1

Given: $n = integer \geq 0$ and $2^n = 3q+1$, for some non-negative integer q:

If $n = 0 = even$ --> $2^0 = 1$ --> remainder upon division 1 by 3 is 1 - OK;
If $n = 1 = odd$ --> $2^1 = 2$ --> remainder upon division 2 by 3 is 2 - not OK;
If $n = 2 = even$ --> $2^2 = 4$ --> remainder upon division 4 by 3 is 1 - OK;
If $n = 3 = odd$ --> $2^3 = 8$ --> remainder upon division 8 by 3 is 2 - not OK;
...

So we can see the pattern of reminders 1-2-1-2-.... --> given condition that the remainder is 1 when 2^n is divided by 3 holds true when $n = even$. So n must be non-negative even number: 0, 2, 4, ...

I. n is greater than zero --> not necessarily true, as n can be zero;
II. $3^n = (-3)^n$ --> as n is even then this statement is always true;
III. $\sqrt{2}^n = integer$ --> as n is non-negative even number then this statement is always true.

Answer: E (II and III only).ù

2

It should be $3^{8n+3}+2$.

The units digit of 3 in positive integer power has cyclicity of 4 for the unis digit:
3^1 --> the units digit is **3**;
3^2 --> the units digit is **9**;
3^3 --> the units digit is **7**;
3^4 --> the units digit is **1**;
3^5 --> the units digit is 3 AGAIN;
...

So, the units digit repeats the following pattern {3-9-7-1}-{3-9-7-1}-.... 3^{8n+3} will have the same units digit as 3^3, which is 7 (remainder when 8n+3 divided by cyclicity 4 is 3). Thus the last digit of $3^{8n+3}+2$ will be $7+2=9$. Any positive integer with the unis digit of 9 divided by 5 gives the remainder of 4.

Answer: E.

3

Notice that $43^{86} = (40+3)^{86}$. Now, if we expand this expression, all terms but the last one will have 40 as multiple and thus will be divisible by 5. The last term will be 3^{86}. So we should find the remainder when 3^{86} is divided by 5.

Next, $3^{86} = 9^{43}$. 9 in odd power has units digit of 9 hence yields the remainder of 4 upon division by 5 (9 in even power has units digit of 1 hence yields the remainder of 1 upon division by 5).

Answer: E.

4

Since we cannot have more than one correct answers in PS questions, then pick some numbers for x, y, and z and find the reminder when 1000x + 100y + 10z is divided by 9.

Say x=1, y=2, and z=3, then 1000x + 100y + 10z = 1,230 --> 1,230 divide by 9 yields the remainder of 6 (1,224 is divisible by 9 since the sum of its digit is a multiple of 9, thus 1,230, which is 6 more than a multiple of 9, yields the remainder of 6 when divided by 9).

Answer; C.