A Multivariable Super Twisting Sliding Mode Control of Descriptor Systems

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***Abstract*—This paper deals with the control of Multi-input multi-output (MIMO) descriptor system using multivariable super-twisting sliding mode control scheme subjected to unknown bounded disturbances. Based on certain conditions, a descriptor system can be represented in generalized regular canonical form which makes the design procedure systematic as a normal system. A step wise method is shown to calculate the appropriate sliding surface gain matrix which can asymptotically stabilize the reduced order system. A design of multivariate super twisting sliding mode control scheme is introduced which eliminates the need to have the derivative of sliding surfaces with a smooth control action. Finally an example is simulated to validate the proposed design.**

***Index Terms*—Descriptor Systems, Weierstrass Canonical Form, Generalized Regular Form, Super-Twisting Control.**

I. Introduction

To derive the state space realization of a given dynamical systems, the natural choice of states are the descriptor variables like position, velocity, acceleration, temperature, current, voltage etc. Depending upon the relationship between the chosen state variables we can get differential equations as well as algebraic equations which help to represent the static constraints. Most of the time system representation results in an implicit system. These differential algebraic equations naturally arise in many practical applications which includes electric circuits, constrained mechanics, power systems, economic systems, biological systems, chemical processes [1], cyber-physical systems or any other large-interconnected systems [2], [3]. Various names have been given to the descriptor systems, viz. implicit systems, differential algebraic equation (DAE) systems, singular systems, generalized state space, noncanonical systems, degenerated systems, nonstandard and semi-state systems [4]. Huge numbers of literature can be found stating the general advantages of using the descriptor system modelling [5], [6]. A significant research interest has been shown in the domain of analysis and controller design of descriptor systems by many researcher [7], [8].

The problem of design of a robust controller for descriptor systems represents an important and active field of research. Sliding mode control (SMC) [9], [10] technique has been an effective robust control scheme with strong theoretical presence from the several decades and it can be observed from the existing literature that this vary technique is generic for controller design and observation of complex practical systems. The SMC of descriptor systems has been exploited by many researcher to solve the robustness issues and some of the prevailing literature in this direction can be found in [11]– [17].

The attractive features of SMC like fast response, insensitive to matched uncertainties and disturbances and ease of implementation can be achieved by designing a sliding surface and an appropriate choice of switched control which unfortunately may introduce high frequency chatterings. For many practical systems, these chatterings are undesirable and may produce detrimental effect to the system performance. Higher order derivatives of control inputs may be contained in the system since the descriptor system conatins both dynamic as well as non-dynamic modes having dimension less than the system dimension. The states will therefore be impulsive unless each control input is smooth enough. To overcome this problem, an integral sliding mode (ISM) controller is designed based on higher order sliding mode control [10] in [11], [12] by artificially introducing integrators. Impelentation of the control law in [11], [12] requires derivative of the sliding variable. In contrast to that the super-twisting control is a continuous control which does not need the derivative of the sliding variable for output with relative degree one. In [13], a super-twisting control is designed for single-input-single-output descriptor system to enforce a sliding motion on the sliding surface. In [14], investigation of ISM control problems for Markovian jump T-S fuzzy descriptor system via super-twisting algorithm is discussed. Observer-based fuzzy ISM control approach for stabilization of nonlinear descriptor systems is discussed in [15]. A robust non-overshooting descriptor system tracking controller based on an ISM technique is investigated in [16]. The design of ISM controller has the advantage that always there exists a feedback control which is known as nominal control for unperturbed descriptor system which is comparatively ease to design and a sliding mode control is added with this nominal control to retain the performance of the descriptor system with disturbances.

Recently in [17], a generalized regular form which is a counter part of regular form for normal system is introduced which

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helps to design the sliding mode controller in a systematic way. Implementation of this control law requires the higher derivative of the sliding variables and also it is not clear how it can be implemented for MIMO systems. As it is mentioned, to realize the super-twisting control does not require the derivative of sliding variables and recently a multivariable super-twisting is presented in [18] which motivates to design a multivariable super-twisting sliding mode controller for perturbed MIMO descriptor system in a systematic way by transforming the system in generalized regular form.

