Formale Systeme Proseminar

Tasks for Week 12, 21.12.2017

Task 1 Consider the relation $R \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$R = \{(n, n+1) \mid n \in \mathbb{N}\}.$$

(a) Find the relation R^2,

(b) Find the relation R^3,

(c) Can you think of a concise way to describe the reflexive and transitive closure relation R^*?

Task 2 Which of the following relations between $A = \{a, b, c\}$ and $B = \{1, 2\}$ are graphs of functions from A to B? Which are graphs of injective functions, which of surjective?

(a) $R_1 = \{(a, 1), (b, 2)\}$.

(b) $R_2 = \{(a, 1), (b, 1), (b, 2), (c, 1)\}$.

(c) $R_3 = \{(a, 1), (b, 2), (a, 2)\}$.

(d) $R_4 = \{(a, 1), (b, 2), (c, 1)\}$.

Why?

Task 3 Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Give an example of a surjective function $f \colon A \to B$.

Task 4 Give an example of an injective function $f \colon \mathbb{N} \to \mathbb{N}$.

Task 5 Let $X = \{1, 2, 3, 4, 5\}$ and consider the function $c \colon \mathcal{P}(X) \setminus \{\emptyset\} \to X$ defined by $c(Y) = |Y|$ for any $Y \subseteq X$, $Y \neq \emptyset$. Show that c is surjective but not injective.

Task 6 Prove Proposition S3 from the lectures, that is, show that if $f \colon A \to B$ is a surjective function and $B' \subseteq B$ then $f(f^{-1}(B')) = B'$.

Task 7 Prove Lemma B2 from the lectures, that is, show that if $f \colon A \to B$ is a bijective function then it's inverse function $f^{-1} \colon B \to A$ is also bijective.