NEWTON FORWARD INTERPOLATION FOR UNEQUALLY SPACED DATA – DOCUMENTATION

This program performs Newton Forward Interpolation using the divided difference method to estimate the value of a function at a given point X from unequally spaced data points.

OBJECTIVE

To calculate the interpolated value f(X) for a given value X using Newton Forward Interpolation when the data points are not equally spaced.

THEORY

Newton Forward Interpolation for unequal spacing uses divided differences.

For n+1 data points $(x0, y0), (x1, y1), \ldots, (xn, yn)$, the polynomial is:

$$Pn(X) = f[x0] + (X - x0)f[x0, x1] + (X - x0)(X - x1)f[x0, x1, x2] + \ldots + (X - x0)\ldots(X - x\_\{n-1\})f[x0, \ldots, xn]$$

Where $f[xi, \ldots, xj]$ are divided differences, computed as:

$$f[xi] = yi$$

$$f[xi, xi + 1] = \frac{(f[xi + 1] - f[xi])}{(xi + 1 - xi)}$$

$$f[xi, \ldots, xi + k] = \frac{(f[xi + 1, \ldots, xi + k] - f[x\_i, \ldots, x\_i + k - 1])}{(xi + k - xi)}$$

The error of interpolation is given by:

$$Error(X) = f[x0, \ldots, xn](X - x0)(X - x1)\ldots(X - xn - 1)$$

INPUT FORMAT (input.txt)

T

N

x0 x1 … xn

y0 y1 … yn

X

1

4

1 4 5 7 10

2 20 30 56 100

8

OUTPUT (Console + output.txt)

For each test case, the program prints:

1. Test Case Number
2. Number of data points (n)
3. x[] and y[] values
4. Interpolation point X
5. Full divided difference table (n+1 x n+1 including zeros)
6. Interpolated value at X
7. Truncation error

Test Case #1

n: 4

x: 1.0000 4.0000 5.0000 7.0000 10.0000

y: 2.0000 20.0000 30.0000 56.0000 100.0000

X: 8

Difference Table:

2.0000 6.0000 1.0000 0.0000 -0.0123

20.0000 10.0000 1.0000 -0.1111 0.0000

30.0000 13.0000 0.3333 0.0000 0.0000

56.0000 14.6667 0.0000 0.0000 0.0000

100.0000 0.0000 0.0000 0.0000 0.0000

Interpolation: 70.9630

Truncation error: 1.0370

ALGORITHM

1. Read number of test cases T
2. For each test case:

- Read n, x[], y[], X
- Initialize (n+1)x(n+2) difference table with zeros
- Fill first column with y[] values
- Compute divided differences column by column
- Apply Newton Forward formula to compute interpolated value
- Compute true error
- Print inputs, difference table, interpolated value, and error to console and output file

FEATURES

- Handles multiple test cases
- Works with unequally spaced data
- Prints full divided difference table including zeros
- Displays all input and output for clarity
- Outputs results to both console and output.txt