Following the introduction section, organization of the paper is as follows: Section II gives the system description introduction and the assumptions and conditions required for further analysis. It also introduce the various canonical form required for the design of the controller. Section III gives the details procedure for the sliding surface gain matrix calculation from the reduced order dynamics of the given system. Section IV discusses, how the arbitrary poles can be assigned for the system and the required conditions. Finally in Section V a example is taken and the efficacy of the discussed method is verified.

Notations: The following notations are used for simplicity. $\mathbb{C}$ denotes the complex field. $\mathbb{R}^{n\times m}$ ($\mathbb{R}^n$) represents $n\times m$ ($n\times n$) Euclidean space. Where I and 0 represent the identity matrix of dimension $n \times n$ and zero matrix of appropriate dimension respectively. $\|.\|$ stands for the Euclidean norm in case of a vector and the spectral norm of a matrix.

II. System description

Consider an uncertain linear-time invariant (LTI) descriptor system (DS) is represented by

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + B\zeta(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are the state, the input and the output vectors respectively. Here $\zeta(t) \in \mathbb{R}^m$ is a unknown sufficiently differentiable disturbance vector. The disturbances $\zeta(t)$ enter the DS through the same channel as $u(t)$ which satisfies the matching condition. The constant matrices $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$. It is assumed that matrix B is of full column rank with rank$(B) = m$ and matrix C is of full row rank with rank $(C) = p$ and $m \le$ rank$(E) = n_E \le n$. The DS (1) may be represented as $\sum(E, A, B, C)$ and when $E = I$, then the system is said to be a normal system (NS) and is denoted by $\sum(A, B, C)$.

For any acceptable input and consistent initial condition, if a unique solution exists for (1), the DS (1) is called solvable. It is well established fact that the qualitative behavior of (1) is firmly determined by the structure of the matrix pencil $\lambda E - A$, $\lambda \in \mathbb{C}$. To avoid notational complexity, we denote $(E, A) := \lambda E - A$.

Assumption 1: [7] The pencil (E, A) is regular, that is det$(\lambda E - A) \neq 0$.

Moreover det$(\lambda E - A) \neq 0$ is a polynomial and deg det$(\lambda E - A) = n_1 < n$, where the degree of the polynomial is denoted by the function deg [7]. The finite eigenvalue set of the matrix pair (E, A) is represented as $\Lambda(E, A) = \{\lambda_1, \lambda_2, \cdots, \lambda_{n_1}\}$. Given the DS (1), and $\lambda E - A$ to be regular, than there exist two matrices M and N which must be non-singular, such that

$$\begin{aligned} E^* = MEN = &\quad diag(I_{n_1}, \Gamma) \\ A^* = MAN = &\quad diag(\Upsilon, I_{n_2}) \\ B^* = MB = &\quad \begin{bmatrix} B_1^\top & B_2^\top \end{bmatrix}^\top \\ C^* = CN = &\quad \begin{bmatrix} C_1 & C_2 \end{bmatrix} \end{aligned} \tag{2}$$

where $n_1 + n_2 = n$ and the matrix $\Gamma \in \mathbb{R}^{n_2\times n_2}$ is nilpotent with degree h, i.e., $N^h = 0$. With (2) and applying following coordinate transformation

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = M^{-1}x, \quad x_1 \in \mathbb{R}^{n_1}, \quad x_2 \in \mathbb{R}^{n_2}$$

the DS (1) can be represented in Weierstrass canonical form (WCF) [11], [12]

$$\dot{x}_1(t) = \Upsilon x_1(t) + B_1u(t) + B_1\zeta(t) \tag{3a}$$

$$\Gamma\dot{x}_2(t) = x_2(t) + B_2u(t) + B_2\zeta(t) \tag{3b}$$

where $x_1(t)$ and $x_2(t)$ are the states of slow and fast subsystems (3a) and (3b) respectively. The corresponding output vector of DS (1) $y(t)$ is given as, $y(t) = C_1x_1(t) + C_2x_2(t)$. The slow subsystem (3a) is an ordinary differential equation and the solution of this system is well-known. For the fast subsystem (3b), the unique solution exists if and only if the DS (1) is regular. The solution of slow subsystem and fast subsystem is then given by

$$x_1(t) = e^{\Upsilon t}x_1(0) + \int_0^t e^{\Upsilon(t-\tau)}B_1(u(\tau) + \zeta(\tau))d\tau \tag{4a}$$

