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Data Sufficiency

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This book is dedicated to Veritas Prep's instructors, whose enthusiasm and experience have contributed mightily to our educational philosophy and our students' success.

It is also dedicated to the teachers who inspired Veritas Prep's instructors. The lesson that follows was only made possible by a lifelong love of learning and of undertaking educational challenges; we have teachers around the world to thank for that.

Finally and most importantly, this book is dedicated to our thousands of students, who have taught us more about teaching and learning than they will ever know. And to you, the reader, thank you for adding yourself to that group.

Personal Dedications

Veritas Prep is a community of educators, students, and support staff, and these books would not be possible without our cast of thousands. We thank you all, but would like to specifically acknowledge the following people for their inspiration:

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CREATING *Think Like the Testmaker*

Creating is the top of the pyramid in Bloom's Taxonomy. When you have completely mastered the GMAT, you are able to Think Like the Testmaker. You are on top of the pyramid looking down! You don't just have good content knowledge and lots of practice with GMAT problems; you understand how a problem has been made, what makes it hard, and how to break it down. When you Think Like the Testmaker you can:

1. Quickly recognize what the problem is actually asking,
2. Discover hidden information and manipulate it to make it useful,
3. Recognize and see through trap answers, and
4. Create your own plan of attack for any problem.



APPLYING *Skills Meet Strategy*

What makes the GMAT difficult is not so much the underlying skills and concepts, but rather the way those skills and concepts are tested. On the GMAT, what you know is only as valuable as what you can do with that knowledge. The Veritas Prep curriculum emphasizes learning through challenging problems so that you can:

1. Learn how to combine skills and strategies to effectively solve any GMAT problem,
2. Most effectively utilize the classroom time you spend with a true GMAT expert, and
3. Stay focused and engaged, even after a long day in the office.



REMEMBERING *Skillbuilder*

In order to test higher-level thinking skills, testmakers must have some underlying content from which to create problems. On the GMAT, this content is primarily:

- Math curriculum through the early high school level, and
- Basic grammar skills through the elementary school level.

To succeed on the GMAT you must have a thorough mastery of this content, but many students already have a relatively strong command of this material. For each content area, we have identified all core skills that simply require refreshing and/or memorizing and have put them in our *Skillbuilder* section. By doing this:

1. Students who need to thoroughly review or relearn these core skills can do so at their own pace, and
2. Students who already have a solid command of the underlying content will not become disengaged because of a tedious review of material they've already mastered.

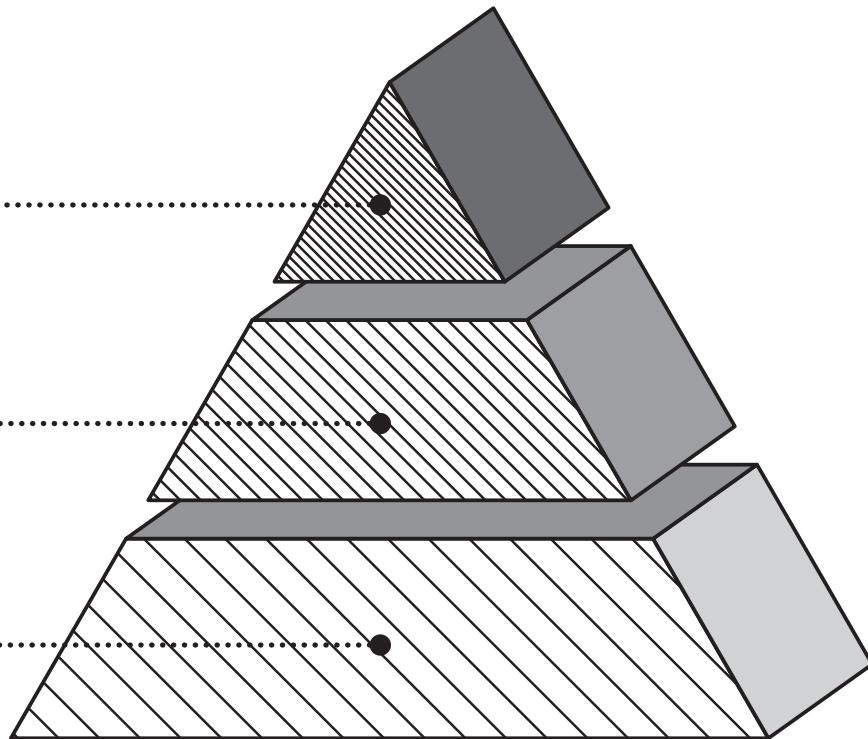




LESSON PREVIEW

The educational philosophy at Veritas Prep is based on the multi-tiered **Bloom's Taxonomy of Educational Objectives**, which classifies different orders of thinking in terms of understanding and complexity. At the base of Bloom's pyramid are "Remembering" and "Understanding," and everything above that - including "Analyzing," "Applying," and "Creating" - is considered Higher-Order Thinking. The GMAT is a self-professed test of Higher-Order Thinking, which is essentially the ability to take information and do something more with it. Consequently, our curriculum will force you to not just remember and understand the content that drives the test but also learn to apply it to the higher-level reasoning required for high GMAT scores.

To achieve a high score on the GMAT, it is essential that you understand the test from the top of the pyramid. On the pages that follow, you will learn specifically how to achieve that goal and how this lesson in particular relates to the **Veritas Prep Pyramid**.



How This Book Is Structured

Our Curriculum Is Designed to Maximize The Value Of Your Time

The Veritas Prep Teaching Philosophy: Learning by Doing

Business schools have long featured the Case Method of education, providing students with real-world problems to solve by applying the frameworks they have studied. The Veritas Prep *Learning by Doing* method is similar. In the lesson portion of this book, you will find that many concepts and strategies are covered by leading with a practice question first, so that you review content and learn strategy while already engaged in the necessity of solving a problem. The Case Method in business school maximizes student engagement and develops higher-order thinking skills, because students must apply and create, not just remember. Similarly, the *Learning by Doing* philosophy maximizes the value of your study.

An important note on *Learning by Doing*: In business school, your goal with a business case is not to simply master the details of a particular company's historical situation, but rather to develop broader understanding of how to apply frameworks to real situations. In this course, you should be certain to reflect on each question not simply through that narrow lens (Did you answer correctly? What key word made the difference?), but rather as an example of larger GMAT strategy (How could the exam bait you with a similar trap? How deeply do you need to understand the content to solve this genre of problem more efficiently?).



Throughout the Veritas Prep curriculum, you will be exposed to several important and recurring themes that you will see in most GMAT problems:



THINK LIKE THE TESTMAKER

- **Abstraction**
- **Reverse-Engineering**
- **Large or Awkward Numbers**
- **Exploiting Common Mistakes**
- **Selling the Wrong Answer and Hiding the Correct Answer**
- **Misdirection**
- **Content-Specific Themes**

REMEMBER: Don't mistake activity for achievement! Focus on recurring themes, not just underlying content.



SKILLS MEET STRATEGY

- **Guiding Principles**
- **Problem-Solving Strategies**
- **Leveraging Assets**

Each book in the Veritas Prep curriculum contains four distinct sections:

1. **Skillbuilder.** We strongly suggest that you **complete each Skillbuilder lesson** at your own pace, and return to the *Skillbuilder* when you recognize a content deficiency through practice tests and GMAT homework problem sets.

The *Skillbuilder* section will:

- Cover content that is **vital to your success on the GMAT**, but is best learned at your own pace outside the classroom.
- Allow you to **review and/or relearn** the skills, facts, formulas, and content of the GMAT. Each student will have his own set of skills that are “rusty” or even brand-new, and will find other items that come back quickly.
- **Vary in length** significantly for each book, based on the number of underlying concepts. (For instance, the Advanced Verbal lesson does not have a Skillbuilder because you are already building on the concepts introduced in three previous lessons.)

2. **Lesson.** The lessons are designed to provide students with maximum value added from GMAT experts by:

- Engaging in challenging problems (*Learning by Doing*), and
- Analyzing those problems for the recurring takeaways.

With each problem, there will be a detailed explanation that will help you understand how the problem is testing a particular concept or series of concepts, what makes the problem hard, and what underlying skills are required to solve it.

When relevant, there will be particular boxes for *Think Like the Testmaker*, *Skills Meet Strategy*, and *Skillbuilder* when you should be focused on particular .

NOTE: When doing in-class and homework problems, you should **do your work below the problem**, and you **should not circle the answer** on the actual question (just note it on the bottom of the page). That way, if you want to redo problems, you can simply cover up your work and proceed as if you had never done it.



3. **You Oughta Know.** The *You Oughta Know* sections will round out each lesson and cover:

- Obscure topics that arise infrequently.
- More advanced topics that are not common on the GMAT but do get tested.

While these uncommon content areas do not warrant in-class time in the Veritas Prep classroom syllabus, it is still important that you have some exposure to these topics before taking the GMAT. Therefore you should **complete these sections before moving to the homework problems.** As with the *Skillbuilders*, the length of these will vary depending on their importance.

4. **Homework Problems.** In many ways, the homework problems are **the most important part of each book.** After refreshing core content in the *Skillbuilder* and then applying that knowledge in the lesson, you must reinforce your understanding with more problems.

Each question is accompanied by a **detailed explanation** at the back of the book, as well as a quick-reference answer key on the last page. A majority of questions are above the 50th percentile in difficulty, and they are arranged in approximate order of difficulty (easiest to most difficult). By completing all of the homework problems, you will learn all of the different iterations of how concepts and skills are tested on the GMAT.

Homework problems are designed to be challenging, so do not despair if you are answering questions incorrectly as you practice! Your goal should be to learn from every mistake. Students can miss a significant percentage of questions in each book and still score extremely high on the GMAT, provided that they learn from each problem. Embrace the challenge of hard problems and the notion that every mistake you make in practice is one that you will know to avoid on the GMAT when every question counts.



SKILLBUILDER

Data Sufficiency questions represent almost half of the Quantitative section, but for many students they make up much more than 50% of their concern. Why? Data Sufficiency questions are unique to the GMAT. Most people have completed thousands of multiple choice math questions in their academic career, but very few people have ever seen a Data Sufficiency question until their GMAT preparation. Data Sufficiency is challenging in large part because it's unique, but keep this in mind: it's unique and challenging for everyone, so by picking up a book and putting in the time to master the genre of Data Sufficiency, you're gaining a substantial competitive advantage.

In this Data Sufficiency lesson, you will learn to master much more than the simple Data Sufficiency structure. This lesson will teach you to:

Skillbuilder

- Know the Data Sufficiency answer choices & the Decision Tree elimination approach
- Understand the two types of Data Sufficiency questions: Yes/No and What is the Value
- Understand what constitutes “sufficiency” and how thin that line can be

Lesson

- Leverage the Data Sufficiency Toolkit to efficiently solve problems
- “Play the Game” of Data Sufficiency, using clues embedded within the question to allocate time effectively and avoid trap answers
- Recognize the Power of Construct Thinking to see the “Reward System” of Data Sufficiency

Homework

- Learn by Doing – build from basic problems, which allow you to practice the Decision Tree and become comfortable with the answer choices, to advanced problems, which will require you to Leverage Assets, Play Devil’s Advocate, and win the game of Data Sufficiency

A NOTE TO SELF-STUDY STUDENTS

This lesson appears halfway through the quantitative section of the Veritas Prep course, at a point at which students have reviewed and practiced the core skills related to Arithmetic, Algebra, Geometry, and basic Statistics, Probability, and Word Problems. This lesson focuses on strategy specific to Data Sufficiency, and will at times assume familiarity with the core math content of the GMAT. Problem explanations will review those core skills, but you will return the most value from this lesson if you've already begun reviewing that core content on your own.

Introduction – The Basics

To begin understanding Data Sufficiency, it's helpful to look at the official directions and answer choices for these problems, courtesy of the Graduate Management Admissions Council:

Directions

This Data Sufficiency problem consists of a question and two statements, labeled (1) and (2), in which certain data are given. You have to decide whether the data given in the statements are sufficient for answering the question. Using the data given in the statements plus your knowledge of mathematics and everyday facts (such as the number of days in July or the meaning of counterclockwise), you must indicate whether:

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked;
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked;
- (C) Both statements (1) and (2) TOGETHER are sufficient to answer the question asked; but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient to answer the question asked;
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed.

You should see from the answer choices that the name of the game is "is the data sufficient?", so let's spend some time discussing what constitutes sufficient information.



Understanding Sufficiency

Data Sufficiency questions hinge on whether a statement is sufficient to answer a question. A statement is sufficient when it guarantees exactly one answer to that question. For example, in the question:

Is integer x positive?

The statement " $x > 9$ " would be sufficient, as any number greater than 9 is also greater than 0 and therefore positive.

The statement " $x^2 > 81$ ", however, would not be sufficient, as there are two potential values of x : 9 (which gives the answer "yes, x is positive") and -9 (which gives the answer "no, x is not positive").

Your job, then, is to determine when a statement is sufficient to provide exactly one answer to the overarching question. Try the following drill questions to get a feel for sufficiency:

1. **Question:** Is integer y positive?

Statement: $y^3 > 27$

2. **Question:** What is the volume of cube W ?

Statement: Cube W has a surface area of 96 square inches.

3. **Question:** What is the volume of rectangular box Z ?

Statement: Rectangular box Z has a surface area of 96 square inches.

4. **Question:** If all apples cost \$A and all pears cost \$P, what is the price of two apples and three pears?

Statement: Together, nine apples and six pears cost \$60.

5. **Question:** If all apples cost \$A and all pears cost \$P, what is the price of two apples and three pears?

Statement: Together, six apples and nine pears cost \$60.

Understanding Sufficiency – Solutions

1. **Sufficient.** Because 3 is an odd exponent, there is no chance of y being a negative integer. The cube root of 27 is 3, so this statement essentially tells you that $y > 3$, meaning all potential values of y are greater than 3. Because all potential values will give the same answer – yes – the statement is sufficient.
2. **Sufficient.** Because all sides of a cube are the same, a cube with a surface area of 96 will have six equal sides with area 16, meaning that each side has a length of 4. And a cube with a side of 4 has an area of 4^3 , which is 64. Because this statement provides exactly one answer to the question – the cube has a volume of 64 – the statement is sufficient.
3. **Not Sufficient.** While this problem may look similar to the previous problem, the fact that this shape is a rectangle and not necessarily a square makes all the difference. There are multiple combinations of length, width, and height that could satisfy that surface area, which for a rectangle equals $2LW + 2LH + 2WH$. Since this problem leaves three variables and can therefore lead to several different answers, the statement is not sufficient.
4. **Not sufficient.** This problem asks you to solve for $2A + 3P$, and gives the fact that $9A + 6P = 60$. These two variables cannot be isolated in just one equation, and even simplifying the equation by dividing all terms by 3 to yield $3A + 2P = 20$ does not allow you to solve for $2A + 3P$. It could be that $A = 6$ and $P = 1$ (which means that $2A + 3P = 15$) or it could be that $A = 4$ and $P = 4$ (which means that $2A + 3P = 20$). Since the given information allows for at least two different answers to the question, this statement is not sufficient.
5. **Sufficient.** While this problem may look similar to #4, there is a key difference: When you simplify $6A + 9P = 60$ by dividing all terms by 3, you get $2A + 3P = 20$. And that's exactly what the question asks you to solve for – what is $2A + 3P$? Since this guarantees that the answer is 20, this statement is sufficient. It is important to pay particular attention to the exact question being asked! While you cannot solve for A or B individually, this question asks for the one particular combination – 2 of A , 3 of B – that you have exactly enough information to solve.

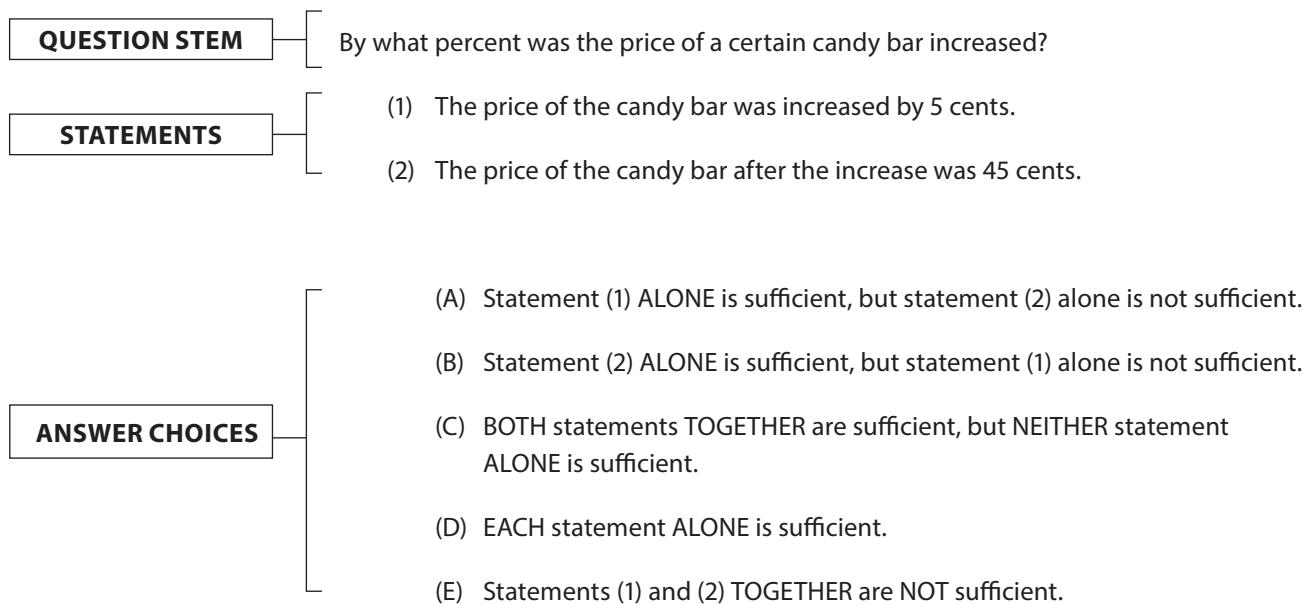


More important than those solutions, however, are these takeaways:

- A statement is sufficient when it guarantees exactly one (and only one) answer to the question.
- This means that in a Yes/No question, you have sufficient information if the answer is “Definitely Yes” or if the answer is “Definitely No”. You do not have sufficient information when the answer is “Sometimes Yes but Sometimes No” (or “Maybe”).
- This means that in a “What is the Value?” question, you have sufficient information when you can pin down exactly one value for the question, but you do not have sufficient information when more than one value is possible.
- Data Sufficiency questions require attention to detail – the drills on the previous page came in pairs, and to the untrained eye each pair might have seemed the same. But subtle differences in what was given or asked – variable squared vs. variable cubed; cube vs. rectangular box; $2A + 3P$ vs. $3A + 2P$ – can make all the difference. The GMAT testmakers prey on this precision in wording frequently, so read carefully.

