SÍ

INCORPORATED

Ecuación de Laplace resuelta con diferencias finitas

INTEGRANTES

PUES LOS QUE YA CONOCEN JAJA

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$$\nabla^2 u = 0$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

LA ECUACIÓN DE LAPLACE

¿QUÉ ES Y CON QUÉ SE COME?

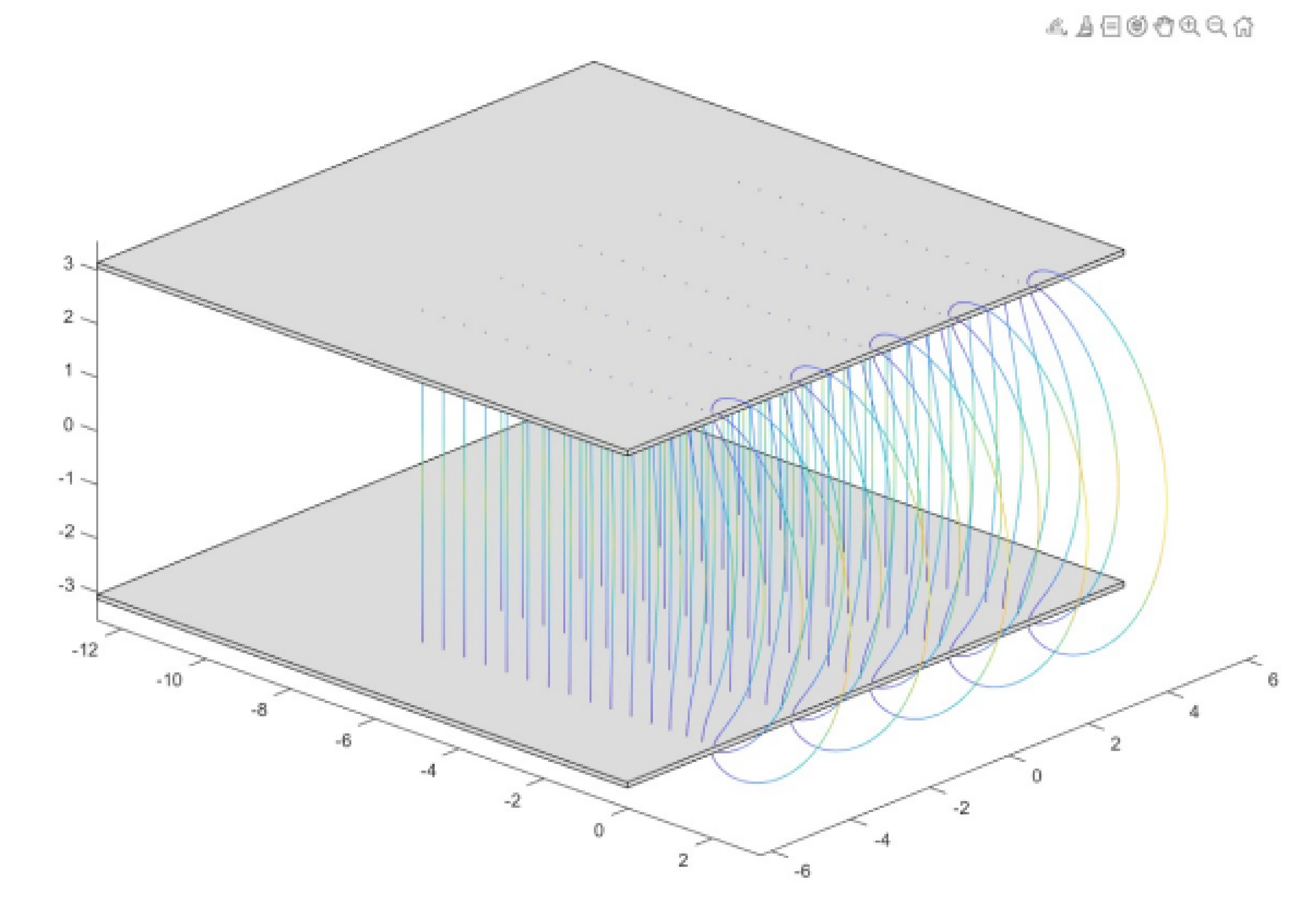
Es básicamente lo de la izquierda

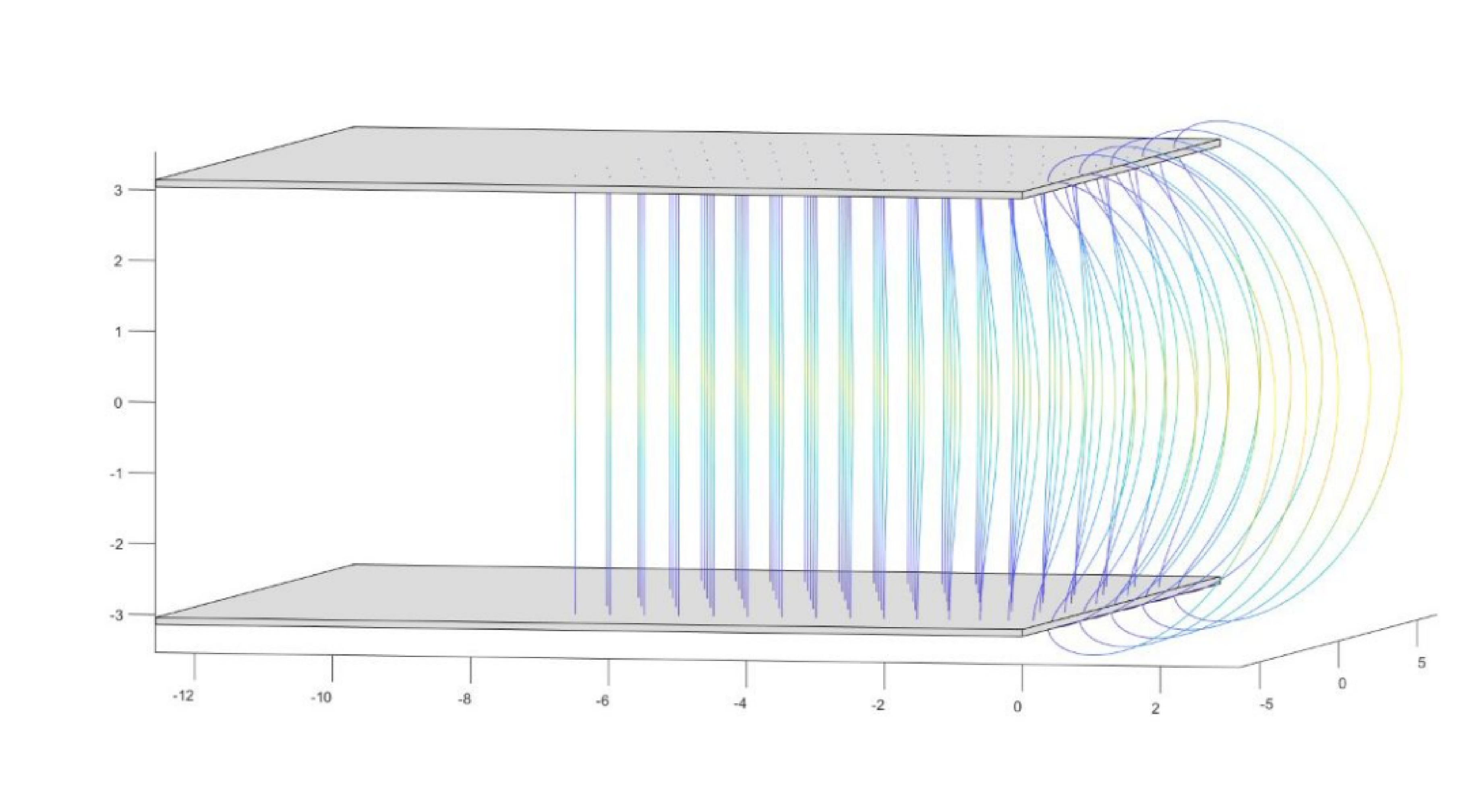
