Mining Time Series: Forecasting

Mining Massive Datasets

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Topic 29

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Sources

- Data Mining, The Textbook (2015) by Charu Aggarwal (chapter 14)
- Introduction to Time Series Mining (2006) tutorial by Keogh Eamonn [alt. link]
- Time Series Data Mining (2006) slides by Hung Son Nguyen

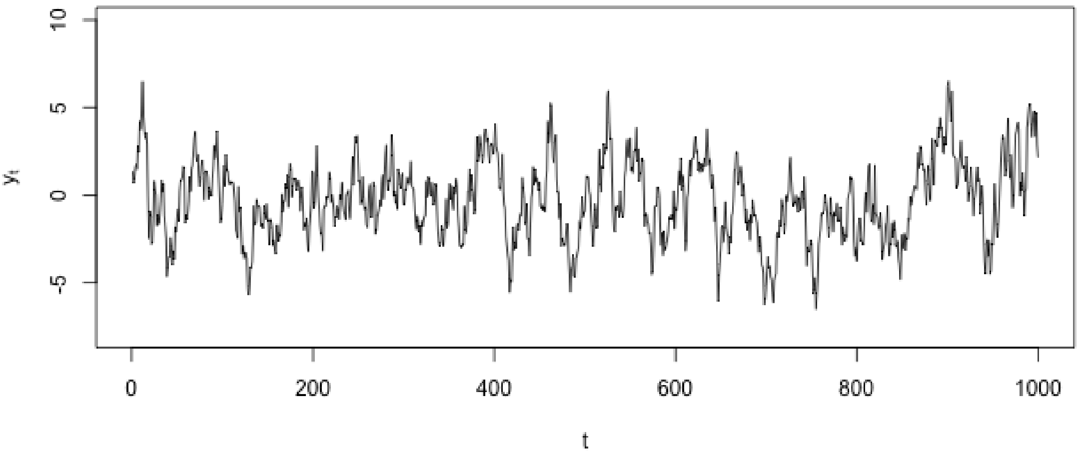
Forecasting

(AR, MA, ARMA, ARIMA, ...)

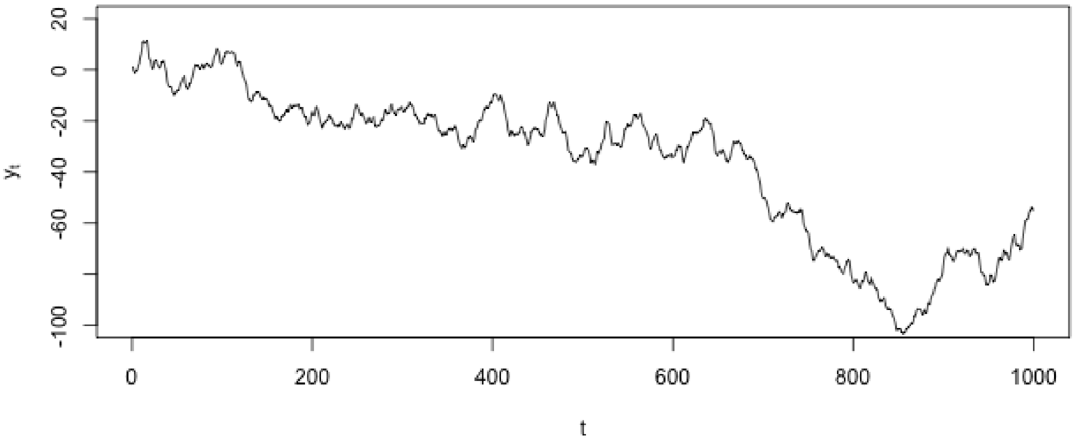
Stationary vs **Non-Stationary** processes

- **Stationary process**
  - Parameters do not change over time
  - E.g., *White noise* has zero mean, fixed variance, and zero covariance between y_t and y_{t+L} for any lag L
- **Non-stationary process**
  - Parameters change over time
  - E.g., price of oil, height of a child, glucose level of a patient, ...

