

In problems that ask you to calculate total average speed, given the speed for onward and return trip, you could apply a formula or a neat trick to get to the answer faster. Lets derive the genral formula first and see how it applies it to couple of examples.

Say, a car travels at  $S_1$  mph on a trip and at  $S_2$  mph on return trip. What is its average speed for the entire trip?

Solution:

\*\*\* Don't fall in the trap of just averaging the 2 speeds. Overall average speed is not  $(S_1+S_2)/2$ . \*\*\*

Total average speed is simply = Total distance/Total time

Lets say,

$D$  = distance travelled by the car in EACH direction

$t_1$  = time spent on onward trip

$t_2$  = time spent on return trip

Thus, the total distance travelled by the car =  $D+D= 2D$

And, by the formula, Speed = Distance/Time

$S_1 = D/t_1 \Rightarrow t_1 = D/S_1$

$S_2 = D/t_2 \Rightarrow t_2 = D/S_2$

Total average speed = Total Distance/Total time =  $2D/(t_1+t_2) = 2D/(D/S_1+D/S_2) = 2S_1*S_2/(S_1+S_2)$

Remember this general formula for a total average speed problems:

Total average speed =  $2S_1*S_2/(S_1+S_2)$

Example:

A car travels at 60 mph on a trip and at 100 mph on return trip. What was its average speed for the entire trip?

Solution:

\*\*\* Total average speed is not  $(60+100)/2 = 80$  \*\*\*

Total average speed =  $2*60*100/(100+60) = 2*60*100/160 = 2*60*5/8 = 60*5/4 = 15*5 = 75$

Alternatively, you may want to check if the following trick saves you some time.

Calculate the ratio of the speeds  $r_1:r_2$ . In our example it is  $60:100 = 3:5$

Then divide the difference between the speeds ( $s_2-s_1$ ) by  $r_1+r_2$  to get one part. In our example  $(100-60)/(3+5) = 5$  is one part

The required answer is  $r_1$  parts away from the lower speed. That is,  $60+r_1*5 = 60+3*5 = 75$  mph

Lets check how it works for  $S_1=20$  mph and  $S_2=40$  mph

Method 1:

Using the formula Total avg speed =  $2S_1*S_2/(s_1+s_2)$

=  $2*20*40/(20+40)$

=  $2*20*40/60$

=  $80/3 = 26.67$  mph

Method 2:

Ratio  $r_1:r_2 = 20:40 = 1:2$

$r_1+r_2 = 3$

1 part =  $(S_2-S_1)/(r_1+r_2) = (40-20)/3 = 20/3 = 6.67$

Total Avg speed is  $r_1$  parts away from smaller speed

Therefore avg speed =  $20+ r_1*6.67 = 20+1*6.67 = 26.67$  mph

$$\text{Average speed} = \text{total distance} / \text{total time taken} = 3d / (d/x + d/y + d/z) = 3xyz / (xy + yz + zx)$$

i.e. Harmonic mean of the individual speeds. You can extend the same to traveling in 4 different speeds and so on.