```
In [1]: %matplotlib notebook
   import numpy as np
   import sympy as sp
   from matplotlib import pyplot as plt
   from matplotlib.animation import FuncAnimation
   from IPython import display
   from mpl_toolkits.mplot3d import Axes3D
   from IPython.display import HTML
```

Simulacion de tres cuerpos

```
In [2]: x,y,z,r,x1,x2,x3,y1,y2,y3,z1,z2,z3=sp.symbols('x,y,z,r,x1,x2,x3,y1,y2,y3,z1,z2,z3')
                       vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,t,m_1,m_2,m_3=sp.symbols('vx1,vx2,vx3,vy1,vy2,v
In [3]: k=39.478
                       p1=sp.Matrix([x1,y1,z1])
                       p2=sp.Matrix([x2,y2,z2])
                       p3=sp.Matrix([x3,y3,z3])
                       v1=sp.Matrix([vx1,vy1,vz1])
                       v2=sp.Matrix([vx2,vy2,vz2])
                       v3=sp.Matrix([vx3,vy3,vz3])
In [3]: F1
In [8]: F2
                         \begin{bmatrix} \frac{3.9478x_1 - 3.9478x_2}{\left(\left|x_1 - x_2\right|^2 + \left|y_1 - y_2\right|^2 + \left|z_1 - z_2\right|^2\right)^{\frac{3}{2}}} & \frac{3.9478x_2 - 3.9478x_3}{\left(\left|x_2 - x_3\right|^2 + \left|y_2 - y_3\right|^2 + \left|z_2 - z_3\right|^2\right)^{\frac{3}{2}}} \\ \frac{3.9478y_1 - 3.9478y_2}{\left(\left|x_1 - x_2\right|^2 + \left|y_1 - y_2\right|^2 + \left|z_1 - z_2\right|^2\right)^{\frac{3}{2}}} & \frac{3.9478y_2 - 3.9478y_3}{\left(\left|x_2 - x_3\right|^2 + \left|y_2 - y_3\right|^2 + \left|z_2 - z_3\right|^2\right)^{\frac{3}{2}}} \\ \frac{3.9478z_1 - 3.9478z_2}{\left(\left|x_1 - x_2\right|^2 + \left|y_1 - y_2\right|^2 + \left|z_1 - z_2\right|^2\right)^{\frac{3}{2}}} & \frac{3.9478z_2 - 3.9478z_3}{\left(\left|x_2 - x_3\right|^2 + \left|y_2 - y_3\right|^2 + \left|z_2 - z_3\right|^2\right)^{\frac{3}{2}}} \end{bmatrix}
Out[8]:
In [9]: F3
Out[9]:
```

Runge Kutta 4to orden

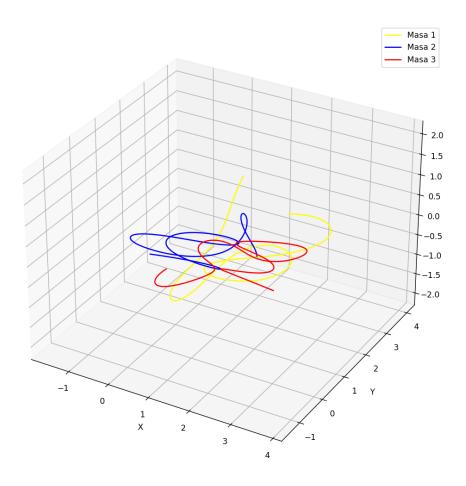
```
In [4]: def d2ydt2(y,f1,f2,f3,g1,g2,g3,h1,h2,h3):
            return np.array([y[9],
                             y[10],
                             y[11],
                             y[12],
                             y[13],
                             y[14],
                             y[15],
                             y[16],
                             y[17],
                             f1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              g1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                             h1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              f2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              g2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              h2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              f3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              g3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              h3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              1])
        def RK42(c1,c2,c3,dt,tf,F1,F2,F3,R,CV,M):
            #Vectorizcion de funciones
            f1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            f2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            f3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            g1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            g2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            g3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            h1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            h2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            h3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
            ts=np.arange(0,tf,dt)
            TT=0#tiempo en completar orbita
            n=len(ts)
            ##############################Posiciones
            ys1=ts*0
            xs1=ts*0
            zs1=ts*0
            ys2=ts*0
            xs2=ts*0
            zs2=ts*0
            ys3=ts*0
            xs3=ts*0
            zs3=ts*0
            #####################################Velocidades
            vvs1=ts*0
            vxs1=ts*0
            vzs1=ts*0
            vys2=ts*0
            vxs2=ts*0
            vzs2=ts*0
```

```
vys3=ts*0
vxs3=ts*0
vzs3=ts*0
avs1=ts*0
axs1=ts*0
azs1=ts*0
ays2=ts*0
axs2=ts*0
azs2=ts*0
ays3=ts*0
axs3=ts*0
azs3=ts*0
##########Condiciones iniciales
xs1[0]=c1[0]
ys1[0]=c1[1]
zs1[0]=c1[2]
vxs1[0]=c1[3]
vys1[0]=c1[4]
vzs1[0]=c1[5]
#############
xs2[0]=c2[0]
ys2[0]=c2[1]
zs2[0]=c2[2]
vxs2[0]=c2[3]
vys2[0]=c2[4]
vzs2[0]=c2[5]
##########
xs3[0]=c3[0]
ys3[0]=c3[1]
zs3[0]=c3[2]
vxs3[0]=c3[3]
vys3[0]=c3[4]
vzs3[0]=c3[5]
for i in range(0,n-1):
    #Runge Kutta
    z0=np.array([xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i]
    k1=d2ydt2(z0,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k2=d2ydt2(z0+(dt*k1)/2,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k3=d2ydt2(z0+(dt*k2)/2,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k4=d2ydt2(z0+dt*k3,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    #Vectores M1
   xs1[i+1]=xs1[i]+(dt/6)*(k1[0]+2*k2[0]+2*k3[0]+k4[0])
   ys1[i+1]=ys1[i]+(dt/6)*(k1[3]+2*k2[3]+2*k3[3]+k4[3])
    zs1[i+1]=zs1[i]+(dt/6)*(k1[6]+2*k2[6]+2*k3[6]+k4[6])
    vxs1[i+1]=vxs1[i]+(dt/6)*(k1[9]+2*k2[9]+2*k3[9]+k4[9])
    vys1[i+1]=vys1[i]+(dt/6)*(k1[12]+2*k2[12]+2*k3[12]+k4[12])
    vzs1[i+1]=vzs1[i]+(dt/6)*(k1[15]+2*k2[15]+2*k3[15]+k4[15])
    axs1[i]=f1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
    ays1[i]=f2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
```

```
azs1[i]=f3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ############Vectores M2
   xs2[i+1]=xs2[i]+(dt/6)*(k1[1]+2*k2[1]+2*k3[1]+k4[1])
   ys2[i+1]=ys2[i]+(dt/6)*(k1[4]+2*k2[4]+2*k3[4]+k4[4])
   zs2[i+1]=zs2[i]+(dt/6)*(k1[7]+2*k2[7]+2*k3[7]+k4[7])
   vxs2[i+1]=vxs2[i]+(dt/6)*(k1[10]+2*k2[10]+2*k3[10]+k4[10])
   vys2[i+1]=vys2[i]+(dt/6)*(k1[13]+2*k2[13]+2*k3[13]+k4[13])
   vzs2[i+1]=vzs2[i]+(dt/6)*(k1[16]+2*k2[16]+2*k3[16]+k4[16])
   axs2[i]=g1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays2[i]=g2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs2[i]=g3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   xs3[i+1]=xs3[i]+(dt/6)*(k1[2]+2*k2[2]+2*k3[2]+k4[2])
   ys3[i+1]=ys3[i]+(dt/6)*(k1[5]+2*k2[5]+2*k3[5]+k4[5])
   zs3[i+1]=zs3[i]+(dt/6)*(k1[8]+2*k2[8]+2*k3[8]+k4[8])
   vxs3[i+1]=vxs3[i]+(dt/6)*(k1[11]+2*k2[11]+2*k3[11]+k4[11])
   vys3[i+1]=vys3[i]+(dt/6)*(k1[14]+2*k2[14]+2*k3[14]+k4[14])
   vzs3[i+1]=vzs3[i]+(dt/6)*(k1[17]+2*k2[17]+2*k3[17]+k4[17])
   axs3[i]=h1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays3[i]=h2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs3[i]=h3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
  if R==True:
   fig = plt.figure(figsize=(10,10))
   ax = fig.add subplot(111, projection='3d')
   #fig2=plt.add(figsize=(10,10))
   #ax.scatter(0,0,0,color='yellow',s=30)
   ax.plot(xs1,ys1,zs1,color='yellow',label='Masa 1')
   ax.plot(xs2,ys2,zs2,color='blue',label='Masa 2')
   ax.plot(xs3,ys3,zs3,color='red',label='Masa 3')
   plt.xlabel('X')
   plt.ylabel('Y')
   #plt.axis('equal')
   #plt.plot(vxs,vys,color='blue',label='Velocidad x vs velocidad y')
   #plt.plot(axs,ays,color='red',label='Aceleracion x vs aceleracion y')
   if CV==True:
      11=50
      ax.quiver(xs[1:-1:11],ys[1:-1:11],axs[1:-1:11],ays[1:-1:11],color='red'
      ax.quiver(xs[1:-1:11],ys[1:-1:11],vxs[1:-1:11],vys[1:-1:11],color='blue
   plt.axis('equal')
   plt.grid('on')
   plt.legend()
   plt.show()
if M==True:
fig3=plt.figure(figsize=(7,7))
```

