Lec 6

Review:

A set that is **either finite** or **has the same cardinality as the set of positive integers $\mathbf{Z}^+$** is called *countable*.
A set that is **not countable** is called *uncountable*.

1. Big O

   |f(n)| <= C*|g(n)| if n > x_0

   ** |x|-|y| <= |x+y| <= |x|+|y|

- $1 + 2 + \cdots + n = O(n^2)$

  $n! = O(n^n)$

  $\log n! = O(n \log n)$

  $\log_a n = O(n)$ for an integer $a \geq 2$

  $n^a = O(n^b)$ for integers $a \leq b$

  $n^a = O(2^n)$ for an integer a

If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$ then
$(f_1 + f_2)(x) = O(\max(|g_1(x)|, |g_2(x)|))$

If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$ then
$(f_1 f_2)(x) = O(g_1(x)g_2(x))$

$f_1(n) = (1.5)^n$

$f_2(n) = 8n^3 + 17n^2 + 111$

$f_3(n) = (\log n)^2$

$f_4(n) = 2^n$

$f_5(n) = \log(\log n)$

$f_6(n) = n^2(\log n)^3$

$f_7(n) = 2^n(n^2 + 1)$

$f_8(n) = n^3 + n(\log n)^2$

$f_9(n) = 100000$

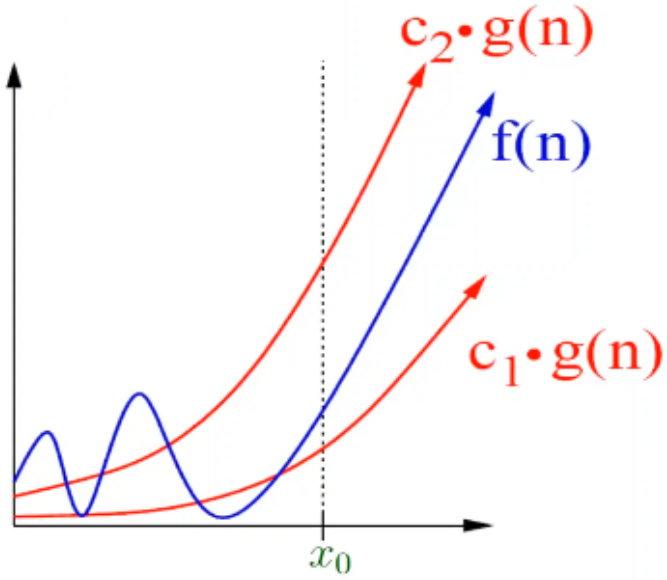
$f_{10}(n) = n!$

2. Big_Omega

   |f(n)| >= C |g(n)| , Whenever n > X_0

3. Big-Theta

Two functions $f(n)$, $g(n)$ have the same order growth if $f(n) = O(g(n))$ and $g(n) = O(f(n))$. In this case, we say that $f(n) = \Theta(g(n))$, which is the same as $g(n) = \Theta(f(n))$.



4. Dealing with Hard Problems

   for NP问题

5. The Class P

6. Certificates and Verifying Certificates

7. The Class NP

Definition The class NP consists of all decision problems such that, for each yes-input, there exists a *certificate* which allows one to verify in polynomial time that the input is indeed a yes-input.

NP -- nondeterministic polynomial-time

8.