Anatomy of a Data Sufficiency question

While the genre of Data Sufficiency is unique, the structure is not – all Data Sufficiency questions are structured exactly the same, with three key elements: the question stem (which may or may not contain important facts), the statements (always two statements), and the answer choices (which are always exactly the same). Consider this example to see what a Data Sufficiency question will look like:





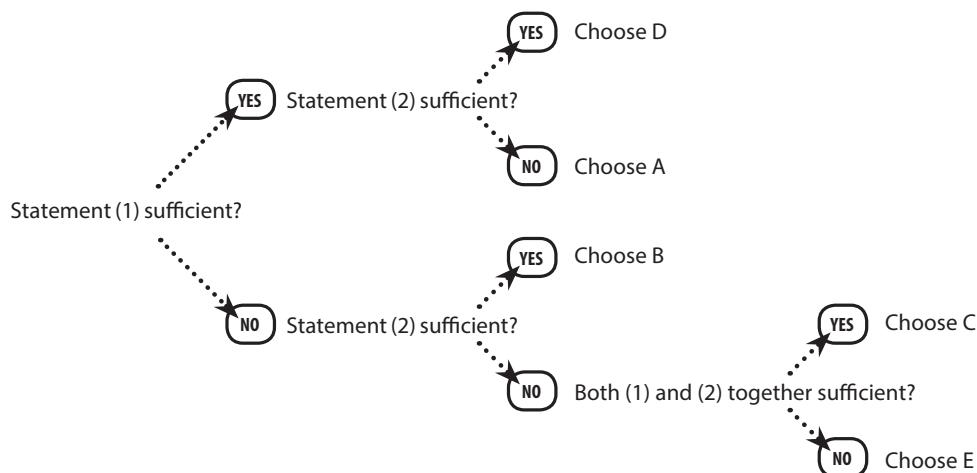
If you're relatively new to Data Sufficiency, the most striking feature of this question is likely the answer choices – they're not numbers that might answer the questions, they're more logical descriptions of when you might be able to answer the question. The good news is that the answer choices are always the same, so you have plenty of opportunities to practice with them. By the time you take the test, you shouldn't even have to glance down at the answer choices because you'll already know them so well, but that takes practice. To internalize the answer choices and have a system to attack them, you should use a system.

For every Data Sufficiency question, ask yourself the following questions (if starting with statement (1)):

- Is the information in statement (1) alone enough to answer the question?
- Is the information in statement (2) alone enough to answer the question?
- Can I answer the question if I combine the information from statements (1) and (2)? (Only ask this of yourself if neither statement alone was enough to answer the question.)

Data Sufficiency Decision Tree

Assess each statement to determine whether it is sufficient or not, and this tree will lead you to the correct answer:



The Decision Tree is a helpful flowchart to ensure that your calculations and decisions lead you to the proper letter choice. To apply this system to the sample question:

The question asks a “What is the Value?” question, asking specifically for the percent increase of the price of a candy bar. The typical components of a percent increase are the change in price and the original price.

Statement 1 provides the percent change, but offers no way to solve for the original price, so statement 1 is not sufficient. This means that you can eliminate choices A and D, leaving only B, C, and E.

Statement 2 ALONE provides the new price, but does not (ALONE) offer any insight as to the original price or the percent change, so statement 2 alone is not sufficient and you can eliminate choice B.

Taking the statements together, you can combine them to see that if the increase is 5 cents and the new price is 45 cents, then the original price is 40 cents. The percent change can be calculated then by dividing the increase (5) by the original price (40) and multiplying by 100. But keep in mind this, too – you don’t have to! Since you know that by finishing these calculations you will arrive at one, exact answer, you’ve already proven that the statements together (but not alone) are sufficient, so you can save yourself the calculation and immediately choose answer choice C.

That’s the Decision Tree in action, and keep in mind that once you’ve used it on a handful of questions it should start to sink in. Many students find it helpful to run through this process quickly by simply jotting down AD / BCE on their noteboard and eliminating one side of the slash after statement 1, then progressing through the elimination from there.

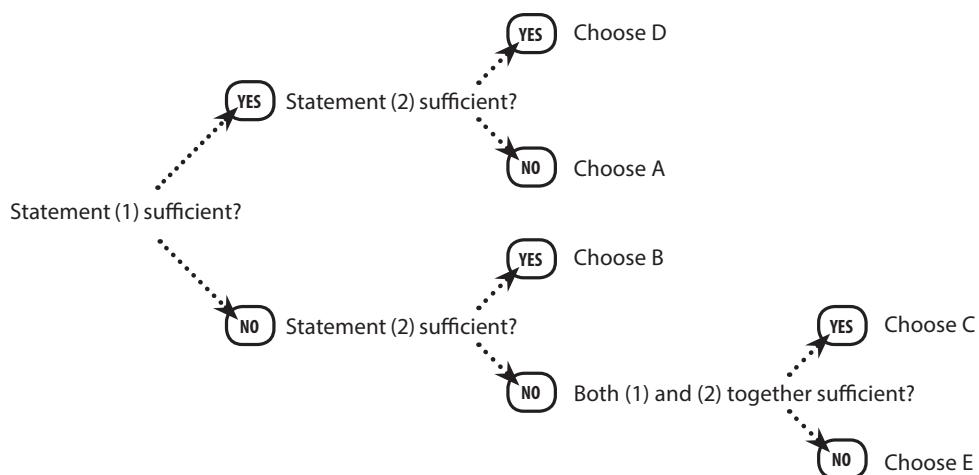


Decision Tree at Work

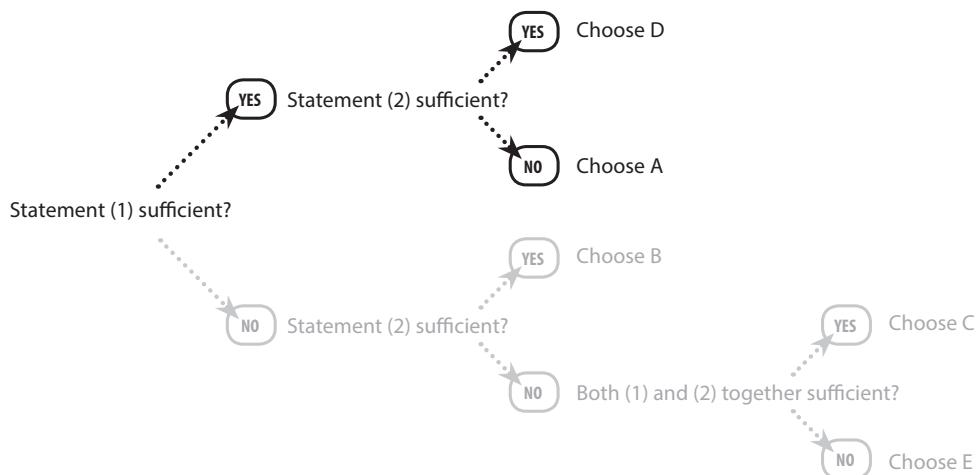
Use the Decision Tree to work toward the correct answer on these problems:

How much is 15% of a certain number?

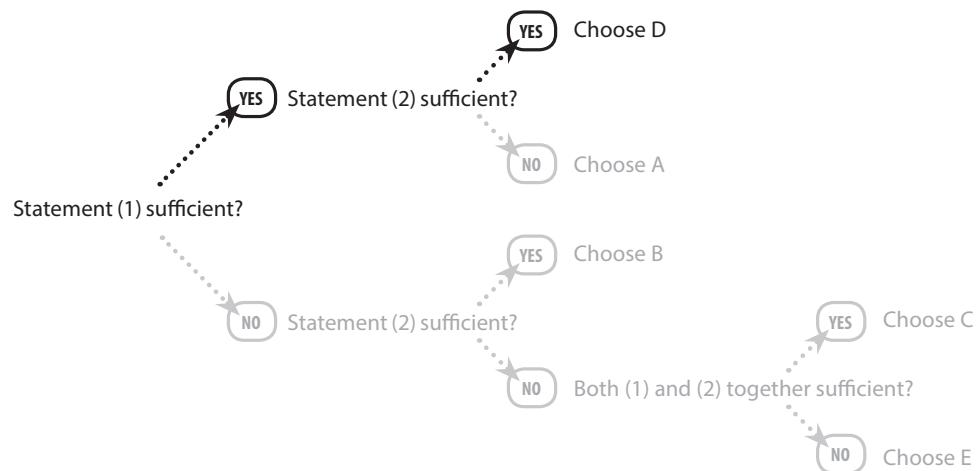
- (1) 5% of the number is 15.
- (2) 30% of twice the number is 180.



Statement 1 is sufficient, as you can set up the equation $.05x = 15$, allowing you to solve for x (the number in question). Eliminate B, C, and E



Statement 2 is sufficient, as you can set up the equation $0.3(2x) = 180$, again allowing you to solve for x. This means that the answer must be D.



Note here, also, that once you had linear equations for the single variable x , you could confidently answer “sufficient” without finishing the math. Since your goal is to determine whether the information is sufficient to find x , you do not have to finish the steps to find x once you’ve proven that you will, indeed, arrive at exactly one value for x .



Anatomy of a Data Sufficiency Question: Note the Question Stem

By the time you take the GMAT, you shouldn't have to read the answer choices – you should know exactly what they mean and have a system to arrive at the correct one (you'll cover that soon). Importantly, though, you should also spend lots of time unpacking the information in the question stem. Everyone pays attention to the statements (as well they should) because the answer choices refer directly to them. The question stem? People often read it too quickly and miss crucial pieces of information embedded within it. Often the most important information is cleverly hidden in the question stem (or explicitly given there), not in the statements. To illustrate this, consider one more "Anatomy of a Data Sufficiency Question" example:

If $xy \neq 0$, what is the value of x ?

- (1) $yx^2 + 4xy + 4y = 0$
- (2) $y = 6$

QUESTION STEM

STATEMENTS

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked;
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked;
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient;
- (D) EACH statement ALONE is sufficient to answer the question asked;
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

ANSWER CHOICES

As you learn about Data Sufficiency, the answer choices and the decision tree should have a disproportionately high share of your attention. But as you consider the three elements of a Data Sufficiency question – Question Stem; Statements; Answer Choices – be sure not to rush through the Question Stem, which is often the “sneakiest” part of the problem.

Here that little caveat that $xy \neq 0$ may not seem altogether useful, but it's the lynchpin of this question. As you work through statement 1, you can factor it to:

$$y(x + 2)(x - 2) = 0$$

Which should leave you with two options to solve that equation. Either $x = -2$, or $y = 0$. But that caveat that $xy \neq 0$ tells you that neither x nor y can equal 0, so once you know that y is not 0, you can divide both sides of the statement 1 equation by y , and arrive simply at:

$(x + 2)^2 = 0$, so statement 1 is sufficient: x must be -2 . You can eliminate B, C, and E, and since statement 2 provides no information about x (later in the lesson you'll learn how to use these obviously-insufficient statements to your advantage), statement 2 is not sufficient and the answer is A.

Your early practice with Data Sufficiency will almost certainly focus on the Decision Tree and mastering the answer choices – this is what we want you to do in the beginning. But a common study mistake of those shooting for 700+ scores who fall short and end up in the high 500s or low 600s is that they overvalue the importance of knowing and eliminating answer choices. The decision tree and the answer choices – which today may seem confusing, but by test day will be as natural as tying your shoes or brushing your teeth – are important but don't neglect the information hidden in the question stem itself.



Sufficiency Drills

As you've noted to this point, Data Sufficiency questions are all about when you have enough information to definitively answer a question. The Decision Tree is a useful tool to help you employ proper process-of-elimination and decision-making once you've begun to assess that sufficiency, but ultimately your success on Data Sufficiency questions will depend on how well you can determine whether statements are sufficient.

So before you dive too deeply into full Data Sufficiency problems and the answer choices, first complete this drill to become more confident with what constitutes sufficiency on these questions.

In the following drills, you will see a single Data Sufficiency question stem for each exercise, followed by several individual statements. Your job: to determine which statements (ALONE) are sufficient, and which are not sufficient. **Please note that, on full Data Sufficiency questions, the statements can be used together and therefore will never contradict each other.** On this drill, however, the goal is simply to practice leveraging one statement at a time to think about sufficiency, so some of the statements may contradict one another.

From this drill, you should focus on:

- What constitutes sufficiency.
- How to leverage information in the question stem and statements to determine whether a statement is sufficient.
- How to avoid assumptions.
- What makes a statement tricky or difficult.

Directions: For each of the following questions, determine which of the statements, ALONE, would be sufficient to answer the question. Please note that there can be multiple sufficient statements for each question.

1. Is $x > 0$?
 - (A) 3^x is an integer
 - (B) $x^2 < x$
 - (C) $x^2 - 3x + 2 = 0$
 - (D) x is a prime number
 - (E) $x^2 - 3x - 4 = 0$

Solutions

- (A) **Not sufficient.** x could be a positive integer (e.g., $3^2 = 9$, an integer, so x could be 2), *but* x could be 0 ($3^0 = 1$, also an integer). If x is 0, the answer is no, but for all other potential values of x (including 2), the answer is yes. Therefore, this statement is not sufficient.
- (B) **Sufficient.** This statement guarantees that x is between 0 and 1. For all negative numbers, their square is positive and therefore greater than x . For 0, its square equals 0 (and is therefore not less than 0). So negatives and 0 are not possible numbers given this statement, proving the answer yes.
- (C) **Sufficient.** Factoring this quadratic, you will find that $(x - 2)(x - 1) = 0$, so $x = 2$ or $x = 1$. Both are positive, so the answer must be yes. Note, also, that you could make this decision quickly, as in the form $ax^2 + bx + c$, if b is negative and c is positive, then the potential values for x must be positive in order to satisfy the equation. (A negative x would mean that all three terms would be positive, making it impossible for their sum to equal 0.)
- (D) **Sufficient.** All prime numbers are, by definition, positive.
- (E) **Not sufficient.** This quadratic factors to $(x - 4)(x + 1) = 0$, meaning that x could be 4 or -1. Because 4 supplies the answer yes and -1 supplies the answer no, we cannot conclude a definitive answer to the question. Note also that this statement looks quite similar to statement (C). Be careful with quadratics and Yes or No questions. You will often need to do some work to determine whether the multiple solutions support the same answer or not.

Exercise takeaways: The first statement shows that “Is $x > 0$?” is a completely different question from “is $x < 0$?”. As zero is the dividing line between positive and negative, it can play an interesting role in Data Sufficiency questions. Statement (A) does not allow for any negative numbers, so you might think that it guarantees that x is positive, but it only promises that x is non-negative, a small but significant distinction. Beware the power of 0 in Data Sufficiency! Also note the similarity and difference between the two quadratics in statements (C) and (E). In Yes or No questions, a statement that allows for multiple values can be sufficient or not sufficient, depending on whether all the values give the same answer to the overall question.



2. In isosceles triangle ABC, what is the measure of angle A?
- (A) Angle B = 30 degrees
 - (B) Angle C = 120 degrees
 - (C) Angle A is one-fourth the measure of angle C
 - (D) Angles A and B add to half the measure of angle C
 - (E) The length of side BC = $2\sqrt{2}$

Solutions

- (A) **Not sufficient.** You know that the sum of $A + B + C$ is 180 degrees, and that either $A = B$, $B = C$, or $A = C$. But this statement does not allow us to determine whether, say, $A = B = 30$ and $C = 120$, or $A = 120$ and $B = C = 30$.
- (B) **Sufficient.** Because the sum of all angles must be 180, it is not possible for both C and another angle to match at 120 each. So if C is 120, then A and B must combine for the other 60, and since two sides must be equal, those are the equal sides, at 30 each.
- (C) **Not sufficient.** This statement still allows for either A to equal B (both 30 degrees, $\frac{1}{4}$ of 120 so that all three angles add up to 180), or for B to equal C (both 80, with A filling in the remaining 20).
- (D) **Sufficient.** This statement guarantees that C is the largest angle, meaning that A must equal B . This allows you to calculate $A + B + C = 180$, and $A = B$, and $C + \frac{1}{2}(C) = 180$. So $\frac{3}{2}(C) = 180$, meaning that $C = 120$, and A and B are each equal 30.
- (E) **Not sufficient.** While this statement may follow the trend of an isosceles right triangle (in which the side ratios are $x, x, x\sqrt{2}$), the problem gives no indication that this is a right triangle—just this one “symptom.”

Exercise Takeaways: Notice that the question stem embeds information (the triangle is isosceles) that needs to be “unpacked” with the various statements. It’s not quite enough information to state an algebraic fact (you know that one angle equals another, but you can’t yet make that an equation), but it’s information that will undoubtedly be important, so you do need to find a way to leverage it with each statement. Also notice that statements (A) and (B) seem to be extremely similar statements, but are wholly different; and that statements (C) and (D) may also look fairly similar, but are substantially different. Data Sufficiency statements are often drawn “to the limit”; they’re often edging up against that line between sufficiency and not, and you need to read carefully and apply the given information in a few ways to ensure that you make the right decision.