$$x_2(t) = -\sum_{i=0}^{h-1} \delta^{i-1}(t)\Gamma^i x_2(0) - \sum_{i=0}^{h-1} \Gamma^i B_2(u^{(i)}(t) + \zeta^{(i)}(t)) \tag{4b}$$

where $x_1(0)$ and $x_2(0)$ are initial states of slow and fast subsystems and δ^i is the i-th derivative of a Dirac impulse and $u^{(i)}(t)$ is the i-th derivative of the $u(t)$. It is observed from the (6) the solution of fast subsystem depends on the linear combinations of $h-1$ derivatives of inputs, which demand the inputs and the unknown disturbances to be $h-1$ differentiable. In order to get a solution without impulsive behaviour, Γ must be a zero matrix which is possible with nilpotency index $h = 1$. With $h = 1$, it is fair enough to assume continuity of ζ and boundedness of its first derivative.

Assumption 2: Let the considered unknown disturbance and its derivative are bounded and satisfies the following conditions:

$$\|\zeta(t)\| \le \phi \tag{5}$$

$$\left\|\dot{\zeta}(t)\right\| \le \varphi \tag{6}$$

where $\phi,\ \varphi \in \mathbb{R}^+$ are positive constants.

Remark 1: The asymptotically stability of DS (1) is guaranteed by the finite eigenvalues of the matrix pair (E, A), that is, the eigenvalues of Υ in (3a), must lie in left half of the complex plane.

Remark 2: The invariant zero of the DS (1) is defined as the values of the parameter α such that the matrix $\begin{bmatrix} \alpha E - A & B \\ C & 0 \end{bmatrix}$ loses rank.

The following sections are devoted to develop the systematic way for designing the multivariable super-twisting sliding mode controller for uncertain MIMO descriptor systems.

III. Multivariable super-twisting sliding mode controller design

In this section we propose a controller design technique based on multivariable super-twisting sliding mode for uncertain MIMO descriptor systems to compensate the unknown disturbances.

Generally designing of a control scheme based on SMC is a two step procedure. The first step is to form a stable sliding manifold such that the system state trajectories will be confined to evolve inside this sliding manifold to exhibit the desired performance and the second step is to design a suitable control law which will force the state trajectories to reach the sliding manifold in finite time and will ensure that they will remain there subsequently.

The sliding manifold is designed as

$$\mathcal{S} = \{x(t) \in \mathbb{R}^n : \sigma(t) = \Lambda E x(t) = 0\} \tag{7}$$

where the sliding variable matrix $\Lambda \in \mathbb{R}^{m\times n}$ is a design matrix such that $\det(\Lambda B) \neq 0$. If a controller exists which produces a sliding motion on the sliding manifold $\mathcal{S}$ despite the presence of disturbances, the states lie on $\mathcal{S}$ and subsequently remain there, i.e. $\sigma(t) = \dot{\sigma}(t) = 0$. The equivalent injection term which is necessary to maintain such motion is given by [9], [10]

$$u_{eq}(t) = (\Lambda B)^{-1}\Lambda A x(t) - \zeta(t) \tag{8}$$

Substituting (8) in (1), the reduced order DS (1) is given by:

$$E\dot{x}(t) = \left[I_n - B(\Lambda B)^{-1}\Lambda\right] A x(t), \quad \Lambda E x(t) = 0 \tag{9}$$

If $E = I_n$, the analysis of the DS(1) will be same as normal system but as matrix E is singular so, analysis of the reduced order dynamics will not be straightforward. For this reason following discussion is made.

Assumption 3: Suppose the following condition is satisfied by the DS (1):

$$rank(\begin{bmatrix} E & B \end{bmatrix}) = rank(E) \tag{10}$$

that is the columns of E should span the columns of B.

