Examples of Grade Contracts

1.

This is a standard insurance contract. Let there be two states of the world: $s \in \{0, 1\}$, and two time periods: $t \in \{0, 1\}$. The state of the world is unknown at $t = 0$ and is revealed at $t = 1$. An insurance contract:

- Buyer pays seller x at $t = 0$
- At $t = 1$, seller pays buyer y if $s = 1$ and 0 if $s = 0$

Example A pays B 5 points. After the exam scores are announced, B pays A 20 points if A's score is ≤ 70 (and pays 0 if A's score is > 70)

2.

An insurance contract can be written on a reference entity that the buyer doesn't own. Real-life example: credit default swaps (CDS)

Example A and B forms a contract about C's score. A pays B 5 points. After the exam scores are announced, B pays A 20 points if C's score is ≤ 70 (and pays 0 if C's score is > 70)[1]

[1] To execute this type of contract, A and B must have C's approval that his or her grade be revealed to them after the exam.

3.

Standard betting contracts.

Example B pays A 10 points if the class average score is ≥ 75. Otherwise, A pays B 10 points.

Example B pays A 10 points if the class standard deviation of scores is ≥ 15. Otherwise, A pays B 10 points.

4.

This is a type of swap contracts. Suppose A receives x_0 if $s = 0$ and x_1 if $s = 1$ ($x_0 < x_1$). Then at $t = 0$, A and B can enter into a swap such that at $t = 1$,

- B pays A x ($x_0 < x < x_1$) regardless of the state of the world
- A pays B x_0 if $s = 0$ and x_1 if $s = 1$

Essentially, A sends his/her return to B in exchange for a fixed return.

Example A and B agree before the exam that, after the exam scores are announced, A will give B an amount of points equal to his/her score and B will give A 80 points[2]. This is an extreme example in which A is guaranteed a score of 80 after the contract is executed.

Example As a less extreme example, a contract can be written such that A will give B an amount of points equal to his/her score and B will give A 90 points if A's score is ≥ 90, 60 points if A's score is ≤ 60, and an amount of points equal to A's score if A's score is $\in (60, 90)$. This guarantees that A's score will be $\in [60, 90]$ after the contract is executed.

[2] For example, if A's score is 60, then B pays A 20 points. If A's score is 90, then A pays B 10 points.

5.

An option contract gives the holder the *right* (but not the obligation) to execute a transaction on or before a certain date (the expiration date).

Example A pays B 5 points before the exam for the *right* to bet with B on the class average score. The expiration date is set at after the exam, but before the exam scores are announced[3].

Example A pays B 10 points before the exam for the *right* to swap his/her score with B. The expiration date is set at after the exam scores are announced.

6.

Combination of different contracts.

Example A forms betting contracts respectively with B and C. A's contract with B: A pays B 10 points if the class average score is ≥ 70. Otherwise, B pays A 10 points; A's contract with C: C pays A 10 points if the class average score is ≥ 60. Otherwise, A pays C 10 points. Then,

$$\text{A's payoff} = \begin{cases} 0 & \text{if class average} \geq 70 \\ 20 & \text{if class average} \in (60, 70) \\ 0 & \text{if class average} < 60 \end{cases}$$

[3] For example, A can buy the following option from B: "This option gives A the *right* to enter into the following bet with B before the exam scores are announced: if the class average score is 75, B will pay A 10 points. Otherwise, A will pay B 10 points."

Then, after taking the exam (but before the exam scores are announced), A can decide whether to actually do the bet with B ("exercise the option").