Itemsets

Mining Massive Datasets
Carlos Castillo
Topic 05

upf. Universitat Pompeu Fabra Barcelona

Sources

- Data Mining, The Textbook (2015) by Charu Aggarwal (Chapters 4, 5) – slides by Lijun Zhang
- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al. (Chapter 6) - slides
- Data Mining Concepts and Techniques, 3rd edition (2011) by Han et al. (Chapter 6)
- Introduction to Data Mining 2nd edition (2019) by Tan et al. (Chapters 5, 6) – slides ch5, slides ch6

Market Basket Analysis

- Understand customers
  - Purchasing habits, sensitivity to price, promotions
- Understand products
  - Co-purchases, fast/slow movers
- Take action: promotions, store layout, …

https://paginas.fe.up.pt/~ec/files_1112/week_04_Association.pdf

Transactions contain items, which can be grouped into itemsets

- Transactions
  - Sets of items bought by customers
- The Goal
  - Determine associations between groups of items bought by customers
- Quantification of the Level of Association
  - Frequencies of sets of items
- The Discovered Sets of Items
  - Large itemsets, frequent itemsets, or frequent patterns

“Transaction” is a general concept

Items	Transactions
Groceries	Grocery cart
University courses	Virtual shopping cart
Guests	Party
Movies	Actor
Symptoms	Patient
Streamed songs	Streaming subscriber
Words	Document
Liked photos	Instagram account

https://web.stanford.edu/class/cs102/lecturenotes/DataMining.pdf

Applications

- Supermarket Data
  - Target marketing, shelf placement
- Text Mining
  - Identifying co-occurring terms
- Generalization to Dependency-oriented Data Types
  - Web log analysis, software bug detection
- Other Major Data Mining Problems
  - Clustering, classification, and outlier analysis

Association rules

- Generated from **frequent itemsets**
- Formulation X⇒Y
  - {Soy latte} ⇒ {Brown Sugar}
  - {Kale, Quinoa} ⇒ {Almond milk}
- Applications
  - Promotion
  - Shelf placement
- Conditional Probability $P(Y|X) = \frac{P(X \cap Y)}{P(X)}$

Association rule mining

- U is a set of d items
- T is a set of n transactions $T_1, T_2, \ldots, T_n$ with $T_i \subseteq U$
- Itemset: a set of items
- k-itemset: a set of k items

  How many different k-itemsets exist? 2^k

Binary representation of a transaction

tid	Set of items	Binary representation
1	Bread, Jam, Juice	110010
2	Tofu, Juice, Tomatoes	000111
3	Bread, Strawberries, Tofu, Juice	101110
4	Tofu, Juice, Tomatoes	000111
5	Strawberries, Juice, Tomatoes	001011

Definitions

- **Support of itemset I**, written $sup(I)$:

  the fraction of transactions in the database $T = \{T_1 \ldots T_n\}$ that contain I as a subset.

- Frequent itemset mining with support minsup:

  Given a set of transactions $T = \{T_1, \ldots, T_n\}$,

  where $T_i \subseteq U$,

  find all itemsets I_j such that sup(I_j) ≥ minsup

Example

tid	Set of items	Binary representation
1	Bread, Jam, Juice	110010
2	Tofu, Juice, Tomatoes	000111
3	Bread, Strawberries, Tofu, Juice	101110
4	Tofu, Juice, Tomatoes	000111
5	Strawberries, Juice, Tomatoes	001011

- sup({Bread, Juice}) = 2/5 = 0.4
- sup({Strawberries, Tomatoes}) = 1/5 = 0.2
- If minsup=0.3, {Bread, Juice} is a frequent itemset

Try it!

Database

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

- Write the support of every 2-itemset or 3-itemset occurring in this database
- Indicate which are frequent itemsets if minsup = 1/2

Properties

- The smaller minsup is, the larger the number of frequent itemsets

Support monotonicity property:

$if\ J \subseteq I,\ sup(J) \geq sup(I)$ WHY?

Properties

- The smaller minsup is, the larger the number of frequent itemsets
- **Support monotonicity property**:
  $$if\ J \subseteq I,\ sup(J) \geq sup(I)$$
- Confusingly, some authors refer to this as the support anti-monotonicity property
- **Downward closure property**: every subset of a *frequent* itemset is also *frequent*

Closed itemset

An itemset is **closed** if all itemsets containing it are less frequent

tid	Set of items
1	Bread, Jam, Juice
2	Tofu, Juice, Tomatoes
3	Bread, Strawberries, Tofu, Juice
4	Tofu, Juice, Tomatoes
5	Strawberries, Juice, Tomatoes

Find a closed itemset in this set of transactions

Closed itemset

An itemset is **closed** if all itemsets containing it are less frequent

tid	Set of items
1	Bread, Jam, Juice
2	Tofu, Juice, Tomatoes
3	Bread, Strawberries, Tofu, Juice
4	Tofu, Juice, Tomatoes
5	Strawberries, Juice, Tomatoes

- Example closed itemset: {Bread, Juice}
- sup({Bread, Juice}) = 2

  sup({Bread, Juice, Jam}) = 1

  sup({Bread, Juice, Strawberries}) = 1

  sup({Bread, Juice, Tofu}) = 1

Maximal itemset

An itemset is **maximal** if
- it is closed and
- it has support ≥ minsup

tid	Set of items
1	Bread, Jam, Juice
2	Tofu, Juice, Tomatoes
3	Bread, Strawberries, Tofu, Juice
4	Tofu, Juice, Tomatoes
5	Strawberries, Juice, Tomatoes

Try it!

