LINEAR ALGEBRA PROJECT ARIF KURU 2104010053

i)

Linear Algebra 2nd Review

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$\alpha = 4$; $\beta = 5$; $\mu = 8$.

$$A = \begin{bmatrix} 4 & 0 & 8 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}; \quad P = \begin{bmatrix} 0,8 & 0,2 & 0,3 \\ 0 & 0,3 & 0,3 \\ 0,2 & 0,5 & 0,4 \end{bmatrix}$$

Column vector $A = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$ for $P = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$.

1)

$N(A) = \{ x \in R^n \mid Ax = 0 \}$

$$\left[\begin{array}{ccc|c} 4 & 0 & 8 & 0 \\ 1 & 1 & 1 & 0 \\ 5 & 2 & 1 & 0 \end{array}\right] \begin{array}{l} -4R_2 + R_1 \to R_1 \\ -5R_2 + R \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & -4 & 4 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 0 \end{array}\right] \quad R_1 \Leftrightarrow R_2 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -4 & 4 & 0 \\ 0 & -3 & -4 & 0 \end{array}\right] \to R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \end{array}\right] \quad -4x_2 = 0$$

$x_2 = 0$

$x_1 = 0$

$N(A) = \{ (0,0,0) \}$

null space of $\begin{pmatrix} 4 & 0 & 8 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}$

NATURAL LANGUAGE | MATH INPUT

Input

null space	$\begin{pmatrix} 4 & 0 & 8 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}$

Result — Step-by-step solution

$\{(0, 0, 0)\}$ (subspace is trivial)

Row-reduced matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

POWERED BY THE **WOLFRAM LANGUAGE**

i)

$$N(P) = \{ x \in \mathbb{R}^n \mid Px = 0 \}$$

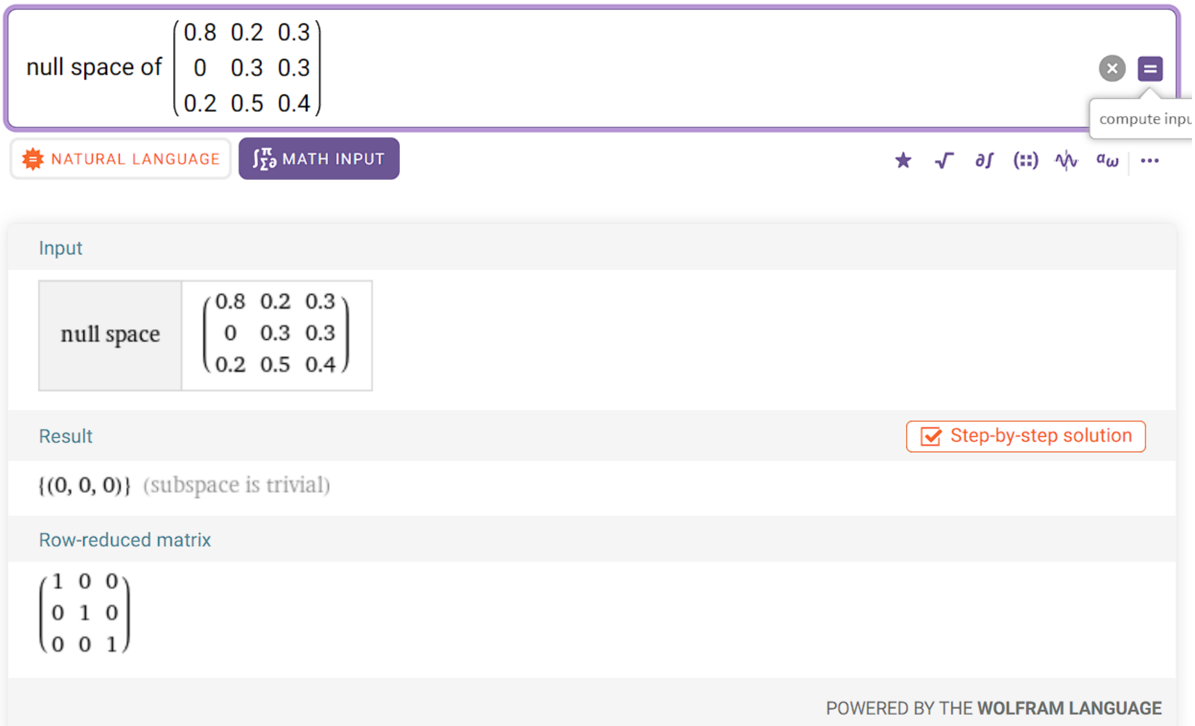
$$\left[\begin{array}{ccc|c} 0{,}8 & 0{,}2 & 0{,}3 & 0 \\ 0 & 0{,}3 & 0{,}3 & 0 \\ 0{,}2 & 0{,}5 & 0{,}4 & 0 \end{array}\right] \quad -4R_3 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 0 & -1{,}8 & -1{,}3 & 0 \\ 0 & 0{,}3 & 0{,}3 & 0 \\ 0{,}2 & 0{,}5 & 0{,}4 & 0 \end{array}\right] \quad 6R_2 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0{,}5 & 0 \\ 0 & 0{,}3 & 0{,}3 & 0 \\ 0{,}2 & 0{,}5 & 0{,}4 & 0 \end{array}\right]$$

