Bachelor Of Engineering In Information Technology

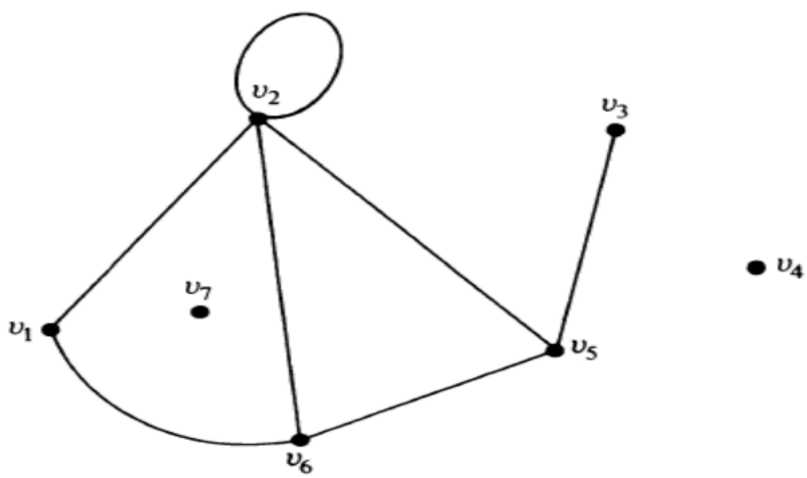
2nd Year 2nd Semester

Subject Name –Graph Theory & Combinatorics (IT/PC/B/T/224)

CO-Wise Assignment

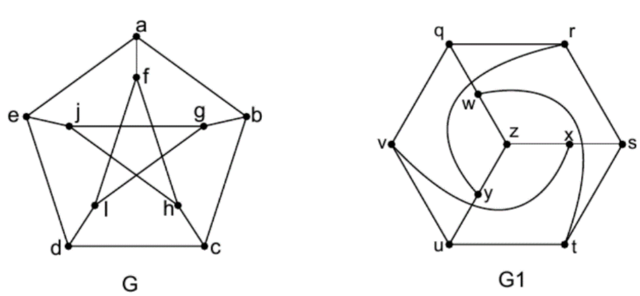
CO1:

1. In the following graph identify isolated vertices and pendant vertex.



2. Define a connected graph. Prove that a connected graph with n vertices has at least (n-1) edges.

3. Define isomorphism of two graphs. Determine whether the two graphs are isomorphic or not.



4. Define a complete graph. In the complete graph with n vertices, where n is an odd number ≥3, show that there are (n-1)/2 edge disjoint Hamilton cycles.

5. a) Is it possible to have simple graphs with the following degree sequences?

If yes,draw the graphs

a) 2,3,3,3,3,3,4,5

b) 1,3,3,4,5,6,6

c) 1,2,3,3,4,5,6

6) Explain digraphs and binary relation on digraphs.

7) Is there any simple graph with degree sequences? (a) (5, 5, 4, 2, 2, 2) (b) (5, 5, 4, 2, 8, 8) (c) (1, 2, 2, 4, 5) (d) (1, 2, 3, 4, 5, 6)

8) If a simple regular graph has n vertices and 36 edges, find the possible values of n.

9) Find the number of vertices in a graph with 20 edges if each vertex degree 2 Show that a connected graph is a circuit if the degree if each vertex is 2.

10) If G is a non-directed graph with 20 edges. If 8 vertices have degree 4 each and rest has degree less than 4, find the minimum number of vertices G can have.

11) The degree of each vertex is 2 5, what should be the number of vertices such that the graph is connected.

12) Find the number of edges in a complete graph with 20 vertices.

13) Examine whether a graph is possible in each case.

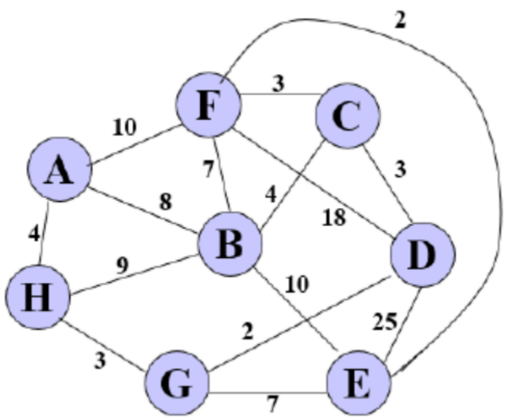
(a) Five vertices with degree 4 each (Possible) (b) Five vertices with degree 3 each (impossible)

(c) Six vertices and 4 edges (Possible) (d) 7 edges and 9 vertices (Possible)

14) Show that there exists a simple graph with 12 vertices and 28 edges such that degree of each vertex in either 3 or 5.

