1

30 sec approach:
Any odd non-prime, greater than 1, can be obtained by the sum of an odd prime and a positive even number. So this set plus the set of odd primes basically makes the set of **all odd numbers greater than 1 in the range**. Now, the set of **all odd numbers greater than 1** together with the set of **all even numbers** makes the set of all numbers from 1 to 30, not inclusive, so total of 28 numbers.

Answer: B.

To illustrate:
# of even numbers in the range is (28-2)/2+1=14: 2, 4, 6, ..., 28;
# of odd primes in the range is 9: 3, 5, 7, 11, 13, 17, 19, 23, and 29;
# of integers which are the sum of a positive multiple of 2 and an odd prime is 5: 9=7+2, 15=13+2, 21=19+2, 25=23+2 and 27=23+4;

Total: 14+9+5=28. You can see that we have all numbers from 1 to 30, not inclusive: 2, 3, 4, 5, 6, ..., 29.

2

The question should read:

The sum of two positive integers is 588 and their greatest common factor is 49. How many such pairs of numbers can be formed?

We are told that the greatest common factor of two integers is 49. So, these integers are $49x$ and $49y$, for some positive integers x and y. Notice that x and y must not share any common factor but 1, because if they do then GCF of $49x$ and $49y$ will be more that 49.

Next, we know that $49x+49y=588$ --> $x+y=12$ --> since x and y don't share any common factor but 1 then (x, y) can be only (1, 11) and (5, 7) (all other pairs (2, 10), (3, 9), (4, 8), (6, 6) do share common factor greater than 1). So, there are only two pairs of such numbers possible: 49*1=49 and 49*11=539 AND 49*5=245 and 49*7=343.

Answer: E.

3

Since x34,586,23y is divisible by 88=8*11 then it's divisible by both 8 and 11.

Now, **in order a number to be divisible by 8 its last three digits must be divisible by 8**, so 23y must be divisible by 8. That's only possibly if y=2 --> 232/8=29. So, our number is x34,586,232.

Next, **in order a number to be divisible by 11 the difference between the sum of the odd numbered digits (1st, 3rd, 5th...) and the sum of the even numbered digits (2nd, 4th...) must be divisible by 11**. So, (x+4+8+2+2)-(3+5+6+3)=x-1 must be divisible by 11, which means that x can only be 1.

Answer: A.

4

Given: $\frac{3{,}070{,}956*n}{720}=integer$.

Factorize the divisor: $720=2^4*3^2*5$.

Now, since 56 (the last two digits of 3,070,956) is divisible by 4, then 3,070,956 is divisible by 4=2^2, and since 956 (the last three digits of 3,070,956) is NOT divisible by 8, then 3,070,956 is NOT divisible by 8=2^3. That means that **n must have 2^2=4 as its factor** (3,070,956 is divisible only by 2^2 so in order 3,070,956*n to be divisible by 2^4 n must have 2^2 as its factor);

Similarly since the sum of the digits of 3,070,956 is 3+0+7+0+9+5+6=30 then 3,070,956 divisible by 3 but not by 3^2, so **n must have 3 as its factor**;

And finally since the units digit of 3,070,956 is 6 then 3,070,956 is not divisible by 5, so **n must have 5 as its factor**.

Therefore the least value of n is $2^2*3*5=60$.

Answer: E.

5

If k is a positive integer, which of the following must be divisible by 24?

(A) (k – 4)(k)(k + 3)(k + 7)
(B) (k – 4)(k – 2)(k + 3)(k + 5)
(C) (k – 2)(k + 3)(k + 5)(k + 6)
(D) (k + 1)(k + 3)(k + 5)(k + 7)
(E) (k – 3)(k + 1)(k + 4)(k + 6)

24=8*3.

Note that the product of two consecutive even integers is always divisible by 8 (since one of them is divisible by 4 and another by 2). Only option B offers two consecutive even numbers for any integer value of k: k-4 and k-2, if k=even or k+3 and k+5 if k=odd.

Also from the following 3 consecutive integers: **(k-4)**, (k-3), **(k-2)** one must be divisible by 3, if it's not k-4 or k-2 then it must be k-3 (if it's k-4 or k-2 option B is divisible by 3 right away). But if it's k-3 then (k-3)+6=**k+3** must also be divisible by 3.

So, option B: (k – 4)(k – 2)(k + 3)(k + 5) is divisible by 8 and 3 in any case.

Answer: B.

6

30 sec approach:
Given: $9^1+(9^2+9^3+9^4+9^5+9^6+9^7+9^8+9^9)$. Notice that in the brackets we have the sum of 8 **odd multiples of 3**, hence the sum in the brackets will be **even multiple of 3** (the sum of 8 odd numbers is even). So, the sum in the brackets is multiple of 6 (remainder is zero). So we are just left with the first term 9, which yields remainder of 3 upon division by 6.

Answer: B.