Data Streams: Bloom Filters

Mining Massive Datasets

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Topic 24

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Sources

- Mining of Massive Datasets (2014) by Leskovec et al. (chapter 4)
  - Slides part 1, part 2
- Tutorial: Mining Massive Data Streams (2019) by Michael Hahsler

Bloom filters

Filtering a data stream

- Suppose we have a large set S of keys
- We want to filter a stream <key, data> to let pass only the elements for which key ∈ S
- Example: key is an e-mail address, we have a total of $|S|=10^9$ allowed e-mail addresses

Naïve solution?

Filtering a data stream

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- Example: key is an e-mail address, we have a total of $|S|=10^9$ allowed e-mail addresses
- Naïve solution? Hash table won't work, too big!

Bloom Filter (1-bit case)

- **Given a set of keys *S***
- Create a **bit array *B[]*** of ***n*** bits
  - Initialize to all ***0*s**
- Pick a **hash function *h*** with range **[*0,n*)**
  - For each member of $s \in S$
    - Hash to one of ***n*** buckets
    - Set that bit to **1**, i.e., ***B[h(s)]* ← *1***
- For each element ***a*** of the stream
  - **Output *a* if and only if B[h(a)] == 1**

Bloom filter creation

Stream processing

Bloom Filter is an approximate filter

- Can it output an element with a key not in S?
- Can it not output an element with a key in S?

Bloom Filter is an approximate filter

- Can it output an element with a key not in S?

  Yes, due to hash collisions $h(x)=h(y)$ when $x \neq y$
- Can it not output an element with a key in S?

  No, because $h(x)$ is always the same for x

Bloom filters are *permissive* (not *strict*)

Bloom filter

- A bloom filter is:
  - An array of n bits, initialized as 0
  - A collection of hash functions $h_1, h_2, \ldots, h_k$
  - A set S of m key values
- The purpose of the bloom filter is to allow all stream items whose key is in S

Bloom filter initialization

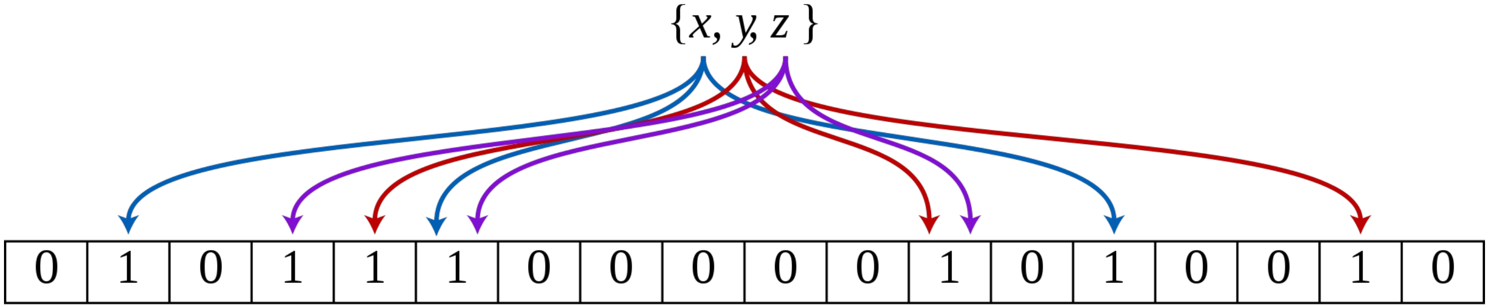
For all positions i in [0, n-1]

B[i] ← 0

For all keys K in S:

For every hash function $h_1, h_2, \ldots, h_k$

$B[h_i(K)] \leftarrow 1$



Bloom filter usage

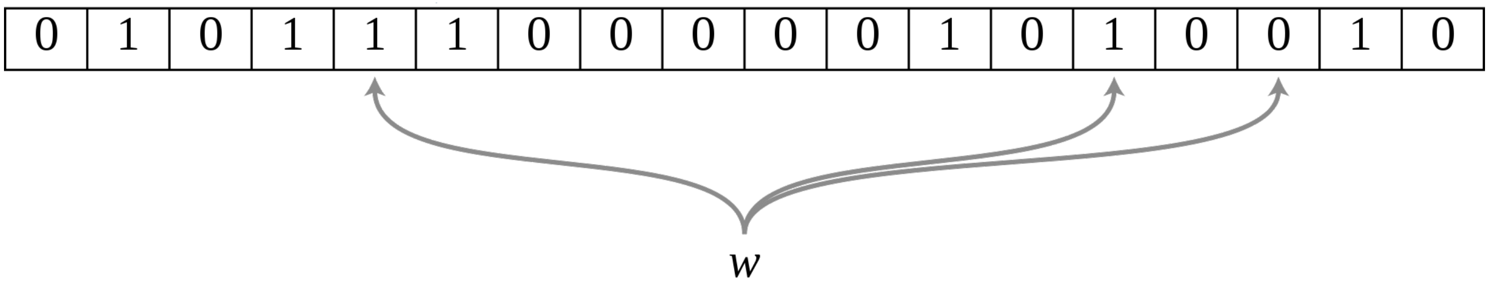
For each input element <key, data>

  allow ← TRUE

  For every hash function $h_1, h_2, \ldots, h_k$

    allow ← allow $\wedge$ $B[h_i(K)] == 1$

  output element if allow == TRUE



Characteristics of Bloom Filters

- Are lax (not strict) and let some items pass
  - May require a second-level check to make filter strict, for instance store output on disk files and then check against hash tables (slower)
- Implementations can be very fast

