1

Given sequence:
x;
$x*r$;
$x*r^2$;
$x*r^3$;
$x*r^4<1,000$ (where x is the first term and r is the constant greater than 1).

To maximize the # of ***non-negative integer*** values possible for x, we should minimize the value of r and since $r=integer>1$ then $r=2$. **(General rule for such kind of problems: to maximize one quantity, minimize the others and to minimize one quantity, maximize the others.)**

Thus, $x*2^4<1,000$ --> $x<\frac{1,000}{16}=62,5$ --> as the first term must be ***a non-negative integer*** then: $x_{max}=62$ and $x_{min}=0$ --> total of 63 values possible for the first term x: {0, 1, 2, ..., 62}.

Answer: D.

2

Let the side of the first square be a, so its area will be $area_1=a^2$;

Next square will have the diagonal equal to a, so its area will be $area_2=\frac{d^2}{2}=\frac{a^2}{2}$;
And so on.

So the areas of the squares will form infinite geometric progression: a^2, $\frac{a^2}{2}$, $\frac{a^2}{4}$, $\frac{a^2}{8}$, $\frac{a^2}{16}$, ... with common ration equal to $\frac{1}{2}$.

For geometric progression with common ratio $|r|<1$, the sum of the progression is $sum=\frac{b}{1-r}$, where b is the first term.

So the sum of the areas will be $sum=\frac{a^2}{1-\frac{1}{2}}=\frac{4^2}{\frac{1}{2}}=32$.

Answer: B.

3

The sequence a_1, a_2, ..., a_n, ... is such that $a_n=4a_{n-1}-3$ for all integers n>1. If a_3=x, then $a_1=$?
A. 4x-3
B. 16x-15
C. (x+3)/4
D. (x+3)/16
E. (x+15)/16

Since, $a_n=4a_{n-1}-3$ then $a_3=4a_2-3$ --> $x=4a_2-3$ --> $a_2=\frac{x+3}{4}$.

Similarly, $a_2=4a_1-3$ --> $\frac{x+3}{4}=4a_1-3$ --> $a_1=\frac{x+15}{16}$.

Answer: E.

Or substitute the value for x, say $x=5$, then $a_3=5=4a_2-3$ --> $a_2=2$ --> $a_2=2=4a_1-3$ --> $a_1=\frac{5}{4}$.

Now, just plug $x=5$ in the answer choices and see which one yields $\frac{5}{4}$: only E.

Answer: E.

Note that for plug-in method it might happen that for some particular number(s) more than one option may give "correct" answer. In this case just pick some other numbers and check again these "correct" options only. For example if you pick $x=1$ then you get three "correct" options A, C and E. Generally -1, 0, and 1 are not good choices for plug-in method.

4

We have a formula to calculate the value of the terms in the sequence starting from x_2: $x_n=2*x_{n-1}-\frac{1}{2}*x_{n-2}$. Hence:

$x_2=2*x_1-\frac{1}{2}*x_0=2*2-\frac{1}{2}*3=\frac{5}{2}$;

$x_3=2*x_2-\frac{1}{2}*x_1=2*\frac{5}{2}-\frac{1}{2}*2=4$.

Answer: C.

5

We have a following sequence: $\sqrt{3}$, 3, $3\sqrt{3}$, 9, ...

Notice that this sequence is a geometric progression with first term equal to $\sqrt{3}$ and common ratio also equal to $\sqrt{3}$.

The question asks: the sum of how many terms of this sequence adds up to $120+121\sqrt{3}$.

The sum of the first n terms of geometric progression is given by: $sum=\frac{b*(r^n-1)}{r-1}$, (where b is the first term, n # of terms and r is a common ratio $\neq 1$).

So, $\frac{\sqrt{3}(\sqrt{3}^n-1)}{\sqrt{3}-1}=120+121\sqrt{3}$ --> $\sqrt{3}(\sqrt{3}^n-1)=120\sqrt{3}+121*3-120-121\sqrt{3}$ --> $\sqrt{3}*\sqrt{3}^n-\sqrt{3}=243-\sqrt{3}$ --> $\sqrt{3}*\sqrt{3}^n=243$ --> $3^{\frac{1+n}{2}}=3^5$ --> $\frac{1+n}{2}=5$ --> $n=9$.

Answer: E.

Shortcut solution:
Alternately you can spot that **every second** term (which also form a geometric progression), 3, 9, 27, ... should add up to 120: $\frac{3(3^k-1)}{3-1}=120$ --> $3^k-1=80$ --> $3^k=81$ --> $k=4$, so the number of terms must be either $2k=8$ (if there are equal number of irrational and rational terms) or $2k+1=9$ (if # of irrational terms, with $\sqrt{3}$, is one more than # rational terms), since only 9 is present among options then it must be a correct answers.