Speeding Up Association Rules Mining

Mining Massive Datasets

Prof. Carlos Castillo

Topic 14

Universitat Pompeu Fabra Barcelona

Sources

- Data Mining, The Textbook (2015) by Charu Aggarwal (Chapters 4, 5) – slides by Lijun Zhang
- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al. (Chapter 6) - slides
- Data Mining Concepts and Techniques, 3rd edition (2011) by Han et al. (Chapter 6)
- Introduction to Data Mining 2nd edition (2019) by Tan et al. (Chapters 5, 6) – slides ch5, slides ch6

Speeding up candidate generation

Level-wise pruning trick

- Let F_k be the set of frequent k-itemsets
- Let C_{k+1} be the set of (k+1)-candidates
- $I \in C_{k+1}$ is frequent only if all the k-subsets of I are frequent
- Pruning
  - Generate all the k-subsets of I
  - If any one of them does not belong to F_k, then remove I

Candidates generation

- A Naïve Approach
  - Check all the possible combinations of frequent itemsets
- An Example of the Naïve Approach
  - itemsets: {abc} {bcd} {abd} {cde}
  - {abc} + {bcd} = {abcd}
  - {bcd} + {abd} = {abcd}
  - {abd} + {cde} = {abcde}
  - ….

Candidates generation (cont.)

- Introduction of Ordering
  - Items in U have a lexicographic ordering
  - Itemsets can be order as strings
- A Better Approach
  - Order the frequent k-itemsets
  - Merge two itemset if the first k-1 items of them are the same

Candidates generation (cont.)

- Example
  - k-itemsets: {abc} {abd} {bcd}
    - {abc} + {abd} = {abcd}
- k-itemsets: {abc} {acd} {bcd}
  - No (k+1) -candidates
- Early stop is possible
  - Do not need to check {abc} +{bcd} after checking {abc} + {acd}
- Do we miss {abcd}?
  - No, due to the Downward Closure Property

Improving computation of support

Naïve support counting

- **Naïve counting:**

  For each candidate $I_i \in C_{k+1}$

  For each transaction T_j in T

  Check whether I_i appears in T_j

- Limitation
  - Inefficient if both $|C_{k+1}|$ and $|T|$ are large

Support counting with a data structure

- A Better Approach
  - Organize the candidate patterns in C_{k+1} in a data structure
- Use the data structure to accelerate counting
  - Each transaction in T_i examined against the subset of candidates in C_{k+1} that might contain T_i

Data structured for support counting based on hashing

Naïve counting:

For each $I_i \in C_{k+1}$

  For all $T_j \in T$

    If $I_i \subseteq T_j$

      Add to $sup(I_i)$

Hashed counting:

For each $T_j \in T$

  For $I_i \in hashbucket(T_j, C_{k+1})$

    If $I_i \subseteq T_j$

      Add to $sup(I_i)$

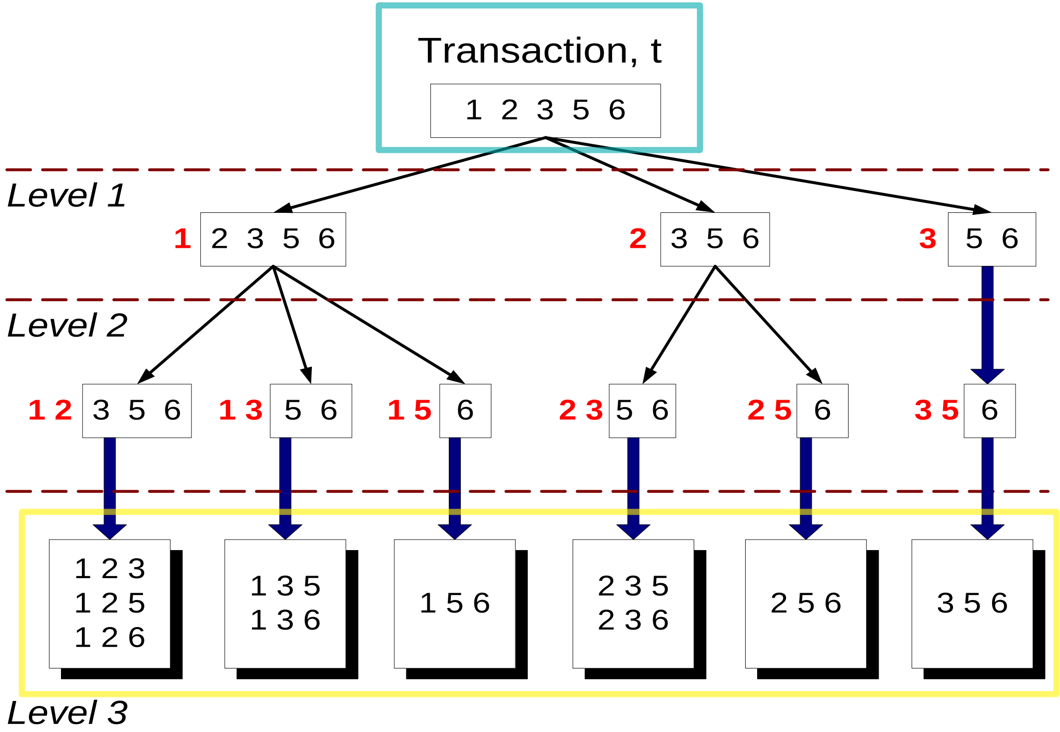
Which candidates are relevant?

