1

Notice that since 3, 4, and 7 are co-prime (they don't share any common factor but 1), then:

x must be a multiple of 4 and 7, so the least value of x is 28 (LCM of 4 and 7);
y must be a multiple of 3 and 7, so the least value of y is 21;
z must be a multiple of 3 and 4, so the least value of z is 12;

So, the least possible value of x + y + z is 28+21+12=61.

Answer: D.

2

Since at most 3 of the pockets are to contain the same number of coins then minimize # of coins in each, so let each contain just 1 coin;

Next, we are told that no two of the remaining 4 pockets should contain an equal number of coins, so they should contain 2, 3, 4, and 5 coins each (also minimum possible);

Total: 1+1+1+2+3+4+5=17.

Answer: C.

3

In a class of 30 students, 2 students did not borrow any books from the library, 12 students each borrowed 1 book, 10 students each borrowed 2 books, and the rest borrowed at least 3 books. If the average number of books per student was 2, what is the maximum number of books any single student could have borrowed?

A. 3
B. 5
C. 8
D. 13
E. 15

"The average number of books per student was 2" means that total of 2*30=60 books were borrowed;
2+12+10=24 students borrowed total of 2*0+12*1+10*2=32 books;
So 60-32=28 books are left to distribute among 30-24=6 students, these 6 are "the rest who borrowed at least 3 books";

To *maximize* the number of books one student from above 6 could have borrowed we should *minimize* the number of books other 5 students from 6 could have borrowed. Minimum these 5 students could have borrowed is 3 books per student, so total number of books they could have borrowed is 5*3=15 books. So the 6th student could have borrowed is 28-15=13 books.

Answer: D.

4

Say the population of the city was 100 in 2005, then in 2007 it would be $100(1+\frac{a}{100})(1+\frac{b}{100})=\frac{ab}{100}+a+b+100=\frac{ab}{100}+120$ (since $a+b=20$).

So, in order to maximize this expression we need to maximize the value of ab, when given that $a+b=20$. Useful property: **for given sum of two numbers, their product is maximized when they are equal**. Hence, ab will be maximized for $a=b=10$ --> $120+\frac{ab}{100}=120+1=121$.

Answer: E

5

A cyclist travels the length of a bike path that is 225 miles long, rounded to the nearest mile. If the trip took him 5 hrs, rounded to the nearest hour, then his average speed must be between:
A. 38 and 50 mph
B. 40 and 50 mph
C. 40 and 51 mph
D. 41 and 50 mph
E. 41 and 51 mph

Length of a path is 225 miles long, rounded to the nearest mile --> $224.5\leq distance<225.5$;
The trip took him 5 hrs, rounded to the nearest hour --> $4.5\leq time<5.5$;

Lowest average rate is $\frac{224.5}{5.5}\approx 40.8$ (take the lowest value of nominator and highest value of denominator);
Highest average rate is $\frac{225.5}{4.5}\approx 50.1$ (take the highest value of nominator and lowest value of denominator);

$40.8<rate<50.1$.

Now, the question is: "the average speed must be between..." hence the range from correct answer choice MUST cover all possible values of rate, so must cover all the range: $40.8<rate<50.1$. Only C does that: $(40)<40.8<rate<50.1<(51)$. D can not be the answer as if $rate=40.9$ or if $rate=50.01$ then these possible values of the average rate are not covered by the range from this answer choice, which is (41-50).

Answer: C.

Alternative approach.
It's based on observing the answer choices. **On the PS section always look at the answer choices before you start to solve a problem. They might often give you a clue on how to approach the question.**
A. 38 and 50 mph
B. 40 and 50 mph
C. 40 and 51 mph
D. 41 and 50 mph
E. 41 and 51 mph

Notice that since the range from A covers entire range from B and D, then B and D are out (if B or D is correct so is A and we cannot have two correct answer, leave the bigger range). Similarly since the range from C covers entire range from E, then E is out too (if E is correct so is C and we cannot have two correct answer, leave the bigger range).

Thus we are left only with two answer choices A (38, 50) and C (40, 51). From here it's much easier to get the correct answer.

6

In order three of the carpenters to complete the maximum fraction of the task we should select three fastest carpenters. So, those three who complete the task in 3, 4, and 5 hours.

In 1 hour they will complete $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{47}{60}$ th of the task. So, in $\frac{3}{4}$ hour (45 minutes) they will complete $\frac{47}{60}*\frac{3}{4}=\frac{47}{80}$ th of the job.

Answer: D.

7

Question should read:
If 5400mn = k^4, where m, n, and k are positive integers, what is the least possible value of m + n?
A. 11
B. 18
C. 20
D. 25
E. 33

Note that m, n, and k are positive integers.

First of all: $5,400=2^3*3^3*5^2$. Now, in order $5,400mn=2^3*3^3*5^2*m*n$ to be equal to the integer in fourth power then mn must complete the powers of 2, 3 and 5 to the fourth power, hence the least value of mn for which $2^3*3^3*5^2*m*n=k^4$ is for $mn=2*3*5^2=150$. In this case $5,400mn=2^3*3^3*5^2*(2*3*5^2)=(2*3*5)^4=k^4$.

So we have that the least value of mn is $2*3*5^2$. Next: in order to minimize $m+n$ we should break $2*3*5^2$ into two multiples which are closest to each other: $2*5=10$ and $3*5=15$, their sum is $10+15=25$.

Answer: D.

8

Say # of part-time employees is p, then # of full-time employees will be $48-p$.

We want to maximize $\frac{p}{3}+\frac{48-p}{4}$ --> $\frac{p}{3}+\frac{48-p}{4}=\frac{p+3*48}{12}=\frac{p}{12}+12$, so we should maximize p, but also we should make sure that $\frac{p}{12}+12$ remains an integer (as it represent # of people). Max value of p for which p/12 is an integer is for $p=36$ (p can not be 48 as we are told that there are some # of full-time employees among 48) --> $\frac{p}{12}+12=3+12=15$.

Answer: D.

Or: since larger share of part-time employees take the subway then we should maximize # of part-time employees, but we should ensure that $\frac{p}{3}$ and $\frac{48-p}{4}$ are integers. So p should be max multiple of 3 for which $48-p$ is a multiple of 4, which turns out to be for $p=36$ --> $\frac{p}{3}+\frac{48-p}{4}=15$.

9

120 prefer X (Group 1);
80 prefer Y (Group 2).

City Y needs 150 people: let **all 80** who prefer Y (entire Group 2) be relocated there, the rest 70 will be those who prefer X from Group 1;
City X needs 50 people: 120-70=**50** from Group 1 will be relocated to X, which they prefer.

So, the highest possible number of employees who will be relocated to the city they prefer is 80+50=130.

Answer: D.

10

We can remove 80 books only from physics or from chemistry. Worst case scenario would be if we remove all books on botany, zoology, geology, 79 books on physics and 79 books on chemistry.

We would have 50(botany)+65(zoology)+50(geology)+79(physics)+79(chemistry)=323 books and still won't have 80 books from the same science. As after this, only books on physics and chemistry will be left then any next book would become the 80th book either on physics or on chemistry.

Answer: E.