CO2:

1. Determine the minimum spanning tree of the following graph and find its rank and Nullity.



2. Sketch two different binary trees on 13 vertices, one having maximum height and other having minimum height.

3. Prove "Every connected graph has a spanning tree".

4. Prove "G is a tree if and only if there is a unique path between any two vertices."

5. Prove that if the edge costs of G are distinct, there is exactly one minimum cost spanning tree. Give an example of a graph G with more than one minimum cost spanning tree.

6) Draw all unlabeled rooted trees of n vertices for n= 1, 2, 3, 4.

7) Draw all trees of n labeled vertices for n = 1, 2, 3, 4, and 5. 3-2. Draw all trees of n unlabeled vertices for n = 1, 2, 3, 4, and 5.

8) Show a tree in which its diameter is not equal to twice the radius. Under what condition does this inequality hold?

9) Prove that a pendant edge (an edge whose one end vertex is of degree one) in a connected graph & is contained in every spanning tree of G. 3-14. Prove that any subgraph g of a connected graph G is contained in some span ning tree of G if and only if g contains no circuit.

10) What is the nullity of a complete graph of n vertices? 3-16. Show that a Hamiltonian path is a spanning tree.

11) Prove that any given edge of a connected graph G is a branch of some spanning tree of G. Is it also true that any arbitrary edge of G is a chord for some spanning tree of G?

12) Suggest a method for determining the total number of spanning trees of a connected graph without actually listing them.

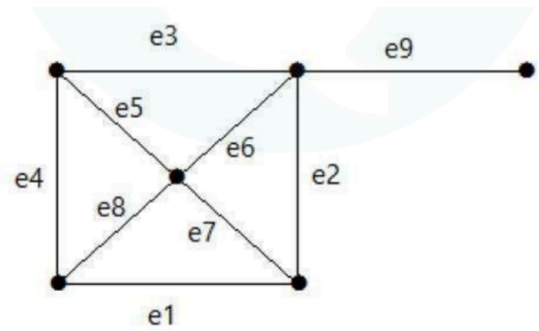
13) Is it possible to have a group of seven people such that each person in the group knows exactly three other people in the group?

14) Prove that in a group of six people, there will be either 3 people who know one another or three people who do not know one another.

15) What is the largest number of vertices in a graph with 35 edges if all vertices are of degree at least 3?

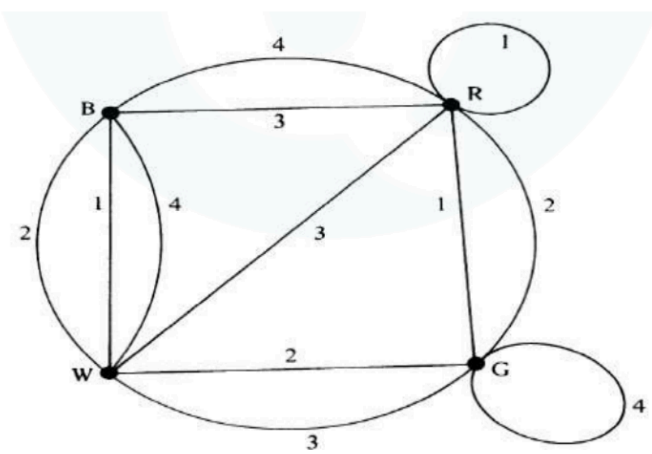
CO3:

1. Define Cut set .Find all cutsets of the graph G given below.

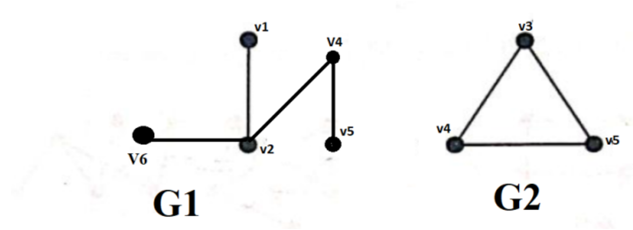


2. Justify using a proper example “A vertex-disjoint sub-graph has also no edge in common ie. it is edge-disjoint also”.

3. Define subgraphs. What are edge disjoint and vertex disjoint subgraphs? Construct
two edge disjoint subgraphs of the graph G.