Stationary process



Non-stationary process



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https://commons.wikimedia.org/w/index.php?curid=41600857

Strictly stationary time series

A **strictly stationary time series** is one in which the distribution of values in any time interval $[a,b]$ is identical to that in $[a+L, b+L]$ for any value of time shift (lag) L

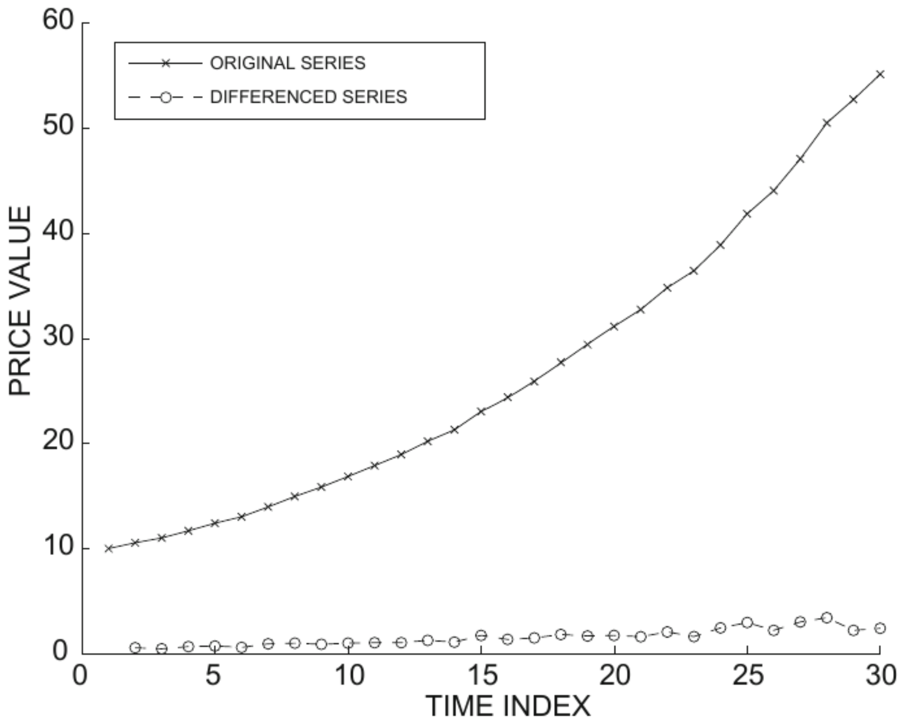
- In this case, current parameters (e.g., mean) are good predictors of future parameters

Differencing

First order differencing

$$y_i' = y_i - y_{i-1}$$

In this first example, if the original series is superlinear, the differenced series is stationary or non-stationary?