- Find the three **maximal** frequent patterns at minsup=0.4
- Tip: first find all frequent patterns at minsup=0.4
  - The total number is 11 (?)

Maximal itemset

An itemset is **maximal** if
- it is closed and
- it has support ≥ minsup

tid	Set of items
1	Bread, Jam, Juice
2	Tofu, Juice, Tomatoes
3	Bread, Strawberries, Tofu, Juice
4	Tofu, Juice, Tomatoes
5	Strawberries, Juice, Tomatoes

- **Maximal** frequent patterns at minsup=0.4

  {Bread, Juice}, {Strawberries, Juice}, {Tofu, Juice, Tomatoes}

- Frequent patterns at minsup=0.4
  - The total number is 11 (?)
  - All of them are subsets of maximal frequent patterns

Maximal itemset

An itemset is **maximal** if
- it is closed and
- it has support ≥ minsup

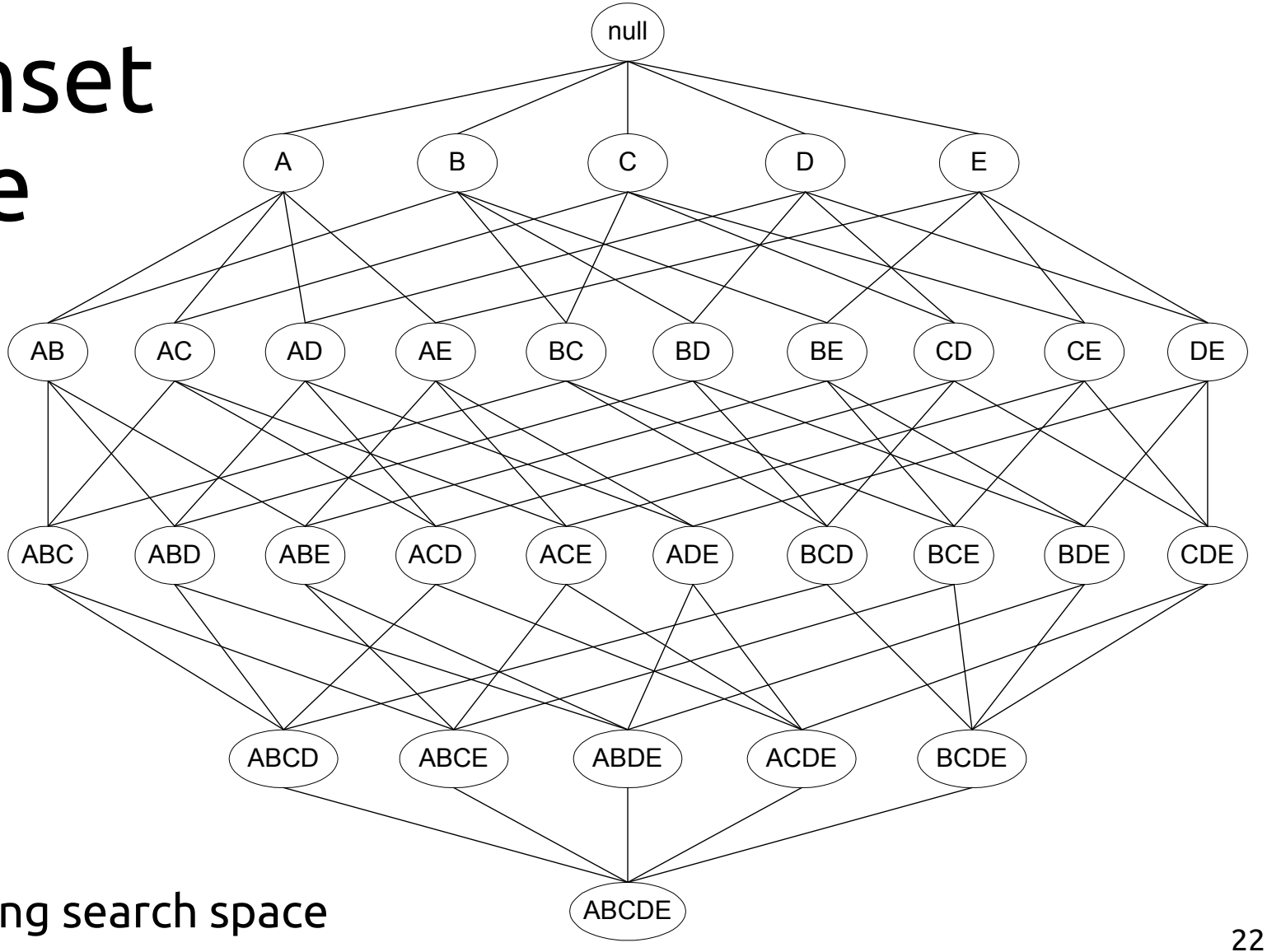
tid	Set of items
1	Bread, Jam, Juice
2	Tofu, Juice, Tomatoes
3	Bread, Strawberries, Tofu, Juice
4	Tofu, Juice, Tomatoes
5	Strawberries, Juice, Tomatoes

