

SOME NOTES:**1. GCD and LCM**

The greatest common divisor (GCD), of two or more non-zero integers, is the largest **positive integer** that divides the numbers without a remainder.

So GCD can only be **positive integer**. It should be obvious as **greatest** factor of two integers can not be negative. For example if -3 is a factor of two integer then 3 is also a factor of these two integers.

The lowest common multiple (LCM), of two integers a and b is the smallest **positive integer** that is a multiple both of a and of b .

So LCM can only be **positive integer**. It's also quite obvious as if we don't limit LCM to positive integer then LCM won't make sense any more. For example what would be the **lowest** common multiple of 2 and 3 if LCM could be negative? There is no answer to this question.

2. DIVISIBILITY QUESTIONS ON GMAT

EVERY GMAT divisibility question will tell you in advance that any unknowns represent **positive integers**.

3. REMAINDER

GMAT Prep definition of the remainder:

If a and d are **positive integers**, there exists unique integers q and r , such that $a = qd + r$ and $0 \leq r < d$. q is called a quotient and r is called a remainder.

Moreover many GMAT books say factor is a "positive divisor", $d > 0$.

I've never seen GMAT question asking the remainder when dividend (a) is **negative**, but if we'll cancel this restriction (and consider $dividend = a < 0$), but leave the other restriction ($0 \leq r < d$), then division of negative integer by positive integer could be calculated as follow:

-8 divided by 6 will result: $0 \leq r < d, a = qd + r \rightarrow 0 \leq r < 6, -8 = (-2) * 6 + 4$.
Hence $remainder = r = 4$.

TO SUMMARIZE, DON'T WORRY ABOUT NEGATIVE DIVIDENDS, DIVISORS OR REMAINDERS ON GMAT.

BACK TO THE ORIGINAL QUESTION:

The integers m and p are such that $2 < m < p$, and m is not a factor of p . if r is the remainder when p is divided by m , is $r > 1$?

Given: $2 < m < p$ and $\frac{p}{m} \neq integer$, which means that $r > 0$. Note here that as $0 < 2 < m < p$ then your example of -8 and 6 is not valid as both m and p are positive.
Question: $r = ?$

(1) the greatest common factor of m and p is 2 \rightarrow both p and m are even (as both have 2 as a factor) \rightarrow even divided by even can give only even remainder (0, 2, 4, ...), since remainder is not zero (as $\frac{p}{m} \neq integer$), then remainder must be more than 1: 2, 4, ... Sufficient.

(2) the least common multiple of m and p is 30 \rightarrow if $m = 5$ and $p = 6$, remainder=1=1, answer to the question would be NO. BUT if $m = 10$ and $p = 15$ remainder=5>1 answer to the question would be YES. Two different answers. Not sufficient.

Answer: A.

2

Remainder when a number is divided by 9 is the same as remainder when the sum of its digits is divided by 9:

$$\text{Remainder} \frac{N}{9} = \text{Remainder} \frac{w+x+y+z}{9}$$

Let's show this on our example:

Our 4 digit number is $1000w+100x+10y+z$. what is the remainder when it's divided by 9?

When $1000w$ is divided by 9 the remainder is $\frac{w}{9}$:

$$\frac{3000}{9} \text{ remainder } 3, \frac{3}{9} \text{ remainder } 3.$$

The same with $100x$ and $10y$.

So, the remainder when $1000w+100x+10y+z$ is divided by 9 would be:

$$\frac{w}{9} + \frac{x}{9} + \frac{y}{9} + \frac{z}{9} = \frac{w+x+y+z}{9}$$

(1) $w + x + y + z = 13 \rightarrow$ remainder $13/9=4$, remainder $N/9=4$. Sufficient.

(2) $N+5$ is divisible by 9 $\rightarrow N+5=9k \rightarrow N=9k-5=4, 13, 22, \dots \rightarrow$ remainder upon dividing this numbers by 9 is 4. Sufficient.

Answer: D.

3

General approach is correct, though the red parts are not.

The last digit of 2^k repeats in pattern of 4 (cyclicity is 4):

$2^1=2 \rightarrow$ last digit is 2;

$2^2=4 \rightarrow$ last digit is 4;

$2^3=8 \rightarrow$ last digit is 8;

$2^4=16 \rightarrow$ last digit is 6;

$2^5=32 \rightarrow$ last digit is 2 again;

Now, when k itself is a multiple of 4 (when there is no remainder upon division k by cyclicity number), then the last digit will be the last digit of 2^4 (4th in pattern), so 6 not 1 (taking 2^0) as you've written.

If k is a positive integer, what is the remainder when 2^k is divided by 10?

Notice that all we need to know to answer the question is the last digit of 2^k .

(1) k is divisible by 10 \rightarrow different multiples of 10 yield different remainders upon division by 4 (for example $10/4$ yields 2 and $20/4$ yields 0), thus we can not get the single numerical value of the last digit of 2^k . Not sufficient.

(2) k is divisible by 4 \rightarrow as discussed, when k is a multiple of 4, the last digit of 2^k equals to the last digit of 2^4 , which is 6. Integer ending with 6 yields remainder of 6 upon division by 10. Sufficient.

Answer: B.

4

A person inherited few gold coins from his father. If he put 9 coins in each bag then 7 coins are left over. However if he puts 7 coins in each bag then 3 coins are left over. What is the number of coins he inherited from his father.

If he puts 9 coins in each bag then 7 coins are left over $\rightarrow c = 9q+7$, so # of coins can be: 7, 16, 25, 34, 43, 52, 61, ...

If he puts 7 coins in each bag then 3 coins are left over $\rightarrow c = 7p+3$, so # of coins can be: 3, 10, 17, 24, 31, 38, 45, 52, 59, ...

General formula for c based on above two statements will be: $c = 63k+52$ (the divisor should be the least common multiple of

above two divisors 9 and 7, so 63 and the remainder should be the first common integer in above two patterns, hence 52). For more about this concept see: [manhattan-remainder-problem-93752.html#p721341](https://www.manhattanreview.com/93752.html#p721341), [when-positive-integer-n-is-divided-by-5-the-remainder-is-90442.html#p722552](https://www.manhattanreview.com/90442.html#p722552), [when-the-positive-integer-a-is-divided-by-5-and-125591.html#p1028654](https://www.manhattanreview.com/125591.html#p1028654)

$c = 63k + 52$ means that # of coins can be: 52, 115, 178, 241, ...

(1) The number of coins lies between 50 to 120 --> # of coins can be 52 or 115. Not sufficient.

(2) If he put 13 coins in one bag then no coin is left over and number of coins being lesser than 200 --> # of coins is a multiple of 13 and less than 200; only 52 satisfies this condition. Sufficient.

Answer: B.