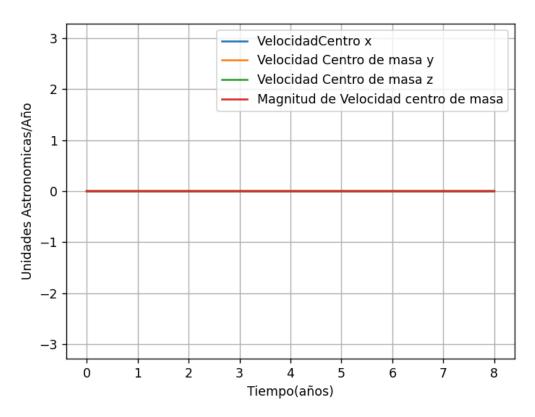
```
plt.plot(ts,np.sqrt(xs**2+ys**2),color='yellow',label='r vs tiempo')
plt.plot(ts,np.sqrt(axs**2+ays**2),color='blue',label='aceleracion neta vs
plt.plot(ts,np.sqrt(vxs**2+vys**2),color='red',label='Velocidad neta vs tie
#plt.plot(ts,vxs,color='red',label='Velocidad')
#plt.plot(ts,axs,color='blue',label='aceleracion')
plt.ylabel('Unidades Atronomicas UA')
plt.xlabel('tiempo(años)')
#plt.axis('equal')
plt.grid('on')
plt.legend()
plt.show()
return xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,v
```

Caso Arbitrario



Velocidad Centro de masa

```
plt.plot(ts,vzz,label='Velocidad Centro de masa z')
plt.plot(ts,np.sqrt(vxx**2+vyy**2+vzz**2),label='Magnitud de Velocidad centro de ma
#plt.ylim([-0.5,0.5])
plt.legend()
plt.ylabel('Unidades Astronomicas/Año')
plt.xlabel('Tiempo(años)')
plt.grid('on')
plt.axis('equal')
```



```
Out[205]: (-0.39999950000000006,
8.3999895,
-5.1164942487929194e-14,
8.559431211857843e-14)

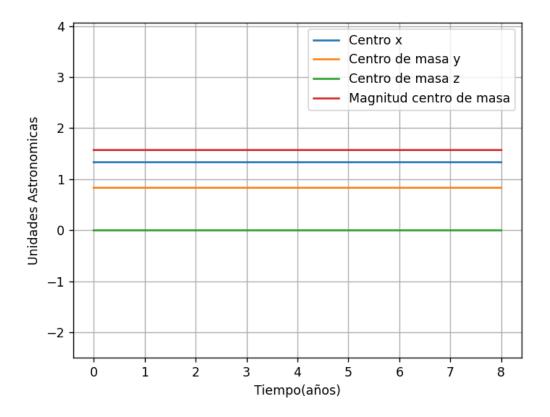
In [43]: np.sqrt(vxx**2+vyy**2+vzz**2)[-100000]-np.sqrt(vxx**2+vyy**2+vzz**2)[0]
```

Out[43]: 7.388945199675211e-14

Centro de Masa

```
In [4]: CX=x1*m_1+x2*m_2+x3*m_3#xs1*m1+xs2*m2+xs3*m3
    CY=y1*m_1+y2*m_2+y3*m_3#ys1*m1+ys2*m2+ys3*m3
    CZ=z1*m_1+z2*m_2+z3*m_3#zs1*m1+zs2*m2+zs3*m3
    M=m_1+m_2+m_3
    CM=sp.Matrix([CX,CY,CZ])*(1/M)
    cx1=sp.lambdify([x1,x2,x3,m_1,m_2,m_3],CM[0])
```

```
cy1=sp.lambdify([y1,y2,y3,m_1,m_2,m_3],CM[1])
cz1=sp.lambdify([z1,z2,z3,m_1,m_2,m_3],CM[2])
```



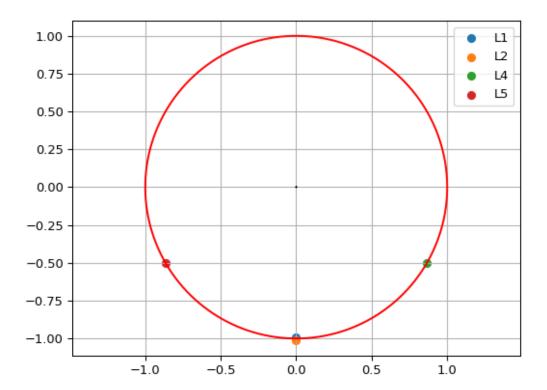
Out[207]: (-0.39999950000000006, 8.3999895, -0.07861650943381412, 1.6509466981100964)

Puntos de Lagrange

```
L1x1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L1[0])
         L1y1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L1[1])
         L2=r2.norm()*(1+(m_2/(3*m_1))**(1/3))*(a.subs(z2,0)/a.subs(z2,0).norm())
         L2x1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L2[0])
         L2y1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L2[1])
         \#L4x=(r2.norm()/2)*((m1-m2)/(m1+m2))
         \#L4y=(sp.sqrt(3)/2)*r2.norm()
         \#L4x = sp. lambdify([x1, x2, x3, y1, y2, y3], L4x)
         \#L4y=sp.lambdify([x1,x2,x3,y1,y2,y3],L4y)
         \#L4x=sp.Matrix([(x1-2*CX+x2)*((m1-m2)/(m1+m2)),,0])
         o=sp.pi/3
         oo=(a.subs(z2,0)/a.subs(z2,0).norm())
         spp=oo[1]*sp.cos(o)+oo[0]*sp.sin(o)
         cpp=oo[0]*sp.cos(o)-oo[1]*sp.sin(o)
         L4=sp.Matrix([a.norm()*((m_1-m_2)/(m_1+m_2))*cpp,a.norm()*spp,0])
         L4=(r2.norm()/2)*sp.sqrt(((m_1-m_2)/(m_1+m_2))**2+3)*sp.Matrix([cpp,spp,0])
         L4x1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L4[0])
         L4y1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L4[1])
         ###L5
         o=-sp.pi/3
         oo=(a.subs(z2,0)/a.subs(z2,0).norm())
         spp=oo[1]*sp.cos(o)+oo[0]*sp.sin(o)
         cpp=oo[0]*sp.cos(o)-oo[1]*sp.sin(o)
         L5=(a.norm()/2)*sp.sqrt(((m_1-m_2)/(m_1+m_2))**2+3)*sp.Matrix([cpp,spp,0])
         L5x1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L5[0])
         L5y1=sp.lambdify([x1,x2,x3,y1,y2,y3,m_1,m_2,m_3],L5[1])
         \#L4y=r2.norm()/2*((m1-m2)/(m1+m2))
         \#1.1x =
In [100... fig6=plt.figure()
         b=0
         plt.axis('equal')
         plt.grid('on')
         plt.scatter(L1x[b],L1y[b],label='L1')
         plt.scatter(L2x[b],L2y[b],label='L2')
         plt.scatter(L4x[b],L4y[b],label='L4')
         plt.scatter(L5x[b],L5y[b],label='L5')
         #plt.plot(cx+L4x,cy+L4y,color='blue')
         #plt.plot(ts,LL)
         LL=np.sqrt(L4x**2+L4y**2)
         plt.plot(xs2,ys2,color='red')
         plt.plot(xs1,ys1,color='black')
         plt.legend()
```

```
#plt.plot(L4x,L4y)
#plt.plot(L5x,L5y)

#plt.scatter(L1x-xs2,L1y-ys2)
```