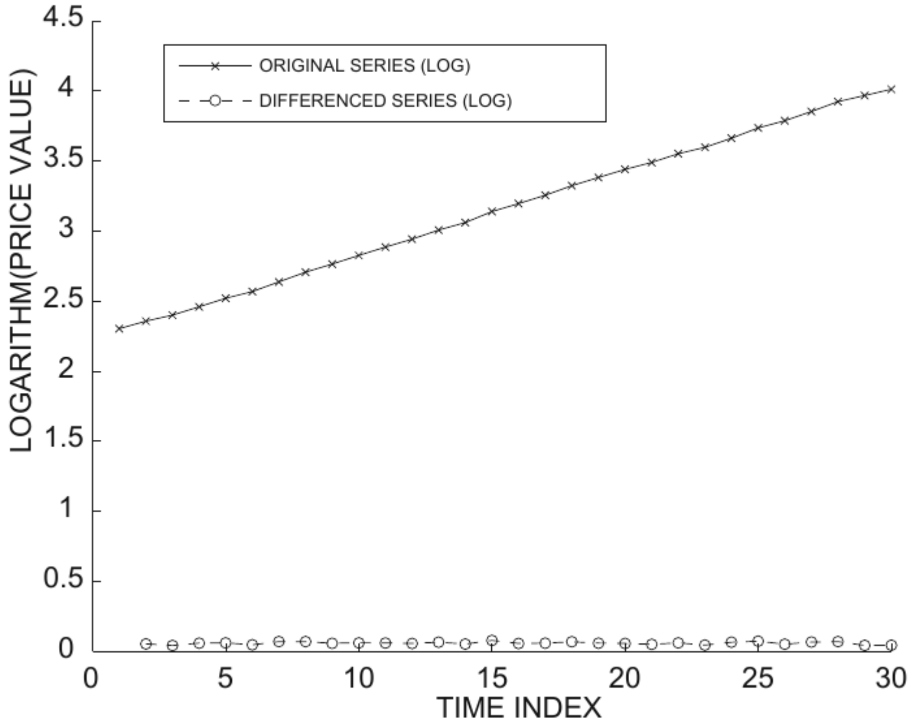


Differencing (cont.)

First order differencing

$$y_i' = y_i - y_{i-1}$$

In this second example, where the series is linear, the differenced series is stationary or non-stationary?



Other differencing operations

- Second-order differencing

$$
\begin{aligned}
y_i'' &= y_i' - y_{i-1}' \\
&= y_i - 2 \cdot y_{i-1} + y_{i-2}
\end{aligned}
$$

- Seasonal differencing (m = 24 hours, 7 days, …)

$$y_i' = y_i - y_{i-m}$$

If you find a differencing that yields a stationary series, the forecasting problem is basically solved.

