1

If $d>0$ and $0<1-\frac{c}{d}<1$, which of the following must be true?

I. $c>0$
II. $\frac{c}{d}<1$
III. $c^2+d^2>1$

A. I only
B. II only
C. I and II only
D. II and III only
E. I, II, and III[/quote]

$0<1-\frac{c}{d}<1$ --> add -1 to all three parts of this inequality --> $-1<-\frac{c}{d}<0$ --> mutliply by -1 and as multiplying by negative flip signs --> $1>\frac{c}{d}>0$.

So we have that: $1>\frac{c}{d}>0$

I. $c>0$ --> as $\frac{c}{d}>0$ and $d>0$, then $c>0$. Always true.

II. $\frac{c}{d}<1$ --> directly given as true.

III. $c^2+d^2>1$ --> if $c=1$ and $d=2$, then YES, but if $c=0.1$ and $d=0.2$, then No, hences this one is not always true.

Answer: C (I and II only).

2

Given: $4x-12\geq x+9$ --> $3x\geq 21$ --> $x\geq 7$.

Only A is always true, as ANY x from the TRUE range $x\geq 7$ will be more than 6.

Answer: A

3

Factor is a "positive divisor" (at least on the GMAT). So, the factors of 4 are 1, 2, and 4 ONLY.

Tips about perfect squares >0:
1. The **number of distinct factors** of a perfect square is ALWAYS ODD. *The reverse is also true: if a number has the odd number of distinct factors then it's a perfect square*;

2. The **sum of distinct factors** of a perfect square is ALWAYS ODD. The reverse is NOT always true: a number may have the odd sum of its distinct factors and not be a perfect square. For example: 2, 8, 18 or 50;

3. A perfect square ALWAYS has an **ODD number of Odd-factors**, and **EVEN number of Even-factors**. *The reverse is also true: if a number has an **ODD number of Odd-factors**, and EVEN number of Even-factors then it's a perfect square*. For example: odd factors of 36 are 1, 3 and 9 (3 odd factor) and even factors are 2, 4, 6, 12, 18 and 36 (6 even factors);

4. Perfect square always has **even powers of its prime factors**. *The reverse is also true: if a number has even powers of its prime factors then it's a perfect square*. For example: $36=2^2*3^2$, powers of prime factors 2 and 3 are even.

According to this, only II and III must be true.

Answer: D.

4

Note that we are asked to determine which MUST be true, not could be true.

$4<\frac{7-x}{3}$ --> $12<7-x$ --> $x<-5$. So we know that $x<-5$, it's given as a fact. Now, taking this info we should find out which of the following inequalities will be true OR which of the following inequalities will be true for the range $x<-5$.

Basically the question asks: if $x<-5$ which of the following is true?

I. $5<x$ --> not true as $x<-5$.

II. $|x+3|>2$, this inequality holds true for 2 cases, (for 2 ranges): 1. when $x+3>2$, so when $x>-1$ or 2. when $-x-3>2$, so when $x<-5$. We are given that second range is true ($x<-5$), so this inequality holds true.

Or another way: ANY x from the range $x<-5$ (-5.1, -6, -7, ...) will make $|x+3|>2$ true, so as $x<-5$, then $|x+3|>2$ is always true.

III. $-(x+5)>0$ --> $x<-5$ --> true.

Answer: D.

5

Given that S+R=B+2, where S, R, and B are times in which Stephanie, Regine, and Brian completed the race.

Min time one could complete the race is 20/8=2.5 hours. Let's see if Brian could have won the race: if he ran at the fastest rate, he would complete the race in 2.5 hours, so combined time needed for Stephanie and Regine would be S+R=B+2=4.5 hours, which is not possible as sum of two must be more than or equal the twice the least time: 2*2.5=5. So Brian could not have won the race.

There is no reason to distinguish Stephanie and Regine so if one could have won the race, another also could. So both could have won the race.

Answer: D.