Imagine 15 candidate itemsets of length 3:

```
{1 4 5}, {1 2 4}, {4 5 7},
{1 2 5}, {4 5 8}, {1 5 9},
{1 3 6}, {2 3 4}, {5 6 7},
{3 4 5}, {3 5 6}, {3 5 7},
{6 8 9}, {3 6 7}, {3 6 8}
```

Now, suppose we look for this transaction:

```
{1 2 3 5 6}
```



Here we depict only the candidates that appear in the transaction (10 out of 15)

Hash tree for itemsets in C_{k+1}

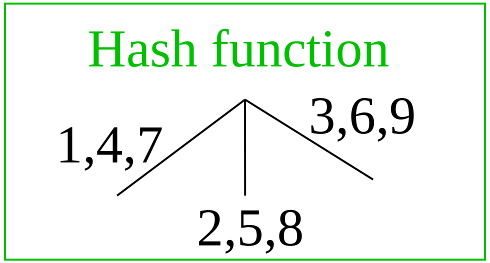
- A tree with fixed degree `r`
- Each itemset in C_{k+1} is stored in a leaf node
- All internal nodes use a hash function to map items to one of the `r` branches (can be the same for all internal nodes)
- All leaf nodes contain a lexicographically sorted list of up to `max_leaf_size` itemsets

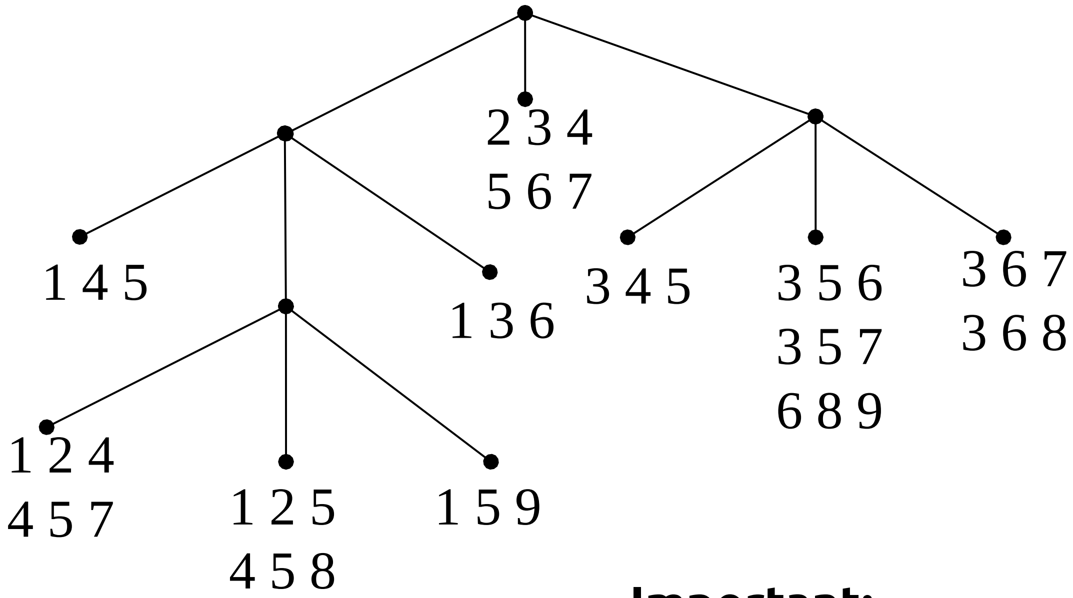
Example hash tree

`r=3  max_leaf_size=3`

Candidate itemsets

```
{1 4 5}, {1 2 4}, {4 5 7},
{1 2 5}, {4 5 8}, {1 5 9},
{1 3 6}, {2 3 4}, {5 6 7},
{3 4 5}, {3 5 6}, {3 5 7},
{6 8 9}, {3 6 7}, {3 6 8}
```

