Pre-Algebra

This part deals with the very basic principles. The domain is rational numbers, and often we are talking about natural numbers. In GMAT it is very important to understand the domain of the variables that are in question. When you see a symbol x, or y, etc, you need to make sure you understand if it is an integer, a rational number, or could it be a fraction, an irrational number, etc.

Positive and Negative

Definition: A positive number is a real number that is greater than zero. A negative number is a real number that is smaller than zero.

Zero is not positive, nor negative.

Even and Odd

Definition: Suppose k is an integer. If there exists an integer r such that $k=2r+1$, then k is an odd number. If there exists an integer r such that $k=2r$, then k is an even number.
Explanation: as long as an integer can be divided by 2, it is an even number.

Zero is an even number.

Useful facts:
Add/subtract two odds or two evens --> even
Add/ Subtract an odd and an even --> odd
Multiplication of odd numbers --> odd
Any even number in a multiplication will always ensure an even product

Factors and Multiples

Definition: The divisors (or factors) of a positive integer are the integers that evenly divide it. A multiple of a number is the product of that number and any other whole number. The proper divisors of the integer n are the positive divisors of n other than n itself.

Explanation: For example, the divisors of 28 are 1, 2, 4, 7, 14 and 28. Of course 28 is also divisible by the negative of each of these, but by "divisors" we usually mean the positive divisors.

Zero is a multiple of every number.

Useful facts/rules:
Any two even numbers in a multiplication will ensure the product be divisible by 4
If 2 numbers have the same factor, then the sum or difference of the two numbers will have the same factor. (e.g. 4 is a factor of 20, 4 is also a factor of 80, then 4 will be a factor of 60 (difference) and also 120 (sum))
Remember to include '1' if you're asked to count the number of factors a number has

The sum of N consecutive integers:
$S=(n+n+N-1)*N/2=n*N+N*(N-1)/2$
$S/N=n+(N-1)/2$
If N is odd, then S would be a multiple of N. If N is even, then S would not be a multiple of N.

Integer is divisible by:
2 - Even integer
3 - Sum of digits are divisible by 3
4 - Integer is divisible by 2 twice. A number is divisible by 4 if the number formed by its last two digits is divisible by 4.
5 - Last digit is 0 or 5
6 - Integer is divisbile by 2 AND 3

8 - Integer is divisible by 2 three times. A number is divisible by 8 if the number formed by the last three digits of the given number are divisible by 8.
9 - Sum of digits is divisible by 9
10 - Last digit is 0
11 - Start with the units digit, add every other digit and remember this number. Form a new number by adding the digits that remain. If the difference between these two numbers is divisible by 11, then the original number is divisible by 11.
eg. Is the number 824472 divisible by 11? Starting with the units digit, add every other number:2 + 4 + 2 = 8. Then add the remaining numbers: 7 + 4 + 8 = 19. Since the difference between these two sums is 11, which is divisible by 11, 824472 is divisible by 11.

LCM and GCD

Definition: The least common multiple of two (or more) nonzero integers is the least positive integer divisible by all of them. This is usually denoted LCM.
The greatest common divisor (archaic: greatest common factor) of two integers a and b is the largest integer that divides them both. This is usually denoted by GCD(a,b).

Two integers are relatively prime if there is no integer greater than one that divides them both (that is, their greatest common divisor is one).

Useful facts:
GCD(a,b)*LCM(a,b) = ab;
LCM(a,b) = ab if and only if a and b are relatively prime.

To find the GCF/LCM, you will need to do prime-factorization. This means reducing a number to its prime-factor form.

E.g. 1

GCF/LCM of 4,18

4 = 2*2
18= 2*3*3
To find the GCF, take the multiplication of the common factors. In this case, GCF = 2.
To find the LCM, take the multiplication of all the factors. In this case, LCM=2*2*3*3=36

E.g. 2

GCF/LCM of 4,24

4 = 2*2
24 = 2*2*2*3
GCF = 2*2 = 4
LCM = 2*2*2*3 = 24

Prime and Composite

Definition: An integer greater than one is called a prime number if its only positive divisors (factors) are one and itself. A positive integer n which can be factored into smaller positive integers (n=ab), neither of which is one, is a composite.

0 and 1 are neither prime nor composite.

Method to determine a prime number:
Find approx square root of the number. Then check if all the prime numbers below the square root are factors of the given number. If none are then the number is prime else not.

e.g. number 91. approx sq root is 10
Prime number below 10 are 2.3.5&7.

91 is not divisible by2,3 or 5. But it is divisible by 7.
Therefore 91 is not prime.