AQUÍ PONES TU PARTE FRANCO O TE SACAMOS DEL EQUIPO :(

50

NUEVOS EMPLEADOS ESTE AÑO

CAPACITORES Y EFECTOS DE BORDE

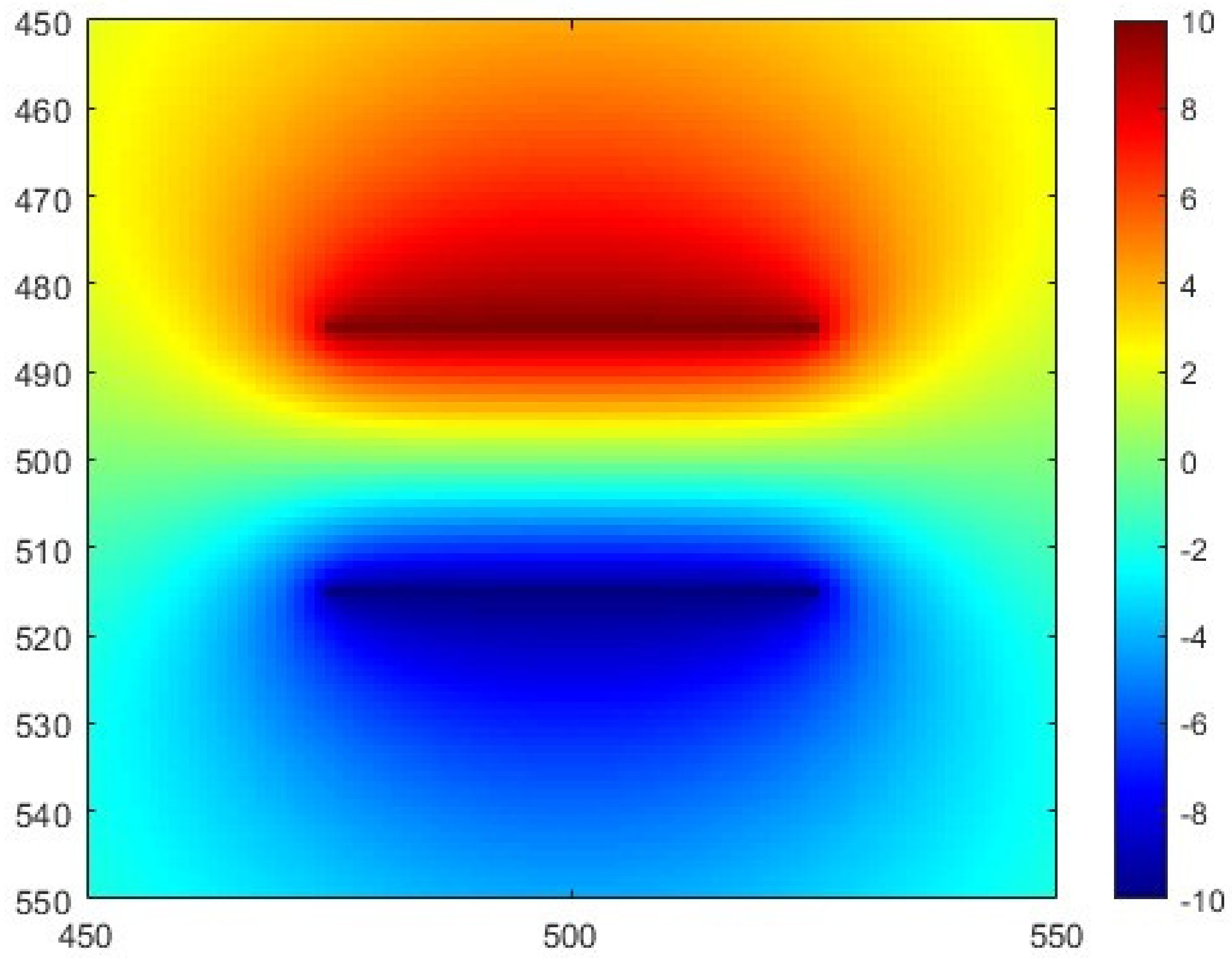




No se crea profe, todos trabajamos en el proyecto ;)

