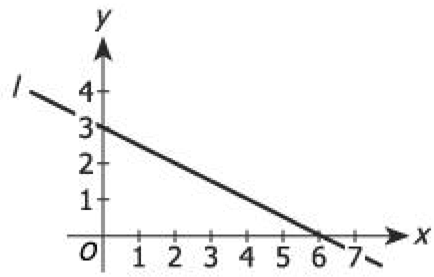
1



All points (x,y) that lie below the line l, shown above, satisfy which of the following inequalities?
A. y < 2x + 3
B. y < -2x + 3
C. y < -x + 3
D. y < 1/2*x + 3
E. y < -1/2*x + 3

First of all we should write the equation of the line l:

We have two points: A(0,3) and B(6,0).

Equation of a line which passes through two points (x_1,y_1) **and** (x_2,y_2) **is** $\frac{y-y_1}{x-x_1}=\frac{y_1-y_2}{x_1-x_2}$

So equation of a line which passes the points A(0,3) and B(6,0) would be: $\frac{y-3}{x-0}=\frac{3-0}{0-6}$ --> $2y+x-6=0$ --> $y=-\frac{1}{2}x+3$

Points below this line satisfy the inequality: $y<-\frac{1}{2}x+3$

OR

The equation of line which passes through the points $A(0,3)$ and $B(6,0)$ can be written in the following way:

Equation of a line in point intercept form is $y=mx+b$, where: m is the slope of the line and b is the y-intercept of the line (the value of y for $x=0$).

The slope of a line, m, is the ratio of the "rise" divided by the "run" between two points on a line, thus $m=\frac{y_1-y_2}{x_1-x_2}$ --> $\frac{3-0}{0-6}=-\frac{1}{2}$ and b is the value of y when $x=0$ --> A(**0**,3) --> $b=3$.

So the equation is $y=-\frac{1}{2}x+3$

Points below this line satisfy the inequality: $y<-\frac{1}{2}x+3$.

Actually one could guess that the answer is E at the stage of calculating the slope $m=-\frac{1}{2}$, as only answer choice E has the same slope line in it.

Answer: E.

2

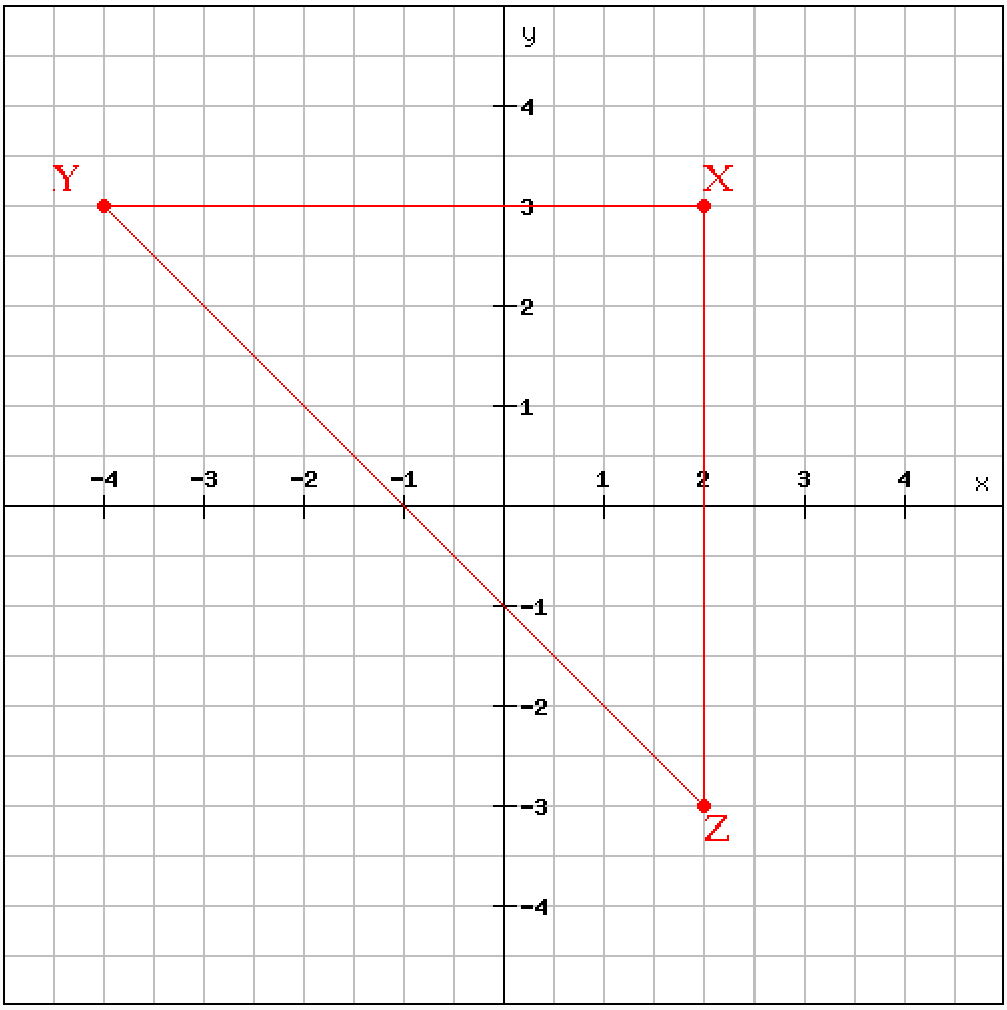
You can use the distance formula to calculate the length of line segment YZ

