LinA Test

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1

$$w_1 = \begin{pmatrix} 4 \\ 0 \\ -2 \\ 4 \\ 0 \end{pmatrix} \quad w_2 = \begin{pmatrix} 5 \\ 1 \\ -2 \\ -3 \\ 1 \end{pmatrix}$$

z.Z: $\lambda_1 w_1 + \lambda_2 w_2 = 0 \Leftrightarrow \lambda_1 = \lambda_2 = 0$

$$\begin{pmatrix} 4 & 5 \\ 0 & 1 \\ -2 & -2 \\ 4 & -3 \\ 0 & 1 \end{pmatrix} \underset{\rightsquigarrow}{\overset{I + 2III}{IV + 2III}} \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ -2 & -2 \\ 0 & -7 \\ 0 & 1 \end{pmatrix} \underset{\rightsquigarrow}{\overset{I \leftrightarrow III}{I : (-2)}} \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & -7 \\ 0 & 1 \end{pmatrix} \begin{matrix} I - II \\ III - II \\ IV + 7II \\ \rightsquigarrow \\ V - II \end{matrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow$ Die einzige Lösung ist $\lambda_1 = \lambda_2 = 0$

$\Rightarrow$ w_1, w_2 sind linear unabhängig

Gram-Schmidt:

$$v_1 = \frac{w_1}{\|w_1\|} = \frac{1}{6} w_1$$

$$\hat{v}_2 = w_2 - \langle v_1, w_2 \rangle v_1$$

$$\hat{V}_2 = W_2 - \langle V_1, W_2 \rangle V_1$$

$$= w_2 - \tfrac{1}{6} \langle w_1, w_2 \rangle \tfrac{1}{6} w_1$$

$$= \begin{pmatrix} 5 \\ 1 \\ -2 \\ -3 \\ 1 \end{pmatrix} - \tfrac{1}{6} \cdot 36 \cdot \tfrac{1}{6} w_1$$

$$= \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$$V_2 = \frac{\hat{V}_2}{\|\hat{V}_2\|} = \tfrac{1}{4} \hat{V}_2$$

Probe: $\langle V_1, V_2 \rangle = 0$ ✓

$$U = \left\{ \tfrac{1}{6} \begin{pmatrix} 4 \\ 0 \\ -2 \\ -4 \\ 0 \end{pmatrix}, \tfrac{1}{4} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

2

$$v = \begin{pmatrix} -5+4 \\ -5+4 \\ -5+5 \\ -5+4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$U_1 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$U_2 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$