$$0{,}5x_3 = 0$$
$$x_3 = 0$$
$$0{,}3x_2 = 0$$
$$x_2 = 0$$
$$0{,}2x_1 = 0$$
$$x_1 = 0$$

$$N(P) = \{(0,0,0)\}$$



null space of $\begin{pmatrix} 0.8 & 0.2 & 0.3 \\ 0 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.4 \end{pmatrix}$
compute input
NATURAL LANGUAGE
MATH INPUT
Input
null space $\begin{pmatrix} 0.8 & 0.2 & 0.3 \\ 0 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.4 \end{pmatrix}$
Result
Step-by-step solution
$\{(0, 0, 0)\}$ (subspace is trivial)
Row-reduced matrix
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
POWERED BY THE WOLFRAM LANGUAGE

ii)

②

$$v_1 = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad v_3 = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$$

$$\det \begin{vmatrix} 4 & 0 & 8 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 4\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 0 + 8\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix}$$

$$4(-1) + 8(-3) = -28$$

Since it's determinant is not equal $\neq 0$ it's linearly independent therefore it's spanning $R3$. and basis for $R3$

linear independence (4, 1, 5), (0, 1, 2), (8, 1, 1)

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | compute input

Input interpretation

linear independence	(4, 1, 5) | (0, 1, 2) | (8, 1, 1)

Result | Step-by-step solution

(4, 1, 5), (0, 1, 2), and (8, 1, 1) are linearly independent

Subspace spanned | Show details

$\mathbb{R}^3$

iii)

③

$$\begin{array}{ccc} \omega_1 & \omega_2 & \omega_3 \end{array}$$

$$\begin{bmatrix} 0.8 & 0.2 & 0.3 \\ 0 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.4 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \mu \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 8 & 2 & 3 & 40 \\ 0 & 3 & 3 & 50 \\ 2 & 5 & 4 & 80 \end{array}\right] -4R_3 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 0 & 2-20 & -13 & -280 \\ 0 & 3 & 3 & 50 \\ 2 & 5 & 4 & 80 \end{array}\right] 6R_2 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 5 & 20 \\ 0 & 3 & 3 & 50 \\ 2 & 5 & 4 & 80 \end{array}\right] \quad 5x_3 = 20, \quad x_3 = 4$$

$3x_2 + 12 = 50$; $x_2 = \frac{38}{3} = 12{,}67$

$2x_1 + (12{,}67)5 + 16 = 80$; $x_1 = 0{,}33$

$$\begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix} = 0{,}33 \begin{bmatrix} 0{,}8 \\ 0 \\ 0{,}2 \end{bmatrix} + 12{,}67 \begin{bmatrix} 0{,}2 \\ 0{,}3 \\ 0{,}5 \end{bmatrix} + 4 \begin{bmatrix} 0{,}3 \\ 0{,}3 \\ 0{,}4 \end{bmatrix}$$

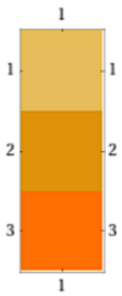
Input

$$0.33\begin{pmatrix}0.8\\0\\0.2\end{pmatrix}+12.67\begin{pmatrix}0.2\\0.3\\0.5\end{pmatrix}+4\begin{pmatrix}0.3\\0.3\\0.4\end{pmatrix}$$

Dimensions — Step-by-step solution

3 (rows) × 1 (column)

Matrix plot — Enlarge | Data | Customize | Plain Text



Transpose — Step-by-step solution

$(3.998 \quad 5.001 \quad 8.001)$

iv)

(a) $v_1 = \begin{bmatrix}4\\1\\5\end{bmatrix}$ $\quad v_2 = \begin{bmatrix}0\\1\\2\end{bmatrix}$ $\quad v$

$$a\begin{bmatrix}4\\1\\5\end{bmatrix} + b\begin{bmatrix}0\\1\\2\end{bmatrix} + cv = 0$$

