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Beobachte $\begin{pmatrix}-5\\2\\1\\0\\0\end{pmatrix}+\begin{pmatrix}-9\\5\\0\\1\\0\end{pmatrix}+\begin{pmatrix}7\\0\\0\\0\\1\end{pmatrix}=\begin{pmatrix}-7\\7\\1\\1\\1\end{pmatrix}$

$$\Rightarrow LIN\left\{\begin{pmatrix}-5\\2\\1\\0\\0\end{pmatrix},\begin{pmatrix}-9\\5\\0\\1\\0\end{pmatrix},\begin{pmatrix}7\\0\\0\\0\\1\end{pmatrix},\begin{pmatrix}-7\\7\\1\\1\\1\end{pmatrix}\right\}$$

$$= LIN\left\{\begin{pmatrix}-5\\2\\1\\0\\0\end{pmatrix},\begin{pmatrix}-9\\5\\0\\1\\0\end{pmatrix},\begin{pmatrix}7\\0\\0\\0\\1\end{pmatrix}\right\}$$

Nun:

$$\begin{pmatrix}-5&-9&7\\2&5&0\\1&0&0\\0&1&0\\0&0&1\end{pmatrix}\rightsquigarrow\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\end{pmatrix}\Rightarrow \mathbb{L}=\left\{\begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}\right\}$$

$$\Rightarrow \left\{\begin{pmatrix}-5\\2\\1\\0\\0\end{pmatrix},\begin{pmatrix}-9\\5\\0\\1\\0\end{pmatrix},\begin{pmatrix}7\\0\\0\\0\\1\end{pmatrix}\right\} \text{ ist Basis von } U.$$

Wir suchen A, b sodass $\{x \mid Ax=b\}=p+U$.

Zunächst sei $C^T=\begin{pmatrix}-5&2&1&0&0\\-9&5&0&1&0\\7&0&0&0&1\end{pmatrix}$

$$\overset{I+III}{\rightsquigarrow}\begin{pmatrix}2&2&1&0&1\\-9&5&0&1&0\\7&0&0&0&1\end{pmatrix}$$

$$\overset{I/2,\ III+II}{\rightsquigarrow}\begin{pmatrix}1&1&1/2&0&1/2\\-9&5&0&1&0\\-2&5&0&1&1\end{pmatrix}$$

$$\overset{II+9I,\ III+2I}{\rightsquigarrow}\begin{pmatrix}1&1&1/2&0&1/2\\0&14&4{,}1/2&1&4{,}1/2\\0&7&1&1&2\end{pmatrix}$$

$$\overset{II-2III,\ I-1/7\,III}{\rightsquigarrow}\begin{pmatrix}1&0&5/14&-1/7&3/14\\0&0&5/2&-1&1/2\\0&7&1&1&2\end{pmatrix}$$

$$\overset{I-1/7\,II,\ III-2/5\,II}{\rightsquigarrow}\begin{pmatrix}1&0&0&0&2/14\\0&0&5/2&-1&1/2\\0&7&0&7/5&9/5\end{pmatrix}$$

$$\overset{III:7,\ II:2.5}{\rightsquigarrow}\begin{pmatrix}1&0&0&0&1/7\\0&1&0&1/5&9/35\\0&0&1&-2/5&1/5\end{pmatrix}$$

$$\Rightarrow \mathbb{L}=\left\{x_1\begin{pmatrix}-1/7\\-1/5\\2/5\\1\\0\end{pmatrix}+x_2\begin{pmatrix}0\\0\\-1\\0\\1\end{pmatrix}\right\}\Rightarrow A=\begin{pmatrix}0&-1/5&2/5&1&0\\-1/7&-9/35&-1/5&0&1\end{pmatrix}$$

Nun ist $b=Ap=\begin{pmatrix}6(-1/5)\\3(-1/7)+6(-9/35)\end{pmatrix}=\begin{pmatrix}-6/5\\-39/35\end{pmatrix}$

$\Rightarrow$ Das LGS ist $\left(\begin{array}{ccccc|c}0&-1/5&2/5&1&0&-6/5\\-1/7&-9/35&-1/5&0&1&-39/35\end{array}\right)$

