Lec 18 Graph

Graph

Definition:

A graph $G = (V, E)$ consists of a nonempty set ***V*** of vertices (or nodes) and a set ***E*** of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to be incident to (or connect its endpoints.

Simple graph vs. multi_graph pseudo_graph

A graph in which at most one edge joins each pair of distinct vertices (vs. multiple edges) and no edge joins a vertex to itself (= loop).

Complete graph Kn

A graph with n vertices that ha s an edge between each pair of vertices

Graph Models

Influence graphs

directed graphs where there is an edge from one person to another if the first person can influence the second one

Collaboration graphs

undirected graphs where two people are connected if they collaborate in some way

Undirected Graphs

Definition:

Two vertices u; v in an undirected graph G are called adjacent (or neighbors) in G if there is an edge e between u and v. Such an edge e is called incident with the vertices u and v and e is said to connect u and v.

Definition:

The set of all neighbors of a vertex v of G = (V; E), denoted by N(v), is called the neightborhood of v. If A is a subset of V, we denote by N(A) the set of all vertices in G that are adjacent to at least one vertex in A.

Definition:

The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes two to the degree of that vertex. The ei degree of the vertex v is denoted by deg(v).

Theorem 1 (Handshaking Theorem)

If $G = (V, E)$ is an undirected graph with m edges, then

$$2m = \sum_{v \in V} deg(v)$$

Proof :

Each edge contributes 2 to $\sum deg$.

$\Longrightarrow 2m = \sum_{v \in V} deg(v)$.

Theorem 2

An undirected graph has an even number of vertices of odd degree.

Proof :

Let V_1 be the vertices of ***even*** degrees and V_2 be the vertices of ***odd*** degree.

$$2m = \sum_{v \in V} \deg(v) = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v)$$

$$\begin{gathered} 2m \ is \ even \ number \\ Then \sum_{v \in V_1} deg(v) \ is \ an \ even \ number \\ and \ deg(v) \ in \ V_1 \ is \ even \ \Rightarrow |V_1| \ is \ even \\ So \sum_{v \in V_2} deg(v) \ must \ be \ a \ even \ number \ because \ sum \ is \ even \\ But \ deg(v) \ in \sum_{v \in V_2} deg(v) \ is \ odd \\ So \ |V_2| \ is \ even \ number \\ \Longrightarrow \ number \ of \ edge \ is \ |V| = \ |V_1| + |V_2| \ is \ even \end{gathered}$$

Directed Graphs

Definition:

An directed graph G = (V; E) consists of V, a nonempty set of vertices, and E, a set of directed edges. Each edge is an ordered pair of vertices. The directed edge (u; v) is said to start at u and end at v.

Definition:

Let (u; v) be an edge in G. Then u is the initial vertex of the edge and is adjacent to v and v is the terminal vertex of this edge and is adjacent from u. The initial and terminal vertices of a loop are the same.

Definition:

The in-degree of a vertex v, denoted by $deg^-(v)$, is the number of edges which terminate at v. The out-degree of v, denoted by$deg^+(v)$, is the number of edges with v as their initial vertex. Note that a loop at a vertex contributes 1 to both the in-degree and the out-degree of the vertex.

Theorem 3

Let G = (V; E) be a graph with directed edges. Then

$$|E| = \sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v)$$

Complete Graphs

N-dimensional Hypercube

An n-dimensional hypercube, or n-cube, Q_n is a graph with 2^n vertices representing all bit strings of length n, where there is an edge between two vertices that differ in exactly one bit position.

$|Q_n| = 2^n \qquad 2m = \sum_{v \in V} deg(v) = n$

Bipartite Graphs

Definition:

A simple graph G is bipartite if V can be partitioned into two disjoint subsets V1 and V2 such that every edge connects a vertex in V1 and a vertex in V2.