1

Jane and Bill working together will paint the wall in $T=\frac{JB}{J+B}$ hours. Now suppose that $J=B$ --> $T=\frac{J^2}{2J}=\frac{J}{2}$, as J and B are even $J=2n$ --> $T=\frac{2n}{2}=n$, as n is an integer, working together Jane and Bill will paint the wall in whole number of hours, meaning that in any case T must be an integer.

(1) They finish painting in 4 hours and 48 minutes, T is not an integer, --> J and B are not equal. Sufficient.

(2) $J+B=20$, we can even not consider this one, clearly insufficient. J and B can be 10 and 10 or 12 and 8.

Answer: A.

2

There are several important things you should know to solve work problems:

1. Time, rate and job in work problems are in the same relationship as time, speed (rate) and distance in rate problems.

$time*speed=distance$ <--> $time*rate=job\ done$. For example when we are told that a man can do a certain job in 3 hours we can write: $3*rate=1$ --> $rate=\frac{1}{3}$ job/hour. Or when we are told that 2 printers need 5 hours to complete a certain job then $5*(2*rate)=1$ --> so rate of 1 printer is $rate=\frac{1}{10}$ job/hour. Another example: if we are told that 2 printers need 3 hours to print 12 pages then $3*(2*rate)=12$ --> so rate of 1 printer is $rate=2$ pages per hour;

So, time to complete one job = reciprocal of rate. For example if 6 hours (time) are needed to complete one job --> 1/6 of the job will be done in 1 hour (rate).

2. We can sum the rates.

If we are told that A can complete one job in 2 hours and B can complete the same job in 3 hours, then A's rate is $rate_a=\frac{job}{time}=\frac{1}{2}$ job/hour and B's rate is $rate_b=\frac{job}{time}=\frac{1}{3}$ job/hour. Combined rate of A and B working simultaneously would be $rate_{a+b}=rate_a+rate_b=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ job/hour, which means that they will complete $\frac{5}{6}$ job in one hour working together.

3. For multiple entities: $\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+...+\frac{1}{t_n}=\frac{1}{T}$, where T is time needed for these entities to complete a given job working simultaneously.

For example if:
Time needed for A to complete the job is A hours;
Time needed for B to complete the job is B hours;
Time needed for C to complete the job is C hours;
...
Time needed for N to complete the job is N hours;

Then: $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+...+\frac{1}{N}=\frac{1}{T}$, where T is the time needed for A, B, C, ..., and N to complete the job working simultaneously.

For two and three entities (workers, pumps, ...):

General formula for calculating the time needed for two workers A and B working simultaneously to complete one job:

Given that t_1 and t_2 are the respective individual times needed for A and B workers (pumps, ...) to complete the job, then time needed for A and B working simultaneously to complete the job equals to $T_{(A\&B)}=\frac{t_1*t_2}{t_1+t_2}$ hours, which is reciprocal of the sum of their respective rates ($\frac{1}{t_1}+\frac{1}{t_2}=\frac{1}{T}$).

General formula for calculating the time needed for three A, B and C workers working simultaneously to complete one job:

$T_{(A\&B\&C)}=\frac{t_1*t_2*t_3}{t_1*t_2+t_1*t_3+t_2*t_3}$ hours.

BACK TO THE ORIGINAL QUESTION:
Machines X and Y produced identical bottles at different constant rates. Machine X, operating alone for 4 hours, filled part of a production lot; then Machine Y, operating alone for 3 hours, filled the rest of this lot. How many hours would it have taken Machine X operating alone to fill the entire production lot?

You can solve this question as Karishma proposed in her post above or algebraically:

Let the rate of X be x bottle/hour and the rate of Y y bottle/hour.
Given: $4x+3y=job$. Question: $t_x=\frac{job}{rate}=\frac{job}{x}=?$

(1) Machine X produced 30 bottles per minute --> $x=30*60=1800$ bottle/hour, insufficient as we don't know how many bottles is in 1 lot (job).

(2) Machine X produced twice as many bottles in 4 hours as Machine Y produced in 3 hours --> $4x=2*3y$, so $3y=2x$ --> $4x+3y=4x+2x=6x=job$ --> $t_x=\frac{job}{rate}=\frac{job}{x}=\frac{6x}{x}=6$ hours. Sufficient.

Answer: B.

3

If Ruth began a job and worked continuously until she finished, at what time of day did she finish the job?

(1) She started the job at 8:15 a.m. and at noon of the same day she had worked exactly half of the time that it took her to do the whole job --> at noon of the same day, so after 3 hours and 45 minutes she started, Ruth had worked **HALF OF THE TIME** that it took her to do the whole job, which means that the total time it took Ruth to do the whole job is 7.5 hours, so she finished the job at 8:15 a.m+7.5 hours=15:45 p.m. Sufficient.

So as you can see we are not concerned about her rate at all.

(2) She was finished exactly 7.5 hours after she had started. Not sufficient, since we don't know at what time Ruth started the job.

Answer: A.

4

An empty swimming pool with a capacity of 75,000 liters is to be filled by hoses X and Y simultaneously. If the amount of water flowing from each hose is independent of the amount flowing from the other hose, how long, in hours, will it take to fi¤ll the pool?

(1) If hose X stopped fi¤lling the pool after hoses X and Y had filled **half the pool**, it would take 21 hours to fi¤ll the pool. After hose X stops, hose Y continues filling the remaining half of the pool alone, and we are told that it needs 21 hours for that, hence to fill the whole pool it needs 21*2=42 hours. We know the rate of Y, though know nothing about the rate of X. Not sufficient.

(2) If hose Y stopped fi¤lling the pool after hoses X and Y had filled **half the pool**, it would take 16 hours to fi¤ll the pool. Reverse case: after hose Y stops, hose X continues filling the remaining half of the pool alone, and we are told that it needs 16 hours for that, hence to fill the whole pool it needs 16*2=32 hours. We know the rate of X, though know nothing about the rate of Y. Not sufficient.

(1)+(2) We know the rates of both hose X and Y, hence we can calculate the time they'll need to fi¤ll the pool of 75,000 liters. Sufficient.

Answer: C.

5

Machines X and Y run simultaneously at their respective constant rates. If machine X produces 400 bolts per hour, how many bolts do machines X and Y produce per hour?

(1) Machine X takes twice as long to produce 400 bolts as it does for machines X and Y, working together, to produce the same number of bolts --> since machine X needs **twice as long** to produce 400 bolts as it does for machines **X and Y, working together**, to produce the same number, then the rates of X and Y are the same: both produce 400 bolts per hour, (working together they need 1/2 hours to produce 400 bolts). Sufficient.

(2) Machines X and Y produce bolts at the same rate. We know the rates of both machines. Sufficient.

Answer: D.