Maximal patterns

{Bread, Juice}, {Strawberries, Juice}, {Tofu, Juice, Tomatoes}

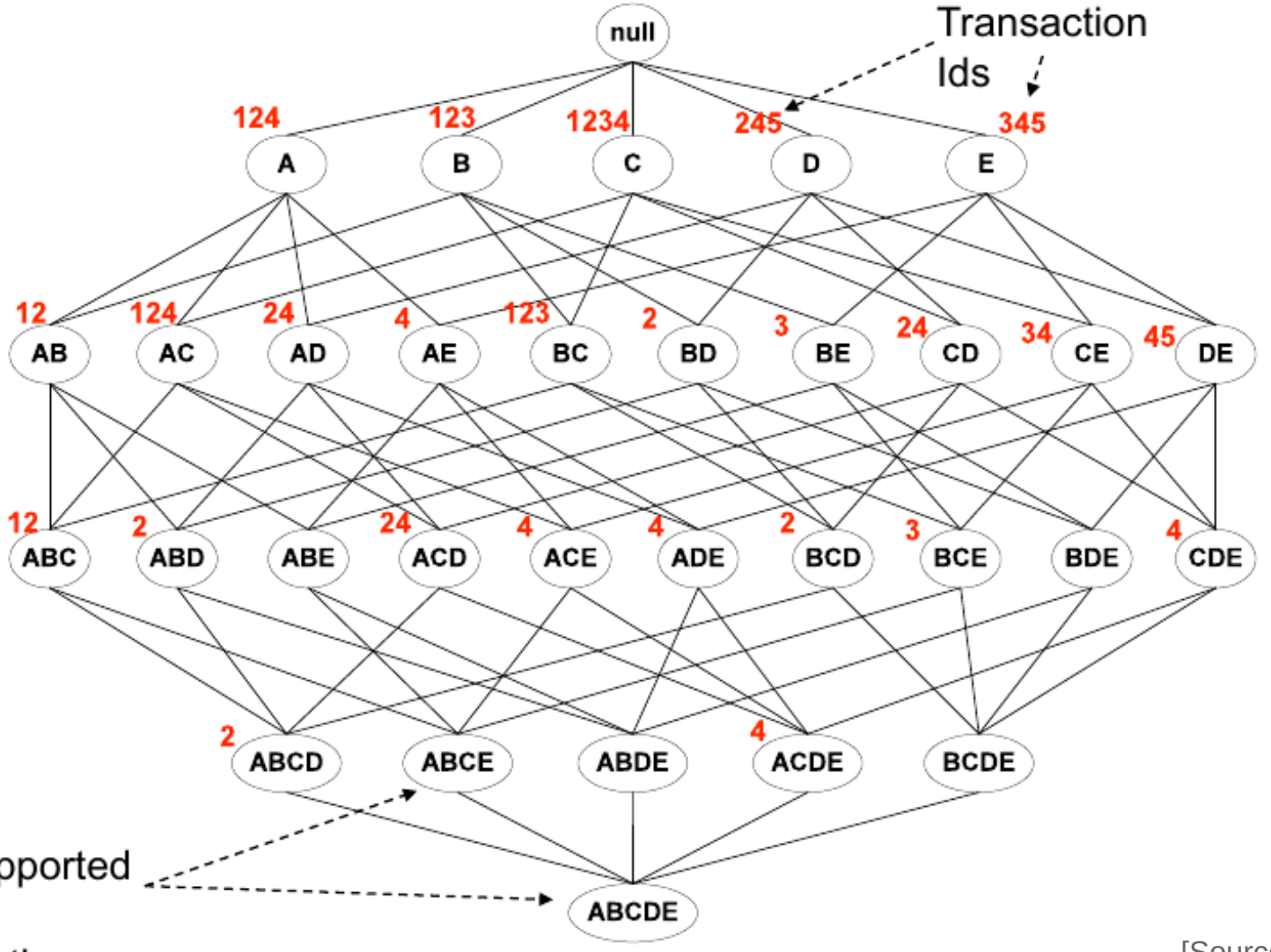
… are **condensed** representations of frequent patterns, but do not retain information about the support of their subsets.