MATLAB

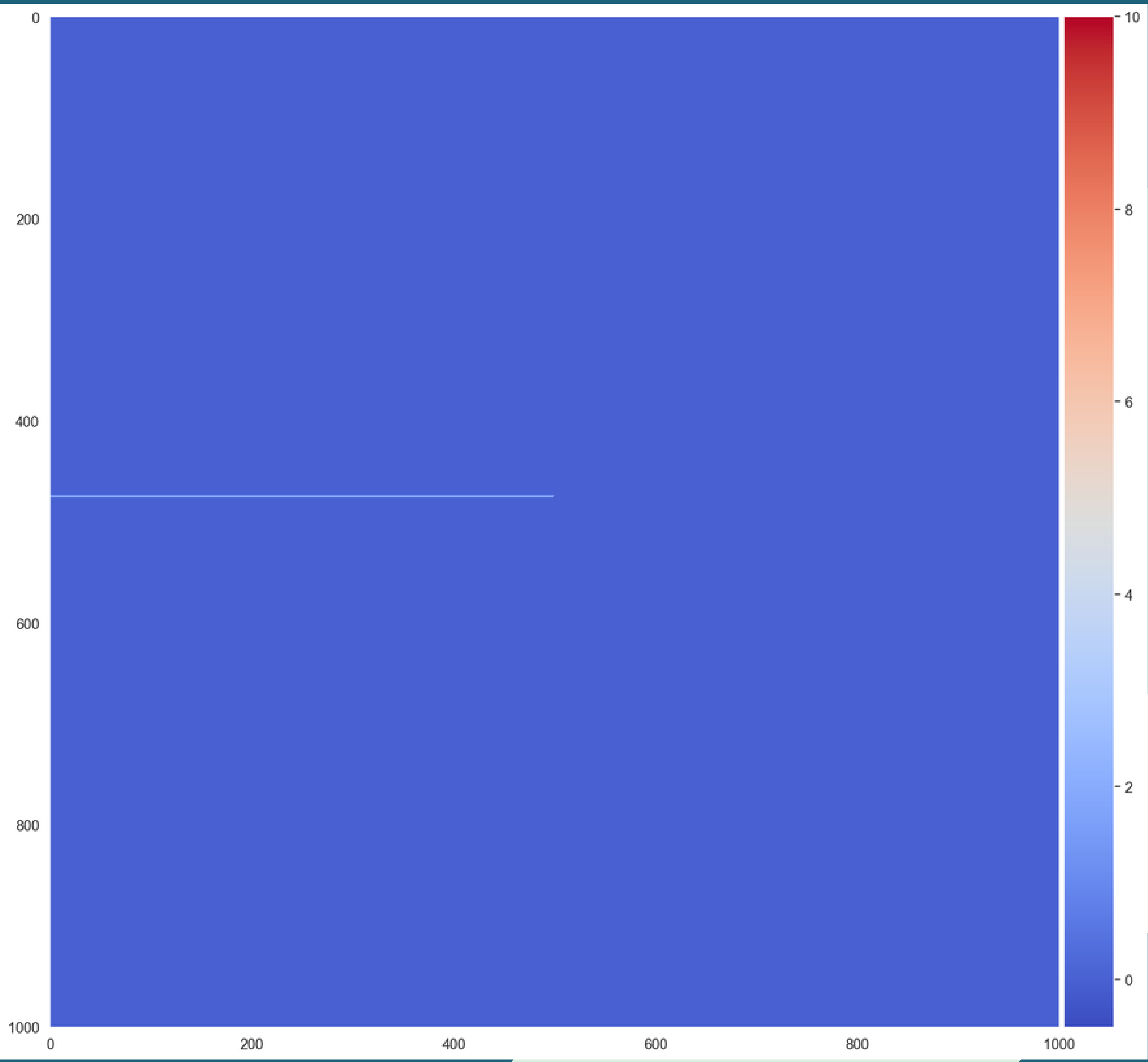
```
%Condiciones de frontera

U(485,475:525) = 10; %V
U(515,475:525) = -10; %V

for k = 1:iter_number
    for i = 2:ny
        for j = 2:nx
            if (i == 485 && 475<=j && j<=525) || (i == 515 && 475<=j && j<=525)

            else
                U(i,j) = Co*(dx^2*(U(i+1,j)+U(i-1,j))+ dy^2*(U(i,j+1)+U(i,j-1)));
            end
        end
    end
end
```


450
460
470
480
490
500
510
520
530
540
550
450
500
550
10
8
6
4
2
0
-2
-4
-6
-8
-10