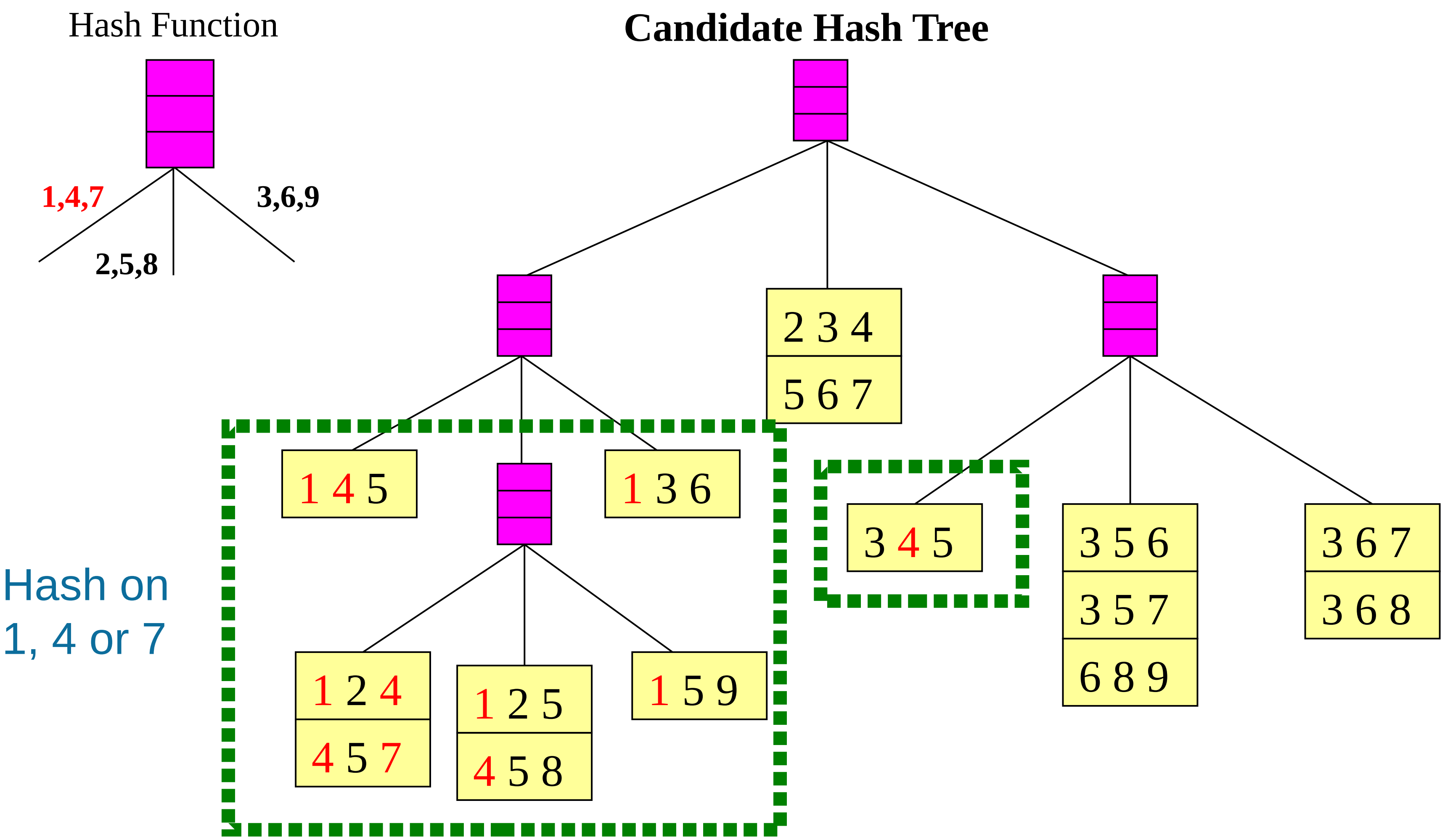




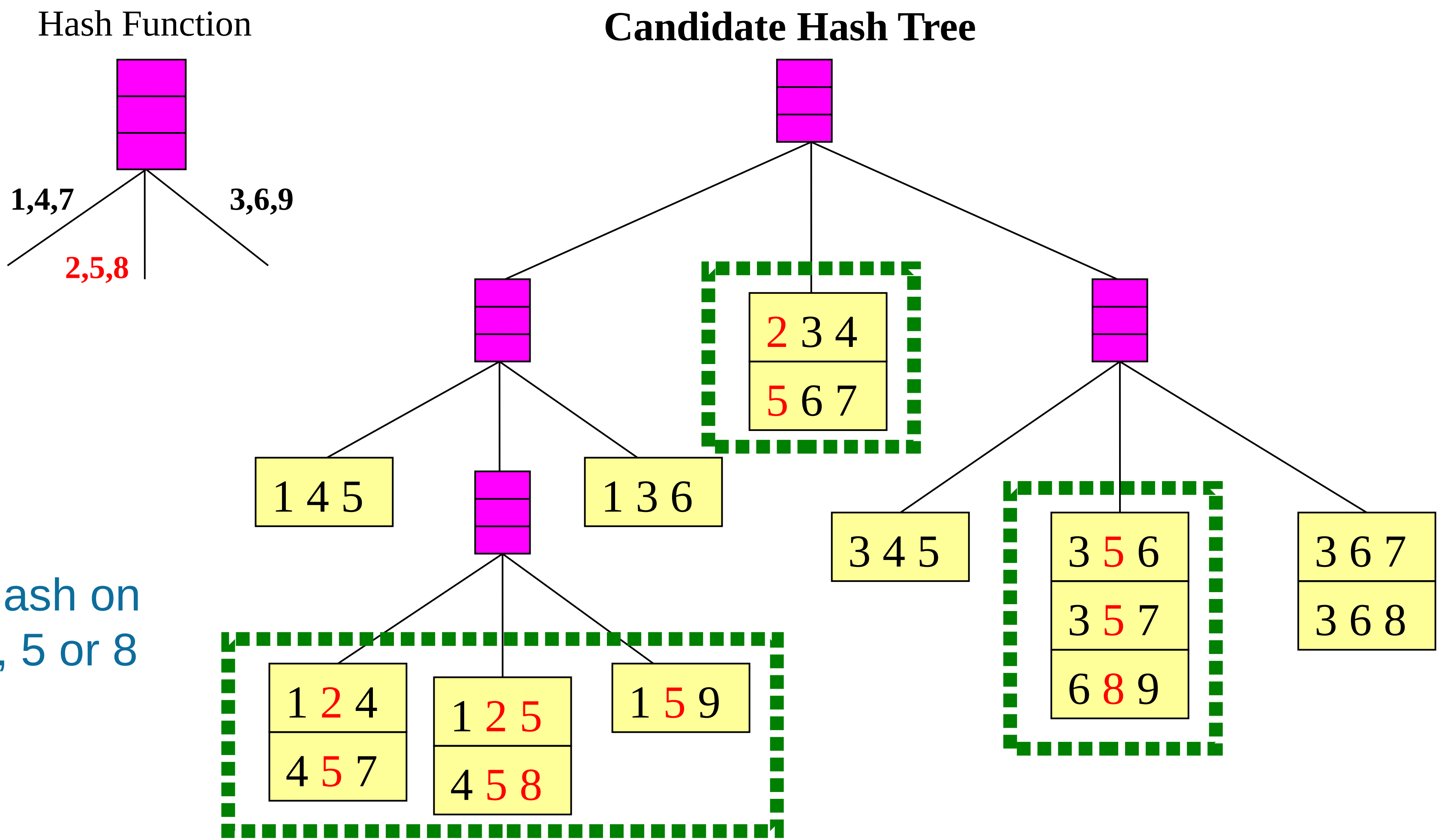
Important: itemsets are sorted!

This example from: Introduction to Data Mining 2nd edition (2019) by Tan et al. (Chapter 5) – slides ch5

Example hash tree (cont.)