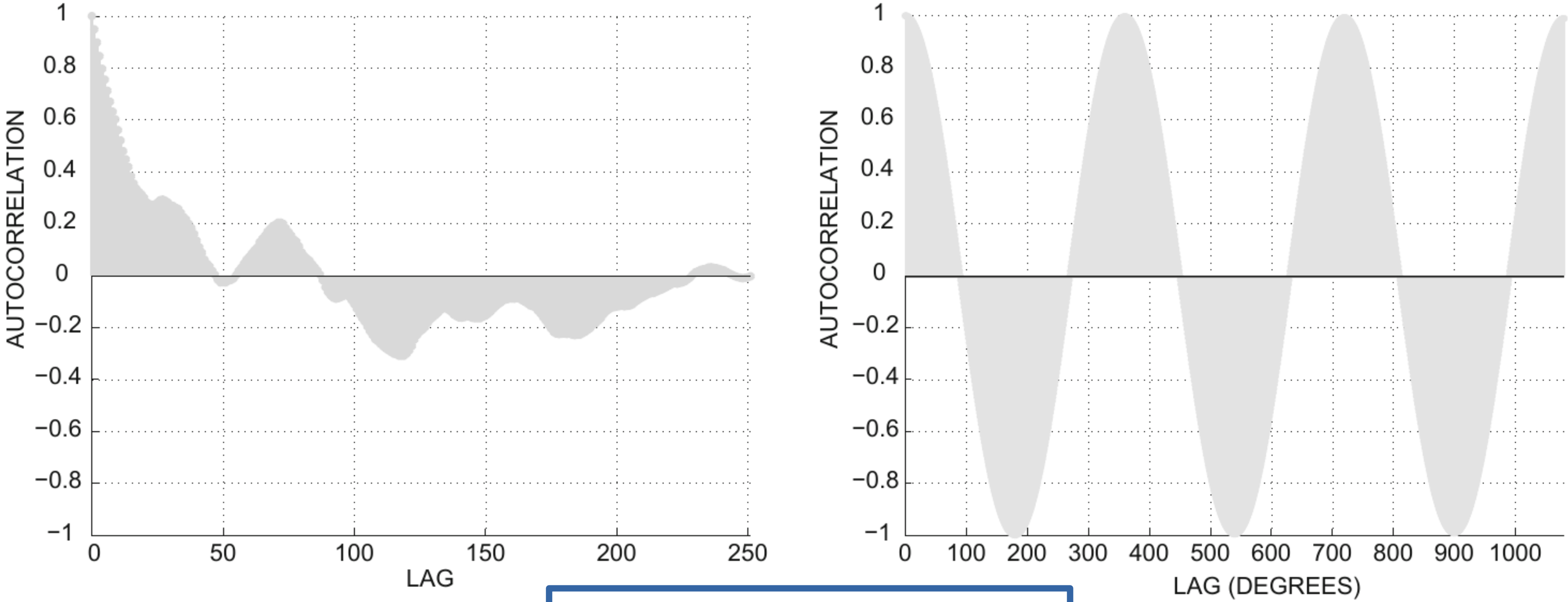
Autoregressive model **AR(p)**

$$\text{Autocorrelation}(L) = \frac{\text{Covariance}_t(y_t, y_{t+L})}{\text{Variance}_t(y_t)}$$

- Autocorrelation lines in [-1,1]
- High absolute values ⇒ predictability
- Autoregressive model of order p, AR(p):

$$y_t^{\text{AR}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + c + \epsilon_t$$

How to decide p? Autocorrelation plots



(a) IBM stock (b) Sine wave

What is a reasonable range for p in these cases?

Finding coefficients and evaluating

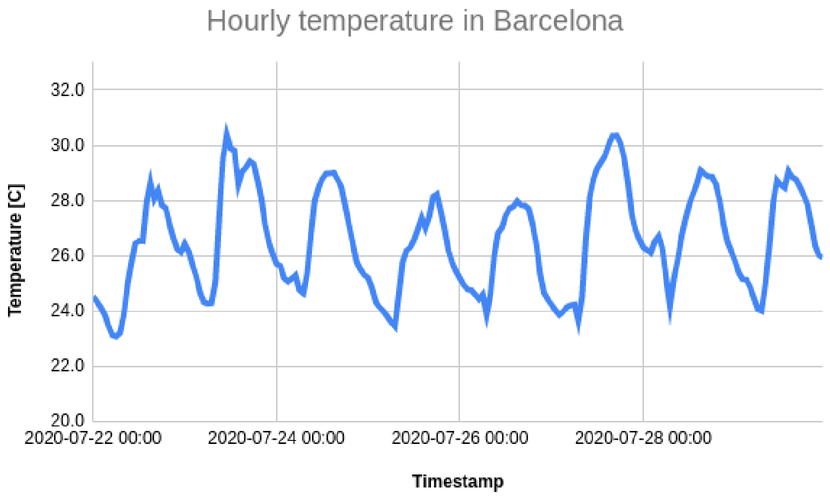
- Each data point is a training element

$$y_t^{\mathrm{AR}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + c + \epsilon_t$$

- Coefficients found by least-squares regression
- Best models have $R^2 \to 1$

$$R^2 = 1 - \frac{\mathrm{Mean}_t(\epsilon_t^2)}{\mathrm{Variance}_t(y_t)}$$

Exercise



- Create a simple auto-regressive model
- Use two lags:

  1 hour

  24 hours

- Compute the predicted series

  (optionally: include it in the plot)

- Compute the maximum error

Answer in
Google Spreadsheet

Moving average model **MA(q)**

- Focus on the variations (shocks) of the model, i.e., places where change was unexpected
- AR(p) model: $y_t^{\text{AR}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + c + \epsilon_t$
- MA(q) model: $y_t^{\text{MA}} = \sum_{i=1}^{q} b_i \cdot \epsilon_{t-i} + c + \epsilon_t$

Autoregressive moving average model **ARMA(p,q)**

- Combines both the autoregressive and the moving average model

$$y_t^{\mathrm{ARMA}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + \sum_{i=1}^{q} b_i \cdot \epsilon_{t-i} + c + \epsilon_t$$

- Select small p, q, to avoid overfitting

Autoregressive integrated moving average model **ARIMA(p,q)**

- Combines both the autoregressive and the moving average model on differenced series

