1

In the xy coordinate plane, line L and line K intersect at the point (4,3). Is the product of their slopes negative?

We have two lines: $y_l=m_1x+b_1$ and $y_k=m_2x+b_2$. The question: is $m_1*m_2<0$?

Lines intersect at the point (4,3) --> $3=4m_1+b_1$ and $3=4m_2+b_2$

(1) The product of the x-intersects of lines L and K is positive. Now, one of the lines can intersect x-axis at 0<x<4 (positive slope) and another also at 0<x<4 (positive slope), so product of slopes also will be positive BUT it's also possible one line to intersect x-axis at 0<x<4 (positive slope) and another at x>4 (negative slope) and in this case product of slopes will be negative. Two different answers, hence not sufficient.

But from this statement we can deduce the following: x-intersect is value of x for $y=0$ and equals to $x=-\frac{b}{m}$ --> so $(-\frac{b_1}{m_1})*(-\frac{b_2}{m_2})>0$ --> $\frac{b_1b_2}{m_1m_2}>0$.

(2) The product of the y-intersects of lines L and K is negative. Now, one of the lines can intersect y-axis at 0<y<3 (positive slope) and another at y<0 (positive slope), so product of slopes will also be positive BUT it's also possible one line to intersect y-axis at y<0 (positive slope) and another at y>3 (negative slope) and in this case product of slopes will be negative. Two different answers, hence not sufficient.

But from this statement we can deduce the following: y-intercept is value of y for $x=0$ and equals to $x=b$ --> $b_1*b_2<0$.

(1)+(2) $\frac{b_1b_2}{m_1m_2}>0$ and $b_1*b_2<0$. As numerator in $\frac{b_1b_2}{m_1m_2}>0$ is negative, then denominator m_1m_2 must also be negative. So $m_1m_2<0$. Sufficient.

Answer: C.

In fact we arrived to the answer C, without using the info about the intersection point of the lines. So this info is not needed to get C.

2

Algebraic approach:

Lines n and p lie in the xy-plane. Is the slope of line n less than the slope of line p?

We have two lines: $y_n=m_1x+b_1$ and $y_p=m_2x+b_2$. Q: $m_1<m_2$ true?

(1) Lines n and p intersect at the point (5,1) --> $1=5m_1+b_1=5m_2+b_2$ --> $5(m_1-m_2)=b_2-b_1$. Not sufficient.

(2) The y-intercept of line n is greater than the y-intercept of line p --> y-intercept is value of y for $x=0$, so it's the value of b --> $b_1>b_2$ or $b_2-b_1<0$. Not sufficient.

(1)+(2) $5(m_1-m_2)=b_2-b_1$, as from (2) $b_2-b_1<0$ (RHS), then LHS (left hand side) also is less than zero $5(m_1-m_2)<0$ --> $m_1-m_2<0$ --> $m_1<m_2$. Sufficient.

Answer: C.

3

$area=\pi r^2$, so we should find the value of radius.

It would be better if you visualize this problem.

(1) Points (-2, 0) and (0,2) lie on the circle --> two points DO NOT define a circle (three points does), hence we can have numerous circles containing these two points, thus we can not find single numerical value of radius. Not sufficient.

Side note: if you put points (-2, 0) and (0,2) on XY-plane you can see that center of the circle must be on the line $y=-x$ (the center of the circle must be equidistant from two pints given).

(2) The radius of the circle is equal to or less than $\sqrt{2}$ --> $r\leq\sqrt{2}$. Clearly insufficient.

(1)+(2) The distance between the 2 points given is $d=\sqrt{2^2+2^2}=2\sqrt{2}$, so it's min length of diameter of the circle passing these points (diameter of a circle passing 2 points can not be less than the distance between these 2 points), thus half of $2\sqrt{2}$ is min length of the radius of the circle --> $r\geq\sqrt{2}$ but as from (2) $r\leq\sqrt{2}$ then $r=\sqrt{2}$ --> $area=\pi r^2=2\pi$. Sufficient.

Answer: C.

4

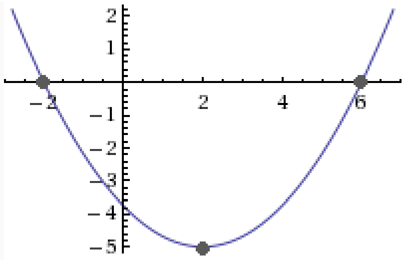
Though it's possible to solve this question algebraically the easiest way will be to visualize it and draw on a paper.