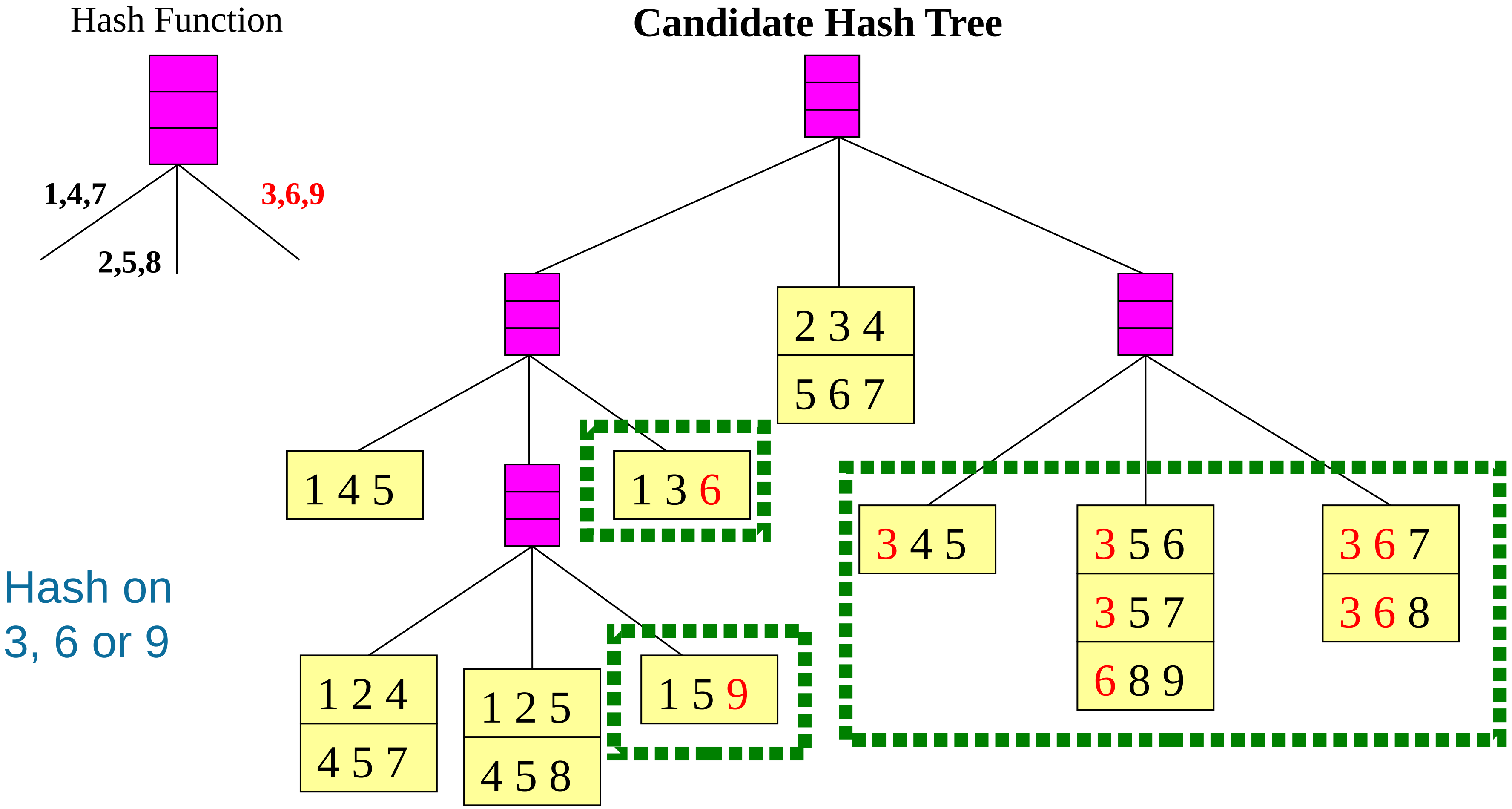


Example hash tree (cont.)