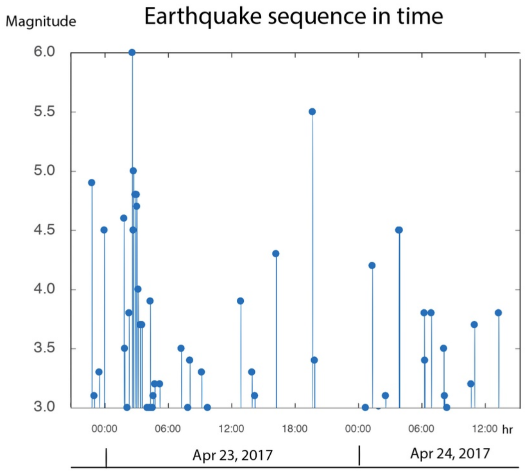
$$y_t^{\text{ARIMA}} = \sum_{i=1}^{p} a_i \cdot (y_{t-i} - y_{t-i-1}) + \sum_{i=1}^{q} b_i \cdot \epsilon_{t-i} + c + \epsilon_t$$

Note: this is an ARIMA(p,1,q) model as we're using first order differencing

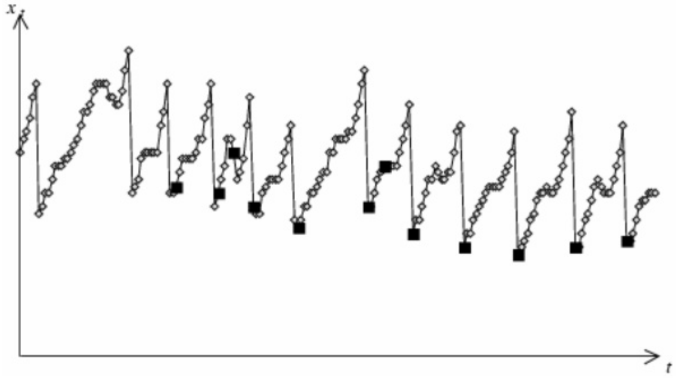
See also: [ARIMA end-to-end project in Python]() by Susan Li (2018)

Event detection
(a simple framework)