PYTHON

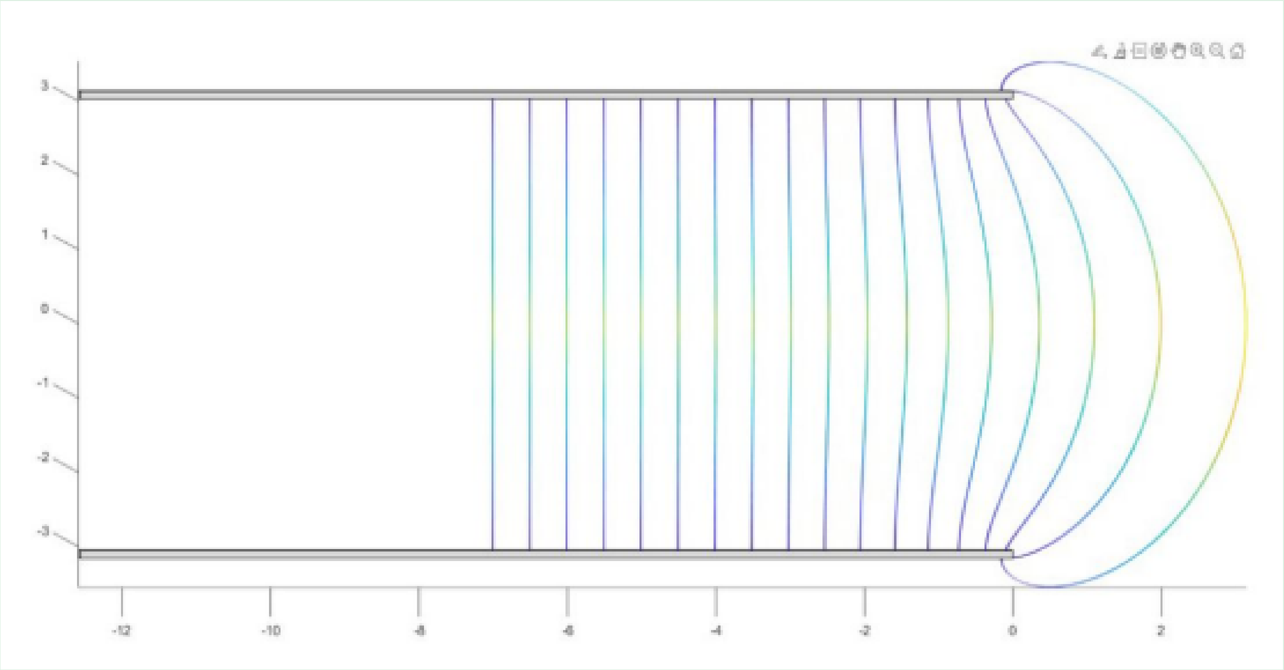
```
a = 10
b = 10

nx = 1000
ny = 1000

dx = a/nx
dy = b/ny

X,Y = np.meshgrid(np.linspace(0,a,nx+1),np.linspace(0,b,ny+1))

Co = 1 / (2*(dx**2+dy**2))
U = np.zeros((ny+1,nx+1))

#Condiciones de frontera

U[475,0:500] = 10#V
#U[-1,:]=-10
#U[:,0]=10
#U[:,-1]=-10
U[525,0:500] = 0 #V

for i in range(1,ny):
    for j in range(1,nx):
        if (i == 475 and j<=500) and (i == 525 and j<=500):
            l=1
        else:
            U[i,j]=Co*(dx**2*(U[i+1,j]+U[i-1,j])+dy**2*(U[i,j+1]-U[i,j-1]))
```