The itemset lattice



$2^{|U|}$ nodes representing search space

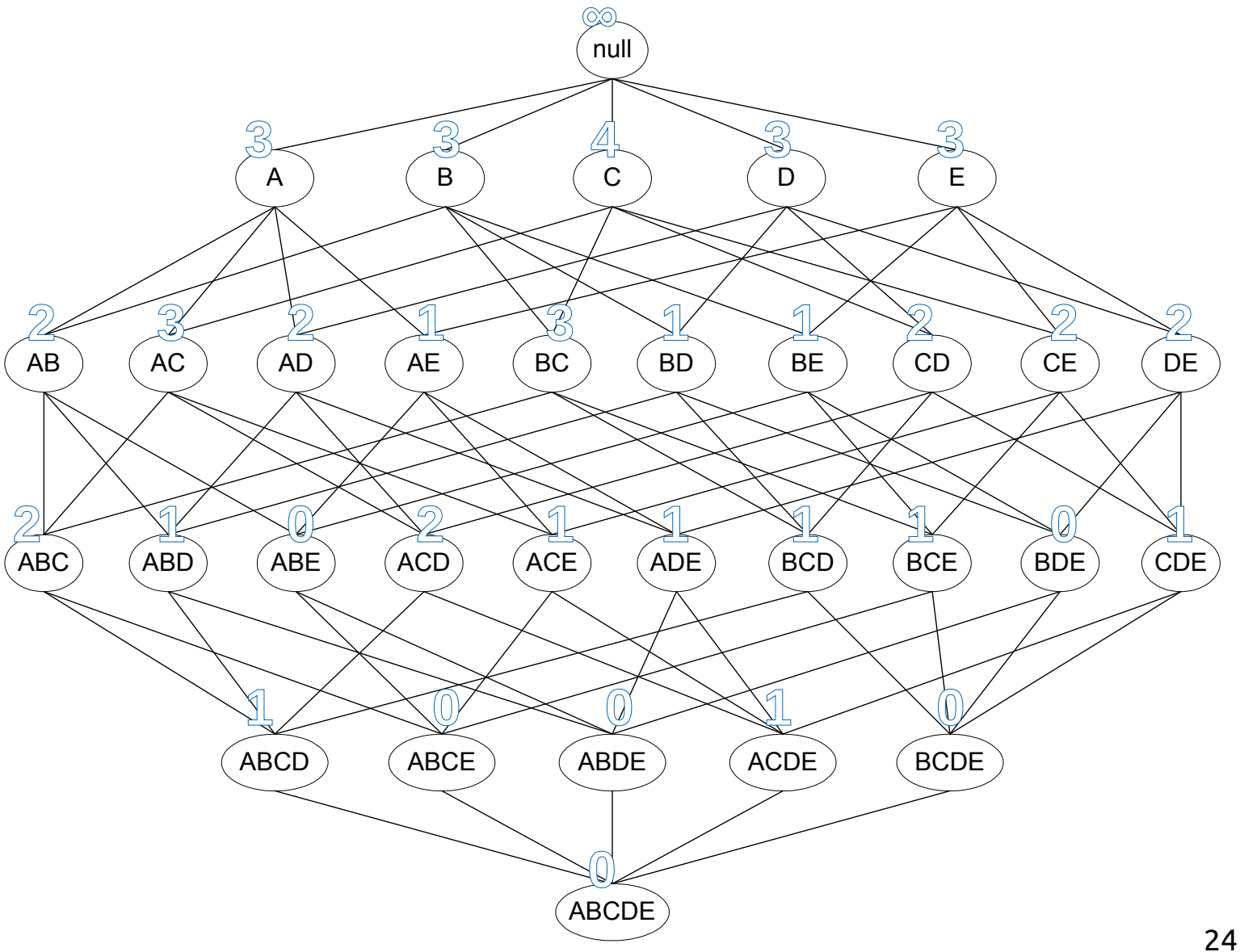
TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



[Source]

