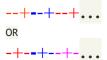
If the OA is B then it should be:

Sequence S consists of 24 nonzero integers. If each term in S after the second is the product of the previous two terms, how many terms in S are negative?

- (1) The third term in S is **positive** --> either all terms are positive, so zero negatives or +--... and not all terms are positive, so more than zero negatives. Not sufficient.
- (2) The fourth term in S is negative --> again two cases:



The same here: in both cases there is a repeated pattern of three terms in which 2 are negative and 1 positive (--+ or -+-) so in both cases out of 24 terms 2/3 will be negative, so there will be 16 negative terms. Sufficient.

Answer: B.

2

An infinite sequence of positive integers is called a perfect sequence. If each term in the sequence is a perfect number, that is, if each term can be expressed as the sum of its divisors, excluding itself. For example, 6 is a perfect number, as its divisors, 1, 2, and 3, sum to 6. Is the infinite sequence S a perfect sequence?

- (1) Exactly one term in S is a prime number --> primes have exactly two divisors 1 and itself, hence no prime is a perfect number, which means that S is not a perfect sequence. Sufficient.
- (2) In sequence S, each term after the first in S has exactly 3 divisors --> a number to have exactly 3 divisors must be square of a prime, for example 3^2=9 has 3 divisors: 1, 3, and 9 (1, p, and p^2). No, such number is a perfect number: 1+3 cannot equal to 9, (1+p cannot equal to p^2 for integer p), which means that S is not a perfect sequence. Sufficient.

Answer: D.

3

A set of nonnegative integers consists of $\{x, x + 7, 2x, y, y + 5\}$. The numbers of this set have four distinct values. What is its average (arithmetic mean)?

Notice couple of things:

- 1. We are told that all numbers in the set are integers;
- 2. We are told that all those integers are non-negative;
- 3. We are told that the set contains four distinct values out of five (so two integers out of 5 are alke and other three are distinct).
- (1) $x \neq 5$. Clearly insufficient.
- (2) 4y + 12 = 6(y + 2) --> 4y + 12 = 6y + 12 --> 2y = 0 --> y = 0. So, we have that our set is $\{x, x + 7, 2x, 0, 5\}$. Since we know that two integers out of 5 are alike then:
- 1. x, x+7 or 2x is 0.

If x=0, then 2x=0 too, so in this case we'll have three alike integers x, 2x, and y. Thus this case is out. If x+7=0, then x=-7 and we know that all integers must be non-negative so this case is out too.

2. x, x+7 or 2x is 5

If x=5, then x+7=12 and 2x=10. This scenario is OK. The set in this case would be: $\{5, 12, 10, 0, 5\}$ If x+7=5, then x=-2 and we know that the integers in the set must be non-negative. Thus this case is out. If 2x=5, then x=5/2 and we know that the numbers in the set must be integers. Thus this case is out.

3. Two out of x, x+7 and 2x are alike.

x=2x is not possible, since in this case x=2x=0 and in this case we'll have three alike integers x, 2x, and y. x=x+7 has no solution.

2x=x+7 --> x=7. This scenario is OK. The set in this case would be: $\{7, 14, 14, 0, 5\}$.

Two cases are possible. Not sufficient.

(1)+(2) Since from (1) $x \ne 5$, then from (2) x=7. So, the set is $\{7, 14, 14, 0, 5\}$. Sufficient.

Answer: C.

4

In the sequence of positive numbers $x_1, x_2, x_3, ...$, what is the value of x_1 ?

 $x_i = \frac{x_{(i-1)}}{2}$ for all integers i > 1 --> we have the general formula connecting two consecutive terms (basically we have geometric progression with common ratio 1/2), but without the value of any term this info is insufficient to find x_1 .

 $x_5 = \frac{x_4}{x_4 + 1} \dots \text{ we have the relationship between } x_5 \text{ and } x_4 \text{, also insufficient to find } x_1 \text{ (we cannot extrapolate the relationship between }} x_5 \text{ and } x_4 \text{ to all consecutive terms in the sequence)}.$

$$x_5 = \frac{x_4}{2} \xrightarrow{\dots} \frac{x_4}{2} = \frac{x_4}{x_4 + 1} \xrightarrow{\dots} x_4 = 1 = x_1 * (\frac{1}{2})^3 \xrightarrow{\dots} x_1 = 8 \text{. Sufficient.}$$

Answer: C.