Algebra

Some Useful Facts / Rules

1) If $a>x$, $b>y$, $c>z$ then $a+b+c+.....>x+y+z......$ and $abc...>xyz.....$

2) $(n!)^2 > n^n$

3) $a/b+b/c+c/d+d/a > 4$

4) $a^2*b +b^2*c + c^2*a >= 3abc$

5) For any positive integer N , $2<=(1 + 1/N)^N <= 3$

6) If a,b,c are positive and not equal then

i) $(a+b+c)(ab+bc+ca) > 9abc$
ii) $(b+c)(c+a)(a+b) > 8abc$

7) If $x > y$, then $x - y$ divides $x^n - y^n$.
$a^n - b^n$ is divisible by $a + b$ if n is even.
$a^n + b^n$ is divisible by $a+b$ if n is odd, and not divisible by $a+b$ if n is even.
a^n-b^n is divisible by $a-b$ whether n is odd or even.

Basic Rules for Inequalities:
(in the example: $a>b>0$, $c>d>0$)

You need to flip signs when both side are multiplied by a negative number:
$-a<-b$, $-c<-d$

You need to flip signs when 1 is divided by both side:
$1/a<1/b$, $1/c<1/d$

You can only add or multiply them when their signs are in the same direction:
$a+c>b+d$
$ac>bd$

You can only apply substractions and divisions when their signs are in the opposite directions:
$a>b$, $d<c$
$a-d>b-c$
$a/d>b/c$
(You can't say $a/c>b/c$. It is WRONG)

Deal with negative numbers:
$-a<-b<0$, $-c<-d<0$
Then
$-a-c<-b-d<0$
$-a-(-d)<-b-(-c)$
However the sign needs to be flipped one more time if you are doing multiplication or division (because you are multiplying/dividing a negative number):
$(-a)*(-c)>(-b)*(-d)$
$(-a)/(-d)>(-b)/(-c)$

For example:
If $x<-4$, $y<-2$, we know that $xy>8$, but we don't know how x/y compare to $(-4)/(-2)=2$ since you can only do division when their signs are in different directions
If $x>-4$ and $y<-2$ then $x/y<2$ but we don't know how xy is compared to 8 since we can only do multiplication when their signs are the same direction.

It is easier to do the derivation, though, if you first change them to postive. For example:
If $x<-4$, $y<-2$, then $-x>4$, $-y>2$, $xy>8$
If $x<-4$, $y<2$, then $-x>4$, $y<2$, $-x/y>2$, $x/y<-2$

Cancelling out "Common Terms" on Both Sides of an Equation

You need to be very careful when you do algebra derivations. One of the common mistakes is to divide both side by "a common term". Remember you can only do this safely if the "common term" is a constant. However you CAN't do it if it contains a variable.

Example:

$x(x-2)=x$
You can't cancel out the x on both side and say $x=3$ is the solution. You must move the x on the right side to the left side.
$x(x-2)-x=0$
$x(x-2-1)=0$
The solutions are: $x=0$ and $x=3$
The reason why you can't divided both sides by x is that when x is zero, you can't divide anything by zero.

Equally important if not more, is that you CAN'T multiple or divide a "common term" that includes a variable from both side of an inequality. Not only it could be zero, but it could also be negative in which case you would need to flip the sign.

Example:

$x^2>x$
You CAN'T divided both sides by x and say $x>1$. What you have to do is to move the right side to the left:
$x^2-x>0$
$x(x-1)>0$
Solution would be either both x and x-1 are greater than zero, or both x and x-1 are smaller than zero. So your solution is: $x>1$ or $x<0$

Example:

$x>1/x$
Again you CAN'T multiply both sides by x because you don't know if x is positive or negative.
What you have to do is to move the right side to the left:
$x-1/x>0$
$(x^2-1)/x>0$
If $x>0$ then $x^2-1>0 \Rightarrow x>1$
If $x<0$ then $x^2-1<0 \Rightarrow x>-1$
Therefore your solution is $x>1$ or $0>x>-1$.
You could also break the original question to two branches from the beginning:
$x>1/x$
if $x>0$ then $x^2>1 \Rightarrow x>1$
if $x<0$ then $x^2<1 \Rightarrow x>-1$
Therefore your solution is $x>1$ or $0>x>-1$.

Absolute values
The way to solve this kind of questions is to break the equation (inequality) into two parts, one is when the value is non negative, the other is when the value is negative.

For example:

$|x-4|<9$

You break it into two parts:
If $x-4>=0$, then $x-4<9$, solve for both you get $x>=4$, $x<13$. So your solution is $4<=x<13$.
If $x-4<0$, then $-(x-4)<9$, ie $x-4>-9$. Solve for both you get $x<4$, $x>-5$. So your solution for this part is $-5<x<4$.
Combine the two solutions, you get $-5<x<13$ as your final solution.

Another example:
$|x+4|>4$
If $x+4>=0$, then $x+4>4$. Solve for both you get $x>=-4$, $x>0$. So your solution is $x>0$.
If $x+4<0$, then $-(x+4)>4$, ie. $x+4<-4$. Solve for both you get $x<-4$, $x<-8$. So your solution is $x<-8$.
You final solution is $x>0$ or $x<-8$.

