Lec 15

Properties of Relations

1. Reflexive Relation

1. A relation R on a set A is called reflexive if (a,a) ∈ R for every element $a \in A$
2. `A relation R is reflexive` if and only if MR(Matrix of R) has 1 in every position on its main diagonal.

2. Irreflexive Relation

1. A relation R on a set A is called irreflexive if (a,a) $\notin$ R for every element a ∈ A.
2. Irreflexive $\neq$ not reflexive
3. A relation R is irreflexive if and only if MR has 0 in **every position on its main diagonal**.

3. Symmetric Relation

1. A relation R on a set A is called symmetric if (b, a) ∈ R whenever (a, b) ∈ R for all a, b ∈ A.
2. A relation R is symmetric if and only if MR is **symmetric**.

   $MR^T = MR$

4. Antisymmetric Relation

1. A relation R on a set A is called antisymmetric if (b, a) ∈ R and (a, b) ∈ R implies a = b for all a, b ∈ A.
2. Antisymmetric $\neq$ not Symmetric
3. if have (b, a) ∈ R and (a, b) ∈ R. a must equal to b;

5. Transitive Relation

1. A relation R on a set A is called transitive if (a, b) ∈ R and (b, c) ∈ R implies (a, c) ∈ R for all a, b, c ∈ A.

Combining Relations

1. **Definition:** Let A and B be two sets. A binary relation from A to B is a subset of a Cartesian product A × B.
2. **Definition:** Let R be a relation from a set A to a set B and S be a relation from B to C. The composite of R and S is the relation consisting of the ordered pairs (a; c) where a ∈ A and c ∈ C and for which there is a b ∈ B such that (a; b) ∈ R and (b, c) ∈ S. We denote the composite of **R and S** by S ∘ R.
3. Combining Relation same as Matrix multiplication.
4. **Power Relation** :

   Let R be a relation on A. The powers R^n , for n = 1, 2, 3, ... , is defined inductively by

   $$R^1 = R \quad and \quad R^{n+1} = R^n \circ R$$

   Theorem : The relation R on a set A is transitive if and only if Rn ∘ R for n = 1, 2, 3, ...

Proof:

if: $R^n \subseteq R$ for n = 1,2,3... $\rightarrow$ R is transitive

Let n = 2 , $R^2 \subseteq R$ $(a,b),(b,c) \in R \quad R^2 = R \circ R$

$(a,c) \in R^2 \subseteq R \Rightarrow R \quad is \quad transitive$

only if :

R is transitive $\rightarrow \quad R^n \subseteq R \quad for \quad n = 1,2,3\ldots$

Mathematical Induction 1: base case: n = 1, $R^1 \subseteq R$

2: i.h. n = k, $R^k \subseteq R$

3: i.s. n = k+1 $R^{k+1} = R^k \circ R$

$(a,b) \in R^{k+1} \quad (x,b) \in R^k \subseteq R \quad (a,b) \in R$

$R^{k+1} \subseteq R$

4: by .....

Number of Reflexive Relations

1. The number of reflexive relations on a set A with |A| = n is $2^{n(n-1)}$.

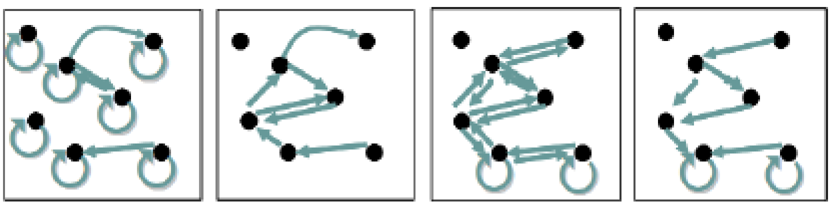
n-ary Relations

Relational Databases ??★★

1. primary key
2. composite key
3. E-R Diagram
4. Selection Operator
5. Projection Operators
6. Join Operator

Some special ways to represent binary relations:

— with a *zero-one matrix*
— with a *directed graph*



reflexive irreflexive symmetric antisymmetric