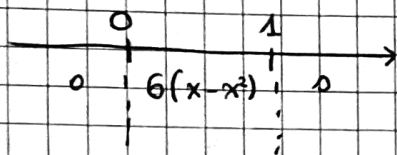


ESERCIZIO V.A. 5

$$f(x) = \begin{cases} 6(x-x^2) & 0 < x < 1 \\ 0 & \text{altrove} \end{cases}$$

$$(i) \quad F(x) = \begin{cases} 0, & \text{se } x < 0 \\ 3x^2 - 2x^3, & \text{se } 0 \leq x < 1 \\ 1, & \text{se } x \geq 1 \end{cases}$$



$$\begin{aligned} \text{Se } 0 \leq x < 1 \quad x \\ F(x) &= \int_0^x 6(t-t^2) dt = \left[6\left(\frac{t^2}{2} - \frac{t^3}{3}\right) \right]_0^x = 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right) = \\ &= 3x^2 - 2x^3 \end{aligned}$$

$$\begin{aligned} \text{Se } x \geq 1 \quad F(x) &= \int_0^1 6(t-t^2) dt = 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right) \Big|_0^1 = \\ &= 3 - 2 = 1 \quad \text{OK} \end{aligned}$$

$$\begin{aligned} (ii) \quad E(x^m) &= \int_0^1 x^m 6(x-x^2) dx = 6 \int_0^1 x^{m+1} dx - 6 \int_0^1 x^{m+2} dx = \\ &= 6 \left[\frac{x^{m+2}}{m+2} \right]_0^1 - 6 \left[\frac{x^{m+3}}{m+3} \right]_0^1 = \\ &= \frac{6}{m+2} - \frac{6}{m+3} = 6 \left(\frac{m+3 - m-2}{(m+2)(m+3)} \right) = \frac{6}{(m+2)(m+3)} \end{aligned}$$

(iii) $Y = X^2 + 1$

$$\begin{aligned}\text{Cov}(X, Y) &= \text{Cov}(X, X^2 + 1) = \text{Cov}(X, X^2) = \\&= E(X \cdot X^2) - E(X) \cdot E(X^2) = \\&= E(X^3) - E(X) E(X^2) = \\&= \frac{6}{5 \cdot 6} - \frac{\cancel{6} \cdot \cancel{2}^1}{\cancel{8} \cdot \cancel{4}^1} \cdot \frac{\cancel{6}^3}{4 \cdot 5} \\&= \frac{1}{5} - \frac{3}{20} = \frac{4-3}{20} = \frac{1}{20}\end{aligned}$$

ESERCIZIO CONGIUNTE 3

2 carte da estrarre
senza reinserimento
da un mazzo
contenente 8 carte
numerate

X = numero più basso

Y = numero più alto

(i)

$(1,2) \quad (1,3) \quad \dots \quad (1,8)$ $8 \cdot 7 = 56$

\vdots

$(8,1) \quad (8,2) \quad \dots \quad (8,7)$

$X \in \{1, \dots, 7\}$

$Y \in \{2, \dots, 8\}$

$X \backslash Y$	2	3	4	5	6	7	8	
1	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\rightarrow 14/56$
2	0	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\rightarrow 12/56$
3	$\frac{2}{56}$ 0	0	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\rightarrow 10/56$
4	0	0	0	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\rightarrow 8/56$
5	0	0	0	0	$\frac{2}{56}$	$\frac{2}{56}$	$\frac{2}{56}$	$\rightarrow 6/56$
6	0	0	0	0	0	$\frac{2}{56}$	$\frac{2}{56}$	$\rightarrow 4/56$
7	0	0	0	0	0	0	$\frac{2}{56}$	$\rightarrow 2/56$
	$\frac{2}{56}$	$\frac{4}{56}$	$\frac{6}{56}$	$\frac{8}{56}$	$\frac{10}{56}$	$\frac{12}{56}$	$\frac{14}{56}$	1

(ii) $p(2,2) = 0 \neq p_X(2) \cdot p_Y(2) = \frac{12}{56} \cdot \frac{2}{56}$

(iii) $E(2X - Y + 1) = 2E(X) - E(Y) + 1 = 2 \cdot 3 - 6 + 1 = 1$

$E(X) = 3$

$E(Y) = 6$

$$E(X \cdot Y) = \frac{2}{56} \cdot (2+3+4+5+6+7+8) +$$

$$+ 2 \cdot \frac{2}{56} \cdot (3+4+5+6+7+8) + 3 \cdot \frac{2}{56} \cdot (4+5+6+7+8)$$

$$+ 4 \cdot \frac{2}{56} \cdot (5+6+7+8) + 5 \cdot \frac{2}{56} \cdot (6+7+8) +$$

$$+ 6 \cdot \frac{2}{56} \cdot (7+8) + 7 \cdot \frac{2}{56} \cdot 8 = \frac{39}{2}$$