The same strategy can apply to square questions.
For example: $(x+4)^2>4$
You could solve it this way:
$x^2+8x+12>0$
$(x+2)(x+6)>0$
$x>-2$ or $x<-6$
Or you can solve it this way:
If $x+4>=0$ then $x+4>2$. Solve for them you get $x>-2$.
If $x+4<0$ then $x+4<-2$. Solve for them you get $x<-6$.

$|y|>|y+1|$
if $y>=0$, $y+1>=0$, $y>y+1$, no solution.
if $y<0$, $y+1<0$, $-y>-(y+1)$, solution is $y<-1$
if $y>=0$, $y+1<0$, $y>-(y+1)$, no solution.
if $y<0$, $y+1>=0$, $-y>y+1$, solution is $-1<=y<-1/2$
So your final solution is $y<-1/2$
You could also solve this question by going the square route.
$y^2>(y+1)^2$
$y^2>y^2+2y+1$
$2y+1<0$
$y<-1/2$

If d is POSITIVE and $|x| < d$, then $-d < x < d$
If d is NEGATIVE and $|x| < d$, then there is no solution

If d is POSITIVE and $|x| > d$, then $x < -d$ OR $x > d$
If d is NEGATIVE and $|x| > d$, then x is all real numbers

Square root

A square root, also called a radical or surd, of x is a number r such that $r^2=x$. The function $r=sqrt(n)$ is therefore the inverse function of $f(x)=x^2$ for $x>=0$.
Eg. if $x<0$, $sqrt (x^2)=-x$

Set and Series

Mean, Median, Mode, Range and Standard Deviation
Mean is the average, median is the middle number, mode is the one that appears the most. Most likely they are not equal to each other. For two sets of numbers, if one set of the three Ms are equal, it means nothing about the other three.
Range is the difference between the smallest and the largest values of a set.
Standard deviation, don't worry about it. It means how much all the numbers vary from one another, basically.

Example:

Set 1 contains {1,1,1,1} ; Set 2 contains {-1,1,-1,1}; Set 3 is the union of Set 1 and Set 2 along with the number 2

Set 1 = {1,1,1,1}; Mean=Median=Mode=1, Range=0
Set 2 = {-1,-1,1,1}; Mean=Median=0, Mode=-1,1 (A set of data can have more than one mode), Range = 2
Set 3 = {-1,-1,1,1,1,1,2}; Mean=4/7, Median=1, Mode=1, Range=3

Sum of a Series

A quick way to find the sum of a series where each preceding term is incremented by the same number would be to find the middle term and multiply it by the number of terms.

The middle term can be found at taking the average of the first and last term. Or for the case of Antmavel's question where there are ten terms, you just need to work up to the 5th and 6th term then find the middle of the these two numbers.

E.g. Sum of 4,8,12,16,20
Middle term: 12
Number of terms: 5
Sum = 12*5=60

Word Problems

Working with Ratios

Ratio questions are very easy to solve if you have mastered the way of thinking.

Basically if you have
a/b=c/d (or a:b=c:d)
then you can immediately derive a variaty of correlated ratios, such as:
a/(a+b)=c/(c+d)
a/(a-b)=c/(c-d)
(a+b)/(a-b)=(c+d)/(c-d)
(a+c)/(b+d)=c/d
(a-c)/(b-d)=c/d
etc

Basically, you can do all kinds of additions and subtractions.

Example:

a/b=3/5 (1)
2a-b=4 (2)
What is a?
From (1) we get a/(2a-b)=3/1, so a=3*4=12
Explanation: a is 3 share, b is 5 share. Two a is 6 share, 2a-b is one share. If one share is 4, then 3 share is 12.

Of course this question can be solved using the more traditional algebra approach:
b=5/3a
substitute in (2)
2a-5/3a=4
1/3a=4
a=12

You can see the two approaches are really the same in nature. However the first approach is very straight forward and does not involve calculation in fractions. Sometimes it can save you lots of time, especially when using this method with word problems such as mixture problems.

Mixture Problems

Example:

A fruit mixture is made up by 25% fruit A and 75% fruit B. Now if the amount of fruit A is doubled, what is their relative share in the new mixture?

A:B=25:75
2A:B=50:75=2:3
The new mixture total quantity is 2A+B
2A:(2A+B)=2:5
B:(2A+B)=3:5
Therefore the new shares are fruit A 40%, fruit B 60%.

Example 2:

In a picnic 60% people ate two hotdogs, 30% people ate one hamburger, and 10% people ate one hotdog. The total number of hotdog and hamburgers consumed is 80. How many hamburgers and hotdogs are consumed?

People:

$T:H:O=6:3:1$ (1)

Food:

$2T+H+O=80$

From (1)

$T:H:O:(2T+H+O)=6:3:1:16$

Therefore

$T:(2T+H+O)=3:8=30:80$ 30 people ate two hotdogs

$H:T=3:6=1:2=15:30$ 15 people ate one hamburger

$O:T=1:6=5:30$ 5 people ate one hotdogs

Total people 50, total hotdogs 65, total hamburgers 15.

Verify, total hotdogs and hamburgers=65+15=80.