1

If x is positive, which of the following could be the correct ordering of 1/x,2x and x^2 ?
I. x^2<2x<1/x
II. x^2<1/x<2x
III. 2x<x^2<1/x

(A) None
(B) I only
(C) III only
(D) I and II only
(E) I II and III

First note that we are asked "which of the following COULD be the correct ordering" not MUST be.

Basically we should determine relationship between x, $\frac{1}{x}$ and x^2 in three areas: $0<1<2<$.

$x>2$

$1<x<2$

$0<x<1$

When $x>2$ --> x^2 is the greatest and no option is offering this, so we know that x<2.
If $1<x<2$ --> $2x$ is greatest then comes x^2 and no option is offering this.

So, we are left with $0<x<1$:
In this case x^2 is least value, so we are left with:

I. $x^2<2x<\frac{1}{x}$ --> can $2x<\frac{1}{x}$? Can $\frac{2x^2-1}{x}<0$, the expression $2x^2-1$ can be negative or positive for $0<x<1$. (You can check it either algebraically or by picking numbers)

II. $x^2<\frac{1}{x}<2x$ --> can $\frac{1}{x}<2x$? The same here $\frac{2x^2-1}{x}>0$, the expression $2x^2-1$ can be negative or positive for $0<x<1$. (You can check it either algebraically or by picking numbers)

Answer: D.

Second condition: $x^2<\frac{1}{x}<2x$

The question is which of the following COULD be the correct ordering not MUST be.

Put 0.9 --> $x^2=0.81$, $\frac{1}{x}=1.11$, $2x=1.8$ --> $0.81<1.11<1.8$. Hence this COULD be the correct ordering.

2

In a room filled with 7 people, 4 people have exactly 1 sibling in the room and 3 people have exactly 2 siblings in the room. If two individuals are selected from the room at random, what is the probability that those two individuals are NOT siblings?

A. 5/21
B. 3/7
C. 4/7
D. 5/7
E. 16/21

As there are 4 people with exactly 1 sibling each: we have two pairs of siblings (1-2; 3-4).
As there are 3 people with exactly 2 siblings each: we have one triple of siblings (5-6-7).

Solution #1:

# of selections of 2 out of 7 - $C^2_7=21$;

# of selections of 2 people which are not siblings - $C^1_2*C^1_2$ (one from first pair of siblings*one from second pair of siblings)+ $C^1_2*C^1_3$

(one from first pair of siblings*one from triple)+ $C1_2*C^1_3$(one from second pair of siblings*one from triple) $=4+6+6=16$.

$P=\frac{16}{21}$

Solution #2:

# of selections of 2 out of 7 - $C^2_7=21$;
# of selections of 2 siblings - $C^2_3+C^2_2+C^2_2=3+1+1=5$;

$P=1-\frac{5}{21}=\frac{16}{21}$.

Solution #3:
$P=2*\frac{3}{7}*\frac{4}{6}+2*\frac{2}{7}*\frac{2}{6}=\frac{4}{7}+\frac{4}{21}=\frac{16}{21}$.

Answer: E.

3

	< 6 hours sleep	>= 6 hours sleep	TOTAL
Tired	75	-	-
Not Tired	-	0.7*X	?
TOTAL	80	X	100

Interns who receive **< than 6 hours sleep** and **Not Tired** = 80 - 75 = 5;

TOTAL interns who receive >= 6 hours sleep = 100 - 80 = 20, so interns who receive **>= 6 hours sleep** and are **Not Tired** = 0.7 * 20 =14;

Interns who are **Not Tired** = 5 + 14 = 19.

Answer: C.

4

It takes 6 days for 3 women and 2 men working together to complete a work.3 men would do the same work 5 days sooner than 9 women.How many times does the output of a man exceed that of a woman?
A. 3 times
B. 4 times
C. 5 times
D. 6 times
E. 7 times

Let one woman complete the job in w days and one man in m days. So the rate of 1 woman is $\frac{1}{w}$ job/day and the rate of 1 man is $\frac{1}{m}$ job/day.

It takes 6 days for 3 women and 2 men working together to complete a work --> sum the rates: $\frac{3}{w}+\frac{2}{m}=\frac{1}{6}$.

3 men would do the same work 5 days sooner than 9 women --> $\frac{m}{3}+5=\frac{w}{9}$.

Solving: $m=15$ and $w=90$. $\frac{w}{m}=6$.

Answer: D.

5

Consider this: we have 5 d's and 3 separators $|$, like: $ddddd|||$. How many permutations (arrangements) of these symbols are possible? Total of 8 symbols (5+3=8), out of which 5 d's and 3 $|$'s are identical, so $\frac{8!}{5!3!}=56$.

