Divergence theorem

$$\iiint_{v} \nabla \cdot \vec{F}\, dv = \oiint_{s} \vec{F} \cdot d\vec{s}$$

Stokes' theorem

$$\oint_{\ell} \vec{F} \cdot d\ell = \iint_{s} \nabla \times \vec{F} \cdot d\vec{s} = \iint_{s} \nabla \times \vec{F} \cdot \overrightarrow{n}\, dx\, dy$$

Donde:

$$\oint_{\ell} \vec{F} \cdot d\ell = \int_{a}^{b} \overrightarrow{F(\ell(t))} \cdot \frac{d\ell}{dt} dt$$

$$\vec{n} = \frac{\partial \vec{F}}{\partial x} \times \frac{\partial \vec{F}}{\partial y}$$

Maxwell's equations (Integral form)

$$\oint_{s} \vec{E} \cdot d\vec{s} = \frac{Q_{net}}{\varepsilon_0}$$

Utilizando el teorema de divergencia:

$$\iiint_{v} \nabla \cdot \vec{E}\, dv = \oiint_{s} \vec{E} \cdot d\vec{s} = \frac{Q_{net}}{\varepsilon_0}$$

$$V\rho = Q_{net}$$

$$\iiint_{V} \rho\, dv = Q_{net}$$

$$\iiint_{v} \nabla \cdot \vec{E}\, dv = \frac{1}{\varepsilon_0} \iiint_{V} \rho\, dv \therefore \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_{s} \vec{B} \cdot d\vec{s} = \oiint_{s} \vec{B} \cdot \vec{n}\, ds = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\oint_{\ell} \vec{E} \cdot d\ell = -\frac{\partial}{\partial t} \iint_{s} \vec{B} \cdot d\vec{s}$$

$$\iint_{s} \nabla \times \vec{\mathrm{E}} \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_{s} \vec{B} \cdot d\vec{s}$$

$$\nabla \times \vec{\mathrm{E}} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_{\ell} \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \iint_{s} \vec{E} \cdot d\vec{s}$$

$$\iint_{s} \nabla \times \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \iint_{s} \vec{E} \cdot d\vec{s}$$

$$j = \frac{I}{A} \therefore \iint_{s} j \, ds = I$$

$$\iint_{s} \nabla \times \vec{B} \cdot d\vec{s} = \mu_0 \iint_{s} j \, ds + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \iint_{s} \vec{E} \cdot d\vec{s}$$

$$\nabla \times \vec{B} = \mu_0 j + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{\mathrm{E}} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 j + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Electromagnetic wave Equation

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \nabla \times \vec{\mathrm{E}} = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right)$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{\mathrm{E}} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla \times \nabla \times \vec{\mathrm{E}} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Utilizando:

$$\nabla \times \nabla \times \vec{\mathrm{F}} = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2 \vec{F}$$

$$\nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla(0) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Wave equation:

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\frac{1}{v^2} = \mu_0 \varepsilon_0 \therefore v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi * 10^{-7} \frac{Vs}{Am} * 8.85 * 10^{-12} \frac{As}{Vm}}} = 3 * 10^8 = c$$

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\begin{bmatrix} \frac{\partial^2 \overrightarrow{E_x}}{\partial x^2} + \frac{\partial^2 \overrightarrow{E_x}}{\partial y^2} + \frac{\partial^2 \overrightarrow{E_x}}{\partial y^2} \\ \frac{\partial^2 \overrightarrow{E_y}}{\partial x^2} + \frac{\partial^2 \overrightarrow{E_y}}{\partial y^2} + \frac{\partial^2 \overrightarrow{E_y}}{\partial y^2} \\ \frac{\partial^2 \overrightarrow{E_z}}{\partial x^2} + \frac{\partial^2 \overrightarrow{E_z}}{\partial y^2} + \frac{\partial^2 \overrightarrow{E_z}}{\partial y^2} \end{bmatrix} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\begin{bmatrix} \dfrac{\partial^2 \overrightarrow{E_x}}{\partial x^2} + \dfrac{\partial^2 \overrightarrow{E_x}}{\partial y^2} + \dfrac{\partial^2 \overrightarrow{E_x}}{\partial y^2} \\ \dfrac{\partial^2 \overrightarrow{E_y}}{\partial x^2} + \dfrac{\partial^2 \overrightarrow{E_y}}{\partial y^2} + \dfrac{\partial^2 \overrightarrow{E_y}}{\partial y^2} \\ \dfrac{\partial^2 \overrightarrow{E_z}}{\partial x^2} + \dfrac{\partial^2 \overrightarrow{E_z}}{\partial y^2} + \dfrac{\partial^2 \overrightarrow{E_z}}{\partial y^2} \end{bmatrix} = \frac{1}{c^2} \begin{bmatrix} \dfrac{\partial^2 \overrightarrow{E_x}}{\partial t^2} \\ \dfrac{\partial^2 \overrightarrow{E_y}}{\partial t^2} \\ \dfrac{\partial^2 \overrightarrow{E_z}}{\partial t^2} \end{bmatrix}$$

Non coupled partial differential equations of second order

$$\frac{\partial^2 \overrightarrow{E_x}}{\partial x^2} + \frac{\partial^2 \overrightarrow{E_x}}{\partial y^2} + \frac{\partial^2 \overrightarrow{E_x}}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 \overrightarrow{E_x}}{\partial t^2}$$

$$\frac{\partial^2 \overrightarrow{E_y}}{\partial x^2} + \frac{\partial^2 \overrightarrow{E_y}}{\partial y^2} + \frac{\partial^2 \overrightarrow{E_y}}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 \overrightarrow{E_y}}{\partial t^2}$$

$$\frac{\partial^2 \overrightarrow{E_z}}{\partial x^2} + \frac{\partial^2 \overrightarrow{E_z}}{\partial y^2} + \frac{\partial^2 \overrightarrow{E_z}}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 \overrightarrow{E_z}}{\partial t^2}$$