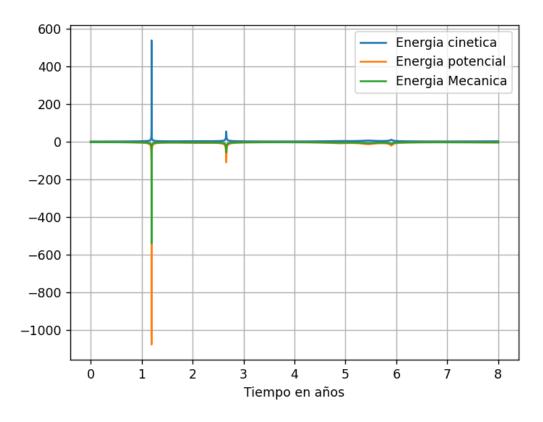
Out[100]: <matplotlib.legend.Legend at 0x2431f9634c0>

conservacion de la energía

$$T_1 = rac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) \ U_1 = rac{k m_1}{r_1}$$

```
In [208... fig6=plt.figure()
    T1=(1/2)*m1*(vxs1**2+vys1**2+vzs1**2)
    T2=(1/2)*m2*(vxs2**2+vys2**2+vzs2**2)
    T3=(1/2)*m3*(vxs3**2+vys3**2+vzs3**2)
    T=T1+T2+T3
    U1=-k*m1*(m2/((p1-p2).norm())+(m3/((p1-p3).norm())))
    U2=-k*m2*(m1/((p2-p1).norm())+m3/((p2-p3).norm()))
    U3=-k*m3*(m2/((p3-p2).norm())+m1/((p3-p1).norm()))
#U1=-k*m1/np.sqrt(xs1**2+ys1**2+zs1**2)
#U2=-k*m2/np.sqrt(xs2**2+ys2**2+zs2**2)
#U3=-k*m3/np.sqrt(xs3**2+ys3**2+zs3**2)
```

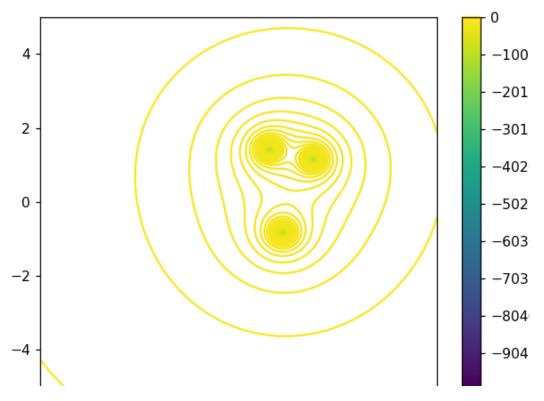
```
U=U1+U2+U3
UUU=U3+U1+U2
UU=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3],UUU)
U=UU(xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3)
plt.plot(ts,T,label='Energia cinetica')
plt.plot(ts,U,label='Energia potencial')
plt.xlabel('Tiempo en años')
plt.plot(ts,T+U,label='Energia Mecanica')
#plt.axis('equal')
#plt.ylim(-1,1)
plt.grid('on')
plt.legend()
###
\#r1=sp.sqrt((x-x1)**2+(y-y1)**2+(z-z1)**2)
\#r2=sp.sqrt((x-x2)**2+(y-y2)**2+(z-z2)**2)
\#r3=sp.sqrt((x3-x)**2+(y3-y)**2+(z3-z)**2)
#L1=p1.cross(m1*v1).norm()
#L2=p2.cross(m2*v2).norm()
#L3=p3.cross(m3*v3).norm()
\#U11 = -k/r1
\#U22 = -k/r2
\#U33 = -k/r3
#UU1=sp.lambdify([x,y,z,x1,y1,z1,vx1,vy1,vz1],U11+L1/(2*m1*r1**2))
#UU2=sp.lambdify([x,y,z,x2,y2,z2,vx2,vy2,vz2],U22+L2/(2*m2*r2**2))
#UU3=sp. Lambdify([x,y,z,x3,y3,z3,vx3,vy3,vz3],U33+L3/(2*m3*r3**2))
```



Out[208]: <matplotlib.legend.Legend at 0x1b2e2b97370>

$$U=rac{km_i}{r_i}$$

```
In [628... fig32=plt.figure()
    xxs=np.linspace(-5,5,1000)
    X,Y=np.meshgrid(xxs,xxs)
    tss=-1
    Z=UU2(X,Y,0,xs2[tss],ys2[tss],zs2[tss],vxs1[tss],vys1[tss],vzs1[tss])+UU3(X,Y,0,xs3)
    cm=plt.contour(X,Y,Z,levels=700)
    plt.colorbar(cm)
```

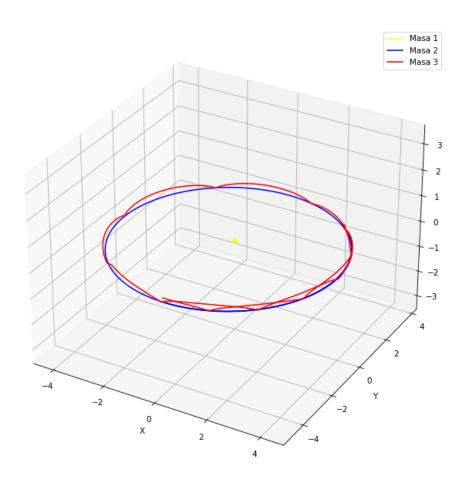


Out[628]: <matplotlib.colorbar.Colorbar at 0x11324f98e20>

Sistema Sol, jupyter, asteroide

```
MM3=np.array([0,-4.7,0.2,2.9,0,0])
dt=0.00002
tf=12
```

In [24]: xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,vzs2,vzs3,ax



SOL, tierra, satelite

```
In [20]: ss=1
    k1=k*ss
    m1=1 # Sol
    m2=3.00273e-6 #tierra
    m3=3.165105302e-27 #James webb telescope
    F1=(-k*m2*(p1-p2)/(p1-p2).norm()**(3)-k*m3*(p1-p3)/(p1-p3).norm()**(3))*(m1/m1)*(k1)
    F2=(-k*m1*(p2-p1)/(p2-p1).norm()**(3)-k*m3*(p2-p3)/(p2-p3).norm()**(3))*(m2/m2)*(k1)
    F3=(-k*m1*(p3-p1)/(p3-p1).norm()**(3)-k*m2*(p3-p2)/(p3-p2).norm()**(3))*(m3/m3)*(k1)
In [21]: MM1=np.array([0,0,0,0,0])*ss
```

```
MM2=np.array([0,-1,0,6.282,0,0])*ss
         MM3=np.array([0,-1.3,0,6.3,0,0])*ss
         dt=0.0001
         tf=1
In [22]: xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,vzs2,vzs3,ax
         cx=cx1(xs1,xs2,xs3,m1,m2,m3)
         cy=cy1(ys1,ys2,ys3,m1,m2,m3)
         cz=cz1(zs1,zs2,zs3,m1,m2,m3)
         L1x=L1x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L1y=L1y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L2y=L2y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L2x=L2x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L4x=L4x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L4y=L4y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L5x=L5x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L5y=L5y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
```