3. Is integer x divisible by 12?

- (A) x is the product of three consecutive positive integers
- (B) x is the product of three consecutive even integers
- (C) x is the product of three consecutive prime numbers
- (D) x^2 is divisible by 36
- (E) x^2 is divisible by 72

Solutions

- (A) **Not sufficient.** x could be the product of 1, 2, and 3 ($x = 6$), but it could also be the product of 12, 13, and 14 (and therefore clearly divisible by 12). This statement does tell us that x is divisible by 6 (in any set of three consecutive integers, at least one will be even and at least one will be divisible by 2), but it is not sufficient to say whether x is divisible by 12.
- (B) **Sufficient.** In any set of three consecutive even integers, at least one will be divisible by 3. Algebraically, you can see this by noting these integers as $2a \cdot (2a + 2) \cdot (2a + 4)$. Factor out the 2s: $2(a) \cdot 2(a + 1) \cdot 2(a + 2)$, and you'll see that you have as factors $2^3 \cdot$ three consecutive integers. Clearly, then, one of those three consecutive integers will be divisible by 3, and you will have at least four factors of 2 in that group. This is sufficient to prove that the product will contain the prime factorization of 12: $2 \cdot 2 \cdot 3$.
- (C) **Sufficient.** This is one of the cases (they're rare but significant) in which the statement is enough to prove "no" as the answer to the question. In order to be divisible by 12, a number needs to have the prime factors $2 \cdot 2 \cdot 3$. Only one prime number, 2, has a factor of 2, so this statement is enough to prove that x will be missing at least one of the required factors of 2.
- (D) **Not sufficient.** This statement tells us that x is divisible by 6, but is not enough to determine whether x is divisible by 12. At a minimum, $x = 2 \cdot 3$, but "is divisible by 6" could include other factors (for example, 12 is divisible by 6).
- (E) **Sufficient.** This is a fairly challenging statement for many examinees. If $x^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot (\text{something})$ and x is an integer, then the odd 2 after you've broken apart the pairs of prime factors *must* have a pair of its own. Otherwise, x wouldn't be an integer, as that 2 would divide into $\sqrt{2}$, an irrational decimal. Remember: That "(something)" in the factorization is there to show that x is divisible by, but not limited to, 72. The definition that x is an integer guarantees that the prime factors of x^2 will form pairs so that x does not carry a radical after that root is taken. Therefore, x^2 must have $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$, meaning that x is divisible by at least $2 \cdot 2 \cdot 3$.

Exercise Takeaways: The first three statements all rely on definitions, and require you to unpack and apply those definitions to more concretely see the parameters of the problem. Many Data Sufficiency questions will offer "light" information that only has substance when you apply what you're given. From statement (C), note that in a Yes or No question, the answer no is sufficient; we'll discuss this more later, as it's only used in smaller percentage of Data Sufficiency problems (don't think that half the time the answer is no), but when it is it can be a hurdle for many test-takers.



4. If $a = 5$, what is value of $b + c$?

- (A) a is the average of the set $\{a, b, c\}$
- (B) $ab + ac = 10$
- (C) a is the median of the set $\{a, b, c\}$
- (D) $ba + bc = 5$
- (E) $a + b = b + c$

Solutions

- (A) **Sufficient.** This statement tells us that $\frac{a+b+c}{3} = 5$, which means that $a + b + c = 15$. We know that $a = 5$, so $b + c$ must equal 10.
- (B) **Sufficient.** Knowing that $a = 5$, this statement tells us that $5b + 5c = 10$. Divide both sides by 5 to find that $b + c = 2$.
- (C) **Not sufficient.** While this statement looks quite similar to statement (A), it is quite different. The median is just the middle number in a set, which only means that one of b or c is 5 or less, and the other is 5 or greater. But the sets could include: {4, 5, 6} and {-10, 5, 7}, for example. This statement is not sufficient.
- (D) **Not sufficient.** While this statement looks similar to statement (B), it is also quite different. Once you plug in 5 for a , our only known variable, you still have one equation with two variables, and cannot manipulate them to get a value for $b + c$. $5b + bc = 5$; which can also be phrased as $b(5 + c) = 5$. Either way, you cannot isolate $b + c$. You might also consider plugging in numbers to find different values. The statement is satisfied, for example, if $b = 1$ and $c = 0$; or if $b = \frac{1}{2}$ and $c = 5$. Because you can get different answers, the statement is not sufficient.
- (E) **Not sufficient.** This statement allows you to solve for c (subtract b from both sides and you'll find that $a = c$, and we know that a is 5). That also means that b could be absolutely anything, so the statement is not sufficient.

Exercise Takeaways: Even when multiple variables are given in a problem, you need not always solve for all variables if the question asks for a combination of them (here, what is $b+c$?). Again, notice the similarity between some of the statements, but how the slight differences make some sufficient and others not. Learn to read carefully.



5. A pet store sells only dogs and cats. If the ratio of dogs to cats is 3:2, how many dogs are in the pet store?
- (A) If the number of cats were to double, the ratio would be 3:4.
- (B) If 3 cats were added, there would be more cats than dogs.
- (C) If 5 cats and 3 dogs were added, there would be more cats than dogs.
- (D) If 4 cats and 6 dogs were added, there would be no change to the ratio.
- (E) If 4 cats were added, there would be twice as many cats as before.

Solutions

- (A) **Not sufficient.** This statement adds no new information; the current totals are $3x$ dogs and $2x$ cats, so doubling the number of cats would just make it $2(2x) = 4x$ cats. We already know that that new ratio would be 3:4.
- (B) **Not sufficient.** But close! With a ratio of 3:2, the possibilities are 3 dogs and 2 cats; 6 dogs and 4 cats; 9 dogs and 6 cats; etc. In each of the first two cases, three more cats would tip the ratio so that there were more cats: 3 dogs and 2+3 cats → more cats. And 6 dogs and 4 + 3 cats = more cats. Because there are two potential solutions, the statement is insufficient.
- (C) **Sufficient.** Using the same ratios as in the previous solution: $3 + 3 \text{ dogs} < 2 + 5 \text{ cats}$, but $6 + 3 \text{ dogs} = 4 + 5 \text{ cats}$. Only with a starting value of 3 dogs and 2 cats does this statement hold true, so the statement guarantees that the store has 3 dogs.
- (D) **Not sufficient.** Like in statement (A), we already know this. We currently have 3 dogs for every 2 cats, so adding 6 dogs and 4 cats simply keeps the ratio the same. ($3x + 6 \text{ dogs and } 2x + 4 \text{ cats} \rightarrow 3(x+2) \text{ dogs and } 2(x+2) \text{ cats, algebraically.}$)
- (E) **Sufficient.** This statement tells us that $2x + 4 = 2(2x)$. (We start with $2x$ for the number of cats; adding four is the same as multiplying by 2.) That lets us solve for the multiplier ($x = 2$), which we can plug back into the total for dogs ($3x$, if $x = 2$ then there are 6 dogs).

Exercise Takeaways: Ratio problems in Data Sufficiency can be quite difficult, as the GMAT authors have quite a few ways to obscure information. Here, nearly all the statements should look to be similar in scope, but notice the difference particularly between statements (C) and (D): Only the numbers themselves change—and not that dramatically—but the answer is completely different. Throughout this drill we have included these subtle changes to statements to show you that you need to read carefully, interpret information effectively, and do some work to unpack the information you're given. In the lesson that follows, you will have plenty of opportunities to practice.



LESSON

Data Sufficiency: Resource Management

Data Sufficiency is a question form unique to the GMAT. While you will not see Data Sufficiency questions on any other exam, you will see plenty on the GMAT, which has increased its pool of Data Sufficiency questions to now constitute close to half of the quantitative section.

What makes Data Sufficiency such an integral part of the GMAT? Data Sufficiency cuts to the core of what MBAs will need to do, **testing primarily your ability to efficiently and effectively manage resources**. To truly understand what Data Sufficiency is all about, view answer choice C (and remember that the answer choices are fixed; they will always say the exact same things):

- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

Data Sufficiency questions require you to know exactly when you have enough information to make a decision. If you use extra information that you don't need, or make a premature decision without enough information, you will be wrong. **To succeed on Data Sufficiency questions, you must maximize the value of each statement to glean as much information as possible, but you must also be careful not to overvalue a statement.** Proper Data Sufficiency technique requires efficient “resource management” of information in each statement.

Fortunately, the question type comes with rules and restrictions that actually work in your favor. As you work through this lesson, you will learn the common mistakes that the question format baits examinees into making, and you will learn the parameters of the question format that will help you astutely play the game as though you are playing chess (or poker) against the authors of the GMAT. To succeed, you will rely on a balanced mix of your logical reasoning skills and the math skills that you have developed thus far.

Data Sufficiency and the Veritas Prep Pyramid

Because this lesson is primarily about the question type itself, the underlying content will not be the focus. Almost all of the takeaways from this lesson relate to the top of the pyramid. If there is one question type on the GMAT in which you must *Think Like the Testmaker* to succeed, it is Data Sufficiency. There will be a series of themes relating to the construct of Data Sufficiency that represent the very top of the pyramid. There will also be a series of guiding strategies from the middle of the pyramid—the Data Sufficiency Toolkit—that will help you decide how to behave on Data Sufficiency questions relating to certain content types. Learning how and when to use these important tools is another key takeaway from this lesson. The following skills and/or takeaways will be particularly highlighted in the different sections of this book:



"Core Skills" from Skillbuilder

- Decision Tree
- Understanding Sufficiency



"Skills Meet Strategy" Takeaways from the Lesson Section

- The Data Sufficiency Toolkit
 1. Manipulate algebraically.
 2. Use conceptual understanding.
 3. Pick numbers and play devil's advocate.
 4. Just do it.
- Leveraging Assets
- Learning by Doing



"Think Like the Testmaker" Takeaways from the Lesson Section

- Construct Thinking
- Misdirection
- Selling the Wrong Answer



SECTION 1: DATA SUFFICIENCY FUNDAMENTALS

How to Approach Each Question

The following is an outline of the core approach that you should use every time you answer a Data Sufficiency question:

1. Read the question carefully and assess all information that is provided (or not provided) in the question stem. Organize this information so that you understand exactly what you will need to sufficiently answer the question. **Note:** **On many Data Sufficiency questions, the most important information is cleverly hidden in the question stem itself.**
2. Avoid careless assumptions. **Do not assume anything that is not explicitly provided in the question stem or the statements that follow.** For instance, do not assume that x and y are integers unless it is explicitly given or can be deduced from the question stem or statements. Unless instructed otherwise, assume that fractions, negatives, and zero are all included in the set of potential values.
3. Make a quick judgment on which statement is easier to assess and start with that one. **The order in which statements are analyzed does not matter.** By starting with the easier statement, you simplify the decision tree and leverage easier information first.
4. Use the decision tree to carefully consider each piece of information separately and then together if necessary. For each question ask yourself the following questions (if starting with statement (1)):
 - Is the information in statement (1) alone enough to answer the question?
 - Is the information in statement (2) alone enough to answer the question?
 - Can I answer the question if I combine the information from statements (1) and (2)? Only ask this of yourself if neither statement alone was enough to answer the question.



The Two Types of Data Sufficiency Questions

- Yes or No
- What Is the Value?

Regardless of the type you are dealing with, your job is the same: If you can use a statement (or statements) to guarantee exactly one answer to the main question, then that information is sufficient. For *Yes or No* questions, if a statement provides a definitive yes or no to the question, it is sufficient. For *What Is the Value?* questions, if a statement provides exactly one value, then it is sufficient.

For Yes or No questions, two common traps exist that tend to work against novice test-takers.

1. A statement allows for multiple values, but all values provide the same answer to the *Yes or No* question.
2. The statement only allows you to obtain the answer of no—but since always no is a consistent answer, that means that the statement is sufficient (even though the answer is not yes.) Remember: You are not trying to prove that the answer is yes—only that it is either yes or no, and not both.

Consider a few drills to highlight these pitfalls and decide if the one statement is sufficient:

Yes or No Drills

Is $x > 5$?

(1) $x^2 = 16$

Is x greater than 5?

(1) $2x - 15 = 17(x - 15) + 171$

In the second drill on the previous page, you saw that it is easy to do too much work on a Data Sufficiency question. **Typically the bigger problem is that people do not do enough work to properly analyze the information given.** Beware of the possibility of information that looks sufficient but upon further inspection is not, as you will see in first *What Is the Value?* example below.

For ***What Is the Value?*** questions, most of the pitfalls relate to assumptions that people make about the problem or restrictions that people miss in the problem. In a *What Is the Value?* question the following mistakes are most common:

1. People assume that values must be integers and/or positive. With that assumption(s), a statement appears to be sufficient when there are actually multiple values possible.
2. People miss restrictions in the problem that *do* guarantee that the numbers involved are, for instance, integers, so they think there are multiple possibilities, when indeed that statement proves one exact value.

1**SECTION 1: DATA SUFFICIENCY FUNDAMENTALS*****The Two Types of Data Sufficiency***

Consider two drills that highlight some of these common pitfalls.

What Is the Value?* Drills*What is the value of x ?**

(1) $x^2 = 9x$

What is the value of x ?

(1) $x^2 - 19x = 5x - 144$

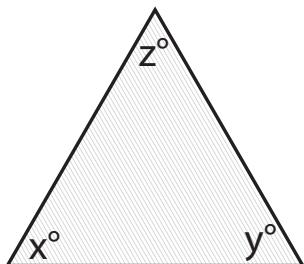
Detailed Solutions to Drills

Yes/No Drill #1 → While x could be 4 or -4, the answer is always “No, x is not greater than 5.” “No” means “sufficient” if it is a consistent conclusion. Remember that no answers are not common in Yes/No questions, but when they are utilized it is usually in tricky cases such as this.

Yes/No Drill #2 → Here you know that you have a linear equation that will simplify to one value of x . Once you have that value, you will know whether or not it is greater than 5. You don’t need to do the work, as you can see that simply taking each step will always produce one exact value and therefore one distinct answer.

What Is the Value? Drill #1 Solution → It is easy to look at this quadratic and assume that x must be 9. However, if you move $9x$ to the left side of the equation, you see that $x^2 - 9x = 0$ and $x(x - 9) = 0$, so $x = 0$ or 9. You can avoid this assumption by properly factoring the quadratic or just remembering about 0, one of the most important numbers on the GMAT.

What Is the Value? Drill #2 Solution → In the first drill, you had to remember that there are two solutions with most quadratic equations. Here you might just be tempted to think that there are two solutions so it won’t be sufficient. However, when you factor the quadratic, you realize that it is a perfect square: $x^2 - 24x + 144 = 0 \rightarrow (x - 12)^2 = 0$ and x is 12, so the statement is sufficient.

**Yes/No Review Problem**

1. Is the triangle above equilateral?
 - (1) $x = y$
 - (2) $z = 50$

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient

(D) EACH statement ALONE is sufficient to answer the question asked

(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

Discussion Questions

1. What is the answer and why?
2. What makes this hard?
3. What is the answer if the question was "Is the triangle above isosceles?"

LEARNING BY DOING**No Is the Same as Yes**

The primary difficulty in this problem is that people forget about the no answer. If a statement proves that the triangle could never be an equilateral triangle, then that statement is sufficient. In statement (1), you learn that two of the angles are identical, so the triangle must be isosceles, but it does not have to be equilateral. Statement (1) provides a maybe answer to the question, so it is not sufficient. People have a tendency to immediately dismiss statement (2) as insufficient because it says nothing about the other angles. But remember: If you know that one angle is 50 degrees, then you are 100% sure that the triangle could never be equilateral. Therefore statement (2) is sufficient by itself, and answer choice B is correct.

Discussion Question Answers:

1. The answer is B as explained above – because it is necessary in an equilateral triangle for ALL angles to be 60 degrees, if just one angle is not 60 degrees you know that this is not an equilateral triangle.
2. What makes this problem difficult is that your mind wants to know something about all three angles. Your inclination might well be “but this statement doesn’t tell me anything about angles x or y!”. Data Sufficiency is made difficult in many ways because of statements like this that don’t tell the entire story, but give just enough information to answer the specific question being asked.
3. This problem is also difficult because, as you saw in the previous drills, the definitive answer to the question is “no”, which in your mind often corresponds directly with “eliminate / cross-out statement 2”. Keep in mind that a definitive “no” answer means that the information is sufficient.
4. If the question asked whether the triangle was isosceles, statement 1 alone would have been sufficient but not statement 2, making the answer A. For an isosceles triangle, only two angles need to be equal, and statement 1 proves that that is true. The lesson? Precision in wording matters. Changing that one word “equilateral” to “isosceles” flipped statement 1 from not sufficient to sufficient and statement 2 from sufficient to not sufficient. Pay careful attention to the specific wording of each question and statement!

**SKILLS MEET STRATEGY**

Understand What Constitutes Sufficiency

Remember: On a Yes or No question, either “definitely yes” or “definitely no” leads to an answer of sufficient. The only answer that is not sufficient is maybe.

**What Is the Value? Review Problem**

2. What is the value of x ?
- (1) $6 < 2x < 10$
- (2) $x^2 = 16$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

Discussion Questions

1. What is the answer?
2. What are the traps?
3. How close is the answer to being answer choice D?
4. What if the question stem were to begin “ x represents the number of children on a field trip?” or “ x denotes the volume, in gallons, of water in a tank?”

LEARNING BY DOING

Don't Make Assumptions

The question on the previous page highlights several traps specific to the Data Sufficiency format. While they are easy to notice here, it is essential that you keep them in mind on test day. Testmakers know that you tend to think via “counting numbers” unless told specifically otherwise (quick: Pick a number 1 through 10. We bet you didn’t pick 6.5, or pi. You probably chose an integer). But on the GMAT, all numbers are possible unless specifically ruled against; that’s why we advise that you play devil’s advocate when picking numbers so that the “trap” numbers, usually nonintegers and negative numbers, work in your favor.

This question has three major traps embedded within it:

1. Statement (1) wants you to assume that x is an integer, but it need not be. x could be 4—but it could also be 4.99.
2. Statement (2) wants you to assume that x is positive, but it could be negative. x could be 4—but it could also be -4.
3. Statement (2) wants you to “remember” statement (1). If you’ve already decided via Statement (1) that x is greater than 3, then you may feel justified in eliminating -4 as an option. But you don’t know Statement (1)—yet. Remember: Answer choice B requires that Statement (2) ALONE is sufficient. You cannot use Statement (1) until you’ve considered Statement (2) by itself.