In [17], it has been shown that if and only if Assumption 3 holds, there exists two non-singular matrices M_R and N_R such that

$$\begin{aligned} E^{**} = M_R E N_R = &\quad diag(E_R, I_m) \\ A^{**} = M_R A N_R = &\quad \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \\ B^{**} = M B = &\quad \begin{bmatrix} 0_{(n-m)\times m} \\ B_2 \end{bmatrix}. \end{aligned} \tag{11}$$

With (11) and applying following coordinate transformation

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = M_R^{-1} x, \quad x_1(t) \in \mathbb{R}^{n-m}, x_2(t) \in \mathbb{R}^m$$

the DS (1) can be represented in generalized regular form (GRF) as

$$E_R \dot{x}_1(t) = A_1 x_1(t) + A_2 x_2(t) \tag{12a}$$
$$\dot{x}_2(t) = A_3 x_1(t) + A_4 x_2(t) + B_2(u(t) + \zeta(t)) \tag{12b}$$

where $E_R, A_1 \in \mathbb{R}^{(n-m)\times(n-m)}$, $A_2 \in \mathbb{R}^{(n-m)\times m}$, $A_3 \in \mathbb{R}^{m\times(n-m)}$, $A_4 \in \mathbb{R}^{m\times m}$ and $B_2 \in \mathbb{R}^{m\times m}$ with rank$(E_R) = n_E - m$ and B_2 is nonsingular. If the system DS (1) is in GRF, the sliding variable matrix Λ in (7) can be partitioned compatibly as

$$\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2 \end{bmatrix} \tag{13}$$

where $\Lambda_1 \in \mathbb{R}^{m\times(n-m)}$ and $\Lambda_2 \in \mathbb{R}^{m\times m}$, then

$$\det(\Lambda B) = \det(\Lambda_2 B_2) = \det(\Lambda_2)\det(B_2)$$

Therefore the sufficient and necessary condition for the matrix ΛB to be nonsingular is that Λ_2 is nonsingular and during design it has been assumed that Λ_2 is nonsingular matrix.

Suppose for a finite time t_s the DS (1) is enforced on the sliding surface $\sigma(t) = 0$ for all $t \geq t_s$, then an ideal sliding mode is said to be taken place for all $t > t_s$, which gives:

$$\begin{aligned} \sigma(t) &= \Lambda E x(t) = 0 \\ &\Rightarrow \begin{bmatrix} \Lambda_1 & \Lambda_2 \end{bmatrix} \begin{bmatrix} E_R & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = 0 \\ &\Rightarrow \Lambda_1 E_R x_1(t) + \Lambda_2 x_2(t) = 0 \end{aligned} \tag{14}$$

and now expressing $x_2(t)$ in terms of $x_1(t)$ results in

$$x_2(t) = -\Lambda_2^{-1}\Lambda_1 E_R x_1(t) \tag{15}$$

Substituting (15) in (12a), dynamics of the reduced order system in terms of sliding surface gain matrix can be written as:

$$E_R \dot{x}_1(t) = (A_1 - A_2\Lambda_2^{-1}\Lambda_1 E_R) x_2(t) \tag{16}$$

Remark 3: From (16) it can be observed that the sliding dynamics of the DS (1) is of reduced order $(n - m)$. Now the task is narrowed down to choose an appropriate sliding variable $\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2 \end{bmatrix}$ such that the eigenvalues of $\left(A_1 - A_2\Lambda_2^{-1}\Lambda_1 E_R\right)$ lies in the left half of complex plane. It follows from [9], if $E = I_n$, the DS will coincide with the normal systems and it is straightforward to stabilize the reduced order system. Furthermore it can also be verified that, the sliding motion poles of the system is determined by the invariant zeros of $(E, A, B, \Lambda E)$.

To stabilize the reduced order system (16), the design procedure is adopted in similar line as in [17]. The following assumptions are made such that with suitable $\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2 \end{bmatrix}$ the finite poles of (16) can be arbitrarily placed which guarantees the asymptotic stability of reduced order system .

Assumption 4: Assume that the DS (1) satisfies the following conditions:

- rank$\left(\begin{bmatrix}\alpha E - A & B\end{bmatrix}\right)$ =rank(A), for $\alpha \in \mathbb{C}^+$ which implies that the DS (1) is R-controllable [7].
- For any value of $\beta \in \mathbb{C}^+$, rank$\left(\begin{bmatrix}\beta A & B \\ E & 0\end{bmatrix}\right)$=$n+m$

