Lecture 12 Counting

Product Rule

Sum Rule

Pigeonhole Principle

Theorem

If there are k+1 objects and k bins,then there is at least one bin with two or more objects.

Generalized

If N objects are placed into k bins, then there is at least one bin containing at least $\lceil N/k \rceil$ objects.

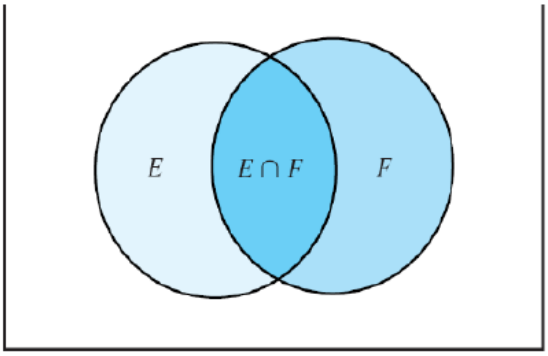
Inclusion-Exclusion Principle

Inclusion-Exclusion Principle: uses a sum rule and then corrects for the overlapping elements.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

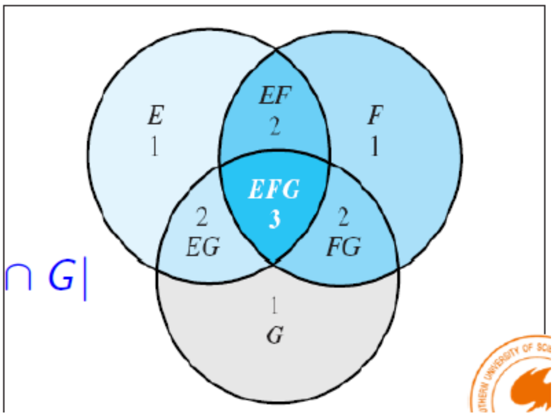
- Two sets

$$|E \cup F| = |E| + |F| - |E \cap F|$$



Three sets

$$\begin{aligned} &|E \cup F \cup G| \\ &= |E| + |F| + |G| \\ &\quad -|E \cap F| - |E \cap G| - |F \cap G| \\ &\quad +|E \cap F \cap G| \end{aligned}$$



推论:

$$|\cup_{i=1}^{n} E_i| = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq n} |E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}|$$

Proof by induction

Base case $(n = 2)$

$$|E \cup F| = |E| + |F| - |E \cap F|$$

Inductive Hypothesis

$$|\cup_{i=1}^{n-1} E_i| = \sum_{k=1}^{n-1} (-1)^{k+1} \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq n-1} |E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}|$$