As you can see now, to solve this you need to have BOTH statements, so answer choice C is correct, but it is easy to think that one or each of the statements ALONE is sufficient.

Discussion Question Answers:

1. As outlined above, the correct answer is C.
2. Statement 1 tries to trap you into thinking that x must be an integer (4), and statement 2 tries to trap you into thinking that x must be positive (eliminating the other potential value, -4) – either because you assume d it, or because you remembered that facet of statement 1.
3. Had the question only allowed for positive integers, each statement alone would be sufficient to prove that x is 4. But as you saw in the traps, this was not the case – nonintegers and negative numbers were perfectly allowable. Make sure that you consider nonintegers, negative numbers, 0, and “numbers nearing infinity” when you’re assessing potential values in Data Sufficiency questions. Often the traps on these problems are built upon our human nature to think primarily about “counting numbers” – small, positive integers.
4. Either of those question stems would have made statement 2 sufficient – neither case allows for a negative number. And the first question stem – “ x represents the number of children...” – would make statement 1 also sufficient, as you cannot have a noninteger number of children. The lesson? Pay attention to the “backstory” on Data Sufficiency questions, as often the seemingly-mundane elements of a word problem will guarantee that the variable in question has to have certain properties (positive, integer, etc.).



SKILLS MEET STRATEGY

Understand What Constitutes Sufficiency

Remember: On a *What Is the Value?* question, a sufficient statement will lead to exactly one and only one value. If a statement leaves the potential for even just a second possible value, it is not sufficient.



Data Sufficiency Fundamentals Summary

As you have seen in this section, it is essential that you understand the following fundamental strategies and rules relating to Data Sufficiency:

- **Use a consistent approach to each Data Sufficiency question.** Always organize the question first and make sure you have leveraged every piece of information that is given (or not given!) in the question before you move to the statements. When you go to the statements, start with the easier statement, as it simplifies the decision tree and allows you to leverage the easiest information first.
- **Completely master the decision tree.** If you are still making mistakes with the decision tree, practice 20–30 easy Data Sufficiency questions to memorize the structure.
- **Understand the two different types of Data Sufficiency questions and what constitutes sufficiency for each one.** For Yes or No questions, a statement is sufficient if it provides either a definitive yes or no answer. For *What Is the Value?* questions, a statement is sufficient if it provides exactly one value for the question.

As tricky as Data Sufficiency can look at first, the way in which they are constructed ensures that you can attack them systematically. As you know, the answer choices are always the same. And you should also know that there are only two types of question stems that the GMAT can ask:

1. **Yes or No Questions**

- A statement gives multiple solutions, but they all give the same answer.
- A statement provides a no answer instead of a yes answer.

2. **What Is the Value? Questions**

- A statement appears to be giving one value because you have assumed properties of the number that were not actually given (positive, integers, etc.).
- Restrictions were placed in the problem that you did not properly leverage (for instance, the problem is asking for the number of children, which must be an integer and cannot be negative).
- **Avoid assumptions.** Every time you approach a Data Sufficiency problem, you must actively consider any assumptions that you may have been baited into making. Avoiding assumptions is perhaps the most important skill in all with Data Sufficiency.



SECTION 2: THE DATA SUFFICIENCY TOOLKIT

With Data Sufficiency, there are numerous ways you might attack a problem: You could consider possible values or think about the problem conceptually; you might manipulate algebraically; you might just do some math. Certain strategies work best for certain types of Data Sufficiency questions, so when you are attacking a Data Sufficiency question, you should be prepared to use any of the following four tools from the Data Sufficiency Toolkit (or some combination thereof), depending on which is the most efficient:

1. Manipulate Algebraically

As the Veritas Prep Algebra lesson highlights, many GMAT problems contain “an inconvenient truth”—information that is sufficient to solve a problem but comes in an inconvenient form that needs to be manipulated. Accordingly, when you see algebra (or when you construct your own equation in a word problem) on a Data Sufficiency question, you should try to make the statement look like the question, or vice versa.

Drills

Is $2x = 3y + 2z$?

$$(1) x - z = \frac{3y}{2}$$

For integers a , b , and c , $\frac{a}{b-c} = 1$. What is the value of $\frac{b-c}{b}$?

$$(1) \frac{a}{b} = \frac{3}{5}$$

SKILLS MEET STRATEGY

*Algebraic Manipulation
and Mirroring*

One of the biggest mistakes that students make with Data Sufficiency is that they always try to solve conceptually or with number picking. Algebraic manipulation and mirroring are the most efficient way to solve *many* Data Sufficiency questions. Why burden your brain with deep conceptual thought or time-consuming number picking when algebraic manipulation can give you a definitive answer? **Importantly, do not forget that algebraic manipulation should be used not only on the statements, but also on the question (and, of course, it might involve both).**

2. Use Conceptual Understanding

Many Data Sufficiency questions hinge on your ability to understand deeply the underlying concepts being tested. For these questions, algebra may not work well, and number picking might be too time-consuming or won't work at all. Many Arithmetic problems are best done with this conceptual approach, and that is especially true in Data Sufficiency. **Importantly, though, if you do not fully understand a problem conceptually, you should try to use algebra or number picking to help you determine sufficiency.**

Drill

What is the ratio of the number of cats in a pet store to the number of dogs in a pet store?

- (1) If there were twice as many cats in the store and five fewer dogs, the ratio of cats to dogs would be 5 to 2.



SKILLS MEET STRATEGY

Conceptual Understanding

One of the keys to success on Data Sufficiency questions is figuring out which of the approaches presented in this section to use on any given problem. There is no magic rule, and you need to be flexible, but generally speaking arithmetic problems are often best solved conceptually (and some can only be solved conceptually).

It is very important to note that attacking a problem conceptually (particularly complicated word problems and inequalities) is a dangerous business. You may think you know a concept deeply, but the problem you are doing involves a particular exception or trick that you don't recognize or understand. As noted, only answer problems conceptually when you are sure that you truly understand the underlying concept or if there is no other way to approach it. If you have any doubt, do some math or number picking to confirm your answer.



3. Play Devil's Advocate and Pick Numbers

Often to solidify your conceptual understanding or even begin a confusing Data Sufficiency problem, you will need to pick numbers. Be careful: Number picking without a goal can be extremely time-consuming and often leads to an incorrect answer. **When you do choose to pick numbers, remember that your goal is to play devil's advocate. Don't pick the same types of numbers over and over, but rather pick the types of numbers (negatives, nonintegers, 0, large numbers) that are likely to give "the other answer" in a Yes or No question.** For instance, if the statement appears to be giving you a yes answer for all the numbers you are picking, you should be actively looking for that one unusual case which would give you a no answer. In a "What is the Value?" question your goal with number picking is simply to show that there are two or more possible solutions from that piece of information, as that automatically results in insufficiency.

Example: Is $xy < 0$?

$$(1) \quad |x-y| > |x|-|y|$$



SKILLS MEET STRATEGY

Number Picking

Number picking should not be used as a "frontline" approach on most Data Sufficiency problems. It is rarely required to solve questions, but when it is, you need to know how to do it effectively (and sometimes it is the best approach). As you learned above, you need to number pick with the purpose of playing devil's advocate. To do that, it is important that you keep in mind five common number properties when testing numbers.

- Positive/negative
- Odd/even
- Nonintegers
- Prime factors
- 0 and 1

4. Just Do It

While a great many (safe to say most) Data Sufficiency problems are logic puzzles, some of them remain math problems in a tricky form. If you don't see a "game" being played, you can begin doing the problem just as a math problem and see where the answers lead you. Often the GMAT will construct these problems so that you have to use one of the above three techniques or you'll waste plenty of precious time. But some of questions are still best solved the old-fashioned way, so sometimes you have to "just do it" as you would in a problem-solving question.

Drill

Is $x > 4$?

$$(1) x^2 - 7x + 6 = 0$$



SKILLS MEET STRATEGY

Some Data Sufficiency Problems Are Just About the Math

While most Data Sufficiency problems have a strong logical basis and require a "game theory" approach (which you will learn shortly), some are just math problems. Certainly, if a problem can be solved conceptually or with good reasoning skills, you should not turn it into a time-consuming, "problem-solving style" math exercise. However, often a purely mathematical approach is required and/or preferred (and safer!), so don't be afraid to just do the math in order to get a definitive answer. This is particularly true for many content areas in Data Sufficiency, such as quadratic equations or inequalities.

**Detailed Solutions to Drills**

- 1a. By taking the equation in the statement and multiplying through by 2, you get $2x - 2z = 3y$, and then adding $2z$ to both sides rephrases the question as a fact. Yes, $2x = 3y + 2z$, so this statement is sufficient. This is called mirroring. Whenever a statement exactly mirrors the question stem, it is sufficient.
- 1b. Note how important it is here to manipulate the question instead of the statement. If you manipulate the given relationship $\frac{a}{b-c}$ to reveal that $a = b - c$, then the question “What is the value of $\frac{b-c}{b}$ ” becomes, more simply, “What is the value of $\frac{a}{b}$ ”? That information is given directly in the statement $\frac{a}{b} = \frac{3}{5}$. Therefore this statement is sufficient to answer the question.
2. Here you should not have to set up the algebra to recognize that, without a baseline number, you cannot tell the impact of five fewer dogs on the ratio. Therefore this statement is not sufficient. If there were over 1,000 dogs, losing five dogs wouldn’t make much of an impact, but if there were only 10 dogs to begin with, reducing the number by five would drastically change the proportion. Generally speaking, adding to or subtracting from a ratio is not sufficient information to gauge its impact on the ratio, unless you have a starting or ending total number. The main exception? If you add or subtract amounts in line with the current ratio (example: The cat/dog ratio is 2:1 and you subtract 2 cats and one dog. The ratio doesn’t change.).
3. In an absolute value problem with multiple variables, performing the algebra takes on too many permutations to be efficient). But picking numbers—particularly in this case, in which the question is asking about a positive/negative number property!—can be quite effective. (However, algebra is the preferred approach for most inequality problems.) If you were to try two positive numbers, say 8 and 6, you would get:

$|8 - 6| = |8| - |6| \rightarrow$ You cannot try those two positive numbers, because they do not satisfy statement (1).

Now your goal is clearer. Can you find a case in which the left-hand side does not equal the right? You may look at the left-hand side and see that, if you make y negative, then you would actually add on the left-hand side (x minus a negative...) whereas on the right, the negative within the absolute value would still represent a positive absolute value. So try a case in which x is negative and y is positive:

$|8 - (-6)| > |8| - |-6| \rightarrow$ This works, as 14 is greater than 8 – 6. So as of now the only numbers we've been able to use are a positive x and a negative y, which give us the answer yes to the overall question.

Now you have one more goal: Can you get the answer no? We've already ruled out positive/positive, recognizing that to make the left-hand side different from the right, y needs to be negative. Let's try negative/negative:

$|(-8) - (-6)| = |-8| - |-6| \rightarrow$ This yields 2 = 2, so we can't use both negative numbers. This means that the answer must be yes, because we can only use a positive x and a negative y. By number picking, we were able to hone in on our goal and choose appropriate numbers quickly.

Seems like it's sufficient, right? Looking back, though, you might have missed one case that will give a no answer. What if they are both positive but y is bigger than x? Then $|(6) - (8)| > |6| - |8| \rightarrow$ so both can be positive and you have a no answer. Therefore this statement is not sufficient. This highlights the difficulty of number picking and why it should generally be used as a last resort.

4. Here you need to know the values of x in order to answer the question, and to get those answers you need to factor out the quadratic. Conceptual understanding can take you only so far (the numbers multiply to a positive 6 but add to a negative 7, so both parentheticals must be in the form x – y), but to ensure a correct answer you should finish the math. The quadratic factors to $(x - 6)(x - 1) = 0$, so $x = 6$ or $x = 1$. Because there is a yes and a no, the statement is not sufficient. Just knowing that you will get two different values for x is not enough to know that this problem is not sufficient. You have to know that at least one value is greater than 4 and at least one value is less than or equal to 4. If the two values for x had been 1 and –6, then this statement would be sufficient, since both of those values are less than 4.



Now that you have seen the Data Sufficiency Toolkit, let's apply the different strategies to some difficult GMAT problems.

Manipulate Algebraically

3. What is the value of $x^2 - y^2$?

- (1) $x + y = 0$
- (2) $x + y = 2x$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING*Manipulate Statements and Questions*

By the time you take the GMAT you should recognize the algebraic “Difference of Squares” rule quickly. When you see the question stem (What is the value of $x^2 - y^2$), you should already see it as two questions: What is the value of $x^2 - y^2$, and what is the value of $(x + y)(x - y)$? With that in mind, statement (1) is clearly sufficient. If $x + y = 0$, then the second, manipulated question reads: “What is the value of $0(x - y)$?” Clearly the answer is 0. With statement (2), manipulating the given equation will also show that it is sufficient. If $x + y = 2x$, then by subtracting x from both sides you find that $y = x$ or $x - y = 0$. Again, this proves that the expression in the question stem equals 0, so answer choice D is correct, because each statement alone is sufficient. Remember that any one algebraic expression or equation can tell you multiple things: $x + y = 7$ also means $x = 7 - y$ and $y = 7 - x$. When something is given to you in one form in Data Sufficiency, look at it in multiple forms to see which one might unlock the problem.

**SKILLS MEET STRATEGY***Don't Forget to Change Questions to Match Statements*

People’s natural inclination in Data Sufficiency is to leave the question alone and try to make the statements match it. One of the most important guiding strategies in Data Sufficiency, particularly on hard problems, is to change the question to match the statements. Often that manipulation is difficult or tedious, but once you do it, the answer is obvious. Of course, as you learned with this problem, don’t forget to properly manipulate any statements as well!

**THINK LIKE THE TESTMAKER****Selling the Wrong Answer**

Every Data Sufficiency question is designed for people to pick one particular incorrect answer choice. On this problem, many people (even those with high-level algebra skills) will pick answer choice A. Why? Because the testmakers have set you up for failure with the way the information is presented. After you manipulate the question, you see clearly that you are on the right track when you go to statement (1). You feel good about yourself because you have quickly recognized the difference of squares and substituted 0 for the $x + y$ portion of the manipulated question. Your natural inclination with statement (2) is simply to take the value of $x + y$ (which is now $2x$) and plug that back into the question, as you did with 0 on the previous statement. That seems to leave all kinds of variables in the expression, so you think the statement is not sufficient and pick answer choice A. The problem, of course, is that you did not also try to manipulate statement (2) and get it in a convenient form. Don't take things at face value in Data Sufficiency; always look to apply all of the strategies outlined in this section.

**SKILLBUILDER**

- Common algebraic equations
- Algebraic manipulation

**Think Conceptually**

4. Is integer k a prime number?
- (1) $k = 10! + m$, where $1 < m < 8$
- (2) k is a multiple of 7
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked;
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked;
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient;
- (D) EACH statement ALONE is sufficient to answer the question asked;
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

LEARNING BY DOING

Understanding the Number Line

On a problem such as this, you must use conceptual understanding of the number line and divisibility to answer the question. Algebraic manipulation will not help you. Number picking will not help you. And certainly just doing it will not help you; calculating the number would be impossible!

The goal from each statement is to prove whether k is prime or importantly *not* prime. It is impossible without computer or calculator assistance to determine whether a large number is actually prime (there are too many numbers you would have to check for divisibility), but it is actually quite easy to prove that a number is *not* prime. If you can prove that k is divisible by anything other than 1 and itself, you have proven that k is *not* prime. **It is on this type of problem that you should be looking to disprove the question and find a no answer.**

In statement (1), you learn that $k = 10! + \text{either } 2, 3, 4, 5, 6, \text{ or } 7$. To review, $10!$ (10 factorial) represents the product of all positive numbers from 1 to 10, inclusive: $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. As you can see, the number $10!$ is a multiple of each of 2, 3, 4, 5, 6, and 7. For demonstration, say that m were 7. $10! + 7$ will then definitely be divisible by 7. If you find any multiple of 7 on the number line and add another 7 to it, it will always still be divisible by 7. Take the number 63, a multiple of 7. If you add 7 to 63, you get 70, another multiple of 7. This is then true for any potential value of m . If m were 2, then $10!$ (an even number) + 2 will remain divisible by 2. If m were 3, then $10!$ (a multiple of 3) + 3 will remain divisible by 3. Statement (1) thus proves that k is not prime, as whatever value of m (2 through 7) we add to $10!$, k will remain divisible by that number, so it *cannot* be a prime number. Statement (1) is sufficient.

Be careful with statement (2). If you carry some information with you from statement (1)—the fact that k is a large multiple of 7—you might think that statement (2) is also sufficient. In other words, you might think, if k is a multiple of 7, then it could never be prime, as it will be divisible by 7. However, remember that there is one multiple of 7 that is prime: 7 itself. Statement (2) is not sufficient, because k could be prime (7) or it could be any of the infinite set of multiples of 7 that are not prime. The answer to this question is answer choice A.

**THINK LIKE THE TESTMAKER****"No" Answers in Data Sufficiency**

In Yes or No Data Sufficiency questions, if a statement is sufficient, it almost always gives an affirmative yes answer. However, in certain types of questions, testmakers will cleverly insert statements that give a definitive no answer. This question is a classic example: You are desperately trying to figure out how to determine that k is prime when you should be trying to show that it is *not* prime. Remember also the equilateral triangle problem from the previous section. Testmakers know when you will forget to disprove things, and they cleverly create problems that exploit this weakness.

SKILLS MEET STRATEGY***Don't Carry Information***

One of the common traps used by testmakers is to create questions in a way that you will carry some small but important piece of information from statement (1) to statement (2). Always make sure that you are properly assessing each statement, independent of the other, with only the information provided in the question stem and that one statement. You will learn shortly the art form of leveraging hints from the other statement, but you must not use any information from the other statement when actually determining sufficiency.

SKILLBUILDER

- Prime numbers
- Divisibility



**Just Do It**

5. Is y a positive number?
- (1) $2x + y > 27$
- (2) $x - 3y < 24$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING*Just Do the Math*

Some inequality problems, particularly those that ask whether a value is > 0 or < 0 , lend themselves well to conceptual understanding. But most of time you will find, as with this problem, that there is too much “action” in the statements to warrant a quick conceptual estimate of what the statements mean. Whenever you are in doubt, it’s generally a good idea to “just do it”—just perform the mathematical operations to solve for a variable or complete a calculation. In many cases you will be able to stop short of the finish line once a few steps have made the picture clearer, but regardless there will be times when you simply need to do the math.

