1

Is |x| + |y| = 0?

Since absolute value is non-negative the from $|x|+|y|=0$ we have that the sum of two non-negative values equals to zero, which is only possible if both of them equal to zero. So, the question basically asks whether $x=y=0$

(1) x + 2|y| = 0. It's certainly possible that $x=y=0$ but it's also possible that $x=-2$ and $y=1$. Not sufficient.

Notice that from this statement $|y|=-\frac{x}{2}$, so $-\frac{x}{2}$ equals to a non-negative value ($|y|$), so $-\frac{x}{2}\geq 0$ --> $x\leq 0$.

(2) y + 2|x| = 0. It's certainly possible that $x=y=0$ but it's also possible that $y=-2$ and $x=1$. Not sufficient.

Notice that from this statement $|x|=-\frac{y}{2}$, so $-\frac{y}{2}$ equals to a non-negative value ($|x|$), so $-\frac{y}{2}\geq 0$ --> $y\leq 0$.

(1)+(2) We have that $x\leq 0$ and $y\leq 0$, hence equations from the statements transform to: $x-2y=0$ and $y-2x=0$. Solving gives $x=y=0$. Sufficient.

Answer: C.

2

If |x+2|=4, what is the value of x ?

$|x+2|=4$ --> $x=2$ or $x=-6$.

(1) x^2 is different from 4 --> $x^2\neq 4$ --> $x\neq 2$ (and $x\neq -2$), so $x=-6$. Sufficient.

(2) x^2 = 36 --> $x=6$ or $x=-6$, so $x=-6$. Sufficient.

Answer: D.

3

If $x\neq 0$, is $\frac{x^2}{|x|}<1$? --> reduce by $|x|$--> is $|x|<1$? or is $-1<x<1$?

Two statements together give us the sufficient info.

Answer: C.

Given: $\frac{x^2}{|x|}<1$

Consider this:

$\frac{x^2}{|x|}=\frac{|x|*|x|}{|x|}=|x|$. It's basically the same as if it were $\frac{x^2}{x}$ --> we could reduce this fraction by x and we would get x, and when x is positive, result is positive and when x is negative, result is negative. Now, $\frac{x^2}{|x|}$ is the ratio of two positive values and the result can not be negative, so we can not get x, we should get $|x|$ to guarantee that the result is positive.

OR:

$x<0$--> then $|x|=-x$ --> $\frac{x^2}{|x|}=\frac{x^2}{-x}=-x<1$ --> $x>-1$;

$x>0$ --> then $|x|=x$ --> $\frac{x^2}{|x|}=\frac{x^2}{x}=x<1$;

So $-1<x<1$.

4

If $x\neq0$, is $|x|<1$?

Is $|x|<1$? --> is - $1<x<1$ ($x\neq0$)?

(1) $x^2<1$ --> $-1<x<1$. Sufficient.

(2) $|x|<\frac{1}{x}$ --> since LHS (|x|) is an absolute value which is always non-negative then RHS (1/x), must be positive (as $|x|<\frac{1}{x}$), so $\frac{1}{x}>0$ --> $x>0$.

Now, if $x>0$ then $|x|=x$ and we have: $x<\frac{1}{x}$ --> since $x>0$ then we can safely multiply both parts by it: $x^2<1$ --> $-1<x<1$, but as $x>0$, the final range is $0<x<1$. Sufficient.

Answer D.

X cannot be negative. Refer to the solution above.

Also if $x<0$ then we have $-x<\frac{1}{x}$ and now if we cross multiply by negative x then we should flip the sign: $-x^2>1$ --> $x^2+1<0$ which cannot be true for any real value of x (the sum of two positive value cannot be less than zero).

5

2. If y is an integer and y = |x| + x, is y = 0?
(1) x < 0
(2) y < 1

Notice that since $y=|x|+x$ then y is never negative. If $x>0$ (so if x is positive) then $y=x+x=2x$ and for $x\leq0$ then (when **x is negative or zero**) then $y=-x+x=0$.

(1) $x<0$ --> $y=|x|+x=-x+x=0$. Sufficient.

(2) $y<1$, as we concluded y is never negative, and we are given that y is an integer, hence $y=0$. Sufficient.

Answer: D.

6

What does |2b| equal?

(1) b^2-|b|-20=0. Solve quadratics for $|b|$: $(|b|)^2-|b|-20=0$ --> $|b|=-4$ or $|b|=5$. Since absolute value cannot be negative then we have that $|b|=5$ and $|2b|=10$. Sufficient.

(2) |2b|=3b+25. Two cases:

If $b \leq 0$ then we would have that $-2b = 3b + 25$ --> $b = -5$.
If $b > 0$ then we would have that $2b = 3b + 25$ --> $b = -25$, but since we are considering the rangeo when $b > 0$ then discard this solution.

So, we have that $b = -5$, hence $|2b| = 10$. Sufficient.

Answer: D.