  E.g., use hardware words for the bit table

Preliminaries for the analysis: targets and darts

- Suppose we throw y darts at x targets
  - All darts will hit one of the targets
- After throwing the darts, how many distinct targets can we expect to hit at least once?
  - Prob. that a given dart will not hit a given target is $(x-1)/x = 1-1/x$
  - Prob. none of the y darts will hit a given target is $(1-1/x)^y = (1-1/x)^{x(y/x)}$
  - Using $(1-\varepsilon)^{1/\varepsilon} \simeq 1/e$ for small ε
  - Prob. none of the y darts will hit a given target is $(1/e)^{y/x}$

Analysis of the 1-bit Bloom Filter

- Each element of S is a dart $|S|=y$
- Each bit in the array is a target $n=x$
- Suppose $y=|S|=10^9$ (1 G) and $x=n=8 \times 10^9$ (8 GB)
- Prob. that a given bit is NOT set to 1 (dart does not hit the target) is $(1/e)^{y/x} = (1/e)^{1/8}$
- Prob. bit is set to 1 is $1 - (1/e)^{1/8} = 0.1175$

    About 12% of bits are set to one in this Bloom Filter

    this is also the false-hit probability in this case

General case

- $|S|=m$ keys, array has n bits
- k hash functions
- Targets $x=n$, darts $y=km$
- Probability that a bit remains 0 is $e^{-km/n}$
- Example:

  We can pick $k=n/m$ to obtain collision probability $1/e = 37\%$

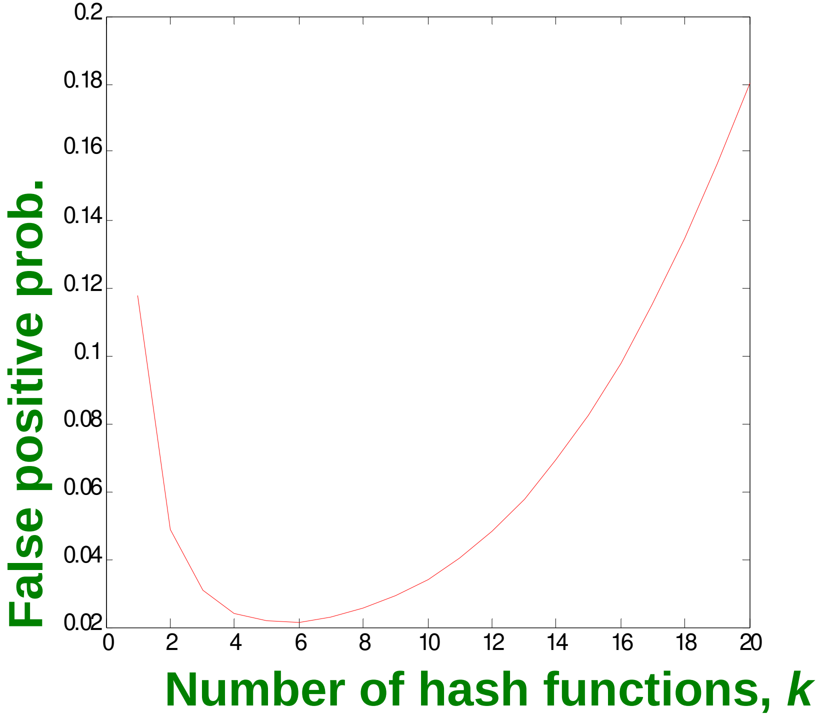
Analysis of a 2-bit Bloom Filter

- Suppose $|S|=10^9$ (1 G) and $n=8 \times 10^9$ (8 GB)
- Suppose we use two hash functions
- Prob. that a given bit is NOT set to 1 (dart does not hit the target) is $(1/e)^{y/x} = (1/e)^{1/4}$
- Prob. a bit is set to 1 is $1 – (1/e)^{1/4}$
- Prob. two bits are set to 1 is $(1 – (1/e)^{1/4})^2 = 0.0493$
- We have a false hit probability of about 5% with two hash functions, while the probability was about 12% with only one

How many hash functions to use?

Too few: test is too unspecific. Too many: table becomes too crowded.

- ***m* = 1 billion, *n* = 8 billion**
  - **k = 1**: $(1 - e^{-1/8})$ = **0.1175**
  - **k = 2**: $(1 - e^{-1/4})^2$ = **0.0493**
- **What happens as we keep increasing *k*?**
  - "Optimal" value of ***k***: ***n/m* ln(2)**
  - **In our case:** Optimal **k = 8 ln(2) = 5.54 ≈ 6**
  - **Error at k = 6**: $(1 - e^{-1/6})^2$ = **0.0235**



Summary

Things to remember

- How to initialize a Bloom filter
- How to use a Bloom filter
- Proofs for 1-bit, 2-bit case

Exercises for TT22-T26

- Mining of Massive Datasets (2014) by Leskovec et al.
  - Exercises 4.2.5
  - Exercises 4.3.4
  - Exercises 4.4.5
  - Exercises 4.5.6