Here, it should be quickly apparent that neither statement alone is sufficient, as each equation allows for any possible value of x . But together, the statements allow you to do the math. Arrange the equations such that the inequalities face the same direction:

$$2x + y > 27$$

$$24 > x - 3y$$

Then manipulate the second equation to get the variables on the same side:

$$3y - x > -24$$

Then double the bottom equation to allow for an elimination of the x term:

$$6y - 2x > -48$$

Then you can combine the inequalities:

$$y + 2x > 27$$

$$6y - 2x > -48$$

$$7y > -21$$

$$y > -3$$

You’ll find that y could be negative (it could be -2 or -1) or positive (all positive numbers are greater than -3), so the statements together are not sufficient, and answer choice E is correct.

**SKILLBUILDER**

- Inequalities
- Algebraic manipulation

**SKILLS MEET STRATEGY***Do the Algebra on Most Inequality Problems*

Inequality problems in Data Sufficiency form are one of the most problematic question types for GMAT students.

Why? Because students try to solve them with awkward, tedious number picking or with suspect conceptual approaches. As for all problems on the GMAT, you must be flexible; sometimes you will use number picking on these problems and sometimes you will use a conceptual approach. But for well over 80% of these problems, they are best solved by algebraic manipulation.

Either you will be able to isolate the required variable or perhaps mirror the inequality that is given in the question stem. Regardless, to use this important strategic approach, you *must* understand deeply how to properly manipulate inequalities. **When you are given inequality problems in Data Sufficiency form, go to algebra first, and mix in number picking and/or conceptual understanding when needed.**

**Play Devil's Advocate/Pick Numbers**

6. If x is a positive integer less than 30, is x odd?
- (1) When x is divided by 3, the remainder is 2.
(2) When x is divided by 5, the remainder is 2.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
(D) EACH statement ALONE is sufficient to answer the question asked
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING

Smart Number Picking

While it's possible to solve this question with a conceptual understanding, it is much easier to put some numbers to work for you. When you do employ numbers, remember that your goal is to play devil's advocate. Your goal is to determine whether x is an odd number, so you will likely start with an odd number that satisfies statement (1). 5 works here, as 5 divided by 3, as $\frac{5}{3} = 1$ remainder 2. So x could be odd. Now that you've found an odd value of x —the answer yes to the overall question—your goal should change. You want to find an even value, because that would show that the statement is not sufficient. If you try everything you can think of and cannot find an even value of x , then you can conclude that it is sufficient. You want to play devil's advocate to ensure that either x must be odd, or conclude that the statement is not sufficient. With that in mind, you might try 8: 8 divided by 3 provides a remainder of 2 ($\frac{8}{3} = 2$ remainder 2). So now you have an even potential x —and the answer no to conclude that statement (1) is not sufficient.

The same process works for statement (2). 7 is an odd number that does the same, so x could still be odd, providing a yes answer. But 12 is an even number that satisfies statement (2), so you can get the answer no, and the answer is thus still maybe. Statement (2) is not sufficient. Taken together, the statements provide a bit more information, as now you know that x provides a remainder of 2 when divided by 3 and when divided by 5. You might recognize 17 as such a number, noting that 15 is the least common multiple of 3 and 5, so 17 will divide out that 15 and leave 2 remaining. Here's where you really need to play devil's advocate: If you chart out the values that work with each statement and look for matches between them, you may well conclude that 17 is the only such value less than 30:

<u>Remainder 2 when divided by 3</u>	<u>Remainder 2 when divided by 5</u>
5	7
8	12
11	17
14	22
17 (MATCH; x could be odd)	27
20	32
23	37
26	26
29	
32 (MATCH; x could be even—but not even and less than 30)	



But still play devil's advocate. Is there any even number that could fit the bill? There is, but it's very hard to find unless you remember what happens when you divide a smaller number by a larger number. 2 also works. When 2 is divided by 3, the quotient is 0 and the remainder is 2. When 2 is divided by 5, the quotient is 0, and the remainder is 5. 2 is the even counterpart, and although it may not be as readily clear as 17, if you force yourself to play devil's advocate and consider the entire range of numbers available to you, you will often find that "catch" upon which correct answers often depend. The correct answer to this problem is E, but the authors of the question are betting that you will forget to consider 2 and therefore fall into the trap of selecting C.



SKILLS MEET STRATEGY

When to Number Pick

Quotient/remainder problems such as this are deeply rooted in divisibility and how the number line works. If you understand those concepts well, you can avoid lots of number picking by leveraging your understanding of repeating patterns with divisibility on the number line. Still, there are many difficult quotient/remainder problems in which number picking is the only reasonable way to efficiently solve the problem. **Remember:** **While number picking is often a tedious and inefficient approach, on some problem types, such as this, it is the best way to show whether certain information is sufficient.**



THINK LIKE THE TESTMAKER

Exploiting Common Mistakes

Clearly this is a very difficult question as it exploits an oddity of arithmetic that even math PhDs might have forgotten: When you divide a smaller number by a larger number, the quotient is always 0 and the remainder is always the dividend. This unusual property of division has been used to create many tricky Data Sufficiency questions, because there is always that one value that people forget (for this problem, it is 2). As you prepare for the GMAT, file away these common mistakes so that you do not make them again if they happen to show on test day. While this is not an extremely common mistake, it is certainly one that has been used to make some particularly difficult questions.



SKILLBUILDER

- Division
- Divisibility and the number line
- Number properties



The Data Sufficiency Toolkit Summary

To succeed on Data Sufficiency, you must break down questions and statements so that you get as much information as possible from them. To do that effectively, you should use one of the four tools in the Data Sufficiency Toolkit:

1. Manipulate algebraically.
2. Use conceptual understanding.
3. Play devil's advocate and pick numbers.
4. Just do it.

These four strategies from the toolkit are summarized in the following list:

Manipulate Algebraically

- Any time you are given algebra that can be manipulated, look to do that strategically. Often you will be able to make the statement mirror the question or vice versa.
- Since algebra “tells the truth,” this is often the safest and most efficient way to show sufficiency. Don’t waste time with number picking or conceptual thinking if you can prove something algebraically.
- Don’t forget that it is often more important to manipulate the question than the statements.

Use Conceptual Understanding

- Conceptual thinking is the best approach for most Arithmetic problems. Because problems about ratios, the number line, percents, etc. are more about the concept than actual calculations, doing math and or algebra can often be avoided.
- If you are not sure conceptually, then you should prove sufficiency by doing some math or picking numbers.
- Testmakers are good at finding exceptions to concepts that you think you understand well, so be careful when solving Data Sufficiency problems on a purely conceptual basis.

Play Devil's Advocate and Pick Numbers

- Number picking is an important strategy, but it is one that should only be used when necessary, as it can be time-consuming and make you error-prone.
- If you have to number pick, make sure that you pick with the purpose of playing devil's advocate and finding the exception. Smart number picking is key!
- Certain questions types and scenarios lend themselves to number picking. Quotient/remainder problems and scenario-driven min/max word problems are great examples of problems in which you should number pick.

Just Do It

- You should treat many Data Sufficiency problems as problem-solving questions and just do the math until you can see the answer.
- Quadratics, inequalities, and many word problems are examples of question types in which you must solve to prove sufficiency.



SECTION 3: LEARNING TO PLAY THE GAME

At this point in the lesson, you have learned the fundamentals of data sufficiency and been given a set of tools to use in assessing statements and questions. But data sufficiency is much more than just a question format with rules. It is a highly sophisticated game in which you are going head-to-head with the testmaker. **Success on Data Sufficiency questions relies heavily on a “game theory” approach in which you leverage any hints given by the testmaker and you learn how to use the question format to your advantage.** To play the game well, it is important to note that there are really only two mistakes you can make on Data Sufficiency questions. To consider this, look at the following variations two drills:

1. **A recipe for mixed nuts includes only whole peanuts and cashews and calls for a strict peanut:cashew ratio of 7:3. How many peanuts are in a bag?**
 - (1) The packaging facility guarantees that each bag will contain no fewer than 95 and no more than 105 nuts.
2. **A recipe for mixed nuts includes only whole peanuts and cashews and calls for a ratio of 7 ounces of peanuts for every 3 ounces of cashews. How many ounces of peanuts are in a bag?**
 - (2) The packaging facility guarantees that each bag will contain no less than 30 ounces and no more than 33 ounces of nuts.

In the first example you have to leverage the fact you are dealing with integers and that there are 10 total parts in the ratio. Because you need a whole-number multiplier, and because the number of total nuts is between 95 and 105, then there must be a total of 100 nuts and thus 70 peanuts. **People get this wrong because they do not leverage all the mathematical assets given in the problem.**

In the second example, particularly after doing the first one, you may think that it is a similar situation. There are still 10 total parts, and the total amount is greater than or equal to 30 but less than 33. It has to be 30 total ounces, right? Not in this case, because you *can* have a fractional multiplier with weight. You do not need to have integer ounces and the multiplier is not known from that information. **People get this wrong because they overvalue information and do not play devil’s advocate.**

These “two wrong answers” are summarized on the following page:



The Two Wrong Answers

1. You think you don't have enough information, but you do.

Examples:

- You miss an inference that you should draw (for example, “ x represents the number of children on a field trip”; you can't have a negative or fractional number of children!).
- One statement implies the other (the classic “C Trap,” which we will discuss in a few pages).

Your strategy: Leverage your assets. Get as much value out of each statement (plus question stem) as you can.

2. You think you have enough information, but you don't.

Examples:

- You assume something that isn't explicitly given (that x is an integer, or that it's positive).
- You include statement (1) in your understanding of statement (2) (or vice versa).

Your strategy: Play devil's advocate. Make sure you're not missing something.

And that's it. That's the testmakers' playbook against you. If you can correctly assess how much information you have, and how much is needed to answer the question, you will be correct each time. That is where the game is truly played. The testmakers will be clever about disguising the amount of information you've truly been given. But you have built-in advantages, too. In the pages that follow, you will learn how to play the game by taking advantage of the Data Sufficiency format and the hints that are given in that format.

Sufficiency Drill: How to Leverage Assets and Play Devil's Advocate

In the preceding sections, you completed several drills with only one statement. **Did you notice that analyzing one statement is actually harder than analyzing two. Why? Because the other statement is often a hint or a piece of information that you can leverage to figure out what is tricky in the problem.** Think about what you learned in Sentence Correction with “Decision Points”: The answer choices are your ally because you can leverage differences between the sentences (which are the hints in Sentence Correction). Remember how hard it was to simply analyze one sentence and decide whether it was correct? The same holds true in Data Sufficiency; it is much harder to decide in a vacuum if a statement is sufficient than when a second statement is present.

To highlight this fact, please assess whether the individual statement is sufficient to answer the question that precedes it. After you have completed these, you will see clearly the advantage of having a second statement:

1. How many integers x exist such that $a < x < b$?

(1) $b - a = 7$

2. A nursery has 30 trees, carrying only pines, oaks, and maples. How many oaks does the nursery have?

(1) The ratio of pines to maples is 14:9.

3. Is $\frac{a}{b} > c$?

(1) $a > bc$

**Two Statements Are Better Than One**

Now let's add a statement (2) to each of the previous drill questions. Care to change any of your answers? To understand construct thinking you must first realize and accept that the statements are actually assets, not liabilities.

1. How many integers x exist such that $a < x < b$?

- (1) $b - a = 7$
- (2) a and b are integers

2. A nursery has 30 trees, carrying only pines, oaks, and maples. How many oaks does the nursery have?

- (1) The ratio of pines to maples is 14:9.
- (2) The ratio of pines to oaks is 2:1.

3. Is $\frac{a}{b} > c$?

- (1) $a > bc$
- (2) $b > 0$

LEARNING BY DOING

Leverage the Statements

In the preceding section, you completed several drills with only one statement. Did you notice that analyzing one statement is actually harder than analyzing two? Why is that? Because the other statement is often a hint or a piece of information that you can leverage to figure out what is tricky in the problem. In many ways this is similar to Sentence Correction, in which you can use the differences between answer choices to alert you to grammatical problems that you might not have seen reading just one sentence (this is the basis for the Veritas Prep “Decision Points” strategy). Remember – Data Sufficiency statements are not created in a vacuum; the author of the question carefully crafts each statement as part of the entire problem, often using a statement to tempt you into a bad decision or to reward you for recognizing a clue. Therefore, if you can learn to recognize clues and traps embedded within “the other” statement, you have a competitive advantage on Data Sufficiency problems.

Consider the first drill example. It’s not at all uncommon for someone to interpret the question and then test a few numbers: If $b = 8$ and $a = 1$, there are six integers in between (2, 3, 4, 5, 6, 7). Maybe try one as a negative to see if that reacts differently: $b = 4$, $a = -3$, and you still have six integers in between (-2, -1, 0, 1, 2, 3).

But if you haven’t played your game of devil’s advocate well enough, statement (2) suggests that you need to consider “what if I didn’t know this information?” Well, if you don’t know that a and b are integers, then you have to account for nonintegers, and if $b = 8.5$ and $a = 1.5$, then there are seven integers (a different answer) in between: (2, 3, 4, 5, 6, 7, and 8). Here, statement (2) should clue you in to a consideration that you might not have made on your own. Answer choice C is correct.

In the second example, you learn from statement (1) that the ratio of pines to maples is 14:9. Since it does not tell you anything about oaks, it can’t be sufficient, right? Remember that you were given a total in the question stem of 30 trees. If the ratio is 14:9 for pines to maples then the multiplier must be 1, as a higher multiplier would make the total over 30 trees. Since you have proved the multiplier is 1, then there must be 7 oaks, and statement (1) is sufficient without needing what is given in statement (2). The correct answer choice is A.

For the third drill, it seems that statement (1) is a simple mirroring of the question and thus sufficient. But remember one of the most important rules for inequalities: You cannot divide or multiply by a variable unless you know the sign of that variable. Thus you need to know that b is either positive or negative so that you can manipulate the statement to match the question. Statement (2) gives you that information, so answer choice C is correct.



SKILLS MEET STRATEGY

Think of Statements as Hints

Perhaps the biggest takeaway in this section is the idea of thinking of statements as hints. Statements are assets, not liabilities, so you need to learn how to leverage information that is given or play devil’s advocate before you pick your final answer. **One of the most important skills on all GMAT questions is your ability to find hints in the question and/or answer choices, and leverage those hints.**

**Statement as Hints**

Consider one more example of how you can use statements to your advantage:

7. What is the value of $m + n$?

(1) $jm + kn + nj + km = 36$

(2) $j + k = 12$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING*Take the Hint!*

If you were given statement (1) alone, it is clearly insufficient, and you would have no idea what to do with it. It is a whole bunch of variables with no clear organization. Statement (2) is also clearly insufficient as the question asks about $m + n$, and the statement tells you about $j + k$. For our purposes, though, it serves as a fantastic clue: If you can get the j and k terms together, you can replace them with a number, leaving just m and n . So that's your catalyst to do some more algebraic manipulation:

$$(1) \quad jm + kn + nj + km = 36$$

Rearrange the order to get like terms together: $jm + km + kn + nj = 36$

Factor out the m and n : $m(j + k) + n(k + j) = 36$ and complete the factoring to show that:

$$(m + n)(j + k) = 36$$

Using the information from statement (2), replace the $(j + k)$ terms with 12: $12(m + n) = 36$

Divide by 12, and we have an answer: $(m + n) = 3$

If you don't use statement (2) as a hint, you could easily miss this and pick answer choice E, as the testmakers want you to do. While it seems like there is not enough information to solve this, there really is, but you will only discover that if you leverage statement (2). Once you do, you will see that answer choice C is correct.

SKILLBUILDER

- Algebraic manipulation
- Factoring

THINK LIKE THE TESTMAKER*Selling the Wrong Answer*

At first glance, it seems that you could never find values for m and n from the information given in the statements; there are just too many unknowns and not enough equations. But if you look closely, you realize that question is not asking for the value of an individual variable, but for the sum of two variables. This should be your first clue that you might be able to do it. As you learned earlier, the second clue is from statement (2), which gives a hint for how to factor statement (1). **In Data Sufficiency, there are many tricks that testmakers can use to make it seem like you don't have enough information, when you really do. Every time you see one of those tricks, you should file it away in your mind.** On this problem, the trick is that when the question stem asks not for individual variables, but for a sum or a difference, you often need less information than you think.



Playing Chess, Not Checkers

At this point you may ask: "But aren't we supposed to only look at one statement at a time?" The simple answer is yes, as a common trap on these questions is that the GMAT gets you to use information before you really have it. The testmakers want to embed something about statement (1) in your mind that lingers when you look at statement (2).

To truly become a master of Data Sufficiency, you need to be able to think in two parts:

1. I'm looking at this statement and only using the information that is specifically given in this statement and in the question stem.
2. *But, I can't ignore that the testmakers have already shown me the other statement, so I should at least use it as a clue.*

In many ways, Data Sufficiency is a chess match between you and the testmakers. So consider this situation: You're playing chess. Your opponent picks up her bishop, moves it five spaces across the board, and sets it on a particular square for a few seconds, letting her finger linger on the piece before deciding to move it back to where it began and select an entirely different move. True, after her move you have to play the game based on where the pieces actually do lie, but you'd be a poor player if you didn't consider "what was she trying to set up with that move?" and "where did she see that she was vulnerable when she ultimately decided to retreat?"

On Data Sufficiency questions, the "other statement" is that move-then-retreat. It tells you something about what the testmakers are setting up. So while you're falling for a beginner's trap if you use the information in statement (1) when you assess statement (2) ALONE, you're falling for almost as novice a trap if you wear blinders to each statement and fail to consider the question as a whole.



Learning To Play The Game Summary

The goal of this section has been to get you to focus on the following core strategies and thought processes.

