1

Tips about the perfect square:
1. The **number of distinct factors** of a perfect square is ALWAYS ODD. *The reverse is also true: if a number has the odd number of distinct factors then it's a perfect square*;

2. The **sum of distinct factors** of a perfect square is ALWAYS ODD. The reverse is NOT always true: a number may have the odd sum of its distinct factors and not be a perfect square. For example: 2, 8, 18 or 50;

3. A perfect square ALWAYS has an **ODD number of Odd-factors**, and **EVEN number of Even-factors**. *The reverse is also true: if a number has an **ODD number of Odd-factors**, and EVEN number of Even-factors then it's a perfect square*. For example: odd factors of 36 are 1, 3 and 9 (3 odd factor) and even factors are 2, 4, 6, 12, 18 and 36 (6 even factors);

4. Perfect square always has **even powers of its prime factors**. *The reverse is also true: if a number has even powers of its prime factors then it's a perfect square*. For example: $36=2^2*3^2$, powers of prime factors 2 and 3 are even.

NEXT:
There is a formula for **Finding the Number of Factors of an Integer**:

First make prime factorization of an integer $n=a^p*b^q*c^r$, where a, b, and c are prime factors of n and p, q, and r are their powers.

The number of factors of n will be expressed by the formula $(p+1)(q+1)(r+1)$. **NOTE:** this will include 1 and n itself.

Example: Finding the number of all factors of 450: $450=2^1*3^2*5^2$

Total number of factors of 450 including 1 and 450 itself is $(1+1)*(2+1)*(2+1)=2*3*3=18$ factors.

Back to the original question:

Is the positive integer N a perfect square?

(1) The number of distinct factors of N is even --> let's say $n=a^p*b^q*c^r$, given that the number of factors of n is even --> $(p+1)(q+1)(r+1)=even$. But as we concluded if n is a perfect square then powers of its primes p, q, and r must be even, and in this case number of factors would be $(p+1)(q+1)(r+1)=(even+1)(even+1)(even+1)=odd*odd*odd=odd\neq even$. Hence n can not be a perfect square. Sufficient.

(2) The sum of all distinct factors of N is even --> if n is a perfect square then (according to 3) sum of odd factors would be odd and sum of even factors would be even, so sum of all factors of perfect square would be $odd+even=odd\neq even$. Hence n can not be a perfect square. Sufficient.

Answer: D.

2

Odd consecutive integers is an evenly spaced set. For any evenly spaced set the mean equals to the average of the first and the last terms, so in our case $mean=10=\frac{x_1+x_n}{2}$ --> $x_1+x_n=20$. Question: $x_1=?$

(1) The range of the n integers is 14 --> the range of a set is the difference between the largest and smallest elements of a set, so $x_n-x_1=14$ --> solving for x_1 --> $x_1=3$. Sufficient.

(2) The greatest of the n integers is 17 --> $x_n=17$ --> $x_1+17=20$ --> $x_1=3$. Sufficient.

Answer: D.

3

Is $m<0$?

(1) $-m=|-m|$ --> first of all $|-m|=|m|$, (for example: $|-3|=|3|=3$), so we have $-m=|m|$, as RHS is **absolute value which is always non-negative**, then LHS, $-m$ must also be non-negative --> $-m\geq 0$ --> $m\leq 0$, so m could be either negative or zero. Not sufficient.

(2) $m^2=9$ --> $m=3=positive$ or $m=-3=negative$. Not sufficient.

(1)+(2) Intersection of the values from (1) and (2) is $m=-3=negative$, hence answer to the question "is $m<$ 0" is YES. Sufficient.

Answer: C.

4

RULE: for x^n-y^n:
x^n-y^n **is ALWAYS divisible by** $x-y$.
x^n-y^n **is divisible by** $x+y$ **when n is even.**

If n is an integer >1, is 3^n-2^n divisible by 35?

(1) n is divisible by 15. --> $3^{15m}-2^{15m}=27^{5m}-8^{5m}$ --> 5m may or my not be even, so insufficient to answer, whether it's divisible by 27+8=35.

(2) n is divisible by 18. --> $3^{18m}-2^{18m}=27^{6m}-8^{6m}$ --> 6m is even, so 3^18m-2^18m=27^6m-8^6m is divisible by 27+8=35. Sufficient.

B.