$v = v_1 + v_2$ so

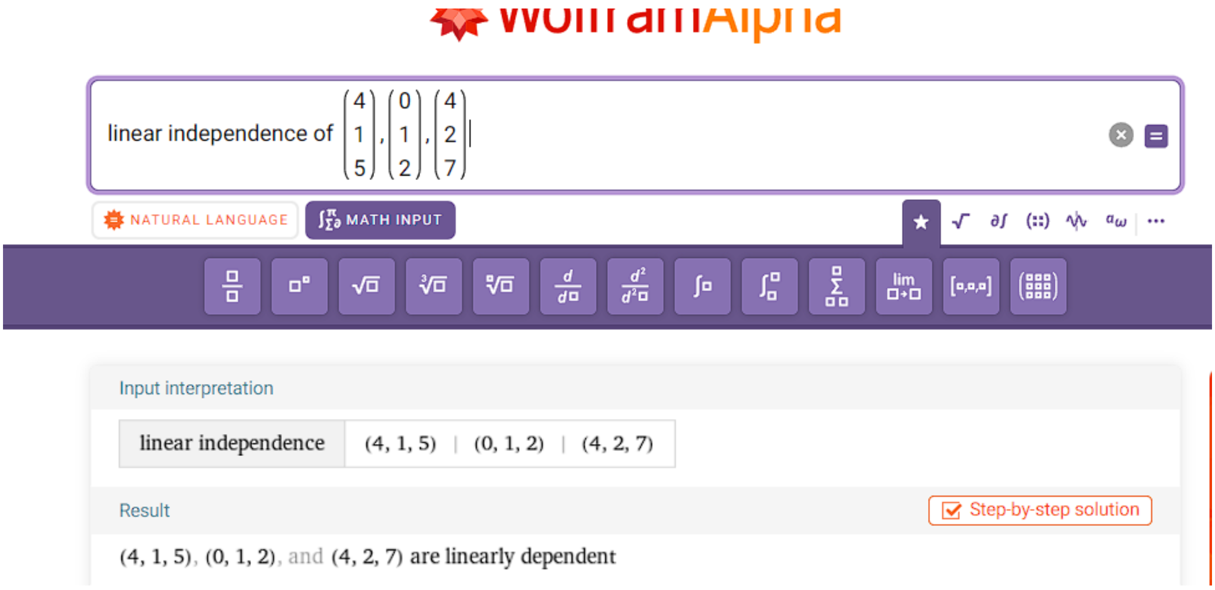
$$(a+c)\begin{bmatrix}4\\1\\5\end{bmatrix} + (b+c)\begin{bmatrix}0\\1\\2\end{bmatrix} = 0$$

$a + c = 0$ and $b + c = 0$

so $a = b = -c$

let $a = 1$; $b = 1$; $c = -1$

$\begin{bmatrix}4\\1\\5\end{bmatrix} + \begin{bmatrix}0\\1\\2\end{bmatrix}$ then $v = \begin{bmatrix}4\\2\\7\end{bmatrix}$



v)

5

$$A = \begin{pmatrix} 4 & 0 & 8 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix} \quad \begin{matrix} -4R_2 + R_1 \to R_1 \\ -5R_3 + R_3 \to R_3 \end{matrix}$$

$$\begin{pmatrix} 0 & -4 & 4 \\ 1 & 1 & 1 \\ 0 & -3 & -4 \end{pmatrix} \quad \frac{-3R_1}{4} + R_3 \to R_3$$

$$\begin{pmatrix} 0 & -4 & 4 \\ 1 & 1 & 1 \\ 0 & 0 & -7 \end{pmatrix} + \frac{4R_3}{7} \to R_1 \quad \begin{pmatrix} 0 & -4 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -7 \end{pmatrix}$$

$\frac{R_3}{7} + R_2 \to R_2$

$\frac{R_1}{4} + R_2 \to R_2$

$R_2 <=> R_1$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -7 \end{pmatrix} \quad \begin{matrix} \frac{R_2}{-4} \to R_2 \\ \frac{R_3}{-7} \to R_3 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

x_1, x_2 and x_3 are free therefore Rank $|A| = 3$

Nullity $(A) = 3 - 3 = 0$

$$\operatorname{rank}\begin{pmatrix}4 & 0 & 8\\ 1 & 1 & 1\\ 5 & 2 & 1\end{pmatrix}$$

NATURAL LANGUAGE | MATH INPUT

Input

$$\operatorname{rank}\begin{pmatrix}4 & 0 & 8\\ 1 & 1 & 1\\ 5 & 2 & 1\end{pmatrix}$$

Result | Step-by-step solution

3

$$\operatorname{nullity}\begin{pmatrix}4 & 0 & 8\\ 1 & 1 & 1\\ 5 & 2 & 1\end{pmatrix}$$

NATURAL LANGUAGE | MATH INPUT

Input interpretation

$$\operatorname{nullity}\begin{pmatrix}4 & 0 & 8\\ 1 & 1 & 1\\ 5 & 2 & 1\end{pmatrix}$$

Result | Step-by-step solution

0