Homework 5 (54 Points)

Solutions

Problem 1 (5 Points)

Read the following two articles by Alesina, et. al.: "Why women should pay less tax," and "Gender-Based Taxation and the Division of Family Chores."

1. The authors propose to reduce income taxes on women and increase income taxes on men. They claim that it is possible to raise taxes on men by less than the reduction on women while also holding tax revenue constant. How is this possible?

   Female labor supply is more elastic.

2. Other than being "optimal" according to the Ramsey principle, what are other benefits of gender-based taxation according to the authors? Briefly summarize their arguments.

   reduce discrimination in hiring, change the traditional division of labour within the family in the long run

3. Do you think gender-based taxation is a good idea? Why or why not?

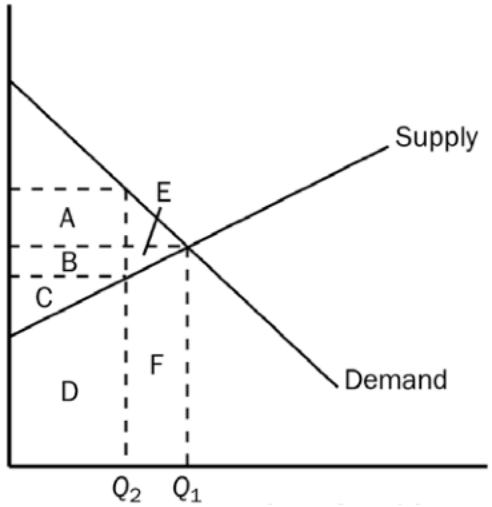
Problem 2 (5 Points)

Read the article "The Incidence of EITC." What are some of the negative unintended consequences of EITC?

There is evidence that EITC increases the labor supply of single mothers, but decreases the labor supply of married women, as credits paid to men allow their wives to leave work. There is little evidence that EITC increases the labor supply of men.

Problem 3 (10 Points)

The following graph illustrates the effect of taxing the market of a certain good. The equilibrium quantity is Q_1 before tax and Q_2 after tax.



1. Identify the following areas both before and after the imposition of the tax[1]: (6 Points)

   (a) total spending by consumers

   `Before tax: B+C+D+E+F; After tax: A+B+C+D`

   (b) total revenue for producers

   `Before tax: B+C+D+E+F; After tax: C+D`

   (c) government tax revenue.

   `Before tax: 0; After tax: A+B`

2. Does the price received by producers rise or fall? Can you tell whether total receipts for producers rise or fall? Explain. (2 Points)

   `Fall. Fall.`

3. Does the price paid by consumers rise or fall? Can you tell whether total spending by consumers rises or falls? Explain. (2 Points)

   `Rise. Whether consumer total spending rises or falls depends on the elasticity of demand.`

[1] For example, government tax revenue after tax is A+B.

Problem 4 (22 Points)

In the Seven Kingdoms of Westeros, people buy and sell Valyrian steel[2] and wildfire. The markets for Valyrian steel and wildfire are described by the following supply and demand equations:

$$\begin{aligned}
\text{Valyrian steel Demand: } Q_D^V &= 400 - 2p^V + p^W \\
\text{Valyrian steel Supply: } Q_S^V &= 40 + p^V \\
\text{Wildfire Demand: } Q_D^W &= 200 - 5p^W + 2p^V \\
\text{Wildfire Supply: } Q_S^W &= 20 + 5p^W
\end{aligned}$$

, where p^V is the price of Valyrian steel, p^W is the price of wildfire, Q_D^V and Q_S^V are respectively the quantity demanded and supplied of Valyrian steel, and Q_D^W and Q_S^W are respectively the quantity demanded and supplied of wildfire.

1. Solve for the equilibrium price and quantity of Valyrian steel and wildfire. (4 Points)

   `Valyrian Steel:` $p^V = 135, Q^V = 175$
   `Wildfire:` $p^W = 45, Q^W = 245$

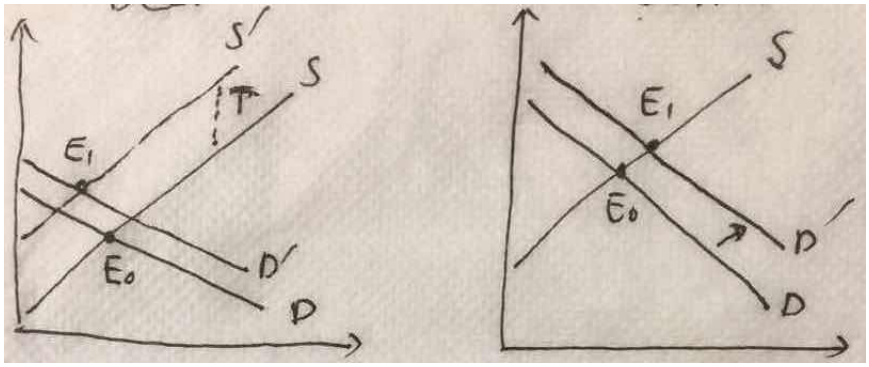
2. Are Valyrian steel and wildfire substitutes, complements, or neither? (2 Points)
   `substitutes`