Event: an important occurrence



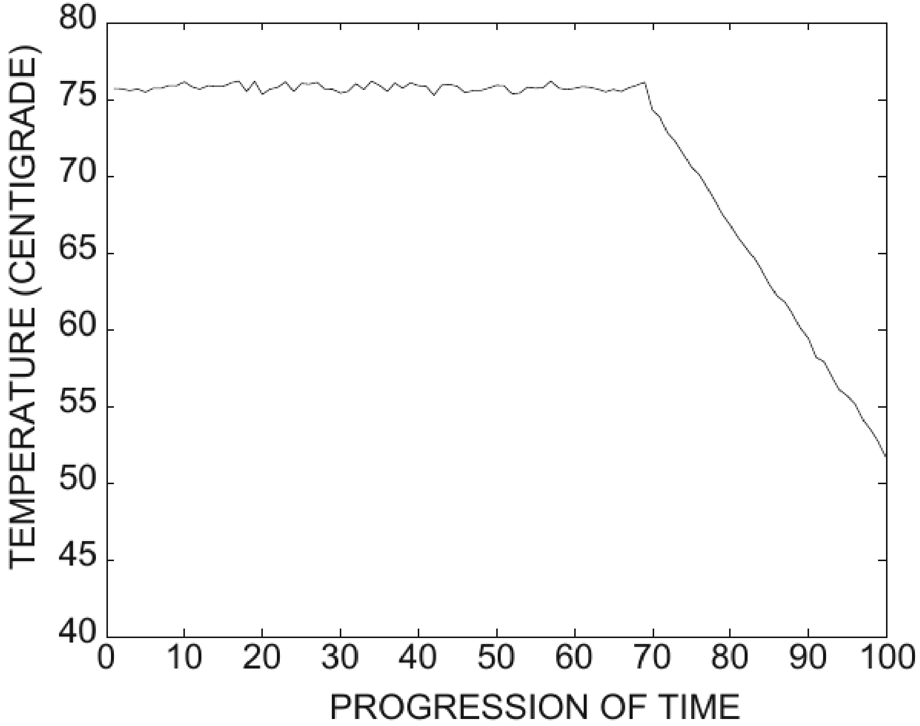
Earthquake or aftershock



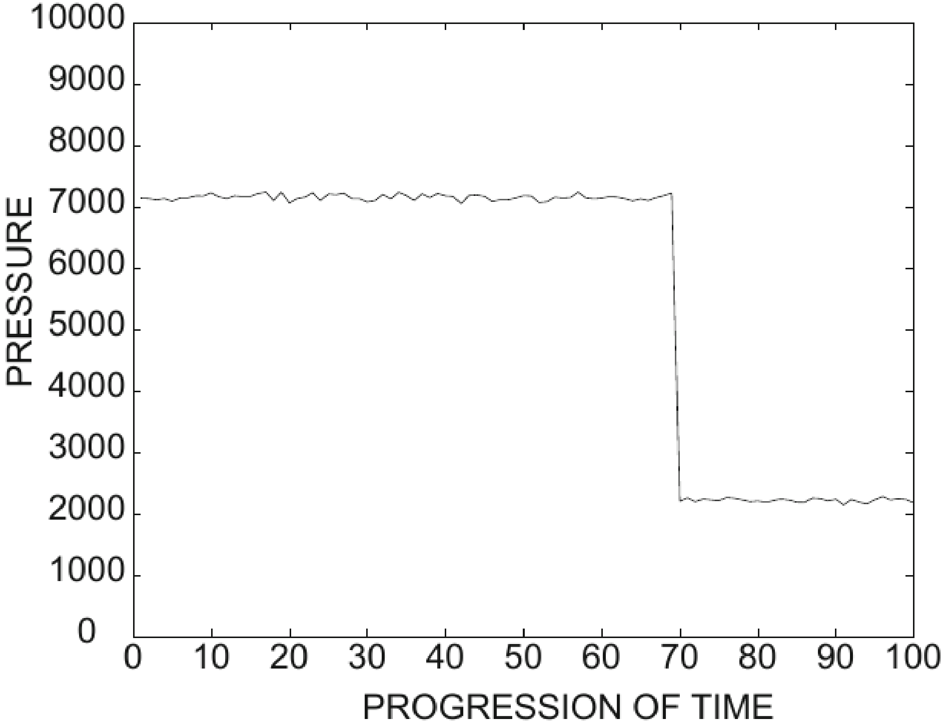
Droplet release

Sudden price change

Example: pipe rupture



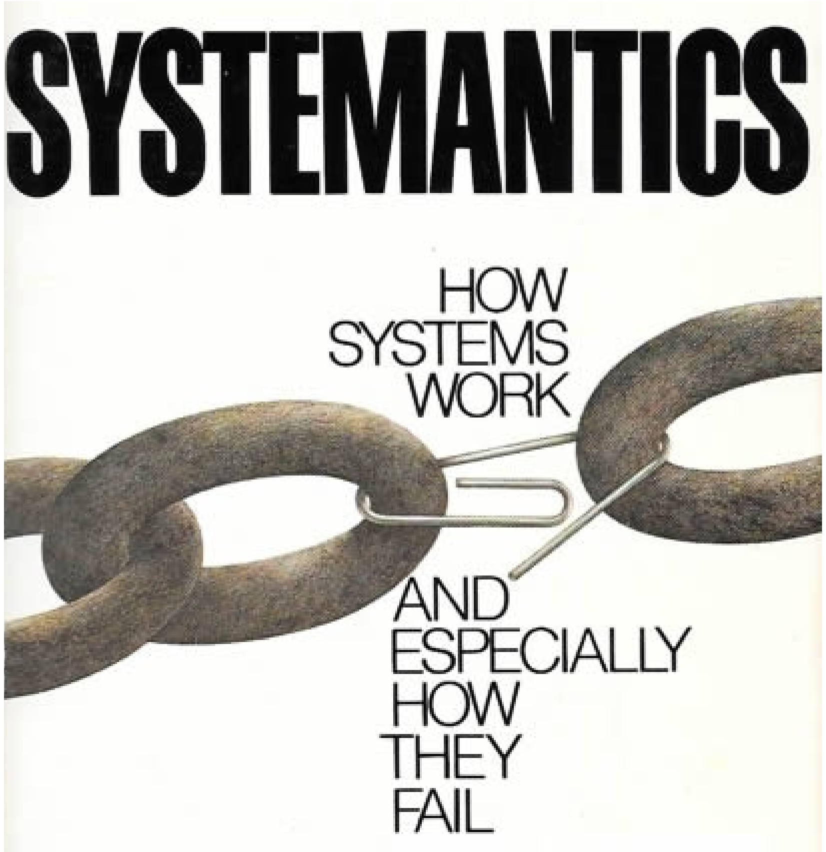
(a) Temperature (pipe rupture scenario)