It follows from [17], if Assumption 4 is satisfied and the system is regular, subsystem (12a) can be always represented as

$$E_R \dot{x}_1(t) = A_1 x_1(t) + A_2 \nu(t) \tag{17}$$

where $\nu(t) \in \mathbb{R}^m$ can be thought as a virtual control input. The system (17) can always be made regular for some appropriate choice of gain K_1 by the following control input [17]

$$\nu(t) = -K_1 E_R x_1(t) + \nu_1(t) \tag{18}$$

where $\nu_1(t)$ is new control input. In this work, it has been assumed that the subsystem (12a) is regular. With this assumption, there always exist two non-singular matrices M_W and N_W such that system (12a) can be represented in the following WCF form

$$\begin{aligned} \dot{x}_1^1(t) &= A_{11} x_1^1(t) + A_{21}\nu(t) \\ J\dot{x}_1^2(t) &= x_1^1(t) + A_{22}\nu(t) \end{aligned} \tag{19}$$

where $x_1(t) = N_W \begin{bmatrix} x_1^1(t) \\ x_1^2(t)\end{bmatrix}$, $x_1^1(t) \in \mathbb{R}^{n1}$, $x_1^2(t) \in \mathbb{R}^{n-m-n1}$ and $J \in \mathbb{R}^{(n-m-n1)\times(n-m-n1)}$ is a nilpotent matrix, $A_{11} \in \mathbb{R}^{n1\times n1}$, $A_{21} \in \mathbb{R}^{n1\times(n-m-n1)}$, $A_{22} \in \mathbb{R}^{(n-m-n1)\times(n-m-n1)}$.

In [17], it is shown that a controller exists of the form

$$\nu(t) = -K_2 x_1^1(t) = -\begin{bmatrix}K_2 & 0_{m\times(n-m-n1)}\end{bmatrix} M_W E_R x_1(t) \tag{20}$$

which can arbitrarily place the finite eigenvalues of the subsystem (12a). Defining $\Lambda_2^{-1}\Lambda_1 = \begin{bmatrix}K_2 & 0_{m\times(n-m-n1)}\end{bmatrix} M_W$, it can be easily verified that the sliding motion (16) is stable asymptotically and its finite eigenvalues can be arbitrarily assigned. With this relation the sliding surface gain matrix Λ can be calculated as follows

$$\Lambda = \Lambda_2 \begin{bmatrix}\Delta + K_1 & I_m\end{bmatrix} M_R \tag{21}$$

where Δ is given by

$$\Delta = \begin{bmatrix}K_2 & 0_{m\times(n-m-n_1)}\end{bmatrix} M_W \tag{22}$$

and Λ_2 is a nonsingular matrix can be selected arbitrarily.

Next a multivariable super-twisting control law will be designed to stabilize the subsystem (12b). The design will be easy if the DS (1) which is represented in GRF can be transformed in the $(x_1, \sigma(t))$ coordinates by two non-singular matrices, $M_2 = \begin{bmatrix} I_{n-m} & 0 \\ \Lambda_1 & \Lambda_2\end{bmatrix}$ and $N_2 = \begin{bmatrix} I_{n-m} & 0 \\ -\Lambda_2^{-1}\Lambda_1 E_R & \Lambda_2^{-1}\end{bmatrix}$, such that DS (1) takes the form

$$E_R \dot{x}_1(t) = \widehat{A}_1 x_1(t) + \widehat{A}_2 \sigma(t) \tag{23a}$$

$$\dot{\sigma}(t) = \widehat{A}_3 x_1(t) + \widehat{A}_4 \sigma(t) + \widehat{B}_2(u(t) + \zeta(t)) \tag{23b}$$

where $\widehat{A}_1 = A_1 - A_2\Lambda_2^{-1}\Lambda_1 E_R$, $\widehat{A}_2 = A_2\Lambda_2^{-1}$, $\widehat{A}_3 = \Lambda_1 A_1 + \Lambda_2 A_3 - \Lambda_1 A_2 \Lambda_2^{-1}\Lambda_1 E_R - \Lambda_2 A_4 \Lambda_2^{-1}\Lambda_1 E_R$, $\widehat{A}_4 = \Lambda_1 A_2 \Lambda_2^{-1} + \Lambda_2 A_4 \Lambda_2^{-1}$ and $\widehat{B}_2 = \Lambda_2 B_2$.