The formula to calculate the distance between two points (x_1,y_1) and (x_2,y_2) is $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Thus $YZ=\sqrt{(-4-2)^2+(3-(-3))^2}=6\sqrt{2}$.

Answer: B.

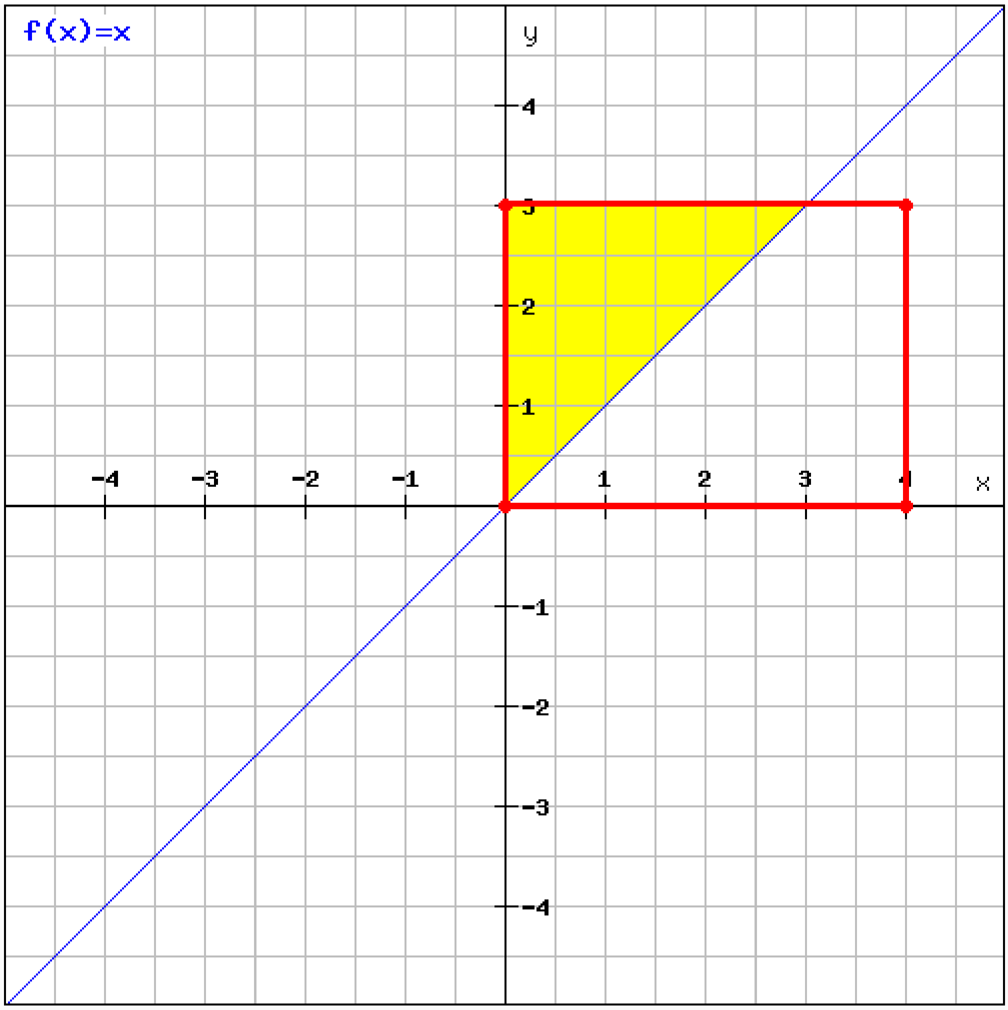
If you are not aware of this formula then you can easily get the answer by drawing the triangle:



