1

APPROACH #1:

Test some small numbers: $\frac{2^2+4^2}{2}=10=3^2+1$ or: $\frac{4^2+6^2}{2}=26=5^2+1$.

APPROACH #2:

Say $54{,}821=x$, then $\frac{54{,}820^2+54{,}822^2}{2}=\frac{(x-1)^2+(x+1)^2}{2}=x^2+1=54{,}821^2+1$.

APPROACH #3:

The units digit of $54{,}820^2+54{,}822^2$ is $0+2=4$. Now, since $54{,}820^2+54{,}822^2$ must be a multiple of 4, then $\frac{54{,}820^2+54{,}822^2}{2}$ must have the units digit of 2. Only answer choice D fits.

Answer: D.

2

In order the answer to be A, the question should read:

If $f(x)=\frac{125}{x^3}$, what is the value of $f(5x)*f(x/5)$ in terms of f(x)?

Say $x=1$, then:

$f(x)=f(1)=\frac{125}{1^3}=125$;

$f(5x)=f(5)=\frac{125}{5^3}=1$;

$f(\frac{x}{5})=f(\frac{1}{5})=\frac{125}{(\frac{1}{5})^3}=125*125$.

$f(5x)*f(\frac{x}{5})=f(5)*f(\frac{1}{5})=125^2$ and since $f(x)=f(1)=125$, then $f(5x)*f(\frac{x}{5})=125^2=(f(x))^2$.

Answer: A.

3

Original question is:
Given that x^4 - 25x^2 = -144, which of the following is NOT a sum of two possible values of x?
A. -7
B. -1
C. 0
D. 3
E. 7

Factor $x^4-25x^2+144=0$ --> $(x^2-16)*(x^2-9)=0$ --> $x^2=16$ or $x^2=9$ (alternately you could solve $x^4-25x^2+144=0$ for x^2 to get the same values for it) --> $x=4$ or $x=-4$ or $x=3$ or $x=-3$.

All but option D. could be expressed as the sum of two roots: A. 7=-4-3; B. -1=-3+4; C. 0=3-3 (or 0=4-4); E. 7=3+4.

Answer: D.

As for your question: there are no enough answer choices to show all possible values of a sum of two roots, so there are 4 possible values for a sum and one which is not.

4

Yes, substituting the values for x is probably the best way but if you want, algebraic approach is given below.

The point here is to factor out x-5 from nominator and then to reduce by it:

$$H=\frac{x^3-6x^2-x+30}{x-5}=\frac{x^3-5x^2-x^2-x+25+5}{x-5}=\frac{(x^3-5x^2)-(x^2-25)-(x-5)}{x-5}=\frac{x^2(x-5)-(x-5)(x+5)-(x-5)}{x-5}=$$
$$=\frac{(x-5)(x^2-(x+5)-1)}{x-5}=x^2-x-6.$$

Answer: A.

5

If x^4+y^4=100, then the greatest possible value of x is between
A. 0 and 3
B. 3 and 6
C. 6 and 9
D. 9 and 12
E. 12 and 15

General rule for such kind of problems:
to maximize one quantity, minimize the others;
to minimize one quantity, maximize the others.

So, to maximize x we should minimize y^4. Least value of y^4 is zero. In this case $x^4+0=100$ --> $x^4=100$ --> $x^2=10$ --> $x=\sqrt{10}\approx 3.2$, which is in the range (3,6).

Answer: B.

OR

$x^4+y^4=100$
When you see even powers, first thing that should come to your mind is that the term will be positive or zero.
If you want to maximize x in the sum, you should minimize y^4 so that this term's contribution in 100 is minimum possible. Since it is an even power, its smallest value is 0 when y = 0.

Then x^4 = 100
Since $3^4=81$ and $4^4=256$,x will lie between 3 and 4.

6

$g(f(x))=g(5x^3-2x+8)$, now since $g(y)=6y-4$ then substitute y with $5x^3-2x+8$: $g(f(x))=g(5x^3-2x+8)=6(5x^3-2x+8)-4=30x^2-12x+44$.

Answer: E.

7

Given: $x^2-xy-10-2y^2=0$;

Rearrange $x^2-y^2-y^2-xy-10=0$;

Apply $a^2-b^2=(a-b)(a+b)$: $(x-y)(x+y)-y^2-xy-10=0$ --
> $(x-y)(x+y)-y(y+x)-10=0$;

Factor out $x+y$: $(x+y)(x-y-y)-10=0$;

Since given that $x+y=2$, then we have that $2(x-2y)-10=0$ --> $x-2y=5$.

Answer: D.

8

$10*\frac{x}{x+y}+20*\frac{y}{x+y}=k$

$10*\frac{x+2y}{x+y}=k$

$10*(\frac{x+y}{x+y}+\frac{y}{x+y})=k$

Finally we get: $10*(1+\frac{y}{x+y})=k$

We know that $x<y$

Hence $\frac{y}{x+y}$ is more than 0.5 and less than 1

$0.5<\frac{y}{x+y}<1$

So, $15<10*(1+\frac{y}{x+y})<20$

Only answer between 15 and 20 is 18.

Answer: D (18)

There can be another approach:

We have: $\frac{10x+20y}{x+y}=k$, if you look at this equation you'll notice that it's a weighted average.

There are x red boxes and y blue boxes. Red box weight is 10kg and blue box weight 20kg, what is the average weight of x red boxes and y blue boxes?

k represents the weighted average. As y>x, then the weighted average k, must be closer to 20 than to 10. 18 is the only choice satisfying this condition.

Answer: D (18)