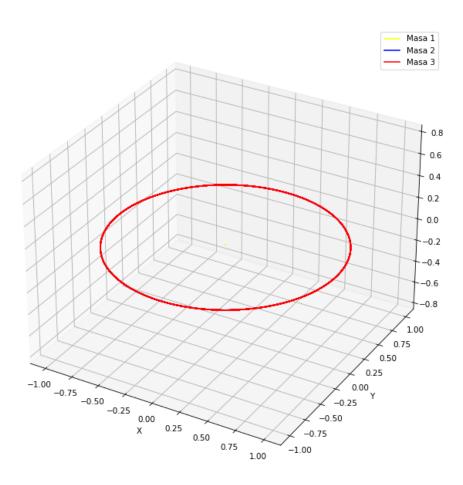


Punto de lagrange L2

```
In [25]: MM2=np.array([0,-1,0,6.282,0,0])
         MM3=np.array([cx[0]+L2x[0],cy[0]+L2y[0],0,6.282,0,0])
         dt=0.0001
         tf=1
In [26]: xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,vzs2,vzs3,ax
         cx=cx1(xs1,xs2,xs3,m1,m2,m3)
         cy=cy1(ys1,ys2,ys3,m1,m2,m3)
         cz=cz1(zs1,zs2,zs3,m1,m2,m3)
         L1x=L1x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L1y=L1y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L2y=L2y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L2x=L2x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L4x=L4x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L4y=L4y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L5x=L5x1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
         L5y=L5y1(xs1,xs2,xs3,ys1,ys2,ys3,m1,m2,m3)
```

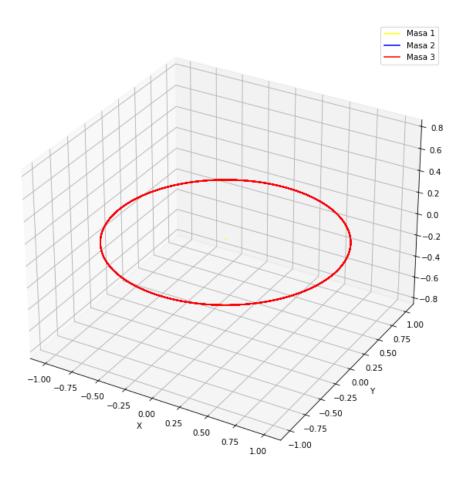


Punto de estabilidad L4



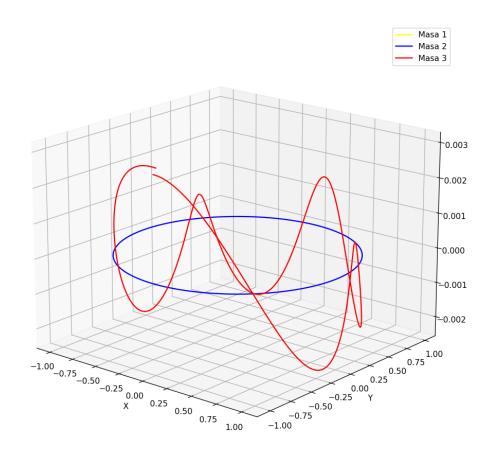
Punto de estabilidad L5

```
In [57]: MM2=np.array([0,-1,0,6.282,0,0])*ss
MM3=np.array([cx[0]+L5x[0],cy[0]+L5y[0],0,3.141,-5.44,0])*ss
dt=0.0001
tf=1
xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,vzs2,vzs3,ax
```



```
In []: fig25=plt.figure()
  plt.plot(ts,np.sqrt(L2x**2+L2y**2),label='Magnitud de L2')
  plt.axis('equal')
  plt.grid('on')
  plt.legend()
  plt.title('Magnitud de L2(Au) vs tiempo(AÑOS)')
  plt.xlabel('tiempo (años)')
```

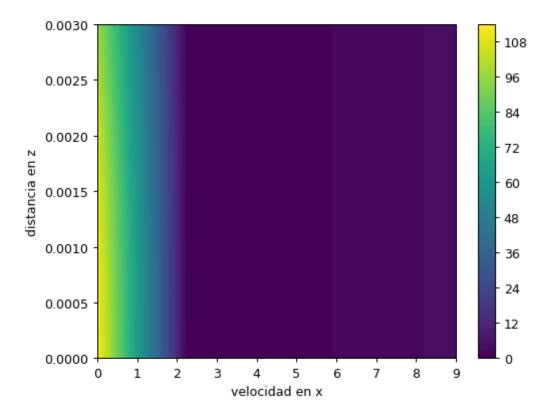
James Webb



Metodo de Disparo

```
dt=0.0005
tf=1/2
for j in range(nn):
    for i in range(nn):
        MM3=np.array([cx[0]+L2x[0],cy[0]+L2y[0],CC2[j,i],CC1[j,i],0,0])
        xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,vzs2
        E[j,i]=np.sqrt(abs(xs3[0]-xs3[-1])**2+abs(ys3[0]-ys3[-1])**2+abs(zs3[0]-zs3
        #if (abs(xs3[0]-xs3[-1])<tol and abs(ys3[0]-ys3[-1])<tol and abs(zs3[0]-zs3
        # CC1=i
        # CC2=j
        # i=nn
        # j=nn
        # break</pre>
```

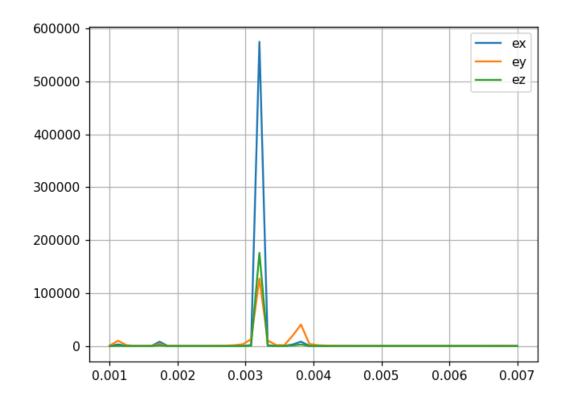
```
In [179... fig55=plt.figure()
    cm=plt.contourf(CC1,CC2,E,levels=100)
    plt.xlabel('velocidad en x')
    plt.ylabel('distancia en z')
    plt.colorbar(cm)
```



Out[179]: <matplotlib.colorbar.Colorbar at 0x200260a3160>

```
ex=cc1*0
ey=cc1*0
for i in range(nn):
    MM3=np.array([cx[0]+L2x[0],cy[0]+L2y[0],cc2[i],0,0,0])
    xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,vzs2,vzs
    ex[i]=abs(xs3[0]-xs3[-1])
    ey[i]=abs(ys3[0]-ys3[-1])
    ez[i]=abs(zs3[0]-zs3[-1])
    if abs(xs3[0]-xs3[-1])
if abs(xs3[0]-xs3[-1])
cC1=cc1(i)
    i=nn
    break
```

```
In [243... fig32=plt.figure()
   plt.plot(cc2,ex,label='ex')
   plt.plot(cc2,ey,label='ey')
   plt.plot(cc2,ez,label='ez')
   #plt.plot(cc1,np.sqrt(ex**2+ey**2+ez**2),label='mag')
   plt.grid('on')
   plt.legend()
```



Out[243]: <matplotlib.legend.Legend at 0x1ffe5246f40>

```
In [244... min(np.sqrt(ex**2+ey**2+ez**2))
```