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$$\left(\begin{array}{ccccc|c} -3 & 6 & -2 & 1 & -8 & 4 \\ 4 & -8 & 5 & 4 & 4 & 9 \\ -2 & 4 & -2 & -2 & 0 & -6 \\ 3 & -6 & -1 & -4 & 5 & -7 \end{array}\right)$$

$$\xrightarrow{IV+III} \left(\begin{array}{ccccc|c} -3 & 6 & -2 & 1 & -8 & 4 \\ 4 & -8 & 5 & 4 & 4 & 9 \\ -2 & 4 & -2 & -2 & 0 & -6 \\ 1 & -2 & -3 & -6 & 5 & -13 \end{array}\right)$$

$$\xrightarrow[III+2IV]{I+3IV,\ II-4IV} \left(\begin{array}{ccccc|c} 0 & 0 & -11 & -17 & 7 & -35 \\ 0 & 0 & 17 & 28 & -16 & 61 \\ 0 & 0 & -8 & -14 & 10 & -32 \\ 1 & -2 & -3 & -6 & 5 & -13 \end{array}\right)$$

$$\xrightarrow{II+2III} \left(\begin{array}{ccccc|c} 0 & 0 & -11 & -17 & 7 & -35 \\ 0 & 0 & 1 & 0 & 4 & -3 \\ 0 & 0 & -8 & -14 & 10 & -32 \\ 1 & -2 & -3 & -6 & 5 & -13 \end{array}\right)$$

$$\xrightarrow[IV+3II]{I+11II,\ III+8II} \left(\begin{array}{ccccc|c} 0 & 0 & 0 & -17 & 51 & -68 \\ 0 & 0 & 1 & 0 & 4 & -3 \\ 0 & 0 & 0 & -14 & 42 & -56 \\ 1 & -2 & 0 & -6 & 17 & -22 \end{array}\right)$$

$$\xrightarrow{I-III} \left(\begin{array}{ccccc|c} 0 & 0 & 0 & -3 & 9 & -12 \\ 0 & 0 & 1 & 0 & 4 & -3 \\ 0 & 0 & 0 & -14 & 42 & -56 \\ 1 & -2 & 0 & -6 & 17 & -22 \end{array}\right)$$

$$\xrightarrow{III-5I} \left(\begin{array}{ccccc|c} 0 & 0 & 0 & -3 & 9 & -12 \\ 0 & 0 & 1 & 0 & 4 & -3 \\ 0 & 0 & 0 & 1 & -3 & 4 \\ 1 & -2 & 0 & -6 & 17 & -22 \end{array}\right)$$

$$\xrightarrow[IV+6III]{I+3III} \left(\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 4 & -3 \\ 0 & 0 & 0 & 1 & -3 & 4 \\ 1 & -2 & 0 & 0 & -1 & 2 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & 4 & -3 \\ 0 & 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \mathbb{L} = \left\{ \lambda_1 \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -4 \\ 3 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \\ 4 \\ 0 \\ 0 \end{pmatrix} \,\middle|\, \lambda_1, \lambda_2 \in \mathbb{R} \right\}$$

$$\Rightarrow U = LIN \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ 3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$P = \begin{pmatrix} 2 \\ -3 \\ 4 \\ 0 \\ 0 \end{pmatrix}$$

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$$G = \begin{pmatrix} 7 \\ 5 \\ -6 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -7 \\ 4 \\ 5 \end{pmatrix}$$

$$H_1 = -8x_1 - 3x_2 - 3x_3 - 7x_4 = 10$$

Wir setzen ein:

$$-8(7+2\lambda) - 3(5-7\lambda) - 3(-6+4\lambda) - 7(3+5\lambda) = 10$$

$$\Leftrightarrow -56 - 16\lambda - 15 + 21\lambda + 18 - 12\lambda - 21 - 35\lambda = 10$$

$$\Leftrightarrow -74 - 42\lambda = 10$$

$$\Leftrightarrow -42\lambda = 84$$

$$\Leftrightarrow \lambda = -2$$

Probe:

$$-8(7-4) - 3(5+14) - 3(-6-8) - 7(3-10) = 10$$

$$\Leftrightarrow -8 \cdot 3 - 3 \cdot 19 - 3 \cdot (-14) - 7(-7) = 10$$

$$\Leftrightarrow -24 - 57 + 42 + 49 = 10 \quad \checkmark$$

$$\Rightarrow G \cap H_1 = \begin{pmatrix} 3 \\ 19 \\ -14 \\ -7 \end{pmatrix}$$

$$H_2 = -3x_1 - x_2 - 4x_3 + 3x_4 = 6$$

$$\Rightarrow -3(7+2\lambda) - 5 + 7\lambda - 4(-6+4\lambda) + 3(3+5\lambda) = 6$$

$$\Leftrightarrow -21 - 6\lambda - 5 + 7\lambda + 24 - 16\lambda + 9 + 15\lambda = 6$$

$$\Leftrightarrow 7 = 6$$

$$\Rightarrow G \cap H_2 = \emptyset$$

$$\Rightarrow G \cap H_3 = G$$