Lesson 60

AIM: How can we use combinations to solve probability problems?

PERFORMANCE STANDARDS: M4, M5, M7

PERFORMANCE OBJECTIVES: The students will be able to…

1. Explain under what circumstances combinations should be used to count the number of elements in an event and in a sample space.
2. Apply the combination formula to solve probability problems.

VOCABULARY: No new terms

MOTIVATION:

Juan, Paul, Jane and Sarah have volunteered to work on a committee that will require only two members. The two members are selected at random. **Ask**: "How many ways may this be done? ($_4C_2 = 6$). What is the probability that Juan and Sarah will be chosen?" ($\frac{1}{6}$)

In response to the first question, students should be asked to write the sample space corresponding to the event. It should be apparent that order does not matter and hence this is a combination problem.

For the second question, students should be asked the definition of probability; the formula P(E) = $\frac{n(E)}{n(S)}$ should be recalled.

DEVELOPMENT:

1. Three cards are drawn from a deck of 52 cards without replacement.
    Ask: " What is the probability that all are Aces?" As in the previous problem, n(S) should be obtained. In this case, this is $_{52}C_3$ as again the order of drawing the cards is not important. The concept that there are $_4C_3$ combinations of ways of drawing the three Aces is not as apparent.
    It should be stressed that in order to obtain n(E) we are concerned only with the four Ace subset of the deck. The three Aces must be selected from this group of 4. From the definition of probability, P(E) = $\frac{_4C_3}{_{52}C_3}$ = $\frac{4}{22100}$

2. Next consider an urn that has 5 black and 3 white marbles. Four marbles are chosen at random. **Ask**: "What is the probability that two are white and two are black?"
    As in the card problem, order is unimportant and therefore n(S) = $_8C_4$.
    Ask: "How many ways may the two white marbles be drawn? ($_3C_2$).

The black marbles?" ($_5C_2$). Recall here the fundamental counting principle to obtain n(E) = $_3C_2 . _5C_2$. Then P(E) = $\dfrac{_3C_2._5C_2}{_8C_4}$. The class should compute the result.

APPLICATIONS:

1. For the above problem, compute the probability that at least three are black if five marbles are chosen at random. **Ask**: "How many ways could this happen?" (3 black, 2 white or 4 black, 1 white or 5 black, 0 white). Call these events A, B, C. **Ask**: "What is the formula for P(A∪B∪C)?" The fact that these are mutually exclusive should be stressed and therefore:

   P(A∪B∪C) = P(A) + P(B) + P(C) = $\dfrac{_5C_3._3C_2}{_8C_5}$ + $\dfrac{_5C_4._3C_1}{_8C_5}$ + $\dfrac{_5C_5._3C_0}{_8C_5}$

   (An alternative solution could be to add the probabilities of 0 black, 5 white + 1 black, 4 white + 2 black, 3 white and subtract this sum from 1.)

2. Three cards are drawn at random from a standard deck of playing cards without replacement. What is the probability that:
   a) all 3 cards are Aces?
   b) at least one of the cards is a picture card?
   c) there is at most one heart?
   d) each card has a different face value?

MATH A-**TYPE QUESTION**:

   Twenty girls quality for the girls' Baseball team. How many teams of 9 girls each may be formed? For each team of 9 girls, in how many different ways can the 9 starting positions be assigned?

SUMMARY:

1. Describe the steps used to solve probability problems involving combinations. (To compute probabilities, we could use P(E) = $\dfrac{n(E)}{n(S)}$. To find n(E) and n(S) we determine if order is important. If it is not, we use combinations to obtain both expressions.
2. What Mathematics Standards did we address today?

HOMEWORK:

1. Find the probability that if two cards are drawn at random from an ordinary deck
   a) both are sixes ($\frac{_4C_2}{_{52}C_2}$)

   b) both are red ($\frac{_{26}C_2}{_{52}C_2}$)

   c) one is red, the other black ($\frac{_{26}C_1 \cdot _{26}C_1}{_{52}C_2}$)

   d) both are picture cards ($\frac{_{12}C_2}{_{52}C_2}$)

2. An urn contains 3 red, 1 blue and 2 green marbles. Two are selected at random. Find the probability:
   a) both are red ($\frac{_3C_2}{_6C_2}$)

   b) one is red, the other green ($\frac{_3C_1 \cdot _2C_1}{_6C_2}$)

   c) both are blue (0)

3. From a group of 6 men and 3 women a committee of four individuals is selected. What is the probability that the committee:
   a) Includes 2 women and 2 men?
   b) Includes at most one woman?
   c) Includes at least one woman?
   d) Includes at least one man?