- There are only two ways that you can make mistakes in Data Sufficiency:
 1. You think you don't have enough information, but you do.
 2. You think you do have enough information, but you don't.
 - In the first case, the mistake is that you have not leveraged all the assets given in the statement and/or the question stem. **Always maximize all the resources given.**
 - In the second case, the mistake is that you overvalued the information that was given and you did not play "Devil's Advocate" to find the exception or the misleading information. **Always be skeptical of information, particularly when it seems too good to be true.**

In a nutshell, success in Data Sufficiency relies on a mix of these two core skills: leveraging your assets and playing devil's advocate. Once you understand how to use those two skills, you must view the statements as hints. Remember: Statements give you important clues as to how to properly leverage information and how to play devil's advocate.

- Be clear about the rules for Data Sufficiency. **You cannot actually use the information from the other statement when assessing sufficiency in the first, but you can use that information as a guide for how to behave.**
- Playing the game properly and using all assets in the problem is the key to success in Data Sufficiency.



SECTION 4: THE POWER OF CONSTRUCT THINKING

In the previous section you learned how to play the game by leveraging all your assets, playing devil's advocate, and using the statements as hints. In the pages that follow, you will learn to recognize specific Data Sufficiency constructs that appear over and over. Once you unlock what these constructs mean, you will have an advantage in Data Sufficiency that cannot be overstated. The most important of the Data Sufficiency constructs hinges on asking this question:

Why Are You Here?

This construct is present any time you are given a statement that is obviously insufficient. When this scenario presents itself, it is essential that you take notice of this statement and ask: "Why are you here?" In doing that, you should consider two possibilities:

1. **The information is giving you something that you need in order to make the other statement sufficient. (In this case, answer choice C is correct.)** In other words, the information is important and it is a hint to an assumption you may have made in the other statement.

OR

2. **The information is something that you think you need, but you really don't or you already know from the other statement. (In this case, answer choice A or B will be correct.)** In other words, you are being tempted with information that you don't need.

Simply put, if a Data Sufficiency statement is clearly not sufficient, you need to ask: "Why are you here?" As an example of this construct, consider this textbook example from the previous drill problems:

How many integers x exist such that $a < x < b$?

(1) $b - a = 7$

(2) **a and b are integers**

Clearly, statement (2) could never be sufficient by itself, so you should ask why that statement is there. Remember how easy it was in this drill to think that statement (1) was sufficient when that's all you had? However, statement (2) is screaming loudly at you the assumption you probably made in the first. Remember: Statements are also hints, and to do well in Data Sufficiency you should always be trying to leverage those hints. **Just remember when asking “Why are you here?” that you may need the information or you may not. What matters is that you are actively and consciously making that decision.** The following are two different examples of how to leverage statements in this construct.

8. Is x between 0 and 1?

- (1) $x^2 < x$
- (2) x is positive.

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



9. Set J consists of terms {2, 7, 12, 17, a}. Is $a > 7$?
- (1) a is the median of set J.
 - (2) Set J does not have a mode.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING

Clearly Insufficient Statements Are There for a Reason

In each of these problems, you are given pieces of information that could never be sufficient on their own. In example problem #8 it is statement (2): If x is positive, that does not tell you if it is between 0 and 1, so it is clearly insufficient. (In example problem #9, it is also statement (2): If set J does not have a mode, that does not tell you anything about a .) For each one you need to figure out why that information is there, and whether you need the information with other statement or not. In the first problem, the “Why Are You Here?” statement is clever; it’s a trap! Statement (1) already implies that x is positive. Any number squared (x^2) will be greater than (or equal to) 0, so if x is negative then x^2 will always be positive. Negative numbers simply do not satisfy statement (1), so we don’t need statement (2); we already knew that. Here, statement (2) is a trap; the authors of the GMAT know that you tend to feel more comfortable when you have more information. So remember: If one statement ALONE is sufficient, you don’t get to use both. The correct answer choice is A.

In #9, you learn in statement (1) that a is the median of the set, so then the set must look like this in order: {2, 7, a , 12, 17}. It would be very easy from this statement alone to think that it is sufficient. Doesn’t a have to be between 7 and 12 for it to be the middle term in this set? No. Remember that a could be any number between 7 and 12 **inclusive**. If a is 7, then it is still the median of the set, so you actually get a maybe answer to the question from statement (1) alone. (a could be greater than 7, but it does not have to be.) Remember: If you were about to make the mistake of picking answer choice A, you should ask: “Why are they telling me that set J has no mode?” That should help remind you that the median could be a repeating number and that you also need the information in statement (2). Answer choice C is correct.



THINK LIKE THE TESTMAKER

Selling the Wrong Answer

If you missed one of these two questions, it was probably the first one. Why? Because the “**Why Are You Here?**” temptation construct (in which you are tempted with something you don’t need) is much harder psychologically than the traditional “**Why Are You Here?**” construct (in which you need the other information given). In other words, it is much harder to say no to something that fits so nicely with the other statement (but that you don’t need) than it is to say yes to something that you *do* need with the other statement. As a result, many of the harder “**Why Are You Here?**” questions are tempting you with something you don’t need. Remember: “**Why Are You Here?**” questions are mainly about isolating the statement that could never be sufficient on its own and figuring out why the testmakers have put it there. Sometimes you will need that information and sometimes you won’t.



SKILLBUILDER

- Statistics
- Arithmetic number properties



Additional Constructs

On the following pages you will learn several other important Data Sufficiency constructs. As you just saw, the “Why Are You Here?” construct relates primarily to two important strategies that we have emphasized throughout the lesson:

- Leverage assets.
- Play devil’s advocate.

The remaining constructs still hinge on these two essential components, but they also relate much more to another important “mantra” that is essential on the GMAT (and in business!):

“Don’t Be the Sucker.”

In business, if something seems too good to be true, it almost always is! In Data Sufficiency, if an answer choice seems obvious, it is almost certainly the incorrect “sucker” choice. Remember: The GMAT is a very challenging test that is trying to differentiate between highly intelligent and well-educated test-takers. Easy answer choices will only be correct well below the 50th percentile, so don’t pick them!

In many respects, this is just another piece of information to leverage in the game of Data Sufficiency. To highlight this, consider the following construct problem:

Last year, did company X produce more trucks than company Y?

- (1) Last year, company X produced 205,000 trucks and company Y produced 150,000 trucks.
- (2) ??????

What must be true about statement (2) in this question?

Statement (2), regardless of what it is, must be *difficult*, or this problem is well below the 20th percentile! Everyone taking the GMAT can recognize that Statement (1) is sufficient. So, if you look at statement (2) and it appears to be sufficient, it probably isn't. If you look at statement (2) and it does not appear to be sufficient, it probably is. Using this thought process can't give you the answer outright, but it will help you realize that you need to either play devil's advocate on statement (2) or leverage some assets given in the question or the statement. If you simply look at statement (2) and accept it at face value, then you are "the sucker" because it is almost surely counterintuitive!

On the pages that follow, questions will come first followed by a summary of the construct. As you try each question, try to isolate the sucker answer and then leverage that to find the correct answer. Remember: Construct thinking cannot give you the answer outright, but it can help you attack the problem efficiently and avoid cleverly created sucker choices.



10. Julie opened a lemonade stand and sold lemonade in two different sizes, a 52-cent (12 oz.) size and a 58-cent (16 oz.) size. How many 52-cent (12 oz.) lemonade drinks did Julie sell?
- (1) Julie sold a total of 9 lemonades.
- (2) The total value of the lemonade drinks Julie sold was \$4.92.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

LEARNING BY DOING*Beware of the C Trap*

Another common Data Sufficiency construct is one that baits you with an “easy C.” **The testmakers know this about you: You like to make decisions using all available information, so if both statements TOGETHER are clearly sufficient but neither alone jumps out as obviously sufficient, you’re quite likely to take the bait.**

Remember, however, how answer choice C is written: “BOTH statements TOGETHER but neither statement ALONE.” If one statement will do on its own, you’re not allowed to use both. Perhaps Tupac Shakur said it best with his song title “U Can’t C Me” or his lyric from “I Get Around”: “I don’t want it if it’s that easy.” If a statement is too easy, don’t take it without further investigation. It’s likely that either:

- **One statement alone is sufficient if you rearrange the algebra or find a “hidden gem” of information embedded beneath the surface,**

OR

- **Both statements actually say the same thing in different ways (or one cleverly says nothing at all), and you actually don’t have two pieces of information.**

This problem follows the first construct. Here, again, you can use one statement to tell you something about the other. Which statement gives you more information? Currently you have:

$$(1) \quad x + y = 9$$

$$(2) \quad 52x + 58y = 492$$

Statement (2) seems to contain a bit more information, as there are likely to be fewer combinations of $52x$ and $58y$ that will end in exactly 492. But you don’t have to test all the combinations! Look at what statement (1) is telling you: that there are exactly nine lemonades. With that piece of information and statement (2), you can easily solve the problem (two unique linear equations, two variables). Do you really need them to tell you that $x + y = 9$? Could it be anything else?

Say you wanted there to be only eight lemonades sold. By reducing the number sold, to stay at the \$4.92 revenue number, she would have to sell higher-priced items to compensate for the fewer units sold. Even eight of the most expensive lemonades ($8 \cdot 58 = 464$) does not come close to the \$4.92 figure, and any other combination of eight small and large lemonades will reduce the total revenue. So she cannot have sold eight or fewer. Could she have sold 10 or more?



Selling 10, she'd have to sell the cheaper items to have a chance at keeping the revenue at \$4.92. Even 10 of the cheapest ($10 \cdot 52 = 520$) are too many; she cannot sell 10 or more. So without statement (1) even having to tell us that $x + y = 9$, you can come up with statement (1) on our own using just the information in statement (2). Statement (2) alone is sufficient, so answer choice B is correct.

Most importantly, recognize the strategy here: The testmakers already gave you the clue that $x + y = 9$, so you don't have to create that information out of thin air. Statement (1) gave you a target; with just statement (2) by itself, with no other context, you're not that likely to prove that it's sufficient in two minutes or less. But with statement (1) there as a clue, you have a head start on what you should try to prove. That's the Data Sufficiency "game."



THINK LIKE THE TESTMAKER

Don't Be the Sucker

The C trap is a great example of the mantra presented earlier: "Don't be the sucker." If an answer choice is handed to you on a gold platter, don't take it. The beauty of the sucker choice is that it gives you a second chance. In many ways, it is logically identical to the hint in the "Why Are You Here?" construct. **If you are about to pick answer choice C and it seems easy, you should ask yourself whether you can do it from one statement alone (you have not leveraged enough information) or whether you can't do it at all (you have overvalued information and not played devil's advocate).** While the easy answer might be the correct answer, it rarely is and using the mantra of "don't be the sucker" will help you to get many difficult problems correct.



SKILLBUILDER

- Algebraic manipulation
- Mixture problems

11. What is the value of x ?

(1) $6x + 4y = 30$

(2) $x = -\frac{2}{3}y + 5$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



12. The ratio of television sets to radios at an electronics store before a new shipment arrives is 12:7. If no television sets or radios leave the store, and the only television sets and radios that arrive are in the new shipment, what is the ratio of television sets to radios after the new shipment arrives?
- (1) The new shipment contains 132 television sets.
- (2) The new shipment contains 77 radios.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING

No Easy Es/Prove E

In the previous two questions, you have seen examples of problems that come down to answer choice C vs. answer choice E. At first, you may have found problem #12 to be similar to the “lemonade problem” from the earlier example, in that the two statements each gave you a linear equation, leaving you with two equations/two variables. Here, however, statements (1) and (2) express the exact same relationship. Neither is sufficient alone, and neither adds new value to the other, so answer choice E is correct—with a “trap” of answer choice C.

This further cements the point made earlier: When answer choices seem obvious, you need to investigate more closely.

Example problem #12 demonstrates the inverse. In colloquial terms, make sure that you recognize that **answer choice E does not mean “I can’t do it”; it means, more specifically, “It cannot be done.”** Much like you should be leery of an “Easy C,” be just as wary of an “Easy E.” If you accept that Data Sufficiency is, above all else, a test of your ability to manage resources, you should recognize that the reward system is twofold. You should be rewarded for:

- **Cleverly extracting as much value as possible out of each piece of information, and**
- **Effectively playing devil’s advocate when the information looks to point in one direction but has some particular limitation.**

Because of that, when a decision comes down to answer choice C vs. answer choice E, **you must prove answer choice E in order to select it.** Lackadaisically selecting answer choice E is fraught with error. The testmakers have every incentive to reward you for taking information that seemingly isn’t sufficient and for adeptly leveraging it to solve a problem. When answer choice E is correct, it is usually correct because, by selecting it, you have been able to demonstrate a keen ability to say: “Not so fast—this conclusion still has one potential flaw.”

In example #11, that one potential flaw is that the two statements actually say the same exact thing, even though at first glance they are totally different.

In example #12, all signs point to “not sufficient.” Clearly neither is sufficient on its own, and even together you are given an initial ratio and then two hard numbers, and then you are asked to comment on the new ratio. This is almost always impossible. With the initial ratio you don’t know whether the store has 12 TVs or 1,200 TVs, so gauging the impact of adding 132 of them on the ratio is impossible. But that’s where your ability to play the GMAT game comes into play. Answer choice E is the obvious choice here, but



further investigation shows that:

The initial ratio is 12:7.

You add 132, or 12(11) TVs, and 77, or 7(11) radios.

If the initial number of units is $12x$ TVs + $7x$ radios, and you add 12(11) TVs and 7(11) radios, then you're adding:

$12x + 12(11)$ TVs + $7x + 7(11)$ radios

You can factor the 12s and 7s to get:

$12(x+11)$ TVs and $7(x+11)$ radios

The ratio is still 12:7.

Is that a clear decision on the surface? Probably not. For most people, 132 isn't an obvious multiple of 12 (or of 11). But knowing that answer choice E is a little too easy here, and knowing that when answer choice E is correct there's usually a moment of "a-ha, that's why it can't be solved" that is missing here, you know to play with the information for just a few more seconds before reluctantly accepting answer choice E. That extra push toward using the information more efficiently should direct you to the correct answer: answer choice C.

SKILLS MEET STRATEGY

Think Counter-Intuitively

As you have seen in this lesson, Data Sufficiency is about resource management. The problems are difficult because you often think you have enough information, but you don't, and vice versa. Since that is the game, it is incredibly helpful to think counter-intuitively and, in a phrase, **be skeptical**. This is particularly true in questions that come down to answer choice C or answer choice E. If everything in the question seems to point to answer choice E, there is a good chance that it is answer choice C, and you should try to come up with a way to prove sufficiency and show you can do it with both. If it feels like answer choice C, then it's probably answer choice E, and you should try to come up with the exception or the information that would prove answer choice E. While of course you can get too paranoid using this thought process, it is essential to success on harder Data Sufficiency problems.



SKILLBUILDER

- Algebraic manipulation
- Ratios

13. For isosceles triangle LMN, what is the measure of angle N?

- (1) The lengths of all three sides of triangle LMN are the same.
 - (2) Angle L measures 60 degrees.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



14. A street vendor sells only apples and pineapples, and all apples weigh 6.5 ounces, while all pineapples weigh 13 ounces. If she sells twice as many apples as pineapples, how many apples does she sell?
- (1) She sells 8 more apples than pineapples.
- (2) She sells an equal amount, by weight, of apples and pineapples.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING

Dealing with Obviously Sufficient Statements

With the “Why Are You Here?” construct, you learned how to deal with statements that are obviously insufficient; they are hints for the other statement, and you must decide if the statement is important or not. What about problems in which there is a statement that is clearly sufficient—the statement is easy and everyone knows it? As you learned in the introduction to these constructs, it must mean that the other statement is tricky or the problem would be too easy. You can use this fact to your advantage. The other statement must be counter-intuitive, so realize that and attack the problem accordingly.

In example problem #13, statement (1) is clearly sufficient. If all sides are the same, then the triangle is equilateral and all angles must be 60 degrees. The second is a bit counter-intuitive to many, though. Because it only gives the measure of one angle, you might think that it’s not sufficient, as you don’t know whether angle L “matches” angle N, making N 60 degrees also, or whether L is the non-matching side, making N = M when we don’t know about either M or N. But wait! Because the given angle is 60, it doesn’t matter. Either:

$$L = N = 60$$

L = M = 60, so L + M = 120 and N is responsible for the remaining part of 180 (which is 60).

OR

L = 60 and M = N, and M + N = 120 (the portion of 180 not represented by L). In that case, N = 60, as well. So statement (2), although it seems a bit light on information, actually does guarantee that N = 60. Your weapon here is to recognize the construct: Statement (1) required very little work, if any, so statement (2) is where the difficulty simply has to lie. Knowing that should urge you to work a little harder to find a bit of ingenuity or difficulty in statement (2). With that in mind you should derive the above and correctly come to answer choice D.

In problem #14, the word problem format begs you to set up equations. The given information states that, of the fruit that she sold, $A = 2P$. Statement (1) should again be pretty straightforward: If she sells 8 more apples than pineapples, then $A = 8 + P$. Using both equations together, we can plug in $2P$ for A to get $2P = 8 + P$, and solve for P .

Statement (2) also seems to offer the same type of information—a second equation to pair with the given information that $A = 2P$. But wait! Your senses should be heightened for the counter-intuitive statement (2) now that statement (1) has proven to be a bit



too easy. Statement (2), as an equation, is that $6.5A = 13P$. Divide by 6.5, and you'll find that $A = 2P$ —the same equation that we already have! Statement (2) offers no new information and is therefore not sufficient (making answer choice A correct here). Having been on guard for a sneaky other statement after a straightforward first statement, you should be looking for that clever restatement of already-known information and avoid this trap.

NOTE: For ease of teaching, the recent examples used the construct “Statement (1) is obvious; (2) is counterintuitive.” There’s nothing magic about statement (1), and the GMAT could well feature the statements of any of these questions in opposite order, baiting you into handling the easier statement (2) first and then being caught unaware on statement (1). You will see examples of this in the homework to follow; just know that when one statement is a little too easy, there’s a high likelihood that the other has some sneaky difficulty built into it that you should anticipate.