3. The King of Westeros wants to support wildfire producers. To do so, he imposes a price floor of 60 on wildfire. Under this policy, what would be the market prices of Valyrian steel and wildfire? How much wildfire will people buy? (4 Points)

   $p^V = 140, p^W = 60, Q^W = 180$

[2] On sale here: http://www.valyriansteel.com/shop/

4. Suppose that instead of a price floor, the King decides to impose a per-unit tax $T = 70$ on the sellers of Valyrian steel. Draw supply and demand diagrams to show the impact of this tax on these two markets. (2 Points)



A Napkin Diagram
Left: Valyrian Steel; Right: Wildfire

5. Solve for the prices that consumers pay for Valyrian steel and wildfire after tax, the prices received by producers of Valyrian steel and wildfire after tax, and the equilibrium quantities of Valyrian steel and wildfire after tax. (6 Points)

After tax on Valyrian steel, the price that consumers pay for Valyrian steel (p_D^V) is no longer the same as the price received by Valyrian steel producers (p_S^V). We have:

$$\begin{aligned} Q_D^V &= 400 - 2p_D^V + p^W \\ Q_S^V &= 40 + p_S^V \\ Q_D^W &= 200 - 5p^W + 2p_D^V \\ Q_S^W &= 2 + 5p^W \\ p_S^V &= p_D^V - T \end{aligned}$$

$\Rightarrow$

Valyrian Steel: $p_D^V = 160, p_S^V = 90, Q^V = 130$

Wildfire: $p_S^W = 50, Q^W = 270$

6. Calculate tax incidence. Who pays how much for this tax? Who benefits how much from this tax? (4 Points)

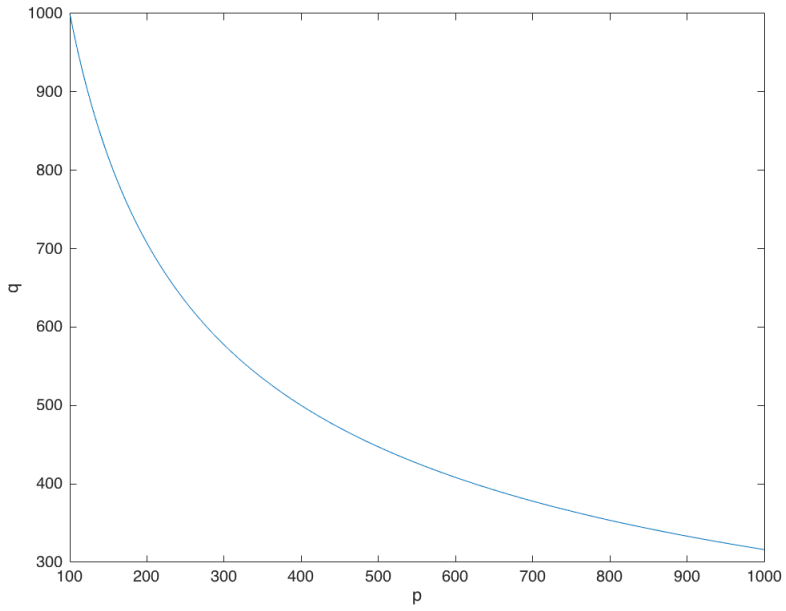
`Valyrian steel producer pays:` $135 - 90 = 45$
`Valyrian steel consumer pays:` $160 - 135 = 25$
`Wildfire consumer pays:` $50 - 45 = 5$
`Wildfire producer gains:` $50 - 45 = 5$

Problem 5 (12 Points)