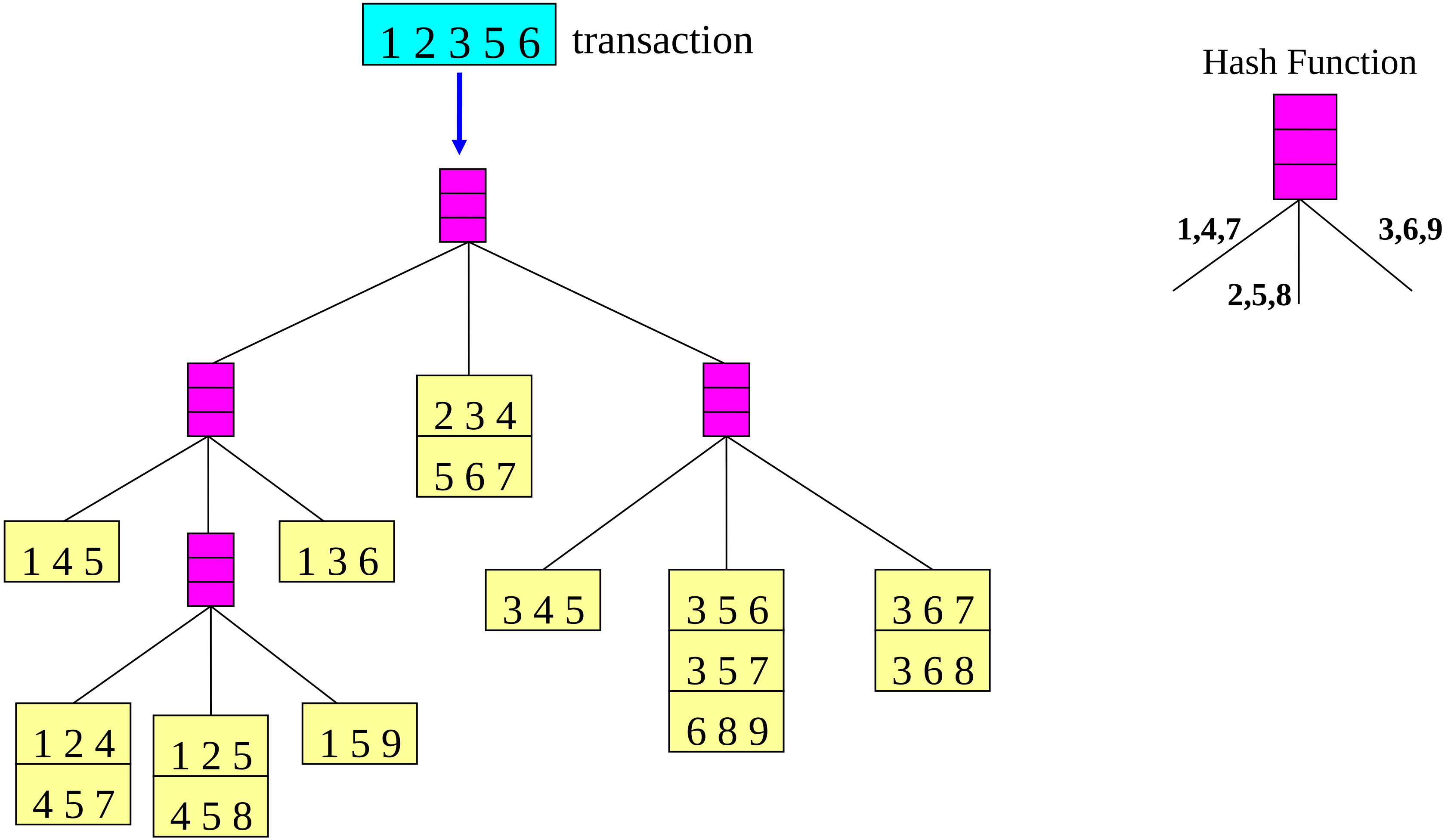


Hash on
2, 5 or 8

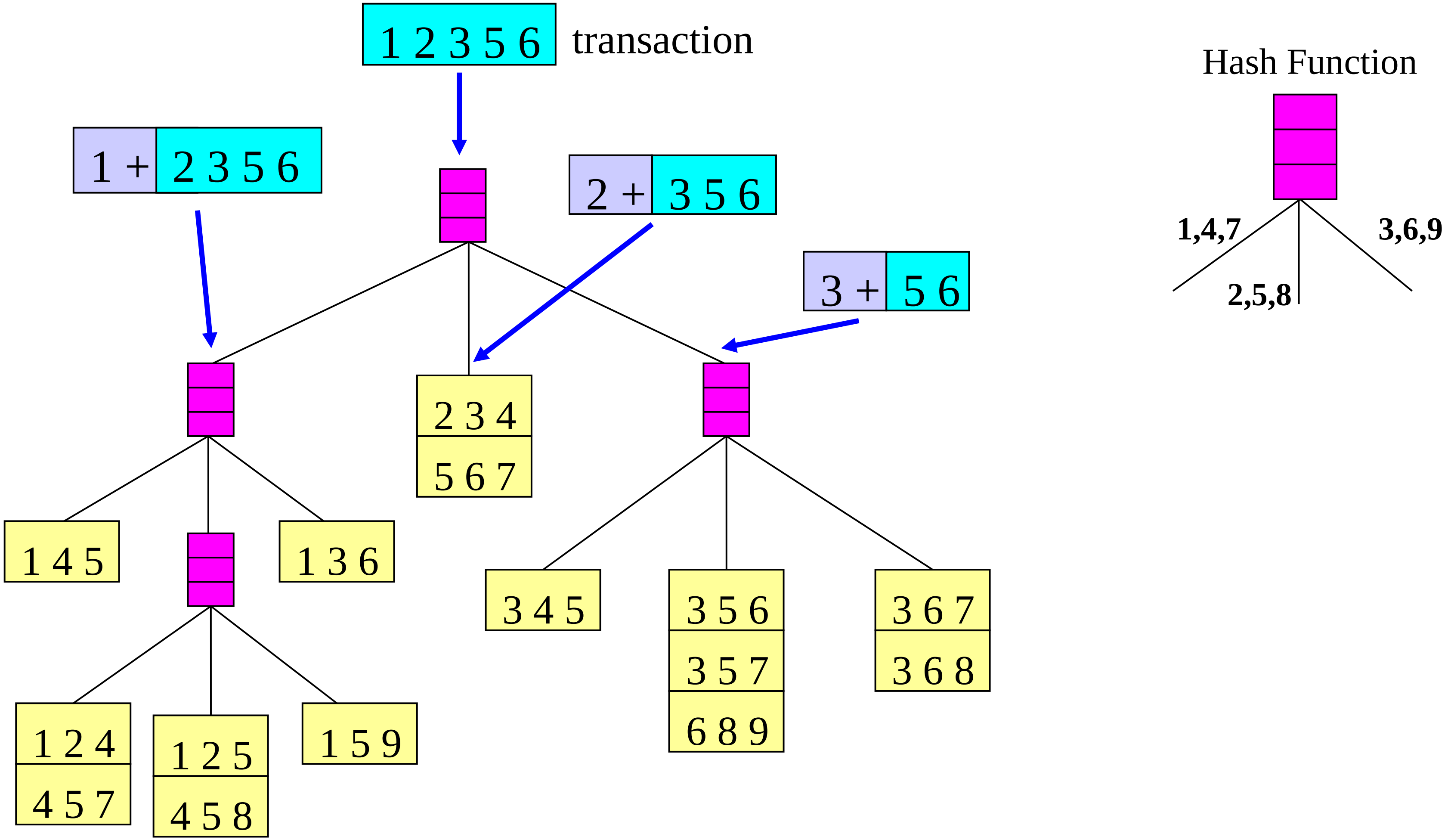
Example hash tree (cont.)



Checking which candidates might be in a transaction



Checking which candidates might be in a transaction



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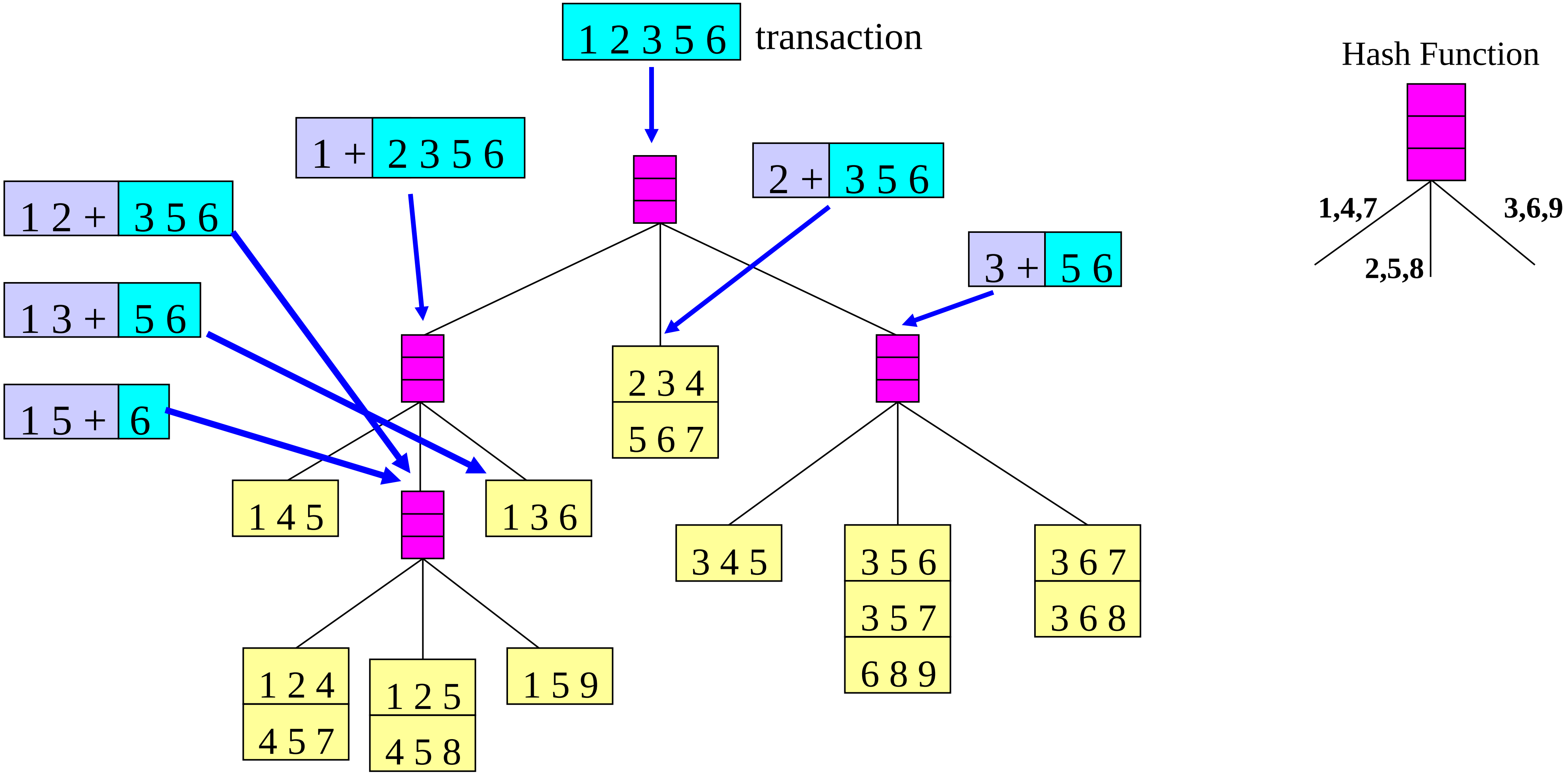