Out[244]: 27.859738860128726

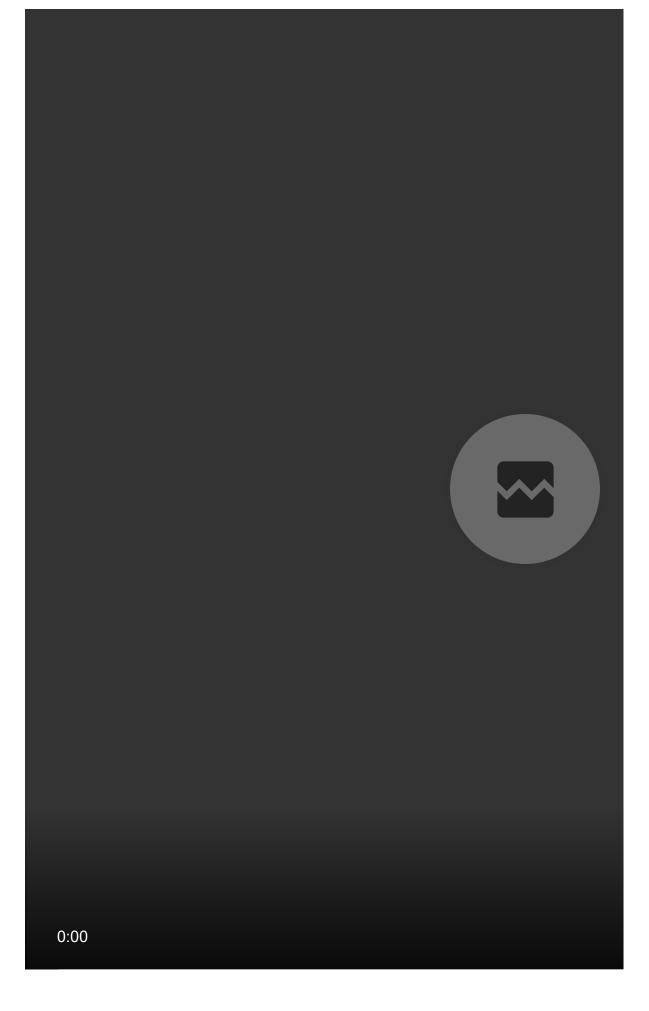
Animacion 2d

```
In [211... fig3 = plt.figure(figsize=(10,10))
         fx = fig3.add_subplot(111)
         fx.set_facecolor('black')
         plt.axis('off')
         nn=len(xs1)
         xx=np.concatenate((xs1,xs2,xs3))
         yy=np.concatenate((ys1,ys2,ys3))
         plt.grid('on')
         plt.axis('equal')
         ii=1/3
         plt.xlim([min(xx)-ii, max(xx)+ii])
         plt.ylim([min(yy)-ii, max(yy)+ii])
         #resorte,=plt.plot([],[],color='black', label='trayectoria')
         #plt.scatter(0,0,color='red')
         #Q=plt.quiver(xs,ys,axs,ays)
         #pp,=plt.plot([],[],'bo',label='Astro')
         #plt.axis('equal')
         plt.legend()
         n=200
         def animate(i):
             fx.clear()
             #plt.axis('off')
             plt.axis('equal')
             plt.xlim([min(xx)-ii, max(xx)+ii])
             plt.ylim([min(yy)-ii, max(yy)+ii])
             ss=60
             fx.scatter(xs1[i-1],ys1[i-1],color='yellow',label='Masa1',s=ss)
             fx.plot(xs1[:i],ys1[:i],color='yellow')
             fx.quiver(xs1[i-1],ys1[i-1],axs1[i-1],ays1[i-1],color='yellow')
             fx.scatter(xs2[i-1],ys2[i-1],color='blue',label='Masa2',s=ss)
             fx.plot(xs2[:i],ys2[:i],color='blue',lw=1)
             fx.quiver(xs2[i-1],ys2[i-1],axs2[i-1],ays2[i-1],color='blue')
             fx.scatter(xs3[i-1],ys3[i-1],color='red',label='Masa 3',s=ss)
             fx.plot(xs3[:i],ys3[:i],color='red',lw=1)
             fx.quiver(xs3[i-1],ys3[i-1],axs3[i-1],ays3[i-1],color='red')
             ##Puntos de Lagrange
             Ls=40
             #fx.scatter((cx+L1x)[i-1],(cy+L1y)[i-1],color='white',label='L1',s=Ls)
             \#fx.scatter((cx+L2x)[i-1],(cy+L2y)[i-1],color='white',label='L2',s=Ls)
             #fx.scatter(L4x[i-1],L4y[i-1],color='white',label='L4',s=Ls)
             #fx.scatter((L5x)[i-1],(L5y)[i-1],color='white',label='L5',s=Ls)
```

```
plt.legend()
    return fig3

fr=np.linspace(0,len(ts),n,dtype=int)
anim=FuncAnimation(fig3,animate,frames=fr,interval=20)
video=anim.to_html5_video()
html=display.HTML(video)
display.display(html)
plt.close()
```

```
No artists with labels found to put in legend. Note that artists whose label star t with an underscore are ignored when legend() is called with no argument. c:\users\arif\appdata\local\programs\python\python39\lib\site-packages\matplotlib \quiver.py:658: RuntimeWarning: divide by zero encountered in double_scalars length = a * (widthu_per_lenu / (self.scale * self.width)) c:\users\arif\appdata\local\programs\python\python39\lib\site-packages\matplotlib \quiver.py:658: RuntimeWarning: invalid value encountered in multiply length = a * (widthu_per_lenu / (self.scale * self.width))
```

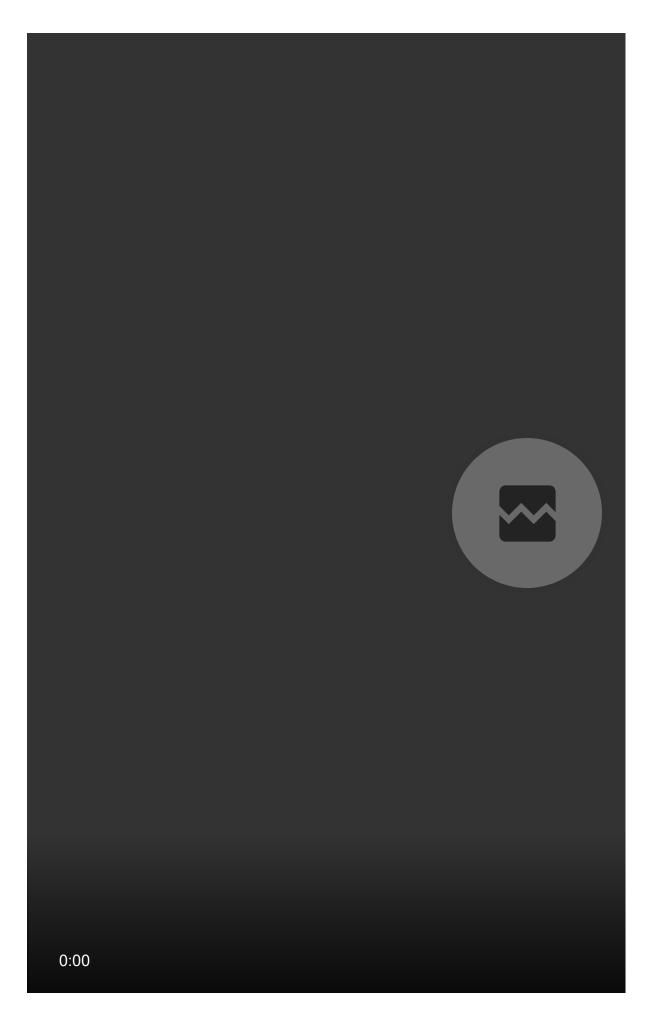


```
C:\Users\Arif\AppData\Local\Temp\ipykernel_5724\2275789233.py:63: MatplotlibDeprec
ationWarning: The close_event function was deprecated in Matplotlib 3.6 and will b
e removed two minor releases later. Use callbacks.process('close_event', CloseEven
t(...)) instead.
   plt.close()
```