As you can see XYZ is not only the right triangle but isosceles right triangle (YX=YZ=6), so it's 45-45-90 right triangle were the sides are always in the ratio $1:1:\sqrt{2}$, thus $YZ=6\sqrt{2}$. Else you can use Pythagorean theorem to get YZ.

Answer: B.

3

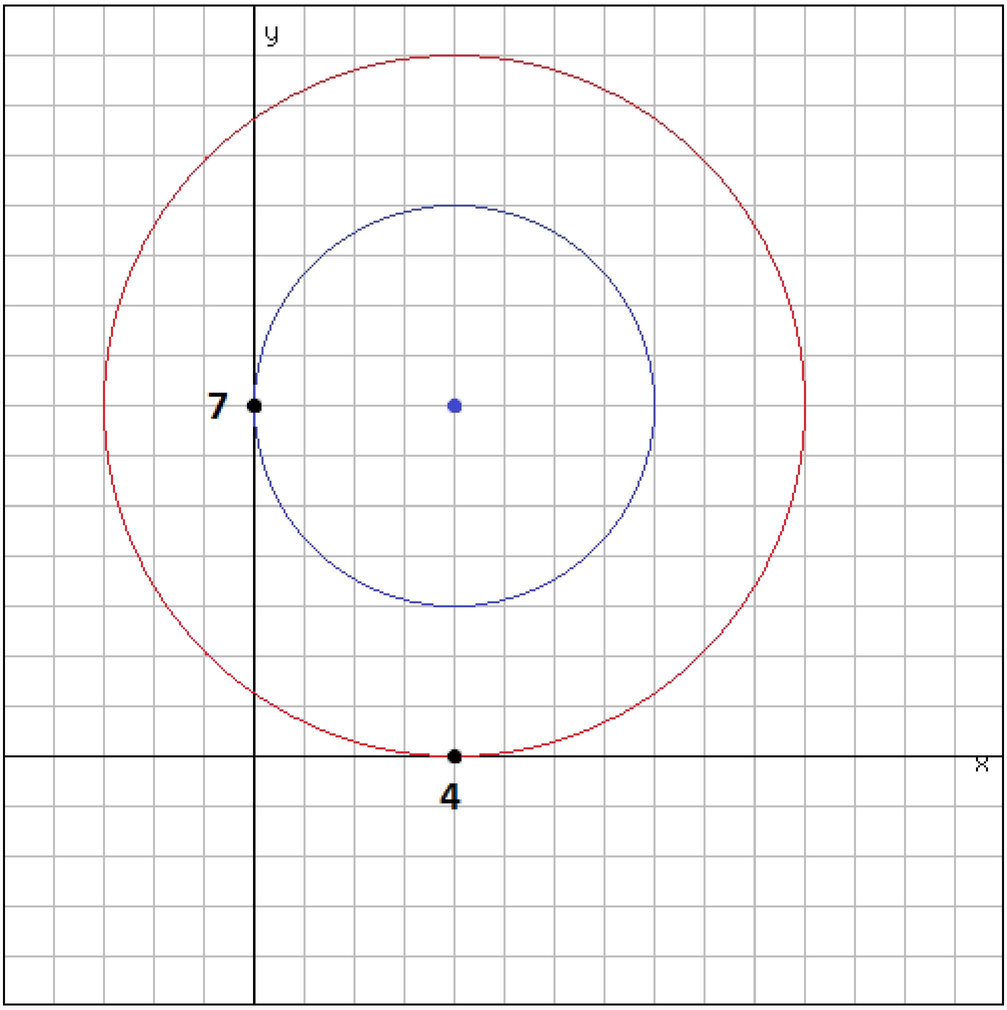


Now, rectangle R has an area of 3*4=12. All point that has y-coordinate greater than x-coordinate lie above the line $y = x$, so in yellow triangle, which has an area of 1/2*3*3=4.5. So, the probability equals to **favorable outcomes/total=yellow triangle/rectangle R=4.5/12=3/8**.

Answer: C.

4

The circles with radius of 4 (blue) and 7 (red):



As you can see if the circle has the radius of 7 some of its points will be in II quadrant so will have negative x coordinate, but we are told that: "each point on the circle k has **non negative coordinates**", so the radius of 7 is not possible (or any other radius more than 4).

So, the max radius is 4, which makes the max area equal to $\pi r^2 = 16\pi$.

Answer: C.

5

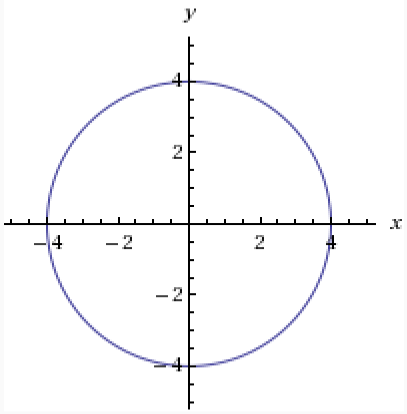
You CAN calculate the distance between any two points with given coordinates on a plane (no matter in which quadrants they are). For example the distance between two points (4,0) and (-4,0) is simply 8.