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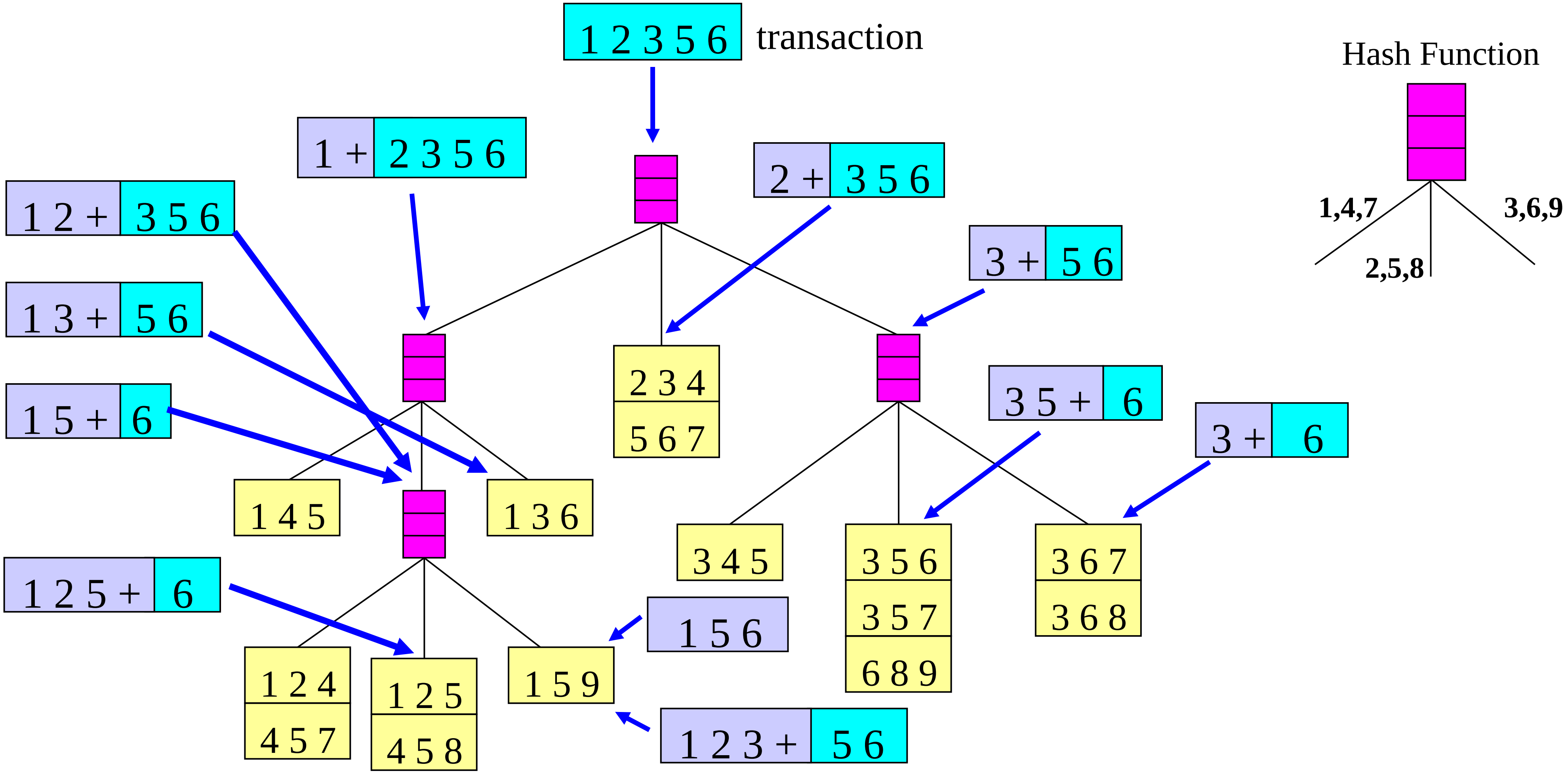
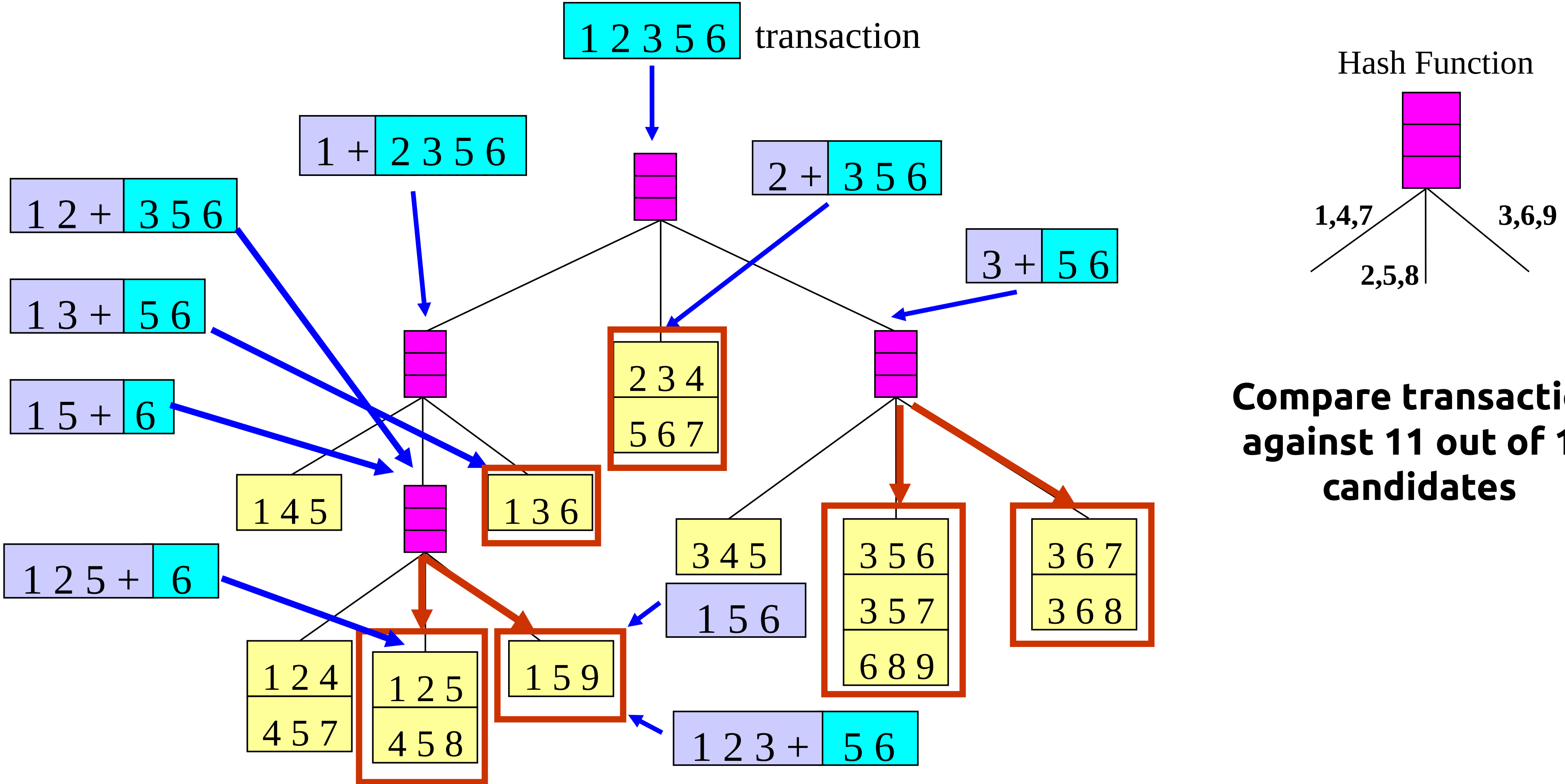