Animacion 3D

```
In [380... fig10 = plt.figure(figsize=(10,10))
         bx = fig10.add_subplot(111, projection='3d')
         #bx.set_facecolor('black')
         plt.axis('equal')
         plt.axis('on')
         nn=len(xs1)
         pch=10
         xx=np.concatenate((xs1,xs2,xs3))
         yy=np.concatenate((ys1,ys2,ys3))
         zz=np.concatenate((zs1,zs2,zs3))
         bx.set xlim3d([min(xx)-min(xx)/pch, max(xx)+max(xx)/pch])
         bx.set_ylim3d([min(yy)-min(yy)/pch, max(yy)+max(yy)/pch])
         bx.set_zlim3d([min(zz)-min(zz)/pch, max(zz)+max(zz)/pch])
         bx.view_init(elev=20, azim=45)
         n=100
         #orbita=ax.plot([],[],[],color='black', label='trayectoria')
         #astro, =ax.plot3D([],[],[], 'ro', label='Astro')
         #astro, = ax.plot([xs],[ys],[ys],'ro',label='Astro')
         #orbita, = ax.plot([], [], [], lw=2,color='black', label='trayectoria')
         def animate(i):
             bx.clear()
             pch=10
             #plt.axis('equal')
             bx.set_xlim3d([min(xx)-1/3, max(xx)+1/3])
             bx.set_ylim3d([min(yy)-1/3, max(yy)+1/3])
             \#bx.set\_zlim3d([min(zz)-1/3, max(zz)+1/3])
             #bx.view_init(elev=20, azim=i/(len(ts)/7.5))
             bx.view_init(elev=15, azim=i/(len(ts)/n))
             plt.xlabel('x')
             plt.ylabel('y')
             #plt.axis('equal')
             #plt.axis('off')
             mim=500
             11=1/5
             norm=True
             ###Trayectoria masa1
             bx.scatter(xs1[i-1],ys1[i-1],zs1[i-1],color='yellow',s=mim*m1/10,label='mas1')
             bx.plot(xs1[:i],ys1[:i],zs1[:i],color='yellow',lw=1)
```

```
#bx.quiver(xs1[i-1],ys1[i-1],zs1[i-1],axs1[i-1],ays1[i-1],azs1[i-1],color='yell
   ###Trayectoria masa2
   bx.scatter(xs2[i-1],ys2[i-1],zs2[i-1],color='blue',s=mim*m2*10000,label='masa2'
   bx.plot(xs2[:i],ys2[:i],zs2[:i],color='blue',lw=1)
   #bx.quiver(xs2[i-1],ys2[i-1],zs2[i-1],axs2[i-1],ays2[i-1],azs2[i-1],color='blue
   ###Trayectoria masa3
   bx.scatter(xs3[i-1],ys3[i-1],zs3[i-1],color='red',s=mim*m3*10000,label='masa3')
   bx.plot(xs3[:i],ys3[:i],zs3[:i],color='red',lw=1)
   #bx.quiver(xs3[i-1],ys3[i-1],zs3[i-1],axs3[i-1],ays3[i-1],azs3[i-1],color='red'
   ###########Puntos de Lagrange
   Ls=20
   # bx.scatter((cx+L1x)[i-1],(cy+L1y)[i-1],0,color='green',label='L1',s=Ls)
   #bx.scatter((cx+L2x)[i-1],(cy+L2y)[i-1],0,color='green',label='L2',s=Ls)
   #bx.scatter(L4x[i-1],L4y[i],0,color='green',label='L4',s=Ls)
   #bx.scatter((L5x)[i],(L5y)[i],0,color='green',label='L5',s=Ls)
   plt.legend()
   return fig10#orbita, #(astro,)
#np.linspace(0,len(ts),n,dtype=int)
#fr=np.arange(0,len(ts),n)
fr=np.linspace(0,len(ts),n,dtype=int)
anim=FuncAnimation(fig10, animate, frames=fr, interval=24)
video=anim.to_html5_video()
html=display.HTML(video)
display.display(html)
plt.close()
```



```
C:\Users\Arif\AppData\Local\Temp\ipykernel_7244\817327584.py:73: MatplotlibDepreca
tionWarning: The close_event function was deprecated in Matplotlib 3.6 and will be
removed two minor releases later. Use callbacks.process('close_event', CloseEvent
(...)) instead.
  plt.close()
```

Runge Kutta Fehlberg

```
In [31]: def d2ydt2(y,f1,f2,f3,g1,g2,g3,h1,h2,h3):
             return np.array([y[9],
                              y[10],
                              y[11],
                              y[12],
                              y[13],
                              y[14],
                              y[15],
                              y[16],
                              y[17],
                              f1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               f2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               f3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               1])
         def RKF(c1,c2,c3,dt,tf,F1,F2,F3,R,CV,M):
             #Vectorizcion de funciones
             flag=1
             f1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             f2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             f3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             g1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             g2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             g3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             h1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             h2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             h3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             ts=np.arange(0,tf,dt)
             TT=0#tiempo en completar orbita
             n=len(ts)
             ##############################Posiciones
             ys1=ts*0
             xs1=ts*0
             zs1=ts*0
             ys2=ts*0
             xs2=ts*0
```

```
zs2=ts*0
#
ys3=ts*0
xs3=ts*0
zs3=ts*0
####################################VeLocidades
vys1=ts*0
vxs1=ts*0
vzs1=ts*0
vys2=ts*0
vxs2=ts*0
vzs2=ts*0
vys3=ts*0
vxs3=ts*0
vzs3=ts*0
ays1=ts*0
axs1=ts*0
azs1=ts*0
ays2=ts*0
axs2=ts*0
azs2=ts*0
ays3=ts*0
axs3=ts*0
azs3=ts*0
##########Condiciones iniciales
xs1[0]=c1[0]
ys1[0]=c1[1]
zs1[0]=c1[2]
vxs1[0]=c1[3]
vys1[0]=c1[4]
vzs1[0]=c1[5]
#############
xs2[0]=c2[0]
ys2[0]=c2[1]
zs2[0]=c2[2]
vxs2[0]=c2[3]
vys2[0]=c2[4]
vzs2[0]=c2[5]
##########
xs3[0]=c3[0]
ys3[0]=c3[1]
zs3[0]=c3[2]
vxs3[0]=c3[3]
vys3[0]=c3[4]
vzs3[0]=c3[5]
for i in range(0,n-1):
   #Runge Kutta
    z0=np.array([xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i]
    k1=d2ydt2(z0,f1,f2,f3,g1,g2,g3,h1,h2,h3)
```

```
k2=d2ydt2(z0+dt*((k1)/4),f1,f2,f3,g1,g2,g3,h1,h2,h3)
   k3=d2ydt2(z0+dt*((3*k2)/32+(9/32)*k2),f1,f2,f3,g1,g2,g3,h1,h2,h3)
   k4=d2ydt2(z0+dt*((1932/2197)*k1-(7200/2197)*k2+(7296/2196)*k3),f1,f2,f3,g1,
   k5=d2ydt2(z0+dt*((439/216)*k1-8*k2+(3680/513)*k3-(845/4104)*k4),f1,f2,f3,g1
   k4=d2ydt2(z0+dt*(-(8/27)*k1+2*k2-(3544/2565)*k3+(1859/4104)*k4-(11/40)*k5),
   #Vectores M1
   xs1[i+1]=xs1[i]+(dt/6)*(k1[0]+2*k2[0]+2*k3[0]+k4[0])
   ys1[i+1]=ys1[i]+(dt/6)*(k1[3]+2*k2[3]+2*k3[3]+k4[3])
   zs1[i+1]=zs1[i]+(dt/6)*(k1[6]+2*k2[6]+2*k3[6]+k4[6])
   vxs1[i+1]=vxs1[i]+(dt/6)*(k1[9]+2*k2[9]+2*k3[9]+k4[9])
   vys1[i+1]=vys1[i]+(dt/6)*(k1[12]+2*k2[12]+2*k3[12]+k4[12])
   vzs1[i+1]=vzs1[i]+(dt/6)*(k1[15]+2*k2[15]+2*k3[15]+k4[15])
   axs1[i]=f1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays1[i]=f2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs1[i]=f3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ############Vectores M2
   xs2[i+1]=xs2[i]+(dt/6)*(k1[1]+2*k2[1]+2*k3[1]+k4[1])
   ys2[i+1]=ys2[i]+(dt/6)*(k1[4]+2*k2[4]+2*k3[4]+k4[4])
   zs2[i+1]=zs2[i]+(dt/6)*(k1[7]+2*k2[7]+2*k3[7]+k4[7])
   vxs2[i+1]=vxs2[i]+(dt/6)*(k1[10]+2*k2[10]+2*k3[10]+k4[10])
   vys2[i+1]=vys2[i]+(dt/6)*(k1[13]+2*k2[13]+2*k3[13]+k4[13])
   vzs2[i+1]=vzs2[i]+(dt/6)*(k1[16]+2*k2[16]+2*k3[16]+k4[16])
   axs2[i]=g1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays2[i]=g2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs2[i]=g3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   xs3[i+1]=xs3[i]+(dt/6)*(k1[2]+2*k2[2]+2*k3[2]+k4[2])
   ys3[i+1]=ys3[i]+(dt/6)*(k1[5]+2*k2[5]+2*k3[5]+k4[5])
   zs3[i+1]=zs3[i]+(dt/6)*(k1[8]+2*k2[8]+2*k3[8]+k4[8])
   vxs3[i+1]=vxs3[i]+(dt/6)*(k1[11]+2*k2[11]+2*k3[11]+k4[11])
   vys3[i+1]=vys3[i]+(dt/6)*(k1[14]+2*k2[14]+2*k3[14]+k4[14])
   vzs3[i+1]=vzs3[i]+(dt/6)*(k1[17]+2*k2[17]+2*k3[17]+k4[17])
   axs3[i]=h1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays3[i]=h2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs3[i]=h3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   if R==True:
   fig = plt.figure(figsize=(10,10))
   ax = fig.add_subplot(111, projection='3d')
   #fig2=plt.add(figsize=(10,10))
   ax.plot(xs1,ys1,zs1,color='yellow',label='Masa 1')
   ax.plot(xs2,ys2,zs2,color='blue',label='Masa 2')
   ax.plot(xs3,ys3,zs3,color='red',label='Masa 3')
   plt.xlabel('X')
   plt.ylabel('Y')
   #plt.axis('equal')
   #plt.plot(vxs,vys,color='blue',label='Velocidad x vs velocidad y')
```

```
#plt.plot(axs,ays,color='red',label='Aceleracion x vs aceleracion y')
   if CV==True:
      11=50
      ax.quiver(xs[1:-1:11],ys[1:-1:11],axs[1:-1:11],ays[1:-1:11],color='red'
      ax.quiver(xs[1:-1:11],ys[1:-1:11],vxs[1:-1:11],vys[1:-1:11],color='blue
   #plt.axis('equal')
   plt.grid('on')
   plt.legend()
   plt.show()
if M==True:
fig3=plt.figure(figsize=(7,7))
   plt.plot(ts,np.sqrt(xs**2+ys**2),color='yellow',label='r vs tiempo')
   plt.plot(ts,np.sqrt(axs**2+ays**2),color='blue',label='aceleracion neta vs
   plt.plot(ts,np.sqrt(vxs**2+vys**2),color='red',label='Velocidad neta vs tie
   #plt.plot(ts,vxs,color='red',label='Velocidad')
   #plt.plot(ts,axs,color='blue',label='aceleracion')
   plt.ylabel('Unidades Atronomicas UA')
   plt.xlabel('tiempo(años)')
   #plt.axis('equal')
   plt.grid('on')
   plt.legend()
   plt.show()
return xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,v
```