The stabilization of subsystem (12b) is equivalent to the stabilization of the sliding dynamics $\sigma(t)$ (23b), because the robust control $u(t)$ will be designed in such a way that in finite time $\sigma(t) = \dot{\sigma}(t) = 0$ despite of the presence of disturbance. It ensures that the m order dynamics $x_2(t)$ became algebraic by the relation (15) and rest of the $n-m$ order dynamics represented by (12a) or (23a) can be stabilized by a suitable choice of Λ given by (21).

The control law $u(t)$ is proposed as

$$u(t) = -\widehat{B}_2^{-1}\left(\widehat{A}_3 x_1(t) + \widehat{A}_4\sigma(t)\right) + u_{MSTC} \tag{24}$$

where multivariable super-twisting control law, u_{MSTC} is as follows

$$\begin{aligned} u_{MSTC}(t) &= -\widehat{B}_2^{-1}\left(K_3 \frac{\sigma(t)}{\|\sigma(t)\|^{\frac{1}{2}}} + K_4\sigma(t) + \eta(t)\right) \\ \dot{\eta}(t) &= -K_5 \frac{\sigma(t)}{\|\sigma(t)\|} - K_6\sigma(t) \end{aligned} \tag{25}$$

with the controller gains K_3, K_4, K_5 and K_6 which has to be computed.

Theorem 1: Consider the subsystem (23b), under Assumption 2. Then the subsystem (23b) with matched disturbance $\zeta(t)$ under the feedback control (24) with gain conditions

$$\begin{aligned} K_3 &> \sqrt{2\delta} \\ K_4 &> 0 \\ K_5 &> 3\delta + \frac{2\delta^2}{K_3^2} \\ K_6 &> \frac{\frac{3}{2}\left(K_3^2 K_5 + 3\delta K_4\right)^2}{K_5 K_3^2 - 2\delta^2 - 3\delta K_3^2} + 2K_4^2 \end{aligned} \tag{26}$$

where $\|\dot{\bar{\zeta}}(t)\| \le \|\widehat{B}_2\|\|\dot{\zeta}(t)\| \le \delta$ and for $\delta > 0$ will make $\sigma(t) = \dot{\sigma}(t) = 0$ in finite time.

Proof 1: Substituting this control (24), the (23a) and (23b) can be written as

$$\begin{aligned} E_R\dot{x}_1(t) &= \widehat{A}_1 x_1(t) + \widehat{A}_2\sigma(t) \\ \dot{\sigma}(t) &= K_3\frac{\sigma(t)}{\|\sigma(t)\|^{\frac{1}{2}}} + K_4\sigma(t) + \eta(t) + \widehat{B}_2\zeta(t)) \\ \dot{\eta}(t) &= -K_5\frac{\sigma(t)}{\|\sigma(t)\|} - K_6\sigma(t). \end{aligned} \tag{27}$$

Assuming $\bar{\zeta}(t) = \widehat{B}_2\zeta(t)$ and $Z(t) = \eta(t) + \bar{\zeta}$, the above equations can be written as

$$\begin{aligned} E_R\dot{x}_1(t) &= \widehat{A}_1 x_1(t) + \widehat{A}_2\sigma(t) \\ \dot{\sigma}(t) &= -K_3\frac{\sigma(t)}{\|\sigma(t)\|^{\frac{1}{2}}} - K_4\sigma(t) + Z(t) \\ \dot{Z}(t) &= -K_5\frac{\sigma(t)}{\|\sigma(t)\|} - K_6\sigma(t) + \dot{\bar{\zeta}}(t). \end{aligned} \tag{28}$$

Using Assumption 2, $\|\dot{\bar{\zeta}}(t)\| \le \|\widehat{B}_2\|\|\dot{\zeta}(t)\| \le \delta$. From, (28), it can be concluded that by choosing the gains K_3, K_4, K_5 and K_6 (26), which introduce a second order sliding mode ensuring the finite time convergence of sliding variable $\sigma(t)$ and its derivative $\dot{\sigma}(t)$ [18]. This ends the proof.

In the next section, the above proposed method is simulated through an example.

IV. Simulation

Consider a MIMO uncertain descriptor system

$$E\dot{x}(t) = Ax(t) + Bu(t) + B\zeta(t) \tag{29}$$

where $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 0 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 0 \\ -1 & 2 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$.

