DIFFERENTIATION USING NEWTON'S FORWARD INTERPOLATION

Objective

The objective of this experiment is to calculate the first and second derivatives of a function at a given point using Newton's Forward Interpolation formula. The program uses equally spaced data points and compares the computed derivatives with the analytical derivatives to determine the error.

Theory

Newton's Forward Interpolation is a numerical method for estimating function values and their derivatives using a set of discrete data points. For a function f(x)f(x)f(x) evaluated at equally spaced points x0,x1,...,xn the derivatives at a point XXX can be approximated as:

Forward Difference Table

$$\Delta 1\text{yi} = \text{yi} + 1 - \text{yi}$$
$$\Delta 2\text{yi} = \Delta 1\text{yi} + 1 - \Delta 1\text{yi}$$
$$\Delta 3\text{yi} = \Delta 2\text{yi} + 1 - \Delta 2\text{yi}$$

Where $yi = f(xi)$

Derivative Formulas

- First Derivative f′(X)

$$f'(X) \approx \frac{\Delta y0 + \frac{(2u-1)\Delta 2y0}{2!} + \frac{(3u2-6u+2)\Delta 3y0}{3!} + \cdots}{h}$$

- Second Derivative f′′(X):

$$f''(X) \approx \frac{\Delta 2y0 + (u-1)\Delta 3y0 + \cdots}{h2}$$

Where:

$$u = \frac{(X - x0)}{h}$$
$$h = x(i+1) - xi$$

Error Calculation

The computed derivatives are compared with analytical derivatives f'(X) and f''(X):

$$\text{Error} = \frac{|Analytical - Numerical|}{Numerical} \times 100$$

Input Format (input.txt)

- Line 1: Number of test cases T
- For each test case:
  - n → number of intervals
  - a b → start and end of interval
  - X → point at which derivatives are computed

Example:

```
4
4
1 2
1.5
5
0 1
0.5
6
1 3
2.5
3
2 4
3.2
```

Output

- For each test case, the program outputs:
- Test case number
- Number of intervals, a, b, and X
- Forward difference table

- First derivative f'(X)
- Second derivative f''(X)
- Percentage errors of first and second derivatives

Example Console / Output.txt format:

```
TEST CASE #1
n: 4, a: 1, b: 2, X: 1.5
Difference table:
       2          1          0          0
       3          2          1          0
       5          4          1          0
       9          6          0          0
       ...
y': 4.123
y'': 5.456
First diff error: 0.0123%
Second diff error: 0.0345%
```

Algorithm / Steps

1. Read number of test cases T.
2. For each test case:

- Read n, a, b, X input file.
- Compute step size $h = (b - a)/n$
- Generate equally spaced points $xi = a + i * h$ and compute $yi = f(xi)$.
- Construct forward difference table.
- Compute first derivative f'(X)) using Newton's formula.
- Compute second derivative f''(X) using Newton's formula.
- Compare with analytical derivatives to compute percentage error.
- Print input, difference table, derivatives, and errors to console and output file.

Features

- Supports multiple test cases
- Computes first and second derivatives numerically
- Generates full forward difference table

- Calculates percentage error with analytical derivatives
- Outputs results to both console and file
- Works for equally spaced intervals