Generally the formula to calculate the distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

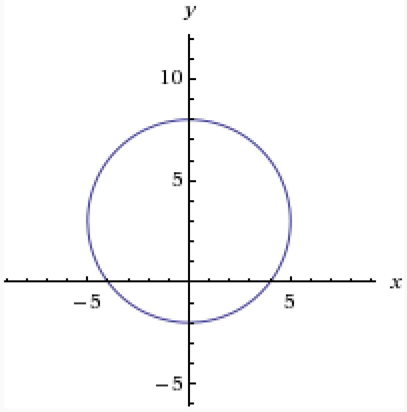
Next, the distance between (4,0) and (-4,0) won't necessarily be the DIAMETER of a circle. The minimum length of a diameter is indeed 8 (so min r=4) but as ANY point on the y-axis will be equidistant from the given points then any point on it can be the center of the circle thus the maximum length of the radius is not limited at all.

The **minimum length** of a diameter is indeed 8 (so min r=4) but as ANY point on the y-axis will be equidistant from the given points then any point on it can be the center of the circle thus **the maximum length of the radius is not limited at all**.

Check 2 possible circles:
Circle with min radius of 4 (equation x^2+y^2=4^2):



Circle with radius of 5 (equation x^2+(y-3)^2=5^2):



Generally circle passing through the points (4, 0) and (– 4, 0) will have an equation $x^2+(y-a)^2=4^2+a^2$ and will have a radius of $r=\sqrt{4^2+a^2}$. As you can see min radius will be for $a=0$, so $r_{min}=4$ and max radius is not limited at all (as a can go to +infinity as well to -infinity).

