1

If the operation # is one of the four arithmetic operations: addition, subtraction, multiplication and division. Is (6#2)#4 = 6#(2#4)?

- (1) 3#2 > 3. # can be either multiplication or addition. In BOTH cases (6#2)#4 = 6#(2#4) is true: (6*2)*4=6*(2*4)=48 and (6+2)+4=6+(2+4)=12. Sufficient.
- (2) 3#1 = 3. # can be either multiplication or division. If it's division the the answer to the question is No and if it's multiplication answer to the question is YES. Two different answers. Not sufficient.

Answer: A.

2

If the symbol @ represents either addition or multiplication, which operation does it represent?

(1) a@b=b@a for all numbers a and b --> @ can be addition (a+b=b+a) as well as multiplication (a*b=b*a). Not sufficient.

(2) a@(b-c)=(a@b)-(a@c) for all numbers a, b, and c --> if @ represents addition we will have $a\mathbb{Q}(b-c)=a+b-c$ which is not equal to $(a\mathbb{Q}b)-(a\mathbb{Q}c)=(a+b)-(a+c)=b-c$, so @ must be multiplication. Sufficient. (Just to check: $a\mathbb{Q}(b-c)=a^*(b-c)=ab-ac$ which is equal to $(a\mathbb{Q}b)-(a\mathbb{Q}c)=ab-ac$

Answer: B.

3

If # denotes one of the four arithmetic operations addition, subtraction, multiplication and division, what is the value of 1 # 2?

- (1) n # 0 = n for all integers n
- (2) n # n = 0 for all integers n

The key here is the bold part of the statements, which tells us that statements MUST be true for all integers.

- (1) n # 0 = n for all integers n --> # may denote both addition and subtraction (as n+0=n and n-0=n is true for all integers n), which gives two different values for 1 # 2. Not sufficient.
- (2) n # n = 0 for all integers n --> # may denote only subtraction to be true for ALL integers (n-n=0 is true for all integers n), though if n=0 it can denote addition and multiplication as well but one value of n can not determine #. So 1 # 2 = 1 2 = -1. Sufficient.

Answer: B.

4

If @ denotes one of two arithmetic operations, addition or multiplication, and if k is an integer, what is the value of 3 @ k?

- (1) 2 @ k = 3 --> @ can only by addition because if it's multiplication then 2*k=3 -- k=3/2, which is not an integer as stated in the stem. So, we have 2+k=3 --> k=1 --> 3@k=3+1=4. Sufficient.
- (2) 1 @ 0 = k --> @ can be both addition and multiplication, since 1+0=1=k=integer and 1*0=0=k=integer, so we also have two values of k: 1 and 0. In this case 3@k=3+1=4 or 3@k=3*0=0, two different answers. Not sufficient.

Answer: A.

5

[x] denotes to be the least integer no less than x. Is [2d] = 0?

[x] denotes to be the least integer no less than x, means that some function [] rounds UP a number to the nearest integer, for example:

$$\begin{bmatrix} 1.5 \end{bmatrix} = 2 \\ \text{since 2 is the least integer which no less than 1.5;} \\ \begin{bmatrix} -0.5 \end{bmatrix} = 0 \\ \text{since 0 is the least integer which no less than -0.5;} \\ \begin{bmatrix} 1 \end{bmatrix} = 1 \\ \text{since 1 itself is the least integer which no less than 1;}$$

(1) [d] = 0 -->
$$-1 < d \le 0$$
 . Now, if $d = 0$ then $[2d] = [0] = 0$ but if $d = -0.5$ then $[2d] = [-1] = -1$. Not sufficient.

or:
$$-1 < d \le 0$$
 ... $-2 < d \le 0$... $[2d] = 0$ (if $-1 < 2d \le 0$) or $[2d] = -1$ (if $-2 < 2d \le -1$). Not sufficient.

(2) [3d] = 0 -->
$$-1 < 3d \le 0$$
 ...> $-\frac{1}{3} < d \le 0$...> $-\frac{1}{3} < d \le 0$. Even if $d = -\frac{1}{3}$ then $[2d] = [-\frac{2}{3}] = 0$ (again since 0 is the least integer which no less than -2/3). Sufficient.

or:
$$-1 < 3d \le 0 ... -\frac{2}{3} < 2d \le 0 ... [2d] = 0$$
. Sufficient.

Answer: B.