1

Let the side of square be s and the side of rectangle a and b

Given: $\frac{a}{b}=\frac{2}{3}$ --> $b=\frac{3a}{2}$. Also: $P=4s=2(a+b)$ --> $2s=a+b=\frac{5a}{2}$ --> $s=\frac{5a}{4}$.

Question: $\frac{ab}{s^2}=?$

$\frac{ab}{s^2}=\frac{3a^2}{2}*\frac{16}{25a^2}=\frac{24}{25}$.

Answer: B.

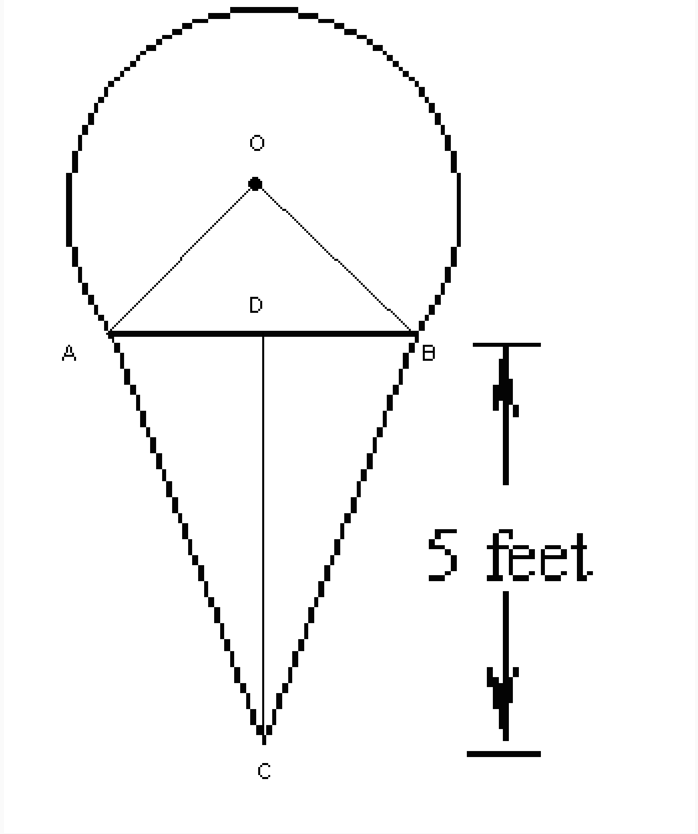
OR: you can pick numbers: let the sides of rectangle be 4 and 6 (ratio 2:3) then the perimeter of the rectangle will be 2(4+6)=20, thus the side of the square will be 20/4=5. Next, the area of the rectangle will be 4*6=24 and the area of the square will be 5^2=25, so the ratio of the areas will be 24/25.

Answer: B.

OR: if we take the side of rectangle to be 2x and 3x (for some positive multiple x), then the perimeter of the rectangle will be 2(2x+3x)=10x, thus the side of the square will be 10x/4=5x/2. Next, the area of the rectangle will be 2x*3x=6x^2 and the area of the square will be (5x/2)^2=25x^2/4, so the ratio of the areas will be 24/25.

Answer: B.

2



As 3/4 of the circumference of a circle is placed on top of a triangle then the perimeter of big arc AB will be 3/4 of a circumference so $\frac{3}{4}*2\pi r=3\pi$, so the answer is either A, B, or C.

Next, as big arc is 3/4 of a circumference then small arc is 1/4 of a circumference or 1/4*360=90 degrees, so $\measuredangle AOB=90$. Now, as $AO=BO=r=2$ then $AB=hypotenuse=2\sqrt{2}$ --> $BD=\frac{AB}{2}=\sqrt{2}$ --> $AC=BC=\sqrt{BD^2+DC^2}=\sqrt{2+25}=3\sqrt{3}$.

The whole perimeter equals to **big arc AB + AC + BC**: $P=3\pi+3\sqrt{3}+3\sqrt{3}=3\pi+6\sqrt{3}$.

Answer: B.

3

Useful property: **a convex quadrilateral can be inscribed in a circle if and only its opposite angles are supplementary** (supplementary angles are two angles that add up to 180°, whereas complementary angles are two angles that add up to 90°). See Central Angle Theorem in Circles chapter of Math Book: math-circles-87957.html.

We can see on the diagram that only I and III meet that requirements.

Answer: C.

4

AC is the edge of the cube. Let's say its length is 'a'.
AB is just the diagonal of a face of the cube i.e. the diagonal of the square whose each side is of length 'a'. Using pythagorean theorem, we know that AB = $\sqrt{2}a$

Now think of the two dimensional triangle ABC (it is right angled at A)
AC = a and AB = $\sqrt{2}a$
Again using pythagorean theorem, $BC^2=a^2+(\sqrt{2}a)^2$
$BC=\sqrt{3}a$

So, $(BC-AB)/AC*100=(\sqrt{3}-\sqrt{2})*100=(1.732-1.414)*100=apprx30\%$

By the way, you would probably be given the value of root 3.

OR

AC is the edge (the side) of a cube, suppose it equals to 1;
AB is the diagonal of a face, hence is equals to $\sqrt{2}$, (either from 45-45-90 triangle properties or form Pythagorean theorem);
BC is the diagonal of the cube itself and is equal to $\sqrt{1^2+1^2+1^2}=\sqrt{3}$;

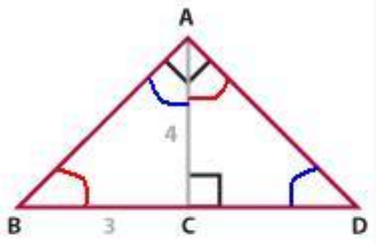
Ratio: $\frac{BC-AB}{AC}=\frac{\sqrt{3}-\sqrt{2}}{1}\approx1.7-1.4=0.3$.

Answer: C.

5

A perpendicular bisector is a line which cuts a line segment into two equal parts at 90°. So AC to be a perpendicular bisector of BD it must not only cut it at 90° (which it does) but also cut it into two equal parts. Now, in order AC to cut BD into two equal parts right triangle ABD must be isosceles, which, as it turns out after some math, it is not.

Complete solution:
In triangle ABC, if BC = 3 and AC = 4, then what is the length of segment CD?
A. 3
B. 15/4
C. 5
D. 16/3
E. 20/3



Important property: **perpendicular to the hypotenuse will always divide the triangle into two triangles with the same properties as the original triangle.**

Thus, the perpendicular AC divides right triangle ABD into two similar triangles ACB and DCA (which are also similar to big triangle ABD). Now, in these three triangles the ratio of the corresponding sides will be equal (corresponding sides are the sides opposite the same angles marked with red and blue on the diagram).

So, $\frac{CD}{AC}=\frac{AC}{BC}$ --> $\frac{CD}{4}=\frac{4}{3}$ --> $CD=\frac{16}{3}$.

Answer: D.