THINK LIKE THE TESTMAKER

Misdirection

In example problem #14, you saw a common trick used by testmakers: provide something that seems important but that you already know. This trick is present in many different GMAT question types (remember those Critical Reasoning Strengthen questions in which an answer choice is just a clever rewording of a premise that already exists in the argument?) and is particularly important in Data Sufficiency. If one of the statements simply repeats information already known from the question stem, it is not sufficient. Also, if a statement gives you something that you already know from the other statement, the answer will never be answer choice C. **Keep your eyes open for statements that are cleverly repeating information that is already given.**



SKILLS MEET STRATEGY

Leverage Assets in the Question Stem

As mentioned previously in this lesson, it is essential that you leverage all the information in the question stem. One of the favorite tricks used by testmakers is to hide important information there, because people tend to forget about it. If you missed example problem #13, it is almost surely because you did not leverage the fact that the triangle is given as isosceles in the question stem. In #14, you need to realize that one of the statements is simply repeating information that is already given, so it cannot be sufficient. In either case, success relies on reading the question carefully and leveraging every piece of information given there.



SKILLBUILDER

- Triangles
- Mixture problems
- Algebraic manipulation

Understanding the Reward System

At this point you have learned to recognize the important constructs, which can all be derived from a look at this chart:

D	Each statement alone is sufficient.
A B	Only one statement alone is sufficient.
C	Neither statement alone is sufficient, but both together are.
E	No combination of statements is sufficient.

If you look a level above and below the “obvious” answer choice, you will probably see the construct that the GMAT author is using. For example:

“Why Are You Here?”	It looks like A or B, but it’s actually C.
“Why Are You Here?—Temptation”	It looks like C, but it’s really A or B.
“The C Trap”	It looks like C, but it’s really A or B.
“...the Other Is Counterintuitive”	It looks like A or B, but is really D.
	It looks like D, but is really A or B.
Prove E	It looks like E, but it’s really C.
No Easy Cs	It looks like C, but it’s really E.

Regardless of the construct, the most important takeaways are these:

Before you pick D, you better make sure it’s not A or B.

Before you pick A, you better make sure it’s not D or C.

Before you pick B, you better make sure it’s not D or C.

Before you pick C, you better make sure it’s not A/B or E.

Before you pick E, you better make sure it’s not C.

These considerations represent the essential building blocks of construct thinking: If you ask yourself those questions every time and leverage the hints given in the construct, your improvement in Data Sufficiency will be dramatic.



What Makes Data Sufficiency So Hard?

15. a, b, c, and d are consecutive integers such that the product $abcd = 5,040$. What is the value of d?
- (1) d is prime
(2) $a > b > c > d$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
(D) EACH statement ALONE is sufficient to answer the question asked
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING*Assumptions and the Power of Suggestion*

This problem is hard. Interestingly, it comes down mainly to a simple assumption that you have learned to avoid throughout this lesson: Don't assume that numbers are positive. So how did you get baited into that on this question? The power of suggestion! Data Sufficiency authors are masters of human psychology; they know that your mind wants to organize information by archiving things you already know, so they know that they can often embed a piece of information deep enough in your psyche that you'll carry it with you even if it's not something you're allowed to use.

In the question stem, it takes some work to unravel the factors of 5,040, but once you do statement (1) works beautifully. 5,040 should jump out at you with two pretty-clear factors: It's definitely divisible by 10 (it ends in 0), and it's definitely divisible by 9 (the sum of the digits is 9). If you take out 9 and 10, you have $9 \cdot 10 \cdot 56$, leaving you with the consecutive factors 7, 8, 9, and 10. Since 7 is the only prime in that set, statement (1) is sufficient.

Now, note your assumptions when you attack statement (2). You've already proven 7, 8, 9, and 10, and probably realized with a bit of trial and error or conceptual understanding that you can't come up with any other (positive) sets of consecutive factors. If you "repackage" 10 as a 2 and a 5, you cannot reallocate those without making one of the other factors much larger and another much smaller (say, 7, 8, 18, and 5). So knowing that 7 is the smallest, you might quickly determine that statement (2) is sufficient as well. But wait! There is one more set of consecutive integers a, b, c, and d that multiply to 5,040: -7, -8, -9, and -10. The problem does not rule out negatives! By asking about only primes in statement 1, the question embedded some lingering assumptions in your mind. It likely got you thinking only in terms of positive numbers and tricked you into taking your eye off the ball. The correct answer is A, but the authors know that many savvy test-takers will fall for trap answer D.

Beware the power of suggestion—and the best way to do this to play devil's advocate. Only use the information that you *know* you have. For everything else you should push the boundaries. Ask "Am I allowed to...?" while trying different concepts or numbers. If you don't find an explicit "no," then err on the side of "yes."

**SKILLBUILDER**

- Prime numbers
- Factors

**THINK LIKE THE TESTMAKER***Misdirection*

There is one other thing that makes this problem hard: Incorrect answer choice D does not feel like a sucker choice. People who are good at game theory and higher-order thinking will only be satisfied if they feel they have done enough work to expose a sucker choice and see that the problem is hard. Because you have to do some clever factoring and a fair bit of work to pick answer choice D, you are satisfied that many other people could not do that and that you have gotten a difficult problem correct. Alas, there is still one more trick lurking, and if you are not truly the "devil's advocate," it is unlikely you will notice that mistake.



The Power of Construct Thinking Summary

Construct thinking itself will not get you to the top of the pyramid. But without it, you are not leveraging one of the most important assets in Data Sufficiency. In analyzing Data Sufficiency constructs you should always consider the following:

- Construct thinking is based mainly in game theory and in understanding how testmakers can make problems hard. **At its core, construct thinking helps you figure you out how and when to leverage information and play devil's advocate. It also teaches you how to recognize and avoid sucker choices.**
- Perhaps the most important construct is what we call “Why Are You Here?” This applies any time there is an **obviously insufficient** statement. Whenever this construct presents itself there are two possibilities:
 1. **The information is giving you something that you need in order to make the other statement sufficient,**

OR

2. **The information is something that you think you need, but you really don't.**
- The “C trap” is another common construct and an important example of “Don’t Be the Sucker.” **Whenever an answer choice is obvious, you should look to either leverage more information or play devil’s advocate.** If the two statements together are obviously sufficient, you should be suspicious of answer choice C and look to answer choice A, B, or E as the likely correct answer.
 - Choices between C and E are particularly hard on the GMAT. Why? Because there are no more hints to leverage from the other statement. Your best hope in choosing between answer choices C and E on hard problems is construct thinking. **Be wary of ever picking answer choice E when it just seems like the information is not sufficient, but you have not proven that is not sufficient. And if it seems to be answer choice C, you should be playing devil's advocate by trying to prove answer choice E.**

- **Any time one of the two statements is easily sufficient, you are being given a very helpful hint: The other statement must be difficult.** It is almost surely counter-intuitive, so you should automatically be skeptical of how the statement appears at face value. If it appears sufficient, then it probably is not and you have not played “devil’s advocate” well enough. If it does not appear sufficient, then it probably is and you have not leveraged your assets well enough.
- **Regardless of whether you have found one of the common constructs, make sure you understand the reward system.** Every answer in Data Sufficiency has a trap on one side (for answer choices D and E) or on both sides (for answer choices A, B, and C). Always check to see that you are not falling for one of those traps before you finalize your answer.



SECTION 5: YOU OUGHTA KNOW

In the following pages, you will see a few more Data Sufficiency structures and rules that, while not core to effective GMAT strategy, will likely prove helpful in your mastery of the discipline and could show up in one of your test questions.

Don't Contradict Yourself

In a Data Sufficiency question, the two statements are factual premises that must be true. Because of this fact, they cannot directly contradict each other, or the entire construct of Data Sufficiency would be illogical. In other words, statement (1) cannot say that $x > 5$ while statement (2) says that $x < 3$. These statements cannot co-exist or the question would be flawed. How can you use this to your advantage?

Very carefully. Remember that you cannot use information from one statement while examining the other statement individually. But as we've discussed previously, you can use clues from one statement to save yourself from bad decisions on the other. If you are completely sure that statement (1) is giving you a yes answer to the question at hand, it would be impossible for statement (2) to give a definitive answer of no. Statement (2) could be ambiguous (an answer of maybe) or give a definitive answer of yes, but it cannot directly contradict the first. Consider a problem:

16. For non-negative integers x , y , and z , is x^z odd?
- (1) The product x^z is odd.
- (2) $x = 2^y$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING*Statements Must Be True*

In this problem, you hopefully found statement (1) to be sufficient, without too much effort. If the product xz is odd, then both integers must be odd. And if x , the base of x^z , is odd, then when raised to any integer it will be odd. Statement (1) guarantees that x^z is odd.

But statement (2) seems to guarantee that x is even, as 2 to any exponent is even. Right? There's one glaring exception: $2^0 = 1$ —an important property for many exponent problems. So while most potential values of x , given statement (2), are even, one does exist where it's odd. Statement (2) is not sufficient, and the correct answer is A.

Here's where you can use some higher-level strategy. If you were certain (as you should be) that statement (1) is sufficient with the answer "odd," then as soon as you see a potential "even" with statement (2) you know that statement (2) is not sufficient! Why? Because the rules of the game dictate that if statement (1) gives you the answer "x must be odd," then statement (2) cannot say "x must be even." The only options are "x must be odd" or "x could be either odd or even." Once you've found that "even" answer, then you know the "maybe" part must be coming, based on the rules of the game.

**SKILLBUILDER**

- Arithmetic number properties
- Exponents



No News Is Good News

If the two statements in a Data Sufficiency problem provide the same information in a different form, or if algebraic transformations show that one of the statements equals the other, then by the rules of the game, the correct answer must be either answer choice D or E. In other words, if you discover that each statement supplies the same information, there are only two possible scenarios: Each statement alone is sufficient, **or** both statements taken together are insufficient. After all, if statement (1) is the same as statement (2), then there's no added value in using them together, and one cannot be sufficient without the other also being sufficient.

Consider this example to see how you can use this concept to your advantage:

17. All attendees at a university gathering are faculty or alumni of the university. Are any of the attendees both faculty and alumni?
- (1) $\frac{3}{5}$ of the attendees are members of the university faculty.
(2) 40% of the attendees are not members of the university faculty.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
(D) EACH statement ALONE is sufficient to answer the question asked
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

LEARNING BY DOING

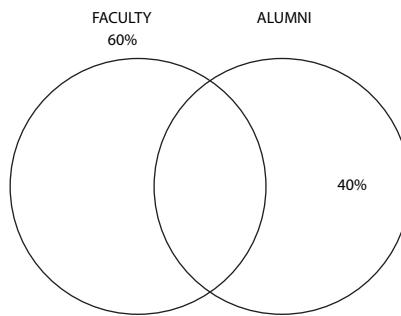
Precision in Wording with Overlapping Sets

In the Skills Meet Strategy box at the right, you'll see that many test-takers quickly pick answer choice C, but the construct of the problem should warn you that C may be a trap.

Two techniques from the Veritas Prep Word Problems lesson can help you conquer this problem and avoid the trap altogether.

Venn Diagram

If you're using a Venn Diagram for this problem, remember that the visual representation of the diagram includes the entire circle for "member of the group" and the crescent shape for "member of this group only". In this case, statement (1) tells you that 60% of the attendees are members of the faculty group, and statement (2) tells you that 40% of the attendees are not. Because there is no "neither" group, this means that those 40% must be in the "alumni only" group. And if you're keeping disciplined accounting with the Venn Diagram, you'll put the 60% Faculty number above the "Group A" circle, and the 40% Alumni Only number inside the "Group B" circle



Notice that this does not allow you to determine the amount of the overlap "both" group using either of the Venn formulas:

$$\text{Group A} + \text{Group B} - \text{Both} + \text{Neither} = \text{Total}$$

In this formula, you know that Group A = 60, Neither = 0, and Total = 100. But you don't have either Group B or Both alone: all you have is that (Group B - Both) = 40, but that doesn't allow you to isolate either variable.

$$\text{Group A Only} + \text{Group B Only} + \text{Both} + \text{Neither} = \text{Total}$$

Here, again, you're left with two variables. You have Group B Only = 40, Neither = 0 and Total = 100, but Group A Only and Both are both variables, and therefore you cannot solve for either. That is why the answer is E.



Matrix Box

Because this situation allows for exactly two categories, each with exactly two options (Alumni and Not Alumni; Faculty and Not Faculty), you could use a Matrix Box to plug in the values from the statements to see if you can solve for the “Both” box (Alumni and Faculty):

	Faculty	Not Faculty	Total
Alumni	??		
Not Alumni			
Total			100%

Using the information from both statements together, you have:

	Faculty	Not Faculty	Total
Alumni	??		
Not Alumni			
Total	60%	40%	100%

Notice that your goal is to find the number in the “Faculty and Alumni” box at the upper left, but given the information from either statement you cannot plug in for any of the alumni boxes, so you cannot solve for that box and therefore the answer must be E.



SKILLS MEET STRATEGY

Using Identical Statements as Construct Thinking

Upon initial inspection, many will look at this problem, see that $\frac{3}{5} = 60\%$, and then notice that 60% and 40% add up to 100%. Therefore, in their minds, 40% of the attendees are alumni, 60% are faculty, and there is no overlap. This looks like a pretty quick “C” answer, which means you need to investigate further to avoid what might well be a trap.

The fact that the two statements are identical should also be a clue. Think about it: if, as statement (1) states, 60% of the attendees are faculty, then of course 40% are not faculty. So statement (2) doesn’t actually add any new information. Note that it does not say “40% of the attendees are **alumni**” but rather just “40% are **not faculty**.” We already know that!

So your two clues that C is a trap are: 1) C came a little too easy, and that’s usually a warning sign. And 2) the statements give the exact same information, meaning the answer must be either D or E. And since neither statement alone is sufficient – neither tells you anything about the alumni as a percentage of the total – the answer must be E.

HOMEWORK

Learning by Doing: The strategies outlined in this lesson take practice, and like any “game” Data Sufficiency more than any other question type requires repetition so that you can get a feel for the Decision Tree, the Data Sufficiency Toolkit, the Reward System, and Construct Thinking. The next 100+ pages include practice problems arranged generally by level of difficulty. The first 25-30 problems are a great opportunity for you to become more comfortable with the Decision Tree and Toolkit. By the last 40-50 problems, you will frequently need to also leverage Construct Thinking and the Reward System. And should you want additional practice, visit the Veritas Prep Question Bank, a free online resource containing hundreds of practice problems arranged by question type.

18. Currently, there are 40 cars in each row of the lot at a car dealership. If the parking spaces are to be widened and lengthened so that only 30 cars fit in each row and fewer rows fit in the lot, how many cars will then fit in the entire lot?
- (1) There will be 3 fewer rows of cars.
(2) Currently there are 10 rows of cars.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
(D) EACH statement ALONE is sufficient to answer the question asked
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

19. Is William taller than Jane?

- (1) William is taller than Anna.
 - (2) Anna is not as tall as Jane.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



20. Is $\frac{11x}{23} < \frac{7x}{13}$?

- (1) x is an integer.
 - (2) $x > 0$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

21. How many years old will Fred be y years from now?
- (1) Doris is 12 years older than Fred.
- (2) The sum of the ages of Doris and Fred is y years.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



22. What is the value of $4x^2 - 3x + 1$?

- (1) $(x - 1)(x + 2) = 0$
 - (2) $x = 1$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
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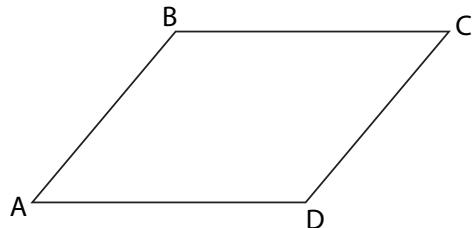
23. What was the final score in a game between Team X and Team Y?

- (1) Team X scored 14 more points during the second half of the game than it scored during the first half.
 - (2) At the end of the first half of the game, the score was tied, but team Y won the game by 4 points.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



24. If \ddagger is an operation, is the value of $b \ddagger c$ greater than 10?

- (1) $x \ddagger y = x^2 + y^2$ for all x and y
 - (2) $b = 2$ and $c = 1$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
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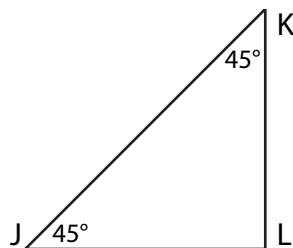
25. In parallelogram ABCD above, what is the measure of angle ADC?

- (1) The measure of angle ADC is greater than 90 degrees.
 - (2) The measure of angle BCD is 70 degrees.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



26. Is the area of circle X greater than the area of circle Y?

- (1) The circumference of circle X is greater than the circumference of circle Z, and the circumference of circle Z is less than the circumference of circle Y.
 - (2) The radius of circle X is greater than the radius of circle Y.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked;
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



27. In triangle JKL shown above, what is the length of segment JL?

(1) $JK = 2\sqrt{2}$

(2) $KL = 2$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



28. Maria wants to send a package overseas. She can either pay shipping company A an amount dependent on the distance of the delivery, or pay shipping company B an amount dependent on the weight of the package. Which option is less expensive?

- (1) Company A charges \$3.00 plus \$0.01 per mile of shipment, and company B charges \$5.00 plus \$8.50 per pound.
 - (2) The person who will receive the packages lives 3,900 miles away from Maria.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

29. A stenographer is tasked with typing a certain letter. How many words long is the letter?
- (1) It will take two minutes less time to type the letter at an average speed of 80 words per minute than at an average speed of 60 words per minute.
 - (2) It will take 6 minutes to type the first half of the letter at an average speed of 40 words per minute.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



30. Is the positive square root of a an integer?

- (1) $a = b^4$ and b is an integer
 - (2) $a = 81$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

31. A certain jewelry store sells gold necklaces in 18-inch and 28-inch lengths, and all necklaces of the same length sell for the same price per necklace regardless of the number of necklaces purchased. What is the price of a 28-inch necklace at this jewelry store?
- (1) The total price of an 18-inch and a 28-inch gold necklace is \$68.
- (2) The total price of two 18-inch necklaces and one 28-inch necklace is \$96.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



32. Is $x > y$?

(1) $x = y + 2$

(2) $\frac{x}{2} = y - 1$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

33. Is the average (arithmetic mean) of a and b less than 40?

- (1) The average (arithmetic mean) of $3a$ and $3b$ is 117.
 - (2) $b = 5a$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



34. Penny does consulting work part-time and is paid on an hourly basis. During week W, how much was the total amount of money Penny earned for her work with companies X and Y?
- (1) During week W, Penny was paid \$36 per hour for work performed for company X, and \$29 per hour for work performed for company Y.
- (2) During week W, Penny worked for a total of 60 hours.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

35. Is x a positive number?

(1) $-5x - 3 > -2x$

(2) x^2 is positive.