The disturbance matrix is defined as $\zeta(t) = \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} = \begin{bmatrix} 0.6\sin(t)) \\ 0.8\cos(5t) \end{bmatrix}$. It can be seen that the DS chosen for simulation satisfies Assumption 1 2, 3 and 4. With this assumptions, the sliding surface gain matrix is designed as $\Lambda = \begin{bmatrix} -4 & 0.7071 & -0.7071 & 0 \\ -4 & -0.7071 & -0.7071 & 0 \end{bmatrix}$ with $K_2 = \begin{bmatrix} 4 & 0 \\ 4 & 0 \end{bmatrix}$. The gain of the controller is found to be $K_3 = 5$, $K_4 = 2$, $K_5 = 2$ and $K_6 = 30$. The initial condition for the DS system is chosen for simulation is $[0.5 \;\; -0.5 \;\; 0 \;\; 0]^\top$.

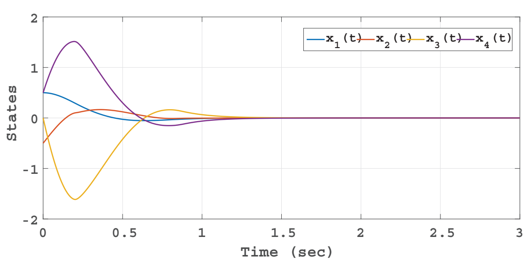

Fig. 1: system states

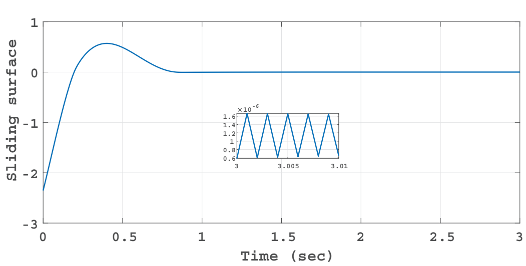

Fig. 2: sliding surface

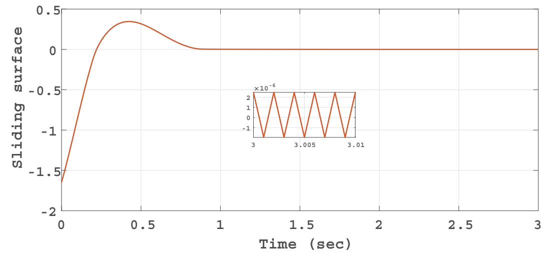

Fig. 3: sliding surface

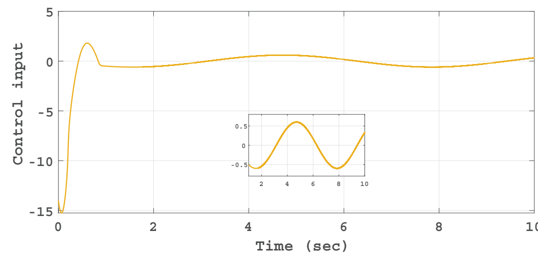

Fig. 4: control input

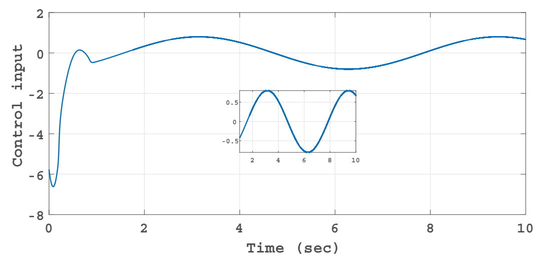

Fig. 5: control input

From the Figure 1 it is clearly seen that the system state trajectories are converging despite of the disturbances. It is evident from the plot that the proposed control law is working effectively. From the Figure 2 and 3 it is seen that the siding variables are converging in finite time to zero and is maintained there for the subsequent time. The smooth control inputs are shown in Figure 4 and 5

V. Conclusion

In this paper, the control of the MIMO descriptor system for matched disturbances is investigated using multivariable super-twisting sliding mode control. A systematic way is discussed to design the sliding surface gain matrix to stabilize the reduced order system and a multivariable super-twisting sliding mode controller is designed which ensures convergence of the sliding variable in finite time which in turn guarantees the asymptotic stability of the given uncertain descriptor system. The simulation results are the evidence of the controller performance.

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