The least time one could complete the race is 20/8=2.5 hours, hence $S+R\geq 5$. Let's see if Brian could have won the race: **best chances to win he would have if he ran at the fastest rate**, so he would complete the race in 2.5 hours, so combined time needed for Stephanie and Regine would be S+R=B+2=4.5 hours, but we know that $S+B\geq 5$, so even if Brian ran at his fastest rate to win the race, given equation S+R=B+2 can not hold true. Hence Brian could not have won the race

6

If x, a, and b are positive integers such that when x is divided by a, the remainder is b and when x is divided by b, the remainder is $a-2$, then which of the following must be true?

A. a is even
B. $x+b$ is divisible by a
C. $x-1$ is divisible by a
D. $b=a-1$
E. $a+2=b+1$

When x is divided by a, the remainder is b --> $x=aq+b$ --> $remainder=b<a=divisor$ (remainder must be less than divisor);
When x is divided by b, the remainder is $a-2$ --> $x=bp+(a-2)$ --> $remainder=(a-2)<b=divisor$.

So we have that: $a-2<b<a$, as a and b are integers, then it must be true that $b=a-1$ (there is only one integer between $a-2$ and a, which is $a-1$ and we are told that this integer is b, hence $b=a-1$).

Answer: D.

7

Given that:

The ratio of cupcakes to children is 104 to 7 --> $\frac{cupcakes}{children}=\frac{104k}{7k}$;

Each child eats exactly x cupcakes --> the number of cupcakes eaten $7kx$ and the number of cupcakes that remain uneaten is $104k-7kx$;

The number of cupcakes that remain uneaten is less than the number of children --> $104k-7kx<7k$ --> $x>13\frac{6}{7}$ --> $x=14$ (notice that x cannot be more than 14 since in this case 7kx>104k, which would mean that more cupcakes were eaten than there were).

Now, if $x=14$, then the number of cupcakes that remain uneaten is $104k-7k*14=6k$, thus the number of uneaten cupcakes must be a multiple of both 2 and 3.

Answer: D.

8

$a-b$ even --> either both even **or** both odd

$\frac{a}{b}$ even --> either both even **or** a is even and b is odd.

As both statements are true --> a and b must be even.

As $\frac{a}{b}$ is an **even integer** --> a must be multiple of 4.

Options A is always even.
Options B can be even or odd.
Options C can be even or odd.
Options D: $\frac{a+2}{2}=\frac{a}{2}+1$, as a is multiple of 4, $\frac{a}{2}$ is even integer --> even+1=odd. Hence option D is always odd.
Options E can be even, odd.

Answer: D.

9

If x is the average (arithmetic mean) of 5 consecutive even integers, which of the following must be true?

I. x is an even integer.
II. x is a nonzero integer.
III. x is a multiple of 5.

(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

First of all notice that we are asked which of the following MUST be true, not COULD be true.

Also notice that **the average of 5 consecutive even integers equals to the median**, so it's **just a middle term**. So, I must always be true: {even, even, **even**, even, even}

Next:
II. x is a nonzero integer. Not necessarily true, consider: {-4, -2, **0**, 2, 4};
III. x is a multiple of 5. Not necessarily true, consider: {0, 2, **4**, 6, 8}.

Answer: A.

10

If y=−m^2, which of the following must be true?

First of all notice m^2 is always non-negative, so -m^2 is non-positive (zero or negative), which means that y is zero when m=0 and y is negative for ANY other value of m.

I. y is negative --> not necessarily true, if m=0 then y=-m^2=0;

II. m is non-negative. m can take ANY value: positive, negative, zero. We don't have any restrictions on its value;

III. If m is negative then y is negative. m is negative means that m is not zero. As discussed above if m is other than zero (positive or negative) then y is negative: y=-(negative^2)=-positive=negative (y=-(positive^2)=-positive=negative). So, this option is always true.

Answer: C (III only).

11

$a+b=a(a+b)$ --> $a(a+b)-(a+b)=0$ --> $(a+b)(a-1)=0$ --> as a and b are positive the $a+b\neq0$, so $a-1=0$ --> $a=1$. Also as a and b are different positive integers then b must be more than $a=1$ --> $a<b$ (b can not be equal to a as they are different and b can not be less than a as b is positive integerand thus can not be less than 1).

So we have that: $a=1$ and $a<b$.

I. a = 1 --> true;
II. b = 1 --> not true;
III. a < b --> true.

Answer: E (I and II only).