Suppose the demand for a good is

$$q = \frac{10000}{\sqrt{p}} \tag{1}$$

1. Plot this demand curve[3]. (2 Points)



2. What is the price elasticity of demand for this good? (2 Points)
   `0.5`

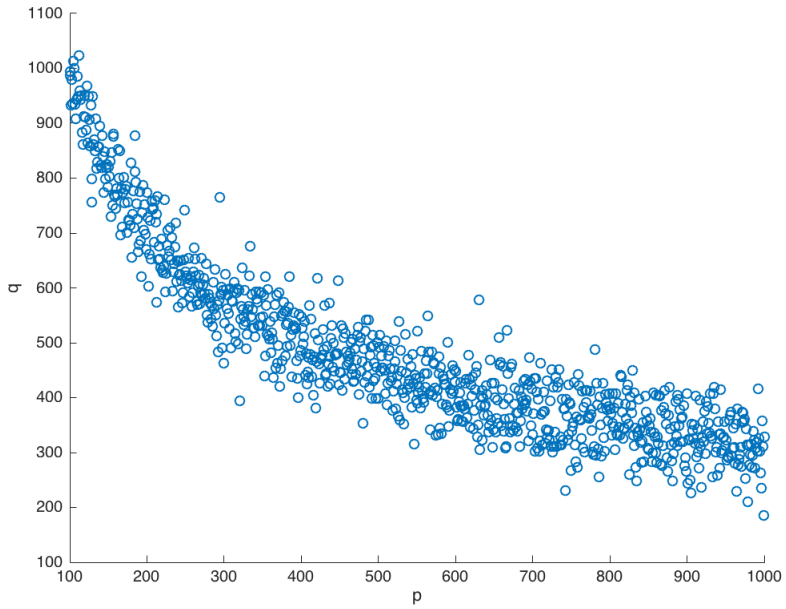
[3]For this question, draw p on the $x-$axis, q on the $y-$axis, and let p range from 100 to 1000.

In reality, the data we observe are often noisy. They may contain measurement error and unobserved variables. In the presence of measurement error, the relationship between observed prices and quantities can be represented by

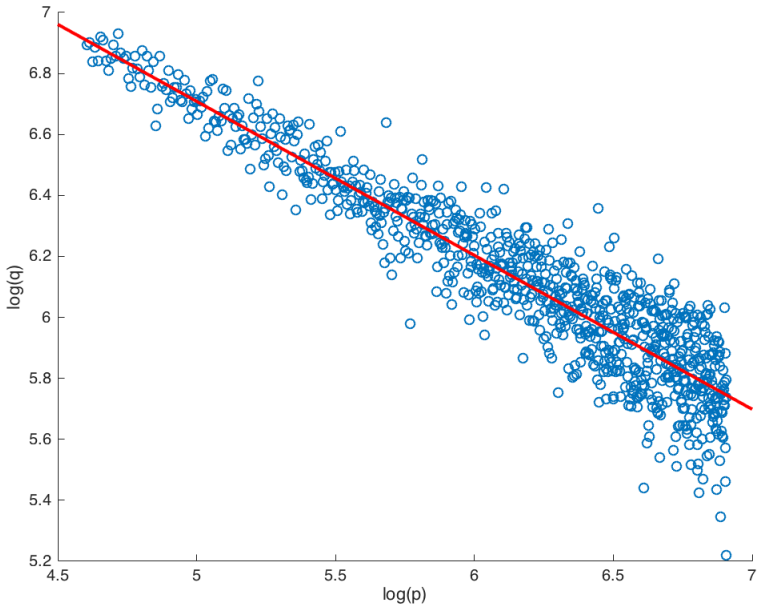
$$q = \frac{10000}{\sqrt{p}} + \epsilon \tag{2}$$

, where ϵ is random noise. "`Demand_data.csv`" contains data generated from (2). Use the data to answer the following questions:

3. Draw a scatter plot of p and q. (2 Points)



4. Draw a scatter plot of $\log(p)$ vs. $\log(q)$ and add a linear best-fit line. (2 Points)



5. Find out the slope of the linear best-fit line by regressing $\log(q)$ on $\log(p)$. (2 Points)
   `-0.5`

6. How does your answer to question 5 compare with your answer to question 2? Why? (2 Points)
   They are equal (in absolute value). This is because $\epsilon_{d,p} = \left|\frac{dq/q}{dp/p}\right| = \left|\frac{d(\ln q)}{d(\ln p)}\right|$.

   Remark. Remember the price elasticity of demand, by definition, measures the relationship between percentage change in p and percentage change in q[4]. This is exactly what a scatter plot of $\log(p)$ and $\log(q)$ shows us. This is because for small r ($|r| \ll 1$), $\log(1+r) \approx r$[5]. Therefore, given a small percentage change in p, say p changes from p to $p(1+r)$[6], $\frac{p(1+r)-p}{p} = r \approx \log(1+r) = \log(p(1+r)) - \log(p)$. Therefore, the scatter plot of $\log(p)$ and $\log(q)$ shows us the relationship between percentage change in p and percentage change in q. Assuming that all data lie on the same demand curve, then (the absolute value of) the slope in the plot of $\log(p)$ vs. $\log(q)$ is by definition the price elasticity of demand.

[4] i.e., given, say, a 1% increase in p, how much will q change in percentage terms?
[5] By Taylor expansion, $\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 \ldots \approx x$ for small x.
[6] say, $r = 0.01$: a 1% increase.