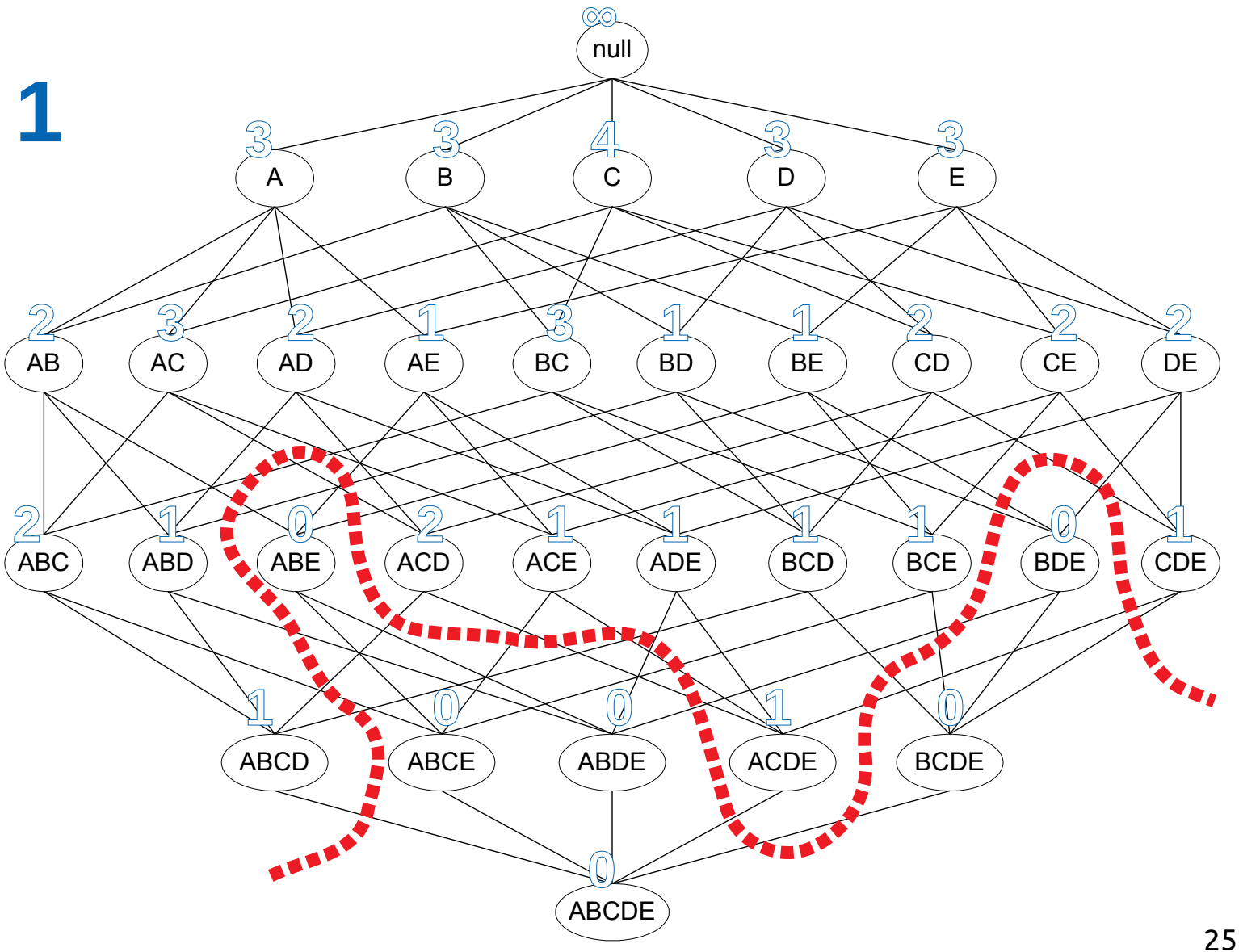
Support of each itemset

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



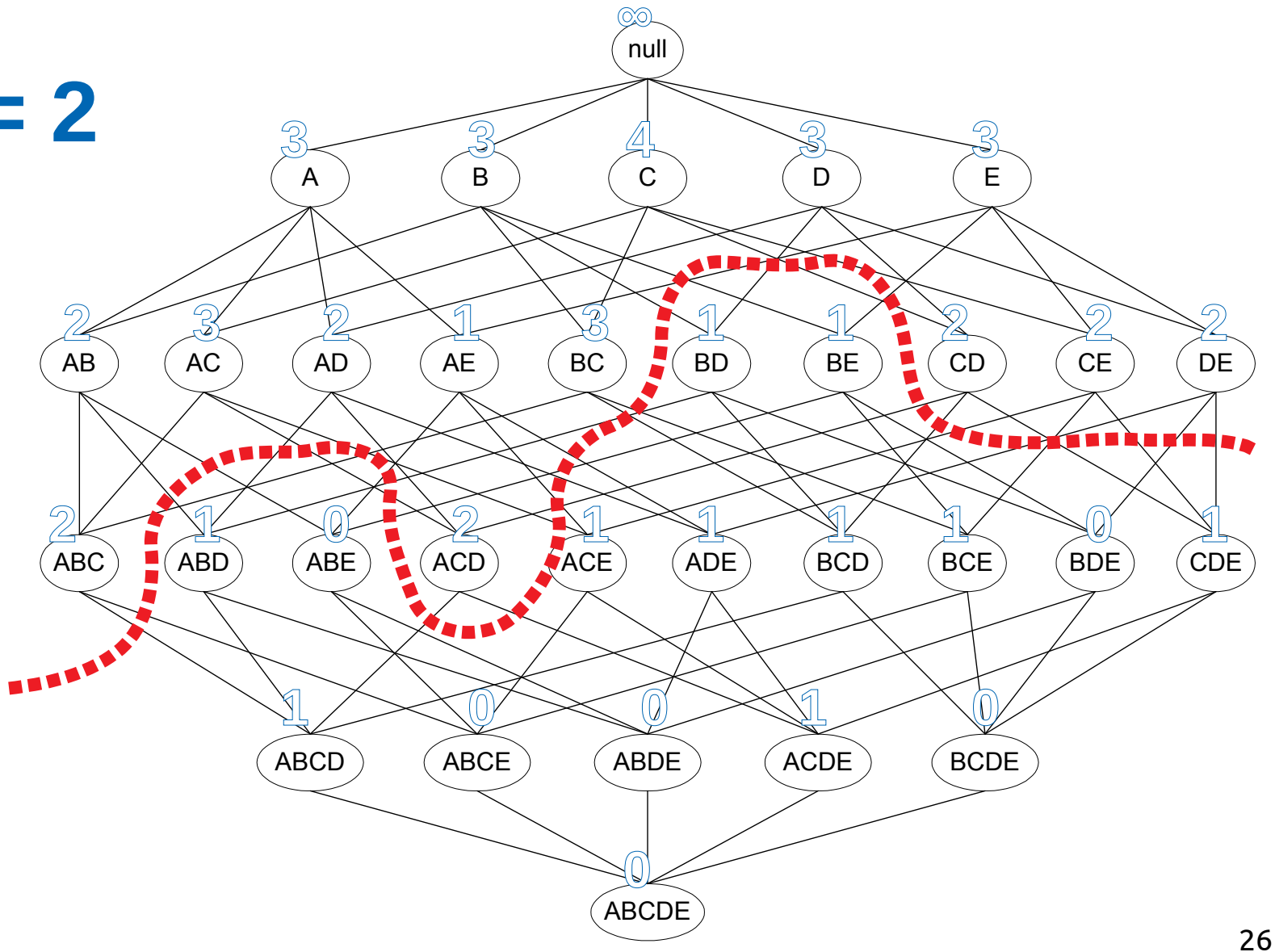
minsup = 1

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



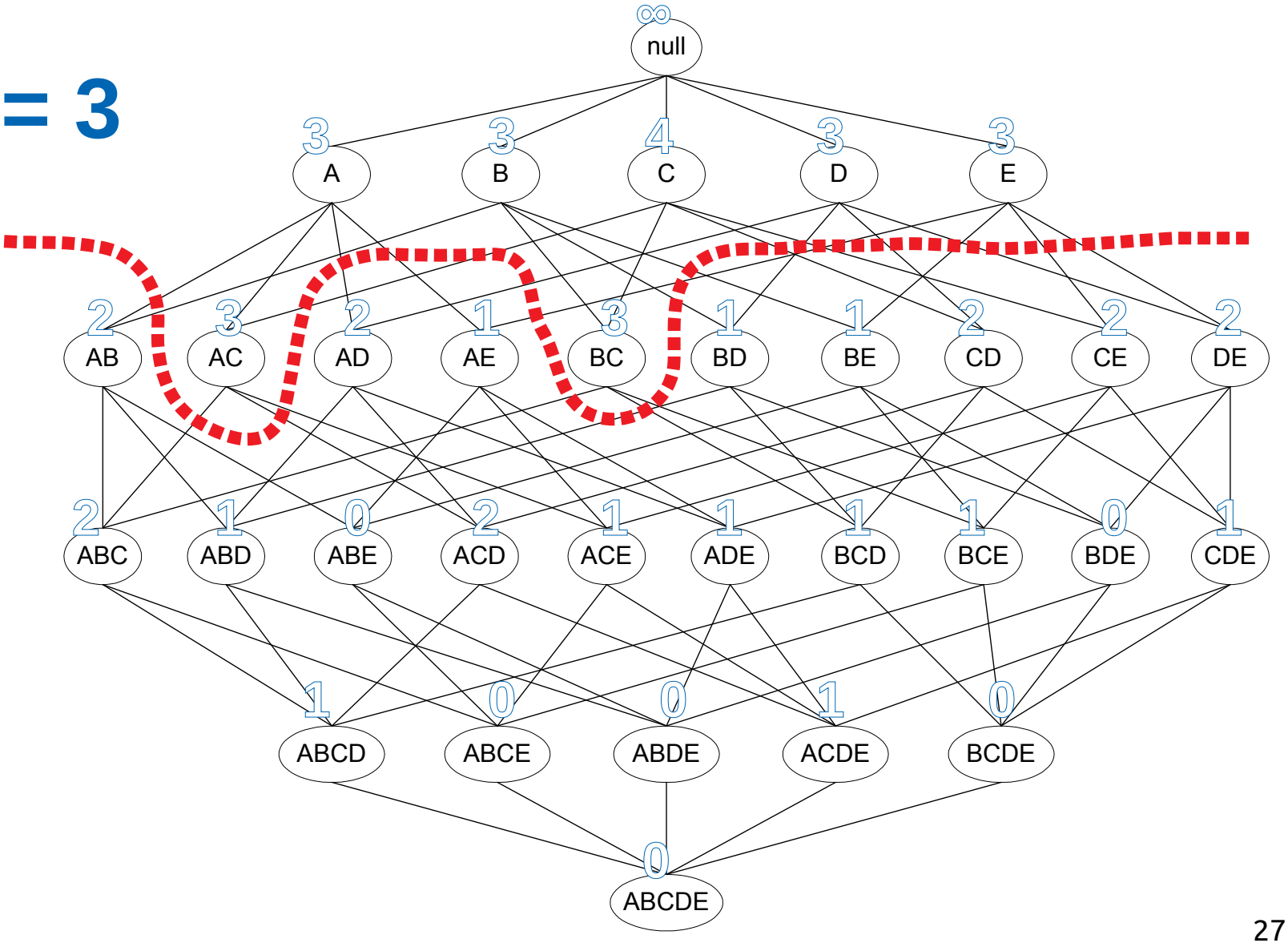
minsup = 2

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



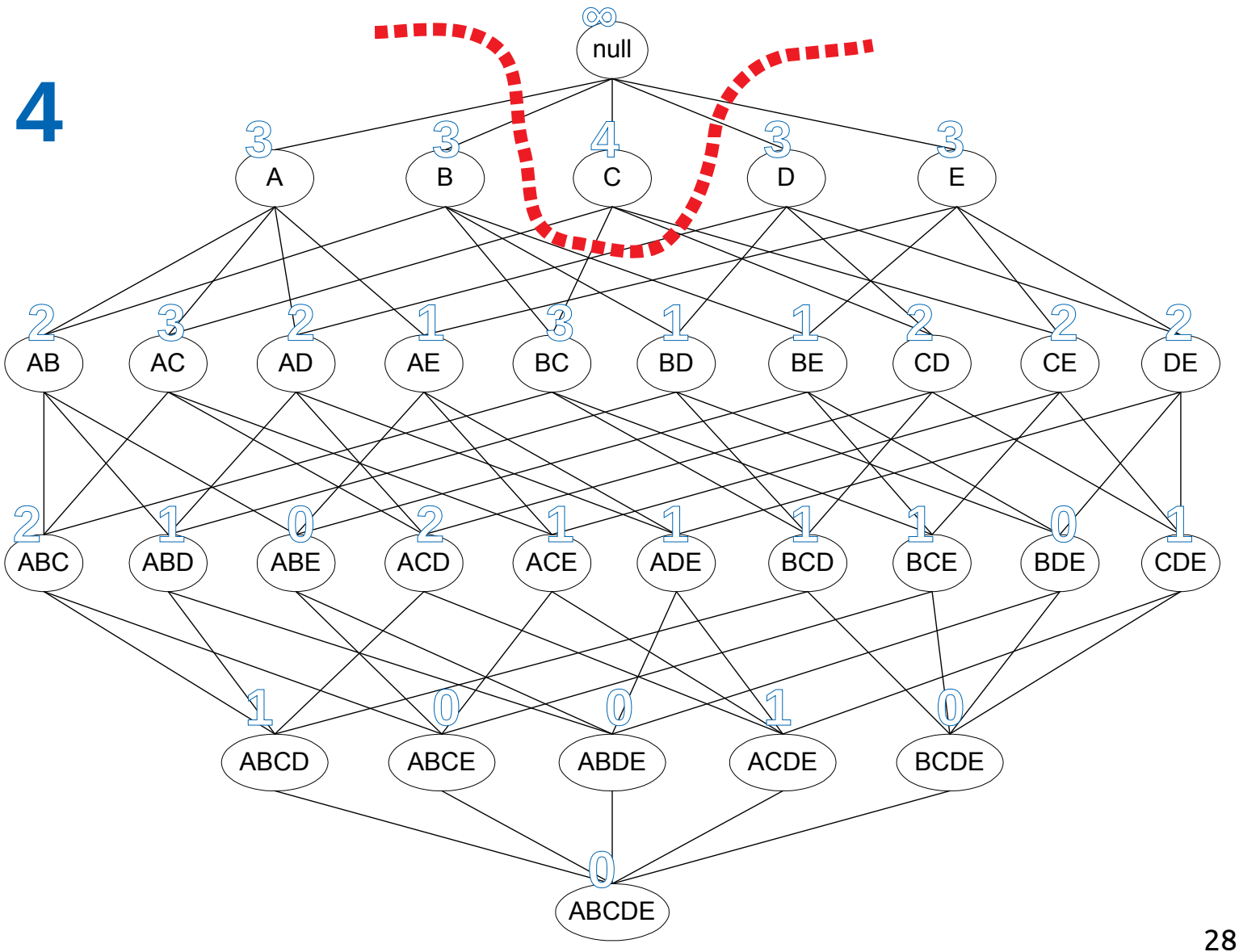
minsup = 3

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



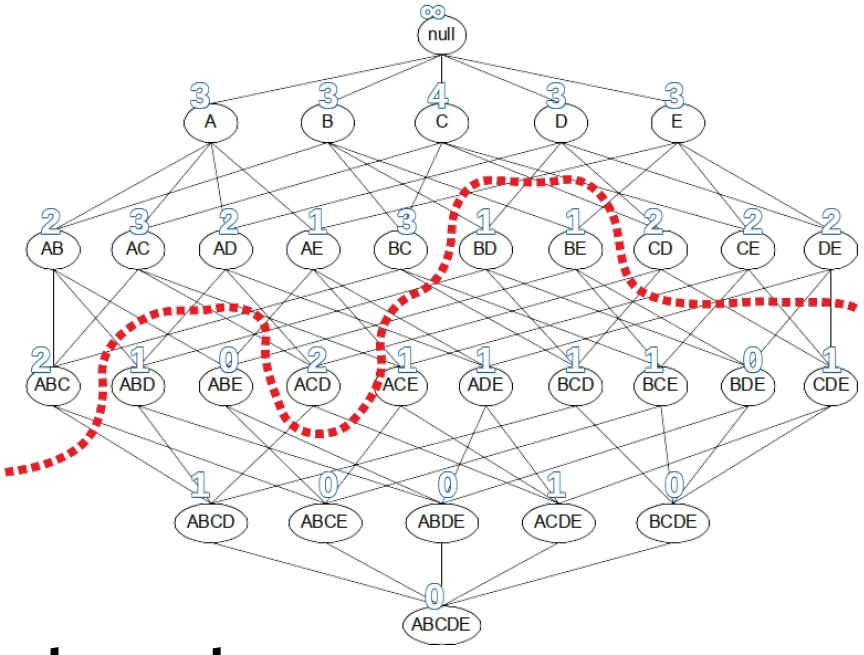
minsup = 4

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



The border is a graph cut and ...

- All itemsets **above** the border are **frequent**
- All itemsets **below** the border are **not frequent**