RK 5to orden

```
In [39]: def d2ydt2(y,f1,f2,f3,g1,g2,g3,h1,h2,h3):
             return np.array([y[9],
                              y[10],
                              y[11],
                              y[12],
                              y[13],
                              y[14],
                              y[15],
                              y[16],
                              y[17],
                               f1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               f2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               f3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               1])
         def RK5(c1,c2,c3,dt,tf,F1,F2,F3,R,CV,M):
             #Vectorizcion de funciones
             f1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             f2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
```

```
f3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
g1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
g2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
g3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
h1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
h2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
h3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
ts=np.arange(0,tf,dt)
TT=0#tiempo en completar orbita
n=len(ts)
###########################Posiciones
ys1=ts*0
xs1=ts*0
zs1=ts*0
ys2=ts*0
xs2=ts*0
zs2=ts*0
ys3=ts*0
xs3=ts*0
zs3=ts*0
####################################VeLocidades
vvs1=ts*0
vxs1=ts*0
vzs1=ts*0
vys2=ts*0
vxs2=ts*0
vzs2=ts*0
vys3=ts*0
vxs3=ts*0
vzs3=ts*0
ays1=ts*0
axs1=ts*0
azs1=ts*0
ays2=ts*0
axs2=ts*0
azs2=ts*0
ays3=ts*0
axs3=ts*0
azs3=ts*0
###########Condiciones iniciales
xs1[0]=c1[0]
ys1[0]=c1[1]
zs1[0]=c1[2]
vxs1[0]=c1[3]
vys1[0]=c1[4]
vzs1[0]=c1[5]
#############
xs2[0]=c2[0]
```

```
ys2[0]=c2[1]
zs2[0]=c2[2]
vxs2[0]=c2[3]
vys2[0]=c2[4]
vzs2[0]=c2[5]
##########
xs3[0]=c3[0]
ys3[0]=c3[1]
zs3[0]=c3[2]
vxs3[0]=c3[3]
vys3[0]=c3[4]
vzs3[0]=c3[5]
for i in range(0, n-1):
    #Runge Kutta
    z0=np.array([xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i]
    k1=d2ydt2(z0,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k2=d2ydt2(z0+(dt*k1)/2,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k3=d2ydt2(z0+dt*(3*k1+k2)/16,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k4=d2ydt2(z0+dt*k3/2,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k5=d2ydt2(z0+dt*(-3*k2+6*k3+9*k4)/16,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k6=d2ydt2(z0+dt*(k1+4*k2+6*k3-12*k4+8*k5)/7,f1,f2,f3,g1,g2,g3,h1,h2,h3)
   k1=7*(k1+k6)
    k3=16*k3
    k2=16*k5
    k4=12*k4
    #Vectores M1
   xs1[i+1]=xs1[i]+(dt/90)*(k1[0]+2*k2[0]+2*k3[0]+k4[0])
   ys1[i+1]=ys1[i]+(dt/90)*(k1[3]+2*k2[3]+2*k3[3]+k4[3])
    zs1[i+1]=zs1[i]+(dt/90)*(k1[6]+2*k2[6]+2*k3[6]+k4[6])
   vxs1[i+1]=vxs1[i]+(dt/90)*(k1[9]+2*k2[9]+2*k3[9]+k4[9])
   vys1[i+1]=vys1[i]+(dt/90)*(k1[12]+2*k2[12]+2*k3[12]+k4[12])
   vzs1[i+1]=vzs1[i]+(dt/90)*(k1[15]+2*k2[15]+2*k3[15]+k4[15])
    axs1[i]=f1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
    ays1[i]=f2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
    azs1[i]=f3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
    ############Vectores M2
   xs2[i+1]=xs2[i]+(dt/90)*(k1[1]+2*k2[1]+2*k3[1]+k4[1])
   ys2[i+1]=ys2[i]+(dt/90)*(k1[4]+2*k2[4]+2*k3[4]+k4[4])
    zs2[i+1]=zs2[i]+(dt/90)*(k1[7]+2*k2[7]+2*k3[7]+k4[7])
   vxs2[i+1]=vxs2[i]+(dt/90)*(k1[10]+2*k2[10]+2*k3[10]+k4[10])
   vys2[i+1]=vys2[i]+(dt/90)*(k1[13]+2*k2[13]+2*k3[13]+k4[13])
   vzs2[i+1]=vzs2[i]+(dt/90)*(k1[16]+2*k2[16]+2*k3[16]+k4[16])
    axs2[i]=g1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
    ays2[i]=g2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
    azs2[i]=g3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   xs3[i+1]=xs3[i]+(dt/90)*(k1[2]+2*k2[2]+2*k3[2]+k4[2])
```

```
ys3[i+1]=ys3[i]+(dt/90)*(k1[5]+2*k2[5]+2*k3[5]+k4[5])
   zs3[i+1]=zs3[i]+(dt/90)*(k1[8]+2*k2[8]+2*k3[8]+k4[8])
   vxs3[i+1]=vxs3[i]+(dt/90)*(k1[11]+2*k2[11]+2*k3[11]+k4[11])
   vys3[i+1]=vys3[i]+(dt/90)*(k1[14]+2*k2[14]+2*k3[14]+k4[14])
   vzs3[i+1]=vzs3[i]+(dt/90)*(k1[17]+2*k2[17]+2*k3[17]+k4[17])
   axs3[i]=h1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays3[i]=h2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs3[i]=h3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
  if R==True:
   fig = plt.figure(figsize=(10,10))
   ax = fig.add subplot(111, projection='3d')
   #ax.set_zlim3d(-0.2,0.2)
   #fig2=plt.add(figsize=(10,10))
   #ax.scatter(0,0,0,color='yellow',s=30)
   ax.plot(xs1,ys1,zs1,color='yellow',label='Masa 1')
   ax.plot(xs2,ys2,zs2,color='blue',label='Masa 2')
   ax.plot(xs3,ys3,zs3,color='red',label='Masa 3')
   plt.xlabel('X')
   plt.ylabel('Y')
   #plt.axis('equal')
   #plt.plot(vxs,vys,color='blue',label='Velocidad x vs velocidad y')
   #plt.plot(axs,ays,color='red',label='Aceleracion x vs aceleracion y')
   if CV==True:
       11=50
       ax.quiver(xs[1:-1:11],ys[1:-1:11],axs[1:-1:11],ays[1:-1:11],color='red'
       ax.quiver(xs[1:-1:11],ys[1:-1:11],vxs[1:-1:11],vys[1:-1:11],color='blue
   #plt.axis('equal')
   plt.grid('on')
   plt.legend()
   plt.show()
if M==True:
fig3=plt.figure(figsize=(7,7))
   plt.plot(ts,np.sqrt(xs**2+ys**2),color='yellow',label='r vs tiempo')
   plt.plot(ts,np.sqrt(axs**2+ays**2),color='blue',label='aceleracion neta vs
   plt.plot(ts,np.sqrt(vxs**2+vys**2),color='red',label='Velocidad neta vs tie
   #plt.plot(ts, vxs, color='red', label='Velocidad')
   #plt.plot(ts,axs,color='blue',label='aceleracion')
   plt.ylabel('Unidades Atronomicas UA')
   plt.xlabel('tiempo(años)')
   #plt.axis('equal')
   plt.grid('on')
   plt.legend()
   plt.show()
return xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,v
```

```
In [18]: def d2ydt2(y,f1,f2,f3,g1,g2,g3,h1,h2,h3):
             return np.array([y[9],
                              y[10],
                              y[11],
                              y[12],
                              y[13],
