1

If k is an integer greater than 1, is k equal to 2^r for some positive integer r?

Given: $k=integer>1$, question is $k=2^r$.

Basically we are asked to determine whether k has only 2 as prime factor in its prime factorization.

(1) k is divisible by 2^6 --> $2^6*p=k$, if p is a power of 2 then the answer is YES and if p is the integer other than 2 in any power (eg 3, 5, 12...) then the answer is NO.

(2) k is <u>not</u> divisible by any odd integers greater then 1. Hence k has only power of 2 in its prime factorization. Sufficient.

Answer: B.

2

If n is a positive integer, is n^2 - 1 divisible by 24?

(1) n is a prime number --> if n=2, then the answer is NO but if n=5, then the answer is YES. Not sufficient.

(2) n is greater than 191. Clearly insufficient (consider n=24^2 for a NO answer and n=17^2 for an YES answer).

(1)+(2) Given that n is a prime number greater than 191 so **n is odd and not a multiple of 3**. n^2-1=(n-1)(n+1) --> out of three consecutive integers (n-1), n and n+1 one must be divisible by 3, since it's not n then it must be either (n-1) or (n+1), so (n-1)(n+1) is divisible by 3. Next, since n is odd then (n-1) and (n+1) are**consecutive even numbers**, which means that one of them must be a multiple of 4, so (n-1)(n+1) is divisible by 2*4=8. We have that (n-1)(n+1) is divisible by both 3 and 8 so (n-1)(n+1) is divisible by 3*8=24. Sufficient.

Answer: C.

For (1)+(2) we have that **n is odd and not a multiple of 3**. Next, (n-1), n and n+1 represent three consecutive integers. Out of ANY three consecutive integers one is always divisible by 3, we know that it's not n, so it must be either n-1 or n+1.

3

Probably the best way of solving would be making the chart of perfect squares and its factors to check both statements, but below is the algebraic approach if needed.

Couple of things:

1. Note that if n is a perfect square powers of its prime factors must be even, for instance: $36=2^2*3^2$, powers of prime factors of 2 and 3 are even.

2. There is a formula for **Finding the Number of Factors of an Integer**:

First make prime factorization of an integer $n=a^p*b^q*c^r$, where a, b, and c are prime factors of n and p, q, and r are their powers.

The number of factors of n will be expressed by the formula $(p+1)(q+1)(r+1)$. **NOTE:** this will include 1 and n itself.

Example: Finding the number of all factors of 450: $450=2^1*3^2*5^2$

Total number of factors of 450 including 1 and 450 itself is $(1+1)*(2+1)*(2+1)=2*3*3=18$ factors.

3. A perfect square ALWAYS has an ODD number of Odd-factors, and EVEN number of Even-factors. For instance odd factors of 36 are 1, 3 and 9 (3 odd factor) and even factors are 2, 4, 6, 12, 18 and 36 (6 even factors).

Back to the original question:

Is the positive integer N a perfect square?

(1) The number of distinct factors of N is even --> let's say $n=a^p*b^q*c^r$, given that the number of factors of n is even --> $(p+1)(q+1)(r+1)=even$. But as we concluded if n is a perfect square then powers of its primes p, q, and r must be even, and in this case number of factors would

be $(p+1)(q+1)(r+1)=(even+1)(even+1)(even+1)=odd*odd*odd=odd\neq even$. Hence n can not be a perfect square. Sufficient.

(2) The sum of all distinct factors of N is even --> if n is a perfect square then (according to 3) sum of odd factors would be odd and sum of even factors would be even, so sum of all factors of perfect square would be $odd+even=odd\neq even$. Hence n can not be a perfect square. Sufficient.

Answer: D.

There are some tips about the perfect square:
• The number of distinct factors of a perfect square is ALWAYS ODD.
• The sum of distinct factors of a perfect square is ALWAYS ODD.
• A perfect square ALWAYS has an ODD number of Odd-factors, and EVEN number of Even-factors.
• Perfect square always has even number of powers of prime factors.

4

"x rounded to the nearest hundred is 500" means $450\leq x<550$ (inequality you've written) BUT it IS NOT the same as "500 is the multiple of 100 that is closest to x", which should be $450<x<550$ as 450 is equidistant from 400 and 500 and we cannot say that 450 rounded to nearest multiple of 100 is 500, it's 500 OR 400. That's why endpoints must be excluded.

If 500 is the multiple of 100 that is closest to X and 400 is the multiple of 100 closest to Y, then which multiple of 100 closest to X + Y ?

"500 is the multiple of 100 closest to X" --> $450<x<550$;
"400 is the multiple of 100 closest to Y" --> $350<y<450$.

(1) x<500 --> $450<x<500$ --> add this inequality to inequality with y --> $800<x+y<950$. If x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

(2) y<400 --> $350<y<400$ --> add this inequality to inequality with x --> $800<x+y<950$. The same here: if x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

(1)+(2) Sum $450<x<500$ and $350<x<400$ --> $800<x+y<900$ --> and again if x+y=810 then closest multiple of 100 is 800 BUT if x+y=860 then closest multiple of 100 is 900. Not sufficient.

Answer: E.