1

$x-y=even$ --> either both even **or** both odd

$\frac{x}{y}=even$ --> either both even **or** x is even and y is odd. Also, this statement implies that $x>y$.

As both statements are true --> both x and y must be even.

As $\frac{x}{y}$ is an **even integer** and y is even too, then x must be multiple of 4 (x/y=even --> x/even=even --> x=even*even).

So, we have that x>y, x is a multiple of 4 and y is even.

Only I and II will give non-prime integers, while III won't be an integer at all.

Answer: D.

2

Notice that we can factor out 3 out of 6!+21 --> 6!+21=3*(2*4*5*6+7), which means that this number is not a prime.

Answer: B.

OR

A prime number has only two factors - 1 and itself.

Without calculating, we cannot say whether 6!-1 or 6!+41 will be prime.

But, I can say that 6!+21 will not be prime. The reason is that 6!+21 = 3(1*2*4*5*6 + 7)
(taking 3 common). This means that whatever, the value of 6!+21, it can be written as the product of two numbers: 3 and something else.
Hence, this number, 6!+21, definitely has 3 as a factor and hence it cannot be prime.
Since a PS question can have only one correct answer, we don't have to worry about the other options. We can say with certainty that they must be prime.

3

Basically the length of an integer is the sum of the powers of its prime factors. For example the length of 24 is 4 because 24=2^3*3^1 --> 3+1=4.

Given: x+3y<1,000. Now, to maximize the length of x or y (to maximize the sum of the powers of their primes) we should minimize their prime bases. Minimum prime base is 2: so if x=2^9=512 then its length is 9 --> 512+3y<1,000 --> y<162.7 --> maximum length of y can be 7 as 2^7=128 --> 9+7=16.

Answer: D.

4

$3{,}150=2*3^2*5^2*7$, now $3{,}150*y$ to be a perfect square y must complete the odd powers of 2 and 7 to even number (perfect square has even powers of its primes), so the least value of y is 2*7=14. In this case $3{,}150y=(2*3^2*5^2*7)*(2*7)=(2*3*5*7)^2=perfect\ square$.

Answer: E.