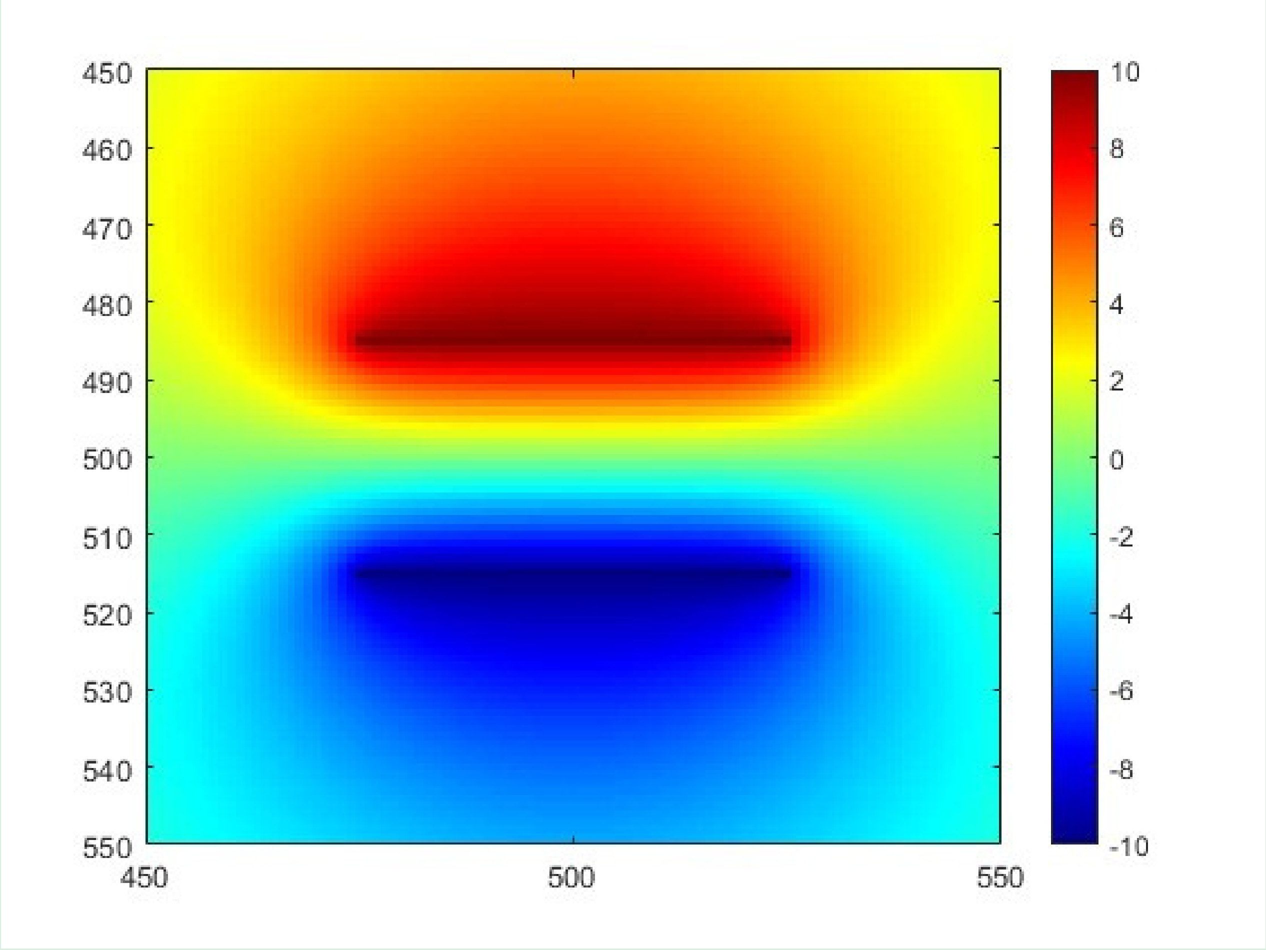
```
fig, ax = plt.subplots(subplot_kw={"projection": "3d"})
#img.plot(U)
#ax.scatter(X,Y,U)
ax.contourf(X,Y,U)
```



COMPARACION SOLUCION FINAL VS MAPEO CONFORME

En mapeo conforme, se calculan las líneas de campo.
En la simulación en matlab se grafica el voltaje en las dos líneas de capacitores.
En ambos se observa el efecto de borde.





CONCLUSION

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¿QUÉ ES EL MAPEO CONFORME?

Es el transformar un problema de una ecuación compleja en un plano "z"a un plano "w" donde son más sencillos los cálculos para regresar el problema después a un plano "z".

2.2 Which properties must conformal mapping satisfy?

1. It must fulfil the Cauchy–Riemann equation
2. It only applies to 2d problems, but a 3d problem with invariance as a third axis can be used.
3. It must be an Analytical function
4. It must Continuous on domain Ω
5. The application of conformal mapping is limited to variables that satisfy the Laplace equation, making it useful for certain fields (such as electric ones) but useless for other fields.

5.1 Calculations

For the conformal mapping used for describing the semi-infinite plates of a capacitor is the following, $w = F(z) = 1 + z + e^z$, where $z = \phi(x, y) + i\psi(x, y)$. If $w = x + iy$, then

$$x = 1 + \phi + e^{\phi} cos(\psi)$$

$$y = \psi + e^{\phi} sin(\psi)$$

Note that, x,y,ϕ and ψ are dimensionless quantities, in order to add physical quantities, $X = \frac{d}{2\pi}(x)$, $Y = \frac{d}{2\pi}(y)$ where d is in meters, as for $\Phi = \frac{V_0}{2\pi}(\phi)$ where V_0 is in volts.

Then the $\mathbf{E} = -\nabla\Phi$, therefore

$$\mathbf{E}_x = -(\frac{V}{d})\frac{\partial\Phi}{\partial x}$$

$$\mathbf{E}_y = -(\frac{V}{d})\frac{\partial\Phi}{\partial y}$$