With these permutations we'll get combinations like: $|dd|d|dd$ this would be 3 digit number 212 OR $|||ddddd$ this would be single digit number 5 (smallest number less than 10,000 in which sum of digits equals 5) OR $ddddd|||$ this would be 4 digit number 5,000 (largest number less than 10,000 in which sum of digits equals 5)...

Basically this arrangements will give us all numbers less than 10,000 in which sum of the digits (sum of 5 d's=5) equals 5.

Hence the answer is $\frac{8!}{5!3!}=56$.

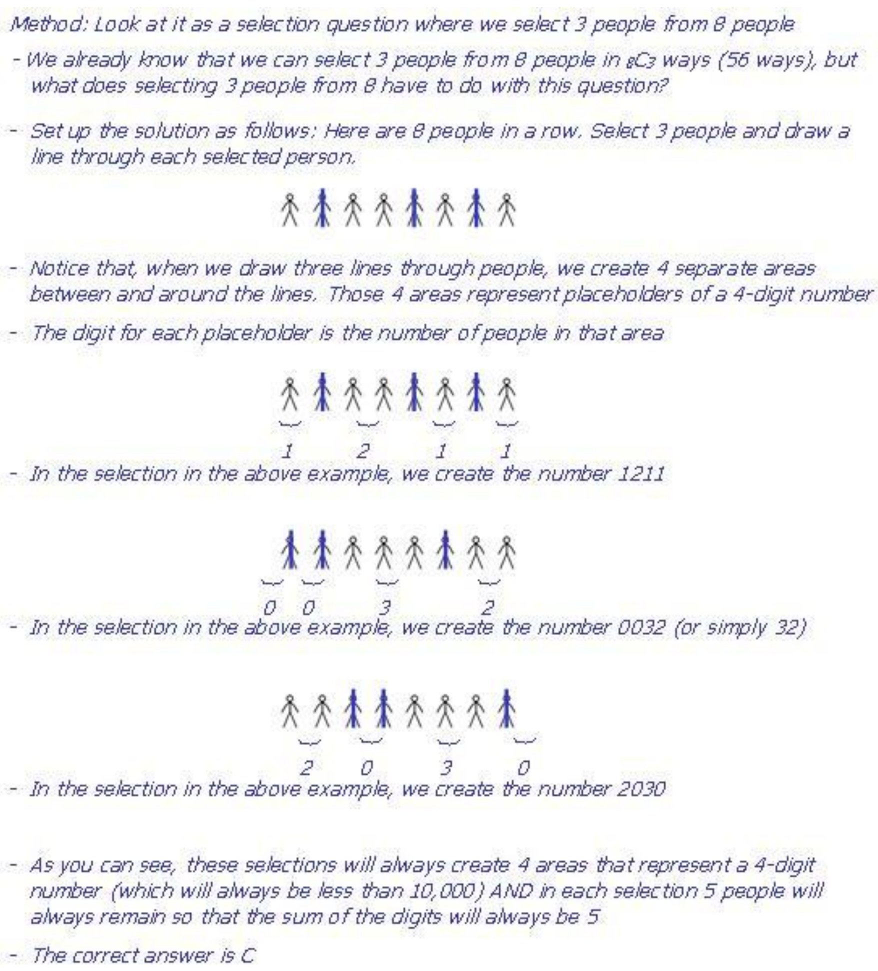
Answer: C (56).

This can be done with direct formula as well:

The total number of ways of dividing n identical items (5 d's in our case) among r persons or objects (4 digt places in our case), each one of whom, can receive **0, 1, 2** or more items (from zero to 5 in our case) is ${}^{n+r-1}C_{r-1}$.

In our case we'll get: ${}^{n+r-1}C_{r-1}={}^{5+4-1}C_{4-1}=8C3=\frac{8!}{5!3!}=56$

Also see the image I found in the net about this question explaining the concept:



Method: Look at it as a selection question where we select 3 people from 8 people

- We already know that we can select 3 people from 8 people in ${}_8C_3$ ways (56 ways), but what does selecting 3 people from 8 have to do with this question?
- Set up the solution as follows: Here are 8 people in a row. Select 3 people and draw a line through each selected person.
- Notice that, when we draw three lines through people, we create 4 separate areas between and around the lines. Those 4 areas represent placeholders of a 4-digit number
- The digit for each placeholder is the number of people in that area

1 2 1 1

- In the selection in the above example, we create the number 1211

0 0 3 2

- In the selection in the above example, we create the number 0032 (or simply 32)

2 0 3 0

- In the selection in the above example, we create the number 2030
- As you can see, these selections will always create 4 areas that represent a 4-digit number (which will always be less than 10,000) AND in each selection 5 people will always remain so that the sum of the digits will always be 5
- The correct answer is C