum_{j=0}^{n}(a + jd) = (n+1)a + d\frac{n(n+1)}{2}$

   Geometric Progression 等比数列

   $S = \sum_{j=0}^{n}(ar^j) = a\sum_{j=0}^{n} r^j = a\frac{r^{n+1}-1}{r-1}$

2. Summations:

$$\sum_{j=1}^{n}(ax_j + by_j) = a\sum_{j=1}^{n} x_j + b\sum_{j=1}^{n} y_j$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_i b_j = \sum_{i=1}^{m} a_i \sum_{j=1}^{n} b_j$$

3. infinite Series

$$\sum_{k=0}^{\infty} x^k = \lim_{n\to\infty} \sum_{k=0}^{n} x^k = \lim_{n\to\infty} \frac{x^{n+1}-1}{x-1} = \frac{1}{1-x}$$

4.

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ ✓ mathematical induction 数学归纳法

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ ✓ "telescoping" $\sum_{k=1}^{n} \left[k^4-(k-1)^4\right] = n^4$

$$\boxed{\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}} \checkmark = 4\sum_{k=1}^{n} k^3 - 6\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1$$

$$4\sum_{k=1}^{n} k^3 = n^4 + 6\sum k^2 - 4\sum k + \sum 1$$

$$= n^4 + n(n+1)(2n+1) - 2n(n+1) + n$$

$$= n^2(n+1)^2 \qquad \frac{n^2(n+1)^2}{4} = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum_{k=1}^{n} k\right.$$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ ✓ mathematical induction 数学归纳法

$\boxed{\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}}$ "telescoping" $\sum_{k=1}^{n} \left[k^3-(k-1)^3\right] = n^3$

$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

$n^3-(n-1)^3$
$(n-1)^3-(n-2)^3$
$\vdots$
2^3-1^3
1^3-0^3

$$\sum_{k=1}^{n} \left[k^3-(k-1)^3\right] = \sum_{k=1}^{n} (3k^2-3k+1)$$

$$= 3\sum_{k=1}^{n} k^2 - 3\sum_{k=1}^{n} k + \sum 1 = n^3$$

$$3\sum_{k=1}^{n} k^2 = n^3 + 3\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = n^3 + 3\frac{n(n+1)}{2} - n$$

$$\| \qquad \frac{n(n+1)(2n+1)}{6} \qquad = \frac{n(n+1)(2n+1)}{2}$$

5. **Every subset of R is countable.**

6. one-to-one correspondence function $\neq$ one-to-one function

7.