1

Is x^2 + y^2 > 100?

(1) 2xy < 100 --> clearly insufficient: if $x=y=0$ then the answer will be NO but if $x=10$ and $y=-10$ then the answer will be YES.

(2) (x + y)^2 > 200 --> $x^2+2xy+y^2>200$. Now, as $(x-y)^2\geq 0$ (square of any number is more than or equal to zero) then $x^2+y^2\geq 2xy$ so we can safely substitute $2xy$ with x^2+y^2 (as x^2+y^2 is at least as big as $2xy$ then the inequality will still hold true) --> $x^2+(x^2+y^2)+y^2>200$ --> $2(x^2+y^2)>200$ --> $x^2+y^2>100$. Sufficient.

Answer: B.

could you please explain why you followed with (x-y)^2 instead of (x+y)^2? Shouldn't (x-y)^2 be distributed as x^2-2xy+y^2 in which case it will be different from the original expression? Also, when you transfer 2xy to the other side from x^2+2xy+y^2, why do you keep it positive?

We have $(x+y)^2>200$ which is the same as $x^2+2xy+y^2>200$.

Now, we need to find the relationship between x^2+y^2 and 2xy.

Next, we know that $(x-y)^2\geq 0$ --> $x^2-2xy+y^2\geq 0$ --> $x^2+y^2\geq 2xy$.

So, we can safely substitute $2xy$ with x^2+y^2 in $x^2+2xy+y^2>200$ (as x^2+y^2 is at least as big as $2xy$ then the inequality will still hold true) --> $x^2+(x^2+y^2)+y^2>200$ --> $2(x^2+y^2)>200$ --> $x^2+y^2>100$.
Sufficient.

Is x^2 + y^2 > 100?

(2) (x + y)^2 > 200
which means: x^2 + y^2 + 2xy > 200

Now you know that 2xy is less than or equal to x^2 + y^2.
If 2xy is equal to (x^2+y^2), (x^2+y^2) will be greater than 100 since the total sum is greater than 200.
If 2xy is less than (x^2+y^2), then anyway (x^2+y^2) will be greater than 100 (which is half of 200).

So statement 2 is sufficient alone.

2

So I'd say the best way for this question would be to try boundary values.

Q: is $2r+3s\leq 6$?

(1) $3r+2s=6$ --> very easy to see that this statement is not sufficient:
If $r=2$ and $s=0$ then $2r+3s=4<6$, so the answer is YES;
If $r=0$ and $s=3$ then $2r+3s=9>6$, so the answer is NO.
Not sufficient.

(2) $r\leq 3$ and $s\leq 2$ --> also very easy to see that this statement is not sufficient:
If $r=0$ and $s=0$ then $2r+3s=0<6$, so the answer is YES;

If $r=3$ and $s=2$ then $2r+3s=12>6$, so the answer is NO.
Not sufficient.

(1)+(2) We already have an example for YES answer in (1) which valid for combined statements:
If $r=2<3$ and $s=0<2$ then $2r+3s=4<6$, so the answer is YES;

To get NO answer try max possible value of s, which is $s=2$, then from (1) $r=\frac{2}{3}<3$ --> $2r+3s=\frac{4}{3}+6>6$, so the answer is NO.
Not sufficient.

Answer: E.

3

If x and y are positive integers such that x = 8y + 12, what is the greatest common divisor of x and y?

Given: $x=8y+12$.

(1) x = 12u, where u is an integer --> $x=12u$ --> $12u=8y+12$ --> $3(u-1)=2y$ --> the only thing we know from this is that 3 is a factor of y. Is it GCD of x and y? Not clear: if $x=36$, then $y=3$ and $GCD(x,y)=3$ but if $x=60$, then $y=6$ and $GCD(x,y)=6$ --> two different answers. Not sufficient.

(2) y = 12z, where z is an integer --> $y=12z$ --> $x=8*12z+12$ --> $x=12(8z+1)$. So, we have $y=12z$ and $x=12(8z+1)$. Now, as z and $8z+1$ do not share any common factor but 1 (8z and 8z+1 are consecutive integers and consecutive integers do not share any common factor 1. As 8z has all factors of z then z and 8z+1 also do not share any common factor but 1). Thus, 12 must be GCD of x and y. Sufficient.

Answer: B.