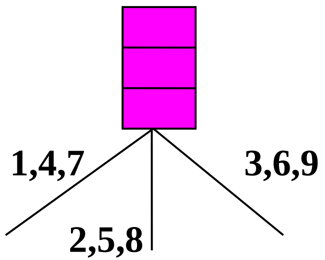
Checking which candidates might be in a transaction

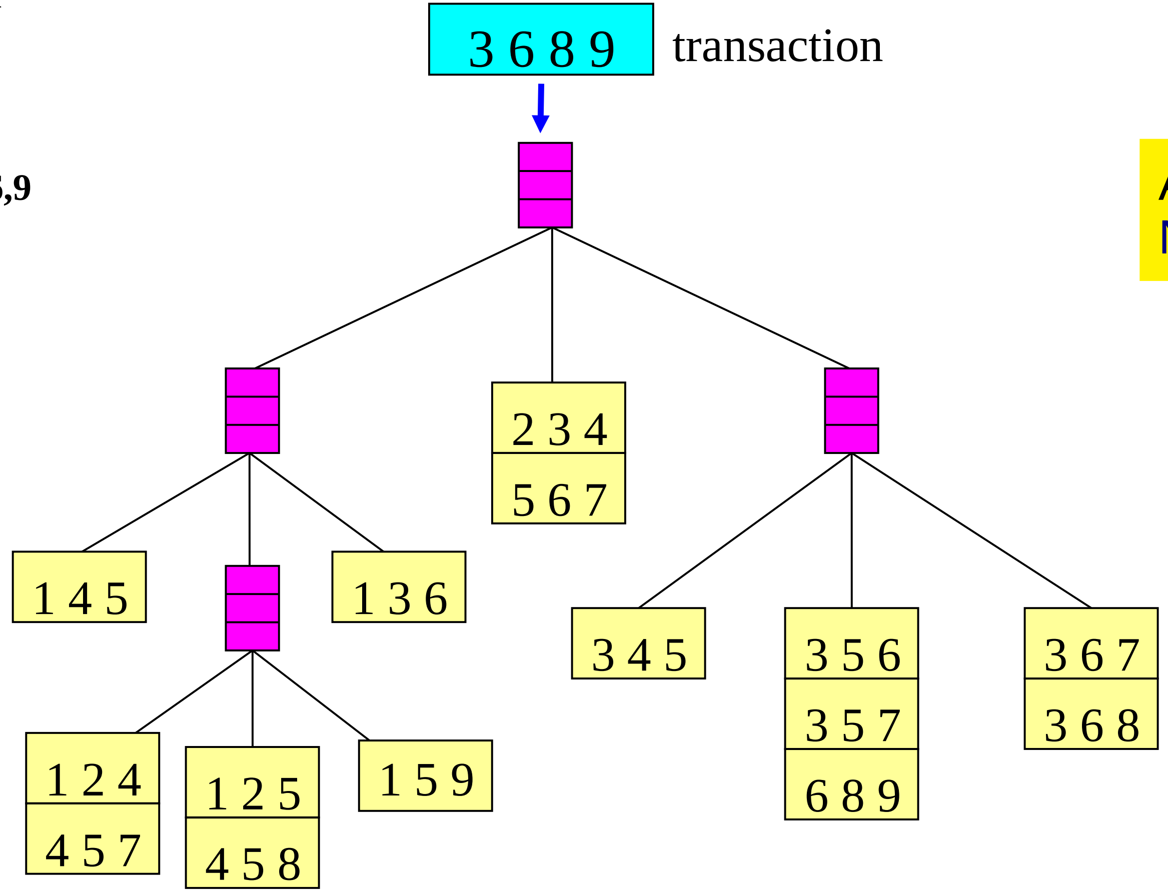


Compare transaction against 11 out of 15 candidates

Exercise: Use the hash tree to determine which candidates might be in this transaction