(1) The vertex of parabola is (2,-5) --> the vertex is in the IV quadrant: if the parabola is downward it'll have negative y-intercept, but if it's upward then it can have positive as well as negative y-intercept. Not sufficient.

(2) The parabola intersects with axis-x at point (-2,0) and (6,0) --> now if the vertex is above x-axis then parabola will have positive y-intercept and if its vertex is below x-axis it'll have negative y-intercept. Not sufficient.

(1)+(2) As from (1) the vertex is below x-axis then from (2) we'll have that parabola must have negative y-intercept. Sufficient.

You can look at the diagram below to see that a parabola passing through the given three points must have negative y-intercept only.



(x from −2 to 6)

Answer: C.

5

Line m passes through the origin. Line l is parallel to line m. What are the equations of the two lines?

Equation of a line in point intercept form is $y=mx+b$, where: m is the slope of the line and b is the y-intercept of the line (the value of y for $x=0$).

From the stem:
Since parallel lines have the same slope, then the slopes of l and m are the same;
Since a line passing through the origin has y-intercept equal to zero then the equation of line m would be $y_m=mx$ and the equation of line l would be $y_l=mx+b$

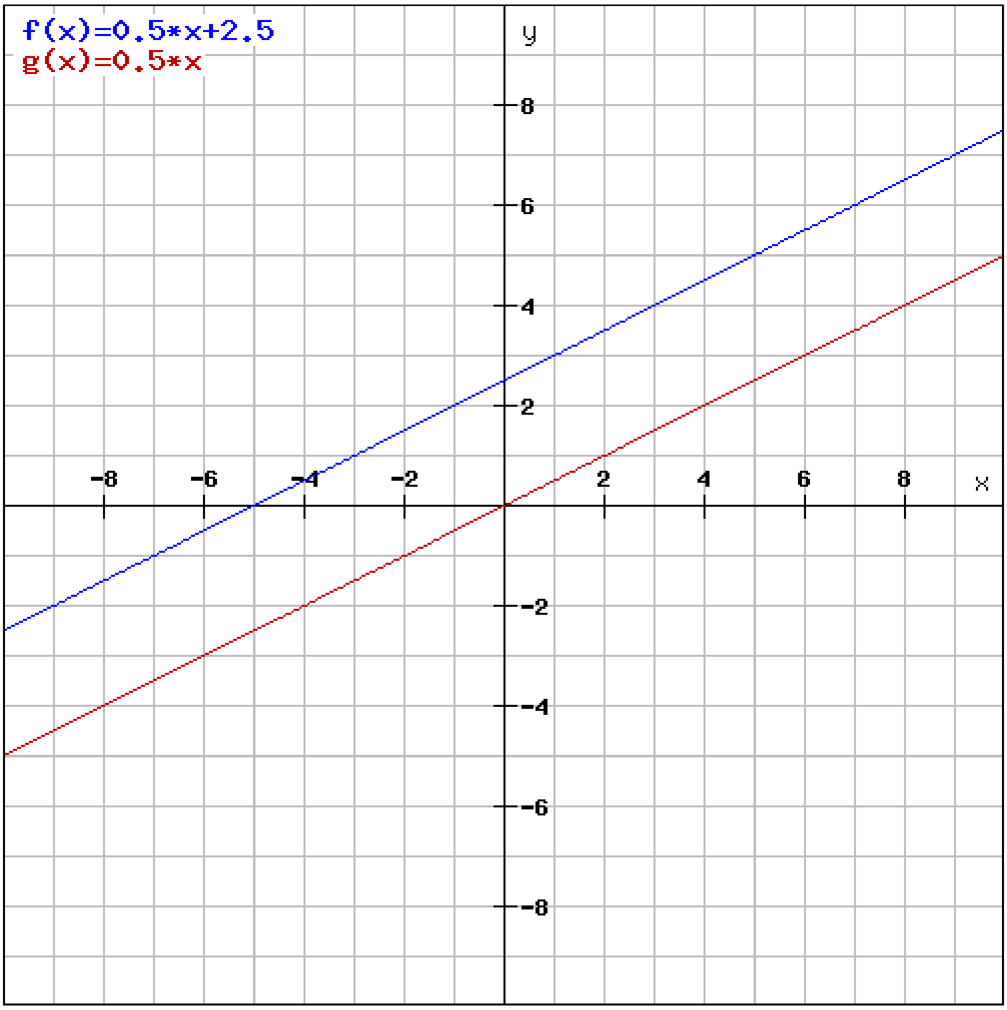
(1) The horizontal distance between the two lines is 5 units --> basically we are told that the x-intercept of line l is either -5 or 5, so we know that line l passes either through the point (-5, 0) or (5, 0). Not sufficient.

