

Gradients and Sobel Filters

- Gradients are a measure of intensity change in an image, and they generally mark object boundaries and changing area of light and dark. If we think back to treating images as functions, $F(x, y)$, we can think of the gradient as a derivative operation $F'(x, y)$. Where the derivative is a measurement of intensity change.
- Sobel filters
 - The Sobel filter is very commonly used in edge detection and in finding patterns in intensity in an image. Applying a Sobel filter to an image is a way of **taking (an approximation) of the derivative of the image** in the x or y direction. The operators for *Sobelx* and *Sobely*, respectively, look like this:

$$S_x = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

$$S_y = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$



Sobel x



Sobel y

- **x vs. y**
 - In the above images, you can see that the gradients taken in both the x and the y directions detect the edges of the brain and pick up other edges. Taking the gradient in the x direction emphasizes edges closer to vertical. Alternatively, taking the gradient in the y direction emphasizes edges closer to horizontal.
- **Magnitude:**
 - Sobel also detects which edges are *strongest*. This is encapsulated by the **magnitude** of the gradient; the greater the magnitude, the stronger the edge is. The magnitude, or absolute value, of the gradient is just the square root of the squares of the individual x and y gradients. For a gradient in both the x **and** y directions, the magnitude is the square root of the sum of the squares.
 - $\text{abs_sobel}_x = \sqrt{(\text{sobel_x})^2}$
 - $\text{abs_sobel}_y = \sqrt{(\text{sobel_y})^2}$
 - $\text{abs_sobel}_{xy} = \sqrt{(\text{sobel_x})^2 + (\text{sobel_y})^2}$
- **Direction:**
 - In many cases, it will be useful to look for edges in a particular orientation. For example, we may want to find lines that only angle upwards or point left. By calculating the direction of the image gradient in the x and y directions separately, we can determine the direction of that gradient!
 - The direction of the gradient is simply the inverse tangent (arctangent) of the y gradient divided by the x gradient:
 - $\tan^{-1}(\text{sobel_y}/\text{sobel_x})$.