Hash Function





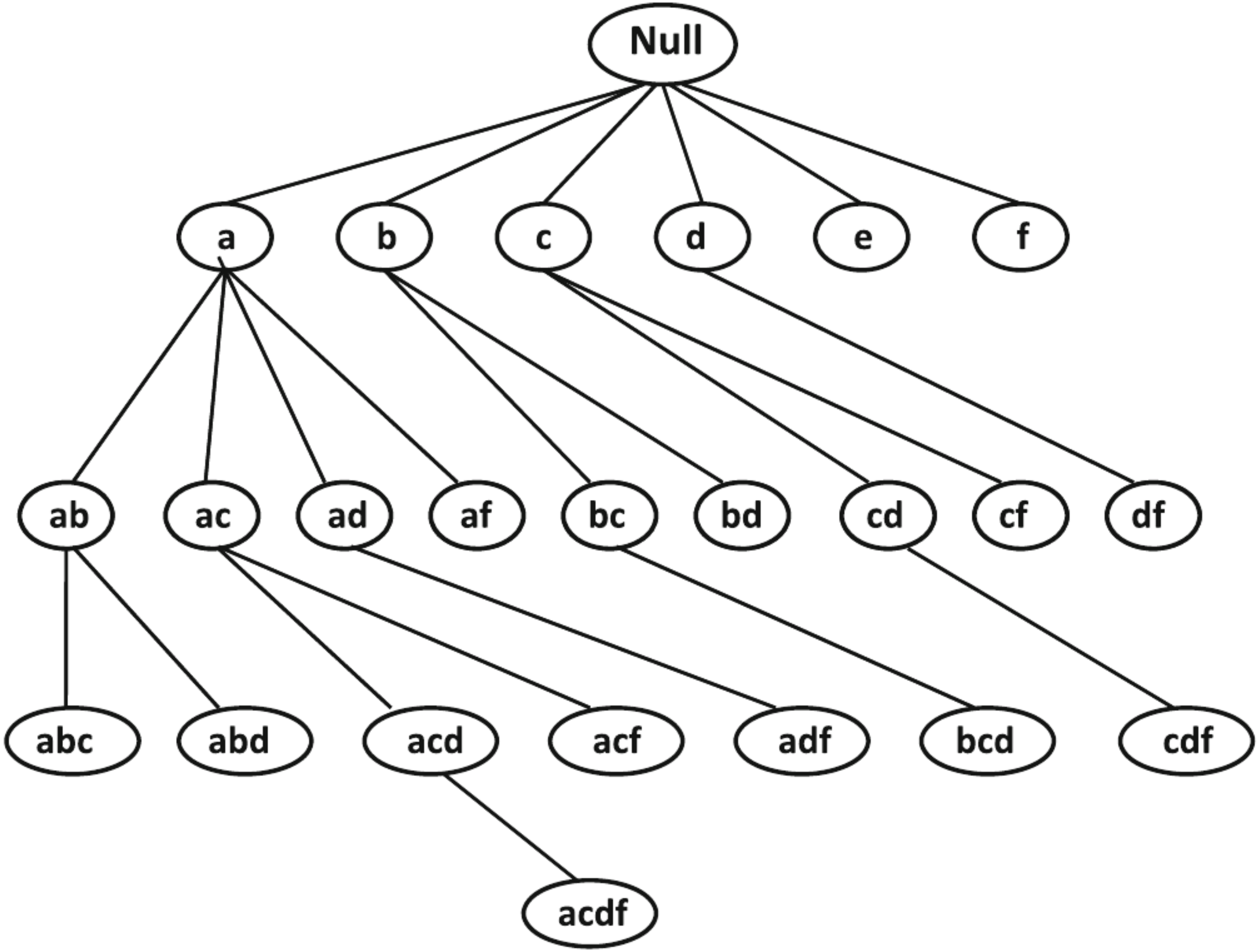
Answer in
Nearpod Draw-it

Enumeration-tree algorithms: Lexicographic tree

- There is a node in the tree for each frequent itemset
- The root of the tree contains the null itemset
- If $I = \{i_1, i_2, \ldots, i_k\}$ then the parent of I in the tree is $\{i_1, i_2, \ldots, i_{k-1}\}$

Example

Note that, unlike the lattice, a parent can only be extended with an item that is lexicographically larger



Enumeration tree algorithm

Algorithm *GenericEnumerationTree*(Transactions: $\mathcal{T}$,
Minimum Support: *minsup*)
begin
  Initialize enumeration tree $\mathcal{ET}$ to single *Null* node;
  while any node in $\mathcal{ET}$ has not been examined **do begin**
    Select one of more unexamined nodes $\mathcal{P}$ from $\mathcal{ET}$ for examination;
    Generate candidates extensions $C(P)$ of each node $P \in \mathcal{P}$;
    Determine frequent extensions $F(P) \subseteq C(P)$ for each $P \in \mathcal{P}$ with support counting;
    Extend each node $P \in \mathcal{P}$ in $\mathcal{ET}$ with its frequent extensions in $F(P)$;
  end
  return enumeration tree $\mathcal{ET}$;
end

Enumeration-tree-based implementation of Apriori

- Apriori constructs the enumeration tree in a breadth-first manner
- Apriori generates candidate (k+1)-itemsets by merging two frequent k-itemsets of which the first k-1 items are the same ⇒ extension in the enumeration-tree

Summary

Things to remember

- Support and confidence on a rule
- Downward closure property
  - every subset of a frequent itemset is also frequent
  - hence, if an itemset X has a subset that is not frequent, X cannot be frequent
- Methods for candidate generation, pruning
- Algorithms for fast support computation

Exercises for this topic

- Data Mining, The Textbook (2015) by Charu Aggarwal
  - Exercises 4.9 → 9-10
- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al.
  - Exercises 6.2.7 → 6.2.5 and 6.2.6
- Introduction to Data Mining 2nd edition (2019) by Tan et al.
  - Exercises 5.10 → 9-12