(2) Line l has a y-intercept of 2.5 --> $b=2.5$, so we know that line l passes through the point (0, 2.5). One point is not enough to determine (fix) a line. Not sufficient.

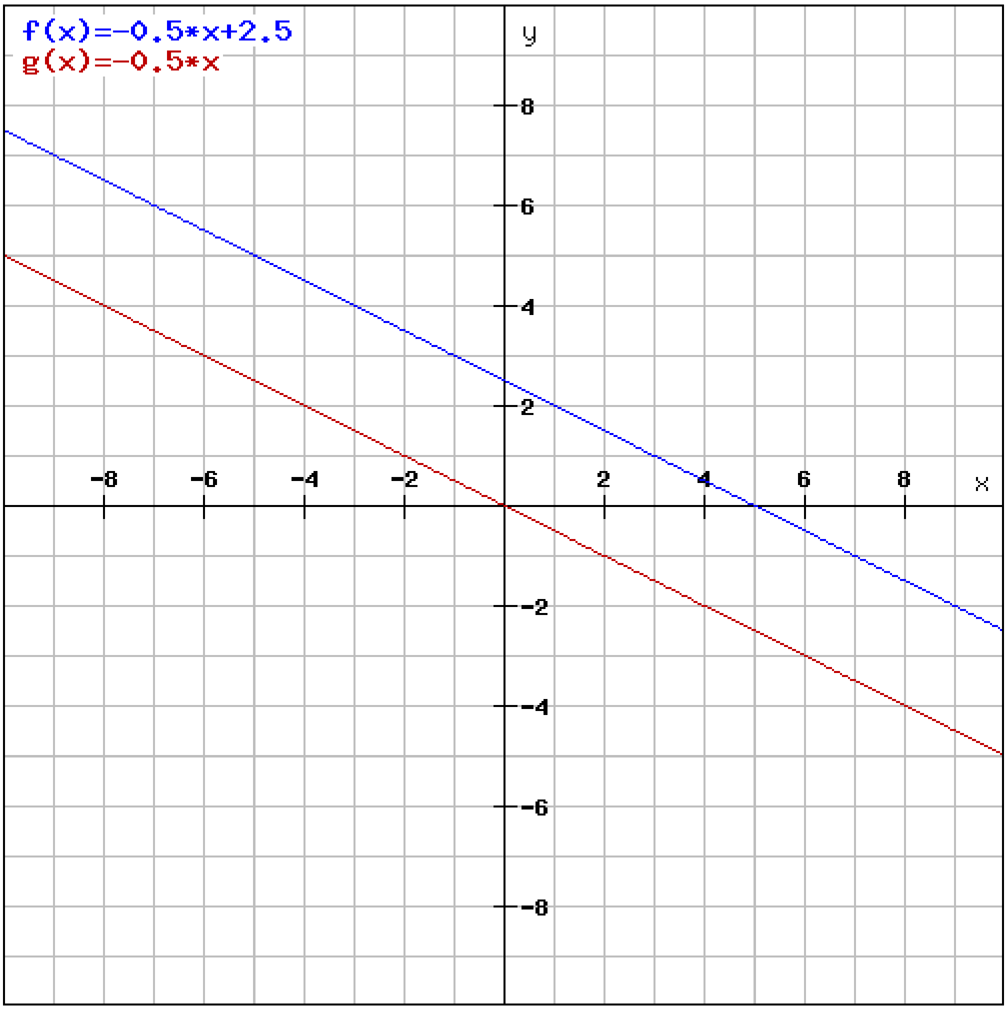
(1)+(2) Now, even take together we cannot determine whether line l passes through the point (-5, 0) or (5, 0). So, we would have two possible points of line l: (-5, 0) and (0, 2.5) OR (5, 0) and (0, 2.5), which means that we would have two possible equations of line l and m: $y_l=0.5*x+2.5$ and $y_m=0.5*x$ OR $y_l=-0.5*x+2.5$ and $y_m=-0.5*x$. Not sufficient.

Answer: E.

To demonstrate.
Case 1:



Case 2:


f(x)=-0.5*x+2.5
g(x)=-0.5*x
y
x
8
6
4
2
-2
-4
-6
-8
-8
-6
-4
-2
2
4
6
8