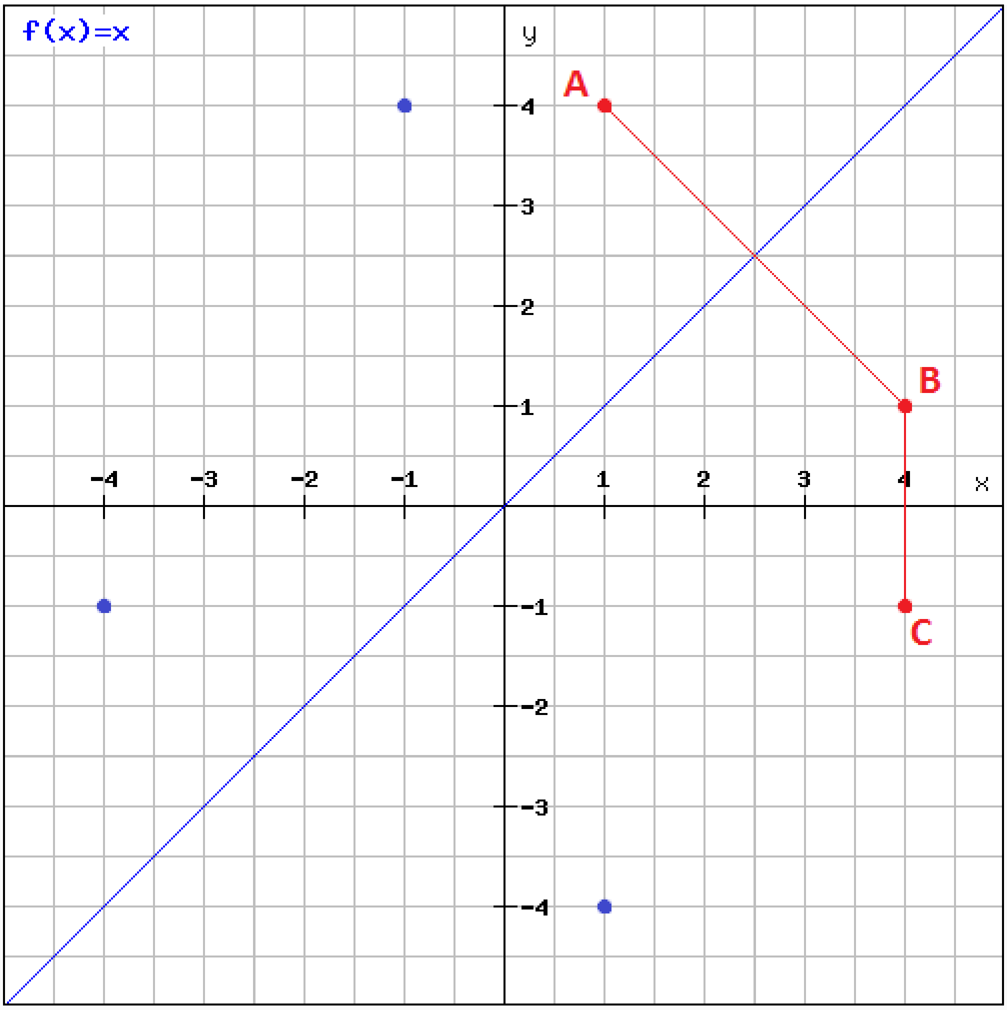
The OA is E.

6

Since the line y=x is the perpendicular bisector of segment AB, then the point B is the mirror reflection of point A around the line y=x, so its coordinates are (4, 1). The same way, since the x-axis is the perpendicular bisector of segment BC then the point C is the mirror reflection of point B around the x-axis, so its coordinates are (4, -1).

Answer: C.

The question becomes much easier if you just draw rough sketch of the diagram:



Now, you can simply **see** that options A, B, and D (blue dots) just can not be the right answers. As for option E: point (4, 1) coincides with point B, so it's also not the correct answer. Only answer choice C remains.

Answer: C.