Formulario CPSM a.a. 2020/21

Fattoriale e Fattoriale discendente

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1 \qquad (n)_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\cdots(n-r+1);$$

Coefficiente Binomiale e formula di ricorrenza

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{(n)_r}{r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}; \qquad \binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r} \qquad 1 \le r \le n;$$

Formula di Vandermonde

$$\binom{n+m}{k} = \sum_{i=0}^{k} \binom{n}{i}\binom{m}{k-i} = \binom{n}{0}\binom{m}{k} + \binom{n}{1}\binom{m}{k-1} + \cdots + \binom{n}{k}\binom{m}{0};$$

Principio di inclusione/esclusione

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) + \ldots + (-1)^{n+1} P(A_1 \cap A_2 \cap \ldots \cap A_n);$$

Per $n=2$: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2);$

Per $n=3$: $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3);$

Formula prodotto

$$P(E_1 \cap \ldots \cap E_n) = P(E_1)\,P(E_2|E_1)\,P(E_3|E_2 \cap E_1)\ldots P(E_n|E_1 \cap \ldots \cap E_{n-1}), \text{ se } P(E_1 \cap \ldots \cap E_{n-1}) > 0;$$

Formula delle alternative

$$P(E) = \sum_{i=1}^{n} P(E|F_i)\,P(F_i), \text{ se } F_i \cap F_j = \emptyset, i \ne j, P(F_i) > 0, \cup F_i = S;$$

Per $n=2$: $P(E) = P(E|F)\,P(F) + P(E|\overline{F})\,P(\overline{F}), \text{ se } 0 < P(F) < 1;$

Formula di Bayes

$$P(F_j|E) = \frac{P(E|F_j)\,P(F_j)}{\sum_{i=1}^{n} P(E|F_i)\,P(F_i)}, \quad j = 1,2,\ldots,n, \text{ se } P(E) > 0, F_i \cap F_j = \emptyset, i \ne j, P(F_i) > 0, \cup F_i = S;$$