(b) Pressure (pipe rupture scenario)

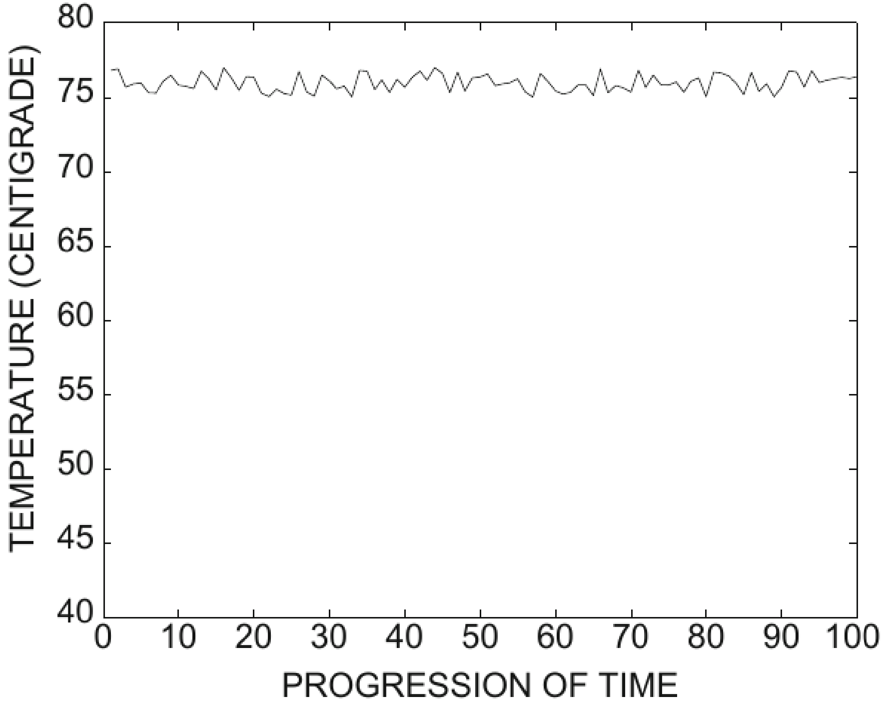
(… but what if sensors fail? …

- “Systems in general work poorly or not at all”
- **“In complex systems, malfunction and even total non-function may not be detectable for long periods, if ever”**