- All **maximal** frequent itemsets are adjacent to the border
- Any border respects the **downward closure** property



Association rules

What is a rule

- A rule is of the form X⇒Y

  X and Y are itemsets

- X is the antecedent, Y is the consequent
- The **confidence** of the rule is:

$$\mathrm{conf}(X \Rightarrow Y) = \frac{\sup(X \cup Y)}{\sup(X)}$$

Confidence of a rule

- The **confidence** of the rule $X \Rightarrow Y$ is:

$$\text{conf}(X \Rightarrow Y) = \frac{\text{sup}(X \cup Y)}{\text{sup}(X)}$$

- This is the conditional probability of X *U* Y occurring in a transaction, given that X occurs in the transaction

Confidence of a rule (cont.)

tid	Set of items
1	Bread, Jam, Juice
2	Tofu, Juice, Tomatoes
3	Bread, Strawberries, Tofu, Juice
4	Tofu, Juice, Tomatoes
5	Strawberries, Juice, Tomatoes

$$\text{conf}(\{\text{tofu}, \text{juice}\} \Rightarrow \{\text{tomatoes}\}) = ?$$

Lift of a rule

- The **lift** of the rule $X \Rightarrow Y$ is:

$$\text{lift}(X \Rightarrow Y) = \frac{\sup(X \cup Y)}{\sup(X)\sup(Y)}$$

- This is the ratio between the observed support and the expected support if X and Y were independent

Try it!

$$\text{conf}(X \Rightarrow Y) = \frac{\sup(X \cup Y)}{\sup(X)}$$

$$\text{lift}(X \Rightarrow Y) = \frac{\sup(X \cup Y)}{\sup(X)\sup(Y)}$$

Rule	Support	Confidence	Lift
$A \Rightarrow D$			
$C \Rightarrow A$			
$A \Rightarrow C$			
$B \,\&\, C \Rightarrow D$			

https://www.saedsayad.com/association_rules.htm

Association rule (minsup, minconf)

- Let X, Y be two itemsets; the rule X⇒Y is an **association rule** of minimum support **minsup** and minimum confidence **minconf** if:

$$sup(X \Rightarrow Y) \geq minsup$$

and

$$conf(X \Rightarrow Y) \geq minconf$$

Summary

Things to remember

- Itemset, k-itemset, transaction
- Support, confidence, lift
- Maximal and closed itemsets
- Association rule of minsup and minconf

Exercises for this topic

- Data Mining, The Textbook (2015) by Charu Aggarwal
  - Exercises 4.9 → 1-3, 5, 7-8
  - Exercises 5.7 → 1-5
- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al.
  - Exercises 6.1.5 → 6.1.1-6.1.7
- Introduction to Data Mining 2nd edition (2019) by Tan et al.
  - Exercises 5.10 → 2-7