                              y[14],
                              y[15],
                              y[16],
                              y[17],
                              f1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                              h1(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               f2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h2(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               f3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               g3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               h3(y[0],y[1],y[2],y[3],y[4],y[5],y[6],y[7],y[8],y[9],y[10],y[1
                               1])
         def RK38(c1,c2,c3,dt,tf,F1,F2,F3,R,CV,M):
             #Vectorizcion de funciones
             f1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             f2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             f3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             g1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             g2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             g3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             h1=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             h2=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             h3=sp.lambdify([x1,x2,x3,y1,y2,y3,z1,z2,z3,vx1,vx2,vx3,vy1,vy2,vy3,vz1,vz2,vz3,
             ts=np.arange(0,tf,dt)
             TT=0#tiempo en completar orbita
             n=len(ts)
             ##############################Posiciones
             ys1=ts*0
             xs1=ts*0
             zs1=ts*0
             ys2=ts*0
             xs2=ts*0
             zs2=ts*0
             ys3=ts*0
             xs3=ts*0
             zs3=ts*0
             #####################################Velocidades
             vys1=ts*0
             vxs1=ts*0
             vzs1=ts*0
             vvs2=ts*0
             vxs2=ts*0
```

```
vzs2=ts*0
#
vys3=ts*0
vxs3=ts*0
vzs3=ts*0
avs1=ts*0
axs1=ts*0
azs1=ts*0
ays2=ts*0
axs2=ts*0
azs2=ts*0
ays3=ts*0
axs3=ts*0
azs3=ts*0
##########Condiciones iniciales
xs1[0]=c1[0]
ys1[0]=c1[1]
zs1[0]=c1[2]
vxs1[0]=c1[3]
vys1[0]=c1[4]
vzs1[0]=c1[5]
#############
xs2[0]=c2[0]
ys2[0]=c2[1]
zs2[0]=c2[2]
vxs2[0]=c2[3]
vys2[0]=c2[4]
vzs2[0]=c2[5]
##########
xs3[0]=c3[0]
ys3[0]=c3[1]
zs3[0]=c3[2]
vxs3[0]=c3[3]
vys3[0]=c3[4]
vzs3[0]=c3[5]
for i in range(0,n-1):
    #Runge Kutta
    z0=np.array([xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i]
    k1=d2ydt2(z0,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k2=d2ydt2(z0+(dt*k1)/3,f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k3=d2ydt2(z0+dt*(-(k1/3)+k2),f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k4=d2ydt2(z0+dt*(k1-k2+k3),f1,f2,f3,g1,g2,g3,h1,h2,h3)
    k2=(3/2)*k2
    k3=(3/2)*k3
    #Vectores M1
    xs1[i+1]=xs1[i]+(dt/8)*(k1[0]+2*k2[0]+2*k3[0]+k4[0])
   ys1[i+1]=ys1[i]+(dt/8)*(k1[3]+2*k2[3]+2*k3[3]+k4[3])
    zs1[i+1]=zs1[i]+(dt/8)*(k1[6]+2*k2[6]+2*k3[6]+k4[6])
   vxs1[i+1]=vxs1[i]+(dt/8)*(k1[9]+2*k2[9]+2*k3[9]+k4[9])
    vys1[i+1]=vys1[i]+(dt/8)*(k1[12]+2*k2[12]+2*k3[12]+k4[12])
    vzs1[i+1]=vzs1[i]+(dt/8)*(k1[15]+2*k2[15]+2*k3[15]+k4[15])
```

```
axs1[i]=f1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays1[i]=f2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs1[i]=f3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ############Vectores M2
   xs2[i+1]=xs2[i]+(dt/8)*(k1[1]+2*k2[1]+2*k3[1]+k4[1])
   ys2[i+1]=ys2[i]+(dt/8)*(k1[4]+2*k2[4]+2*k3[4]+k4[4])
   zs2[i+1]=zs2[i]+(dt/8)*(k1[7]+2*k2[7]+2*k3[7]+k4[7])
   vxs2[i+1]=vxs2[i]+(dt/8)*(k1[10]+2*k2[10]+2*k3[10]+k4[10])
   vys2[i+1]=vys2[i]+(dt/8)*(k1[13]+2*k2[13]+2*k3[13]+k4[13])
   vzs2[i+1]=vzs2[i]+(dt/8)*(k1[16]+2*k2[16]+2*k3[16]+k4[16])
   axs2[i]=g1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays2[i]=g2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs2[i]=g3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   xs3[i+1]=xs3[i]+(dt/8)*(k1[2]+2*k2[2]+2*k3[2]+k4[2])
   ys3[i+1]=ys3[i]+(dt/8)*(k1[5]+2*k2[5]+2*k3[5]+k4[5])
   zs3[i+1]=zs3[i]+(dt/8)*(k1[8]+2*k2[8]+2*k3[8]+k4[8])
   vxs3[i+1]=vxs3[i]+(dt/8)*(k1[11]+2*k2[11]+2*k3[11]+k4[11])
   vys3[i+1]=vys3[i]+(dt/8)*(k1[14]+2*k2[14]+2*k3[14]+k4[14])
   vzs3[i+1]=vzs3[i]+(dt/8)*(k1[17]+2*k2[17]+2*k3[17]+k4[17])
   axs3[i]=h1(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   ays3[i]=h2(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
   azs3[i]=h3(xs1[i],xs2[i],xs3[i],ys1[i],ys2[i],ys3[i],zs1[i],zs2[i],zs3[i],v
  if R==True:
   fig = plt.figure(figsize=(10,10))
   ax = fig.add_subplot(111, projection='3d')
   bx.set_zlim3d(-1, 1)
   #fig2=plt.add(figsize=(10,10))
   ax.scatter(0,0,0,color='yellow',s=30)
   ax.plot(xs1,ys1,zs1,color='yellow',label='Masa 1')
   ax.plot(xs2,ys2,zs2,color='blue',label='Masa 2')
   ax.plot(xs3,ys3,zs3,color='red',label='Masa 3')
   plt.xlabel('X')
   plt.ylabel('Y')
   plt.axis('equal')
   #plt.plot(vxs,vys,color='blue',label='Velocidad x vs velocidad y')
   #plt.plot(axs,ays,color='red',label='Aceleracion x vs aceleracion y')
   if CV==True:
       11=50
       ax.quiver(xs[1:-1:11],ys[1:-1:11],axs[1:-1:11],ays[1:-1:11],color='red'
       ax.quiver(xs[1:-1:11],ys[1:-1:11],vxs[1:-1:11],vys[1:-1:11],color='blue
   plt.axis('equal')
   plt.grid('on')
   plt.legend()
   plt.show()
```

```
if M==True:
fig3=plt.figure(figsize=(7,7))
   plt.plot(ts,np.sqrt(xs**2+ys**2),color='yellow',label='r vs tiempo')
   plt.plot(ts,np.sqrt(axs**2+ays**2),color='blue',label='aceleracion neta vs
   plt.plot(ts,np.sqrt(vxs**2+vys**2),color='red',label='Velocidad neta vs tie
   #plt.plot(ts, vxs, color='red', label='Velocidad')
   #plt.plot(ts,axs,color='blue',label='aceleracion')
   plt.ylabel('Unidades Atronomicas UA')
   plt.xlabel('tiempo(años)')
   #plt.axis('equal')
   plt.grid('on')
   plt.legend()
   plt.show()
return xs1,xs2,xs3,ys1,ys2,ys3,zs1,zs2,zs3,vxs1,vxs2,vxs3,vys1,vys2,vys3,vzs1,v
```

In []: