

1

The tens and units digits of a three-digit integer must be 0: $X00$ (no digit gives the same digit when divided by 2 but 0: $0/2=0$). Now, apart from this, X must be even because $300/2=150$, not the same tens and units digits. Hence there are following three-digit integers possible: 200, 400, 600, and 800.

Answer: E.

2

The average age of a group of 5 members is 20 years. Two years later, a new member joins the group. The average age of the group becomes 22 years. What is the age of the new member ?

- A. 20 years
- B. 21 years
- C. 22 years
- D. 23 years
- E. 24 years

The average age of a group of 5 members is 20 years \rightarrow the sum of the ages is $5 \cdot 20 = 100$;
Two years later the sum of the ages of these 5 members would be $100 + 5 \cdot 2 = 110$;

Now, say the age of the new member is x years, so the sum of the ages of a new 6 member group is $110 + x$. Since given that the average age of this group of 6 members is 22 years then: $22 \cdot 6 = 110 + x \rightarrow x = 22$.

Answer: C.

3

Jim is twice as old as Stephanie $\rightarrow J = 2S$;

Stephanie four years ago, was three times as old as Kate $\rightarrow S - 4 = 3(K - 4) \rightarrow K = (S + 8) / 3$ (it would be better if it were "Stephanie four years ago, was three times as old as Kate was four years ago");

Five years from now, the sum of their ages will be 51 $\rightarrow (J + 5) + (S + 5) + (K + 5) = 51 \rightarrow (2S + 5) + (S + 5) + ((S + 8) / 3 + 5) = 51 \rightarrow S = 10$.

Answer: B.

4

Total charges of company A: $575 + 0.04 \cdot 575 + 82.5 = 680.5$;

Total charges of company B: $530 + 0.03 \cdot 530 + 93 = 638.9$;

If we deal with company B, which offers the lower price, we can save $680.5 - 638.9 = 41.6$ dollars.

Answer: A.

5

D (23) is not a trap answer, it's a correct answer.

Jane started baby-sitting when she was 18 years old. Whenever she baby-sat for a child, that child was no more than half her age at the time. Jane is currently 32 years old, and she stopped baby-sitting 10 years ago. What is the current age of the oldest person for whom Jane could have baby-sat?

- A. 20
- B. 21
- C. 22
- D. 23
- E. 24

Check two extreme cases:

Jane = 18, child = 9, years ago = $32 - 18 = 14 \rightarrow$ child's age now = $9 + 14 = 23$;

Jane = 22, child = 11, years ago = $32 - 22 = 10 \rightarrow$ child's age now = $11 + 10 = 21$.

Answer: D.

6

$$\frac{0.99999999}{1.0001} - \frac{0.999999991}{1.0003} = \frac{1-10^{-8}}{1+10^{-4}} - \frac{1-9 \cdot 10^{-8}}{1+3 \cdot 10^{-4}}$$

Now apply $a^2 - b^2 = (a+b)(a-b)$:

.

Answer: D.

7

Original question reads:

The value of $(2^{-14} + 2^{-15} + 2^{-16} + 2^{-17})/5$ is how many times the value of 2^{-17} ?

$$\frac{\frac{1}{5} \cdot (2^{-14} + 2^{-15} + 2^{-16} + 2^{-17})}{2^{-17}} = \frac{\frac{1}{5} \cdot (\frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{1}{2^{17}})}{\frac{1}{2^{17}}}$$

We need to find the value of: .

Now, .

$$\frac{\frac{1}{5} \cdot (\frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{1}{2^{17}})}{\frac{1}{2^{17}}} = \frac{2^{17}}{5} \cdot (\frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{1}{2^{17}}) = \frac{1}{5} \cdot (2^3 + 2^2 + 2 + 1) = \frac{1}{5} \cdot 15 = 3$$

Answer: C.

8

Given that $M = \frac{1}{201} + \frac{1}{202} + \frac{1}{203} + \dots + \frac{1}{300}$. Notice that $1/201$ is the largest term and $1/300$ is the smallest term.

If all 100 terms were equal to $1/300$, then the sum would be $100/300=1/3$, but since actual sum is more than that, then we have that $M > 1/3$.

If all 100 terms were equal to $1/200$, then the sum would be $100/200=1/2$, but since actual sum is less than that, then we have that $M < 1/2$.

Therefore, $1/3 < M < 1/2$.

Answer: A.

9

Yes factoring out 10^2 and 10^3 and then reducing the fraction by 10^2 is one way to deal with this question:

$$\frac{10^8 - 10^2}{10^7 - 10^3} = \frac{10^2(10^6 - 1)}{10^3(10^4 - 1)} = \frac{10^6 - 1}{10 \cdot (10^4 - 1)}$$

Now, $10^6 - 1$ is very close to 10^6 and $10^4 - 1$ is very close to 10^4 , hence $\frac{10^6 - 1}{10 \cdot (10^4 - 1)} \approx \frac{10^6}{10 \cdot 10^4} = \frac{10^6}{10^5} = 10$.

Answer: B.

Or else you can notice that we need *approximate* value of a fraction. Now, 10^8 is much, much, much bigger than 10^2 . So subtracting 10^2 from 10^8 will be very close to 10^8 , basically 10^2 is negligible in this case. The same for 10^7 and 10^3 .

$$\frac{10^8 - 10^2}{10^7 - 10^3} \approx \frac{10^8}{10^7} = 10$$

so

10

We need to find the speed of the car in **miles per hour**. So, we should convert feet in miles and seconds in hours.

20 feet is $\frac{20}{5280}$ miles;

0.5 second is $\frac{0.5}{3600} = \frac{0.5}{60^2}$ hours;

The speed of the car therefore is
$$\frac{\text{distance}}{\text{time}} = \frac{(\frac{20}{5280})}{(\frac{0.5}{60^2})} = (\frac{20}{5280}) * (\frac{60^2}{0.5})$$

Answer: A.