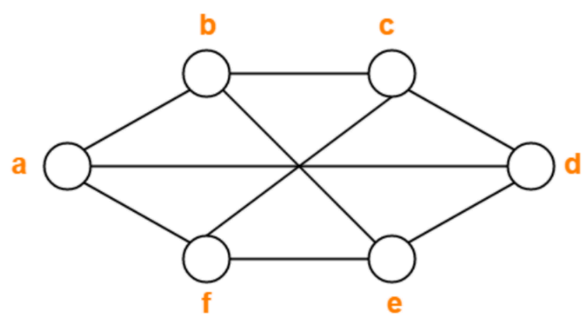
4. Find the intersection of the given two graphs G1 and G2.



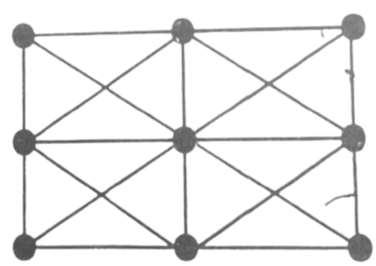
5) Prove that any circuit in a graph G must have at least one edge in common with a chord set.

CO4:

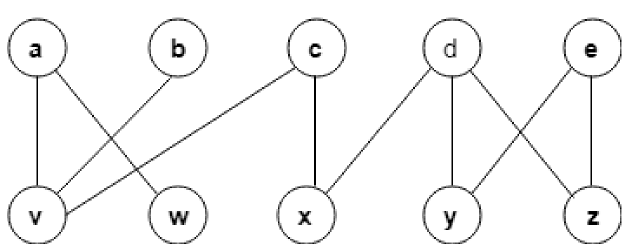
1. Suppose G has n vertices and chromatic number k. Prove that G has at least (k 2) edges.
2. Find chromatic number of the following graph.



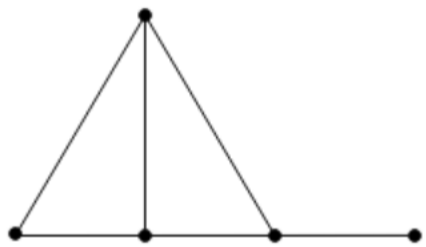
3. Define Chromatic Number.
4. prove that "A graph is 2 colourable if it is bipartite and every cycle in G has even length".
5. Find the chromatic number of the graph given below. (Mention all steps properly)



6. Find out how many vertices can be matched using maximum matching in the bipartite graph algorithm of the following graph given below ?



7. Compare **perfect matching, maximum matching and maximal matching** with a proper example.
8. Find the chromatic number of the graph given below. (Mention all steps properly)



9. Prove that two colors are necessary and sufficient to paint all #vertices ($n \geq 2$) of a tree, such that no edge in the tree has both of its end vertices of the same color.

CO5:

1. A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)?

2. How many ways are there to select three unordered elements from a set with five elements when repetition is allowed?

3. Suppose that a department contains 10 men and 12 women. How many ways are there to form a committee with five members if it must have more women than men?

4. Explain Pigeonhole Principle with an example.

5. Explain the principle of inclusion-exclusion with an example.

6. The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain exactly one vowel?

7. How many bit strings contain exactly eight 0s and 10 1s if every 0 must be immediately followed by a 1?

8. How many ways are there to select six students from a class of 25 to serve on a committee?

9. How many ways are there to select six students from a class of 25 to hold six different executive positions on a committee?

CO6:

1. Find the generating functions of the following sequences in closed form given below:

$$< 1, 1, 1, 1, 1, 1, \ldots >$$

2. Find the sequence generated by the following generating functions:

$$\frac{1}{1-4x}$$

3. Let a_r denote the number of subsets of $\{1, 2, \ldots, r-1, r\}$ which do not contain two consecutive numbers. Determine a_r.

4. Find the generating function for each of the following sequences by relating them back to a sequence with known generating function.

a) 2,4,6,8,10,….
b) 0,0,0,2,4,6,8,10,….
c) 1,5,25,125,….
d) 1,−3,9,−27,81,….
e) 1,0,5,0,25,0,125,0,….
f) 0,1,0,0,2,0,0,3,0,0,4,0,0,5,….