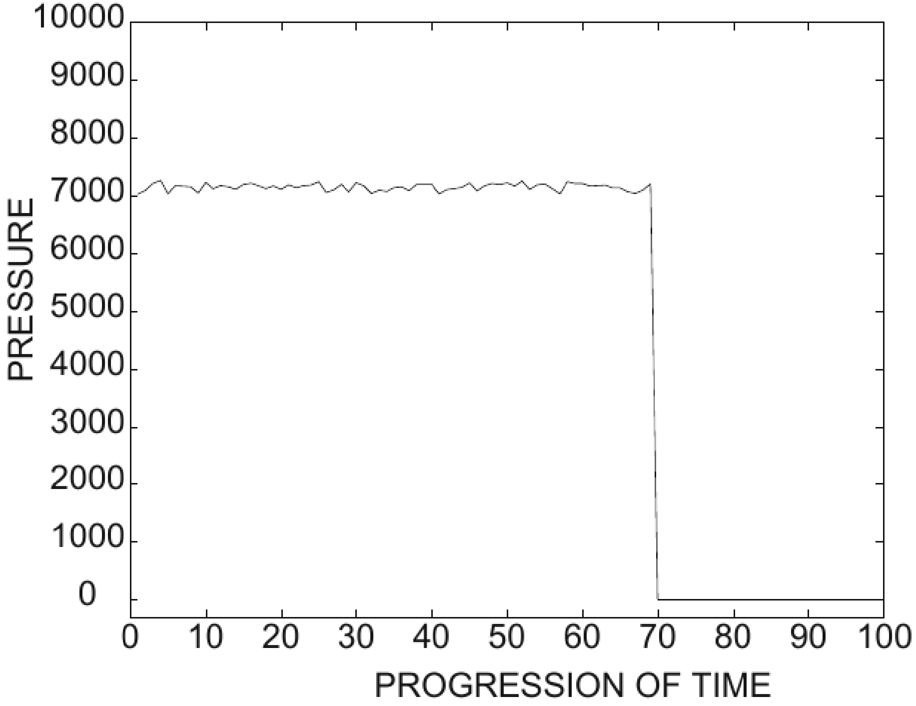


Gall, John. Systemantics: the underground text of systems lore:
how systems really work and especially how they fail. Ann Arbor, MI, 1975.

… can we detect failure? …)



(c) Temperature (sensor failure scenario)



(d) Pressure (sensor failure scenario)

A general scheme for event detection in multivariate time series

- Let $T_1, T_2, \ldots, T_r$ be times at which an event has been observed in the past
- (Offline) Learn coefficients $\alpha_1, \alpha_2, \ldots, \alpha_d$ to distinguish between event times and non-event times
- (Online) Observe series and determine deviation of every stream i at timestamp t as z_t^i
- (Online) Compute composite alarm level $Z_t = \sum_{i=1}^{d} \alpha_i \cdot z_t^i$

Learning discrimination coefficients

$$\alpha_1, \alpha_2, \dots, \alpha_d$$

$$\boxed{Z_t = \sum_{i=1}^{d} \alpha_i \cdot z_t^i}$$

- Average alarm level for events

$$Q^{\text{event}}(\alpha_1, \dots, \alpha_d) = \frac{1}{r} \sum_{i=1}^{r} Z_{T^i}$$

- Average alarm level for non-events (we assume most points are non-events)

$$Q^{\text{normal}}(\alpha_1, \dots, \alpha_d) = \frac{1}{N} \sum_{i=1}^{N} Z_t$$

Learning discrimination coefficients $\alpha_1, \alpha_2, \ldots, \alpha_d$ (cont.)

- For events $$Q^{\text{event}}(\alpha_1, \ldots, \alpha_d) = \frac{1}{r}\sum_{i=1}^{r} Z_{T^i}$$
- For non-events $$Q^{\text{normal}}(\alpha_1, \ldots, \alpha_d) = \frac{1}{N}\sum_{i=1}^{N} Z_t$$

Maximize $$Q^{\text{event}}(\alpha_1, \ldots, \alpha_d) - Q^{\text{normal}}(\alpha_1, \ldots, \alpha_d)$$

subject to $$\sum_{i=1}^{d} \alpha_i^2 = 1$$

Use any off-the-shelf iterative optimization solver

Summary

Things to remember

- Time series forecasting
- Event detection

Exercises for TT27-TT29

- Data Mining, The Textbook (2015) by Charu Aggarwal
  - Exercises 14.10 → 1-6