Formulario CPSM a.a. 2020/21

Variabili aleatorie discrete

Distribuzione di Bernoulli

$p(x) = p^x (1-p)^{1-x}, \quad x = 0, 1;\ E(X) = p;\ Var(X) = p(1-p);$

Distribuzione Binomiale

$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n;\ E(X) = np;\ Var(X) = np(1-p);$

Distribuzione di Poisson

$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots;\ E(X) = \lambda;\ Var(X) = \lambda;$

Distribuzione Geometrica

$P(X = n) = (1-p)^{n-1} p, \quad n = 1, 2, \ldots;\ E(X) = 1/p;\ Var(X) = (1-p)/p^2;$

Distribuzione Ipergeometrica

$$P(X = k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}, \quad k = 0, 1, \ldots, n;\ E(X) = n\tfrac{m}{N};\ Var(X) = n\,\tfrac{m}{N}\left(1 - \tfrac{m}{N}\right)\left(1 - \tfrac{n-1}{N-1}\right);$$

Distribuzione Uniforme discreta

$P(X = k) = \frac{1}{N}, \quad k = 1, 2, \ldots, N;\ E(X) = \frac{N+1}{2};\ \mathrm{Var}(X) = \frac{N^2-1}{12};$

Variabili aleatorie assolutamente continue

Distribuzione Uniforme

$$f_X(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{altrimenti.} \end{cases}, \quad F(x) = \begin{cases} 0 & x < \alpha \\ \dfrac{x-\alpha}{\beta-\alpha} & \alpha \le x < \beta \\ 1 & x \ge \beta \end{cases} \quad E[X] = \tfrac{\alpha+\beta}{2},\ \mathrm{Var}(X) = \tfrac{(\beta-\alpha)^2}{12};$$

Distribuzione Normale

$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty, \quad \mu \in \mathbb{R}, \quad \sigma^2 > 0; \quad E(X) = \mu, \quad Var(X) = \sigma^2.$

Distribuzione Esponenziale

$$f(x) = \begin{cases} \lambda \mathrm{e}^{-\lambda x} & x \ge 0 \\ 0 & \text{altrimenti} \end{cases}, \quad F(x) = \begin{cases} 1 - \mathrm{e}^{-\lambda x} & x \ge 0 \\ 0 & \text{altrimenti} \end{cases};$$

$E(X) = 1/\lambda, \quad Var(X) = 1/\lambda^2$

Vettori Aleatori

$F(x,y) = P\left(X \leq x, Y \leq y\right), \quad x, y \in \mathbb{R}; \quad \lim_{y \to +\infty} F(x,y) = F(x); \quad \lim_{x \to +\infty} F(x,y) = F(y);$

$p(x,y) = P\left(X = x, Y = y\right),$

Valore Atteso

$E[g(X,Y)] = \sum_y \sum_x g(x,y)p(x,y);$

Covarianza

$Cov(X,Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y];$

Varianza di somme di variabili aleatorie

$\mathrm{Var}\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n \mathrm{Var}(X_i) + 2\sum_{i<j} Cov(X_i, X_j);$

Coefficiente di correlazione

$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}; \quad -1 \leq \rho(X,Y) \leq 1$