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



36. Julie runs a small art museum. How many paintings does her museum have?

- (1) Her next acquisition of 4 Monet paintings will increase the number of paintings in the museum's collection by 10%.
 - (2) The museum currently exhibits 3 impressionist paintings for every 2 non-impressionist paintings.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

37. If n is an integer, then n is divisible by how many positive integers?

- (1) n is the product of a prime number and a non-prime positive integer.
 - (2) n and 20 are each divisible by the same number of positive integers.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



38. What is the remainder when integer k is divided by integer j?

- (1) j and k each have the same number of unique factors.
 - (2) $1 < j < k < 10$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

39. If production on line A increased 5% from 2006 to 2007, and if production on line B increased 10% in the same period, how many units did line A produce in 2006?
- (1) The two lines combined produced 100,000 units in 2006.
- (2) The two lines combined produced 107,500 units total in 2007.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



40. In a retail store, the average (arithmetic mean) sale for month M was d dollars. Was the average (arithmetic mean) sale for month J at least 20 percent higher than that for month M?
- (1) For month M, total revenue from sales was \$3,500.
- (2) For month J, total revenue from sales was \$6,000.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

41. If x and y are positive integers, is the product xy divisible by 9?

- (1) The product xy is divisible by 6.
 - (2) x and y are perfect squares.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



42. A certain company paid bonuses of \$125 to each of its executive employees and \$75 to each of its non-executive employees. If 100 of the employees were non-executives, how many of the employees were executives?
- (1) The company has a total of 120 employees.
- (2) The total amount that the company paid in bonuses to its employees was \$10,000.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

43. If $x = y^2$, what is the value of $y - x$?

- (1) $x = 4$
- (2) $x + y = 2$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



44. Exactly how many show dogs does Sheila have?

- (1) Of Sheila's show dogs, exactly 3 have won prizes of at least \$500.
 - (2) Of Sheila's show dogs, exactly 40% have not won a prize of \$500 or more.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

45. Each M-type unit will increase the base memory capacity of a certain computer by 3 megabytes. What is the base memory capacity, in megabytes, of the computer?
- (1) 2 M-type memory units will increase the computer's base memory by 300 percent.
- (2) The memory capacity of the computer after 2 M-type memory units are added to the base memory capacity is 1.6 times the memory capacity after 1 M-type memory unit is added to the base memory capacity.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



46. Is $a^2 > 3a - b^4$?

- (1) $3a - b^4 = -5$
 - (2) $a > 5$ and $b > 0$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

47. If x is a positive integer, is x even?

- (1) $5x$ is even.
- (2) $x - 5$ is odd.

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



48. Is $x^2 > x^3$?

(1) $x^3 > 0$

(2) $x \neq 1$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

49. In a taste test conducted for coffee brands X and Y, if each of the 1,600 people polled voted for either X or Y (but not both), what percent of the males voted for brand Y?
- (1) Forty percent of the females polled voted for brand X.
- (2) Sixty-five percent of the males voted for brand X.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



50. Jeff's average (arithmetic mean) monthly utility bill for the months of June, July, and August was \$75. During which month did he receive the highest bill?
- (1) The lowest bill was received in July.
- (2) The total of his bills for July and August was \$160.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

51. What is the value of two-digit integer x ?

- (1) The product of the two digits is 14.
 - (2) x is divisible by 9.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



52. If $x + y = 6$, then what does $x - y$ equal?

- (1) $x^2 - y^2 = 12$
 - (2) $2y + x = 8$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

53. If the two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?
- (1) Each step is $\frac{3}{4}$ foot high.
- (2) Each step is 1 foot wide.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



54. If a , b , and c are integers, is $2a - b + c$ greater than $a - b - 2c$?

- (1) a is positive.
 - (2) c is positive.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

55. A beer company spent \$100,000 last year on hops, yeast, and malt. How much of the total expenditure was for hops?
- (1) The expenditure for yeast was 20% greater than the expenditure for malt.
- (2) The total expenditure for yeast and malt was equal to the expenditure for hops.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



56. If $xyz \neq 0$, what is the value of $\frac{x^5y^4z^2}{z^2y^4x^2}$?

- (1) $x = 1$
 - (2) $y = 1$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

57. A certain 5-liter solution of floor cleaner consists of x liters of concentrated cleaner and y liters of water. How many liters of concentrated cleaner does the solution contain?

(1) $\frac{x}{y} = \frac{1}{8}$

(2) $\frac{x}{5} = \frac{1}{9}$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



58. If a rope is cut into three pieces of unequal length, what is the length of the shortest of these pieces of rope?
- (1) The combined length of the longer two pieces of rope is 12 meters.
(2) The combined length of the shorter two pieces of rope is 11 meters.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
(D) EACH statement ALONE is sufficient to answer the question asked
(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

59. If a , b , and c are integers, is the number $3(a + b) + c$ divisible by 3?

- (1) $a + b$ is not divisible by 3.
 - (2) c is divisible by 3.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



60. What is the value of $x - y$?

(1) $x - y = y - x$

(2) $x - y = x^2 - y^2$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

61. Is n an integer less than 5?

(1) $5n$ is a positive integer.

(2) $\frac{n}{5}$ is a positive integer.

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



62. Every man in a certain class either belongs to group A, belongs to group B, or belongs to both groups. 20% of group A consists of men and 65% of group B consists of men. What percentage of the two groups together is made up of men?
- (1) Group A contains 50 people.
- (2) Group B contains 100 people.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

63. If a and b are consecutive negative integers, is b greater than a ?

- (1) $a + 1$ and $b - 1$ are consecutive negative integers.
 - (2) b is an odd number.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



64. Is x^2 equal to xy ?

- (1) $x^2 + y^2 = -(x^2 - y^2)$
 - (2) $x = y$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

65. If $2.5 < x < 4.5$, is the tenths digit of the decimal representation of x equal to 4?

(1) $x + 0.006 < 4.5$

(2) $x + 0.06 > 4.5$

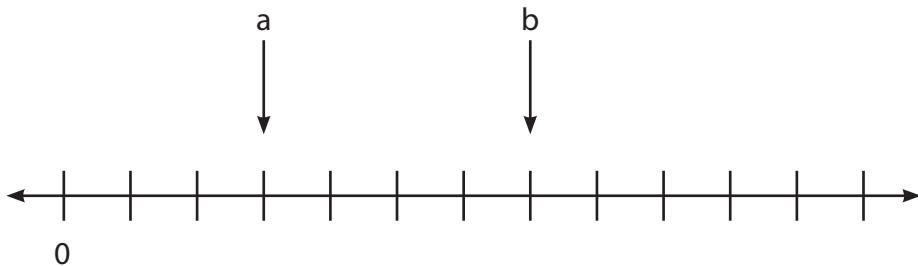
(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient

(D) EACH statement ALONE is sufficient to answer the question asked

(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



66. If the successive tick marks shown on the number line above are equally spaced and if a and b are the numbers designating the end points of intervals as shown, what is the value of b ?

(1) $a = \frac{1}{4}$
(2) $b - a = \frac{1}{3}$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

67. A certain ice cream factory makes two flavors of ice cream: chocolate and caramel. Each flavor comes in two varieties: one with over 20 percent milk and one with under 20 percent milk. Did more than $\frac{2}{5}$ of all the ice cream made in July contain over 20 percent milk?
- (1) Exactly 80 percent of the caramel ice cream made in July contained over 10 percent milk, and of this amount, $\frac{1}{4}$ contained over 20 percent milk.
- (2) Exactly 1,000 gallons of the chocolate ice cream made in July contained over 20 percent milk.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



68. If $x \neq 0$, is y greater than 0?

- (1) $xy = 24$
 - (2) $x - y = 5$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

69. If $\frac{a}{3} = \frac{4}{b}$, is a less than b?

(1) $b \geq 4$

(2) $b \leq 5$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



70. Does $x = 2$?

- (1) x is a number such that $x^2 - 3x + 2 = 0$.
 - (2) x is a number such that $x^2 - x - 2 = 0$.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

71. Is y less than 2.4?

(1) $y < 2.5$

(2) $y < 2.3$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



72. Ted owns an electronics business and is calculating the price of a certain group of microwaves in the inventory he purchased. If the total price of m equally priced microwaves was \$18,000, what was the price per microwave in the inventory?
- (1) If the price per microwave in Ted's inventory had been \$10 more, the total price of the m microwaves would have been \$5,000 more.
- (2) If the price per microwave in Ted's inventory had been \$9 less, the total price of the m microwaves would have been 25 percent less.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

73. A store sold 6 bicycles with an average sale price of \$1,000. What was the price of the most expensive bicycle?
- (1) The median price was \$1,000.
- (2) The range of prices was \$600.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



74. What is the value of $j - k$?

- (1) $2j + 4n = 2k + 4n - 6$
 - (2) $n = 3$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

75. If $20x + 30y = 280$, what is the value of x ?

(1) $4x = 56 - 6y$

(2) $y^2 = 16$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



76. If y is an integer, is y^2 divisible by 4?

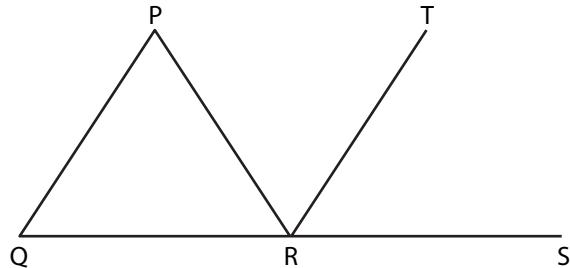
- (1) y is even.
 - (2) y^3 is divisible by 4.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

77. Is $xy < 8$?

(1) $x < 2$ and $y < 4$

(2) $0 < x < \frac{1}{2}$ and $y^2 < 225$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



78. In the figure above, QRS is a straight line and $QR = PR$. Is it true that lines TR and PQ are parallel?

- (1) Length $PQ =$ Length PR
 - (2) Line TR bisects angle PRS .
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

79. If each of the 20 bolts of fabric on a shelf is either 100 percent cotton, 100 percent wool, or a mixture of cotton and wool, how many bolts contain both cotton and wool?
- (1) Of the 20 bolts, 18 contain some wool and 14 contain some cotton.
- (2) Of the 20 bolts, 6 are 100 percent wool.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



80. If x , y , and z are lengths of three sides of a triangle, is $x < 3$?

- (1) $z = y + 3$
 - (2) $y = 3$ and $z = 6$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

81. Is the positive integer x an even number?

(1) If x is divided by 3, the remainder is 2.

(2) If x is divided by 5, the remainder is 2.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient

(D) EACH statement ALONE is sufficient to answer the question asked

(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



82. The surface area of a rectangular field was changed so that the length of one of the dimensions was reduced by 10 feet and the length of the other dimension was increased by 20 feet. What was the surface area before these changes were made?
- (1) After the changes were made, the surface area was 2,500 square feet.
- (2) The length and width of the field were equal after the changes were made.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

83. In a certain store, item X sells for 10 percent less than item Y. What is the ratio of the store's revenue from the sales of item X to that from the sales of item Y?
- (1) The store sells 20 percent more units of item Y than of item X.
 - (2) The store's revenue from the sales of item X is \$6,000 and from the sales of item Y is \$8,000.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



84. During a three-year period, the profits of company X changed by what percent from the second year to the third year?
- (1) The increase in profits of company X from the first year to the second year was the same as the increase from the first year to the third year.
- (2) For company X, the profits for the first year were \$13,800 and the profits for the third year were \$15,900.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

85. A pyramid-shaped box to protect a plant is constructed with 4 lateral faces and an open bottom. What is the lateral area of the box?
- (1) The base of the pyramid is a polygon with all sides of equal length, and the perimeter of the base is 1 meter.
- (2) The lateral faces are isosceles triangles that have the same size and shape.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



86. If x and y are integers and their sum is 23, is $y \geq 9$?

- (1) $x - 6 < 9$
 - (2) $x^3 = 2,744$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

87. A farmer has a total of 60 pigs, cows, and horses on his farm. How many pigs does he have?
- (1) The ratio of horses to cows is 2:9.
- (2) He has more than 36 cows.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



88. What is the value of $x + y$?

(1) $\frac{4x^2 - 4y^2}{2(x+y)} = 2x - 2y$

(2) $3x + 2y = 24$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

89. What is the value of the two-digit number x ?

- (1) The sum of the two digits is 4.
 - (2) The difference between the two digits is 2.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
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90. Is $\sqrt{7ab}$ an integer?

- (1) $a = 7$
 - (2) b is equal to an integer raised to the third power.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

91. Is $xy > 24$?

- (1) $y - 2 < x$
- (2) $2y > x + 8$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



92. Is ab a prime number?

- (1) a is a prime number.
 - (2) b is not a prime number.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
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93. A rectangular floor that is 4 meters wide is to be completely covered with non-overlapping square tiles, each with side of length 0.25 meter, with no portion of any tile remaining. What is the least number of such tiles that will be required?
- (1) The length of the floor is three times the width.
- (2) The area of the floor is 48 square meters.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



94. Is $x > 3$?

- (1) The sum of x and the square of x is 12.
- (2) $x^2 > 9$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
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95. If a , b , and c are distinct positive integers where $a < b < c$ and $\sqrt{abc} = c$, what is the value of a ?

(1) $c = 8$

(2) The average of a , b , and c is $\frac{14}{3}$.

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
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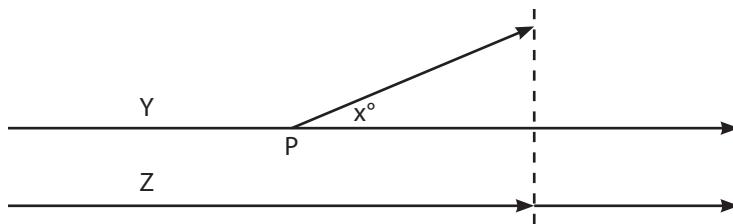


96. If x is an integer, what is the value of x ?

(1) $\frac{1}{5} < \frac{1}{1+x} < \frac{1}{2}$

(2) $(x - 3)(x - 4) = 0$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
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97. Cars Y and Z travel side-by-side at the same rate of speed along parallel roads as shown above. When car Y reaches point P, it forks to the left at angle x° , changes speed, and continues to stay even with car Z as shown by the dotted line. The speed of car Y beyond point P is what percent of the speed of car Z?

- (1) The speed of car Z is 50 miles per hour.
 - (2) $x = 45$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
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98. For integers a , b , and c , $\frac{a}{b-c} = 1$. What is the value of $\frac{b-c}{b}$?

(1) $\frac{a}{b} = \frac{3}{5}$

(2) a and b have no common factors greater than 1.

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

99. What is the value of $x - y$?

(1) $(x + y)^2 = 4xy$

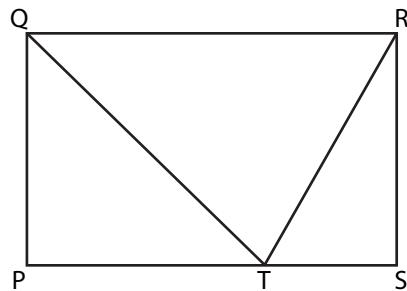
(2) $x^2 - y^2 = 0$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



100. For positive integer a , is the product $(a)(a + 1)(a + 2)$ divisible by 48?

- (1) a is even.
 - (2) $4a$ is divisible by 32.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
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101. In rectangular region $PQRS$ above, T is a point on side PS . If $PS = 4$, what is the area of region $PQRS$?

- (1) $\triangle QTR$ is equilateral.
 - (2) Segments PT and TS have equal length.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



102. If $xy \neq 0$, is $\frac{1}{x} + \frac{1}{y} = 16$?

- (1) $x + y = 16xy$
- (2) $x = y$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

103. How many students in the senior class take both French and Spanish?

- (1) In the senior class, 100 students take French and 125 students take Spanish.
 - (2) There are 200 students in the senior class.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



104. Is $x > 0$?

(1) $x^2 = 9x$

(2) $|x| = -x$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

105. What is the average of the terms in set J?

- (1) The sum of any three terms in set J is 21.
 - (2) Set J consists of 12 total terms.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
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CHALLENGE PROBLEMS

106. Line M is tangent to a circle, which is centered on point $(3, 4)$. Does Line M run through point $(6, 6)$?
- (1) Line M runs through point $(-8, 6)$.
- (2) Line M is tangent to the circle at point $(3, 6)$.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

107. When positive integer x is divided by 7 the quotient is q and the remainder is 1. What is the remainder when x is divided by 10?
- (1) When x is divided by 5 the quotient is q and the remainder is 1.
- (2) x is less than 50.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



108. Of the 60 families in a certain neighborhood, 38 have a cat. How many families in this neighborhood have a dog?
- (1) 28 of the families in this neighborhood have a cat but not a dog.
- (2) The number of families in the neighborhood who have a dog and a cat is the same as the number of families who have neither a cat nor a dog.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

109. If $x \neq 0$, is $\frac{5x - 2}{3} - \frac{5x - 1}{4} > 0$?

- (1) $x > 1$
- (2) $x = |x|$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



110. If x and y are positive integers, what is the greatest common factor of x and y ?

- (1) When x is divided by y , the remainder is 1.
 - (2) $x^2 - 2xy + y^2 = 1$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

111. Is $(y - 10)^2 > (x + 10)^2$?

(1) $-y > x + 5$

(2) $x > y$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked;
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked;
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient;
- (D) EACH statement ALONE is sufficient to answer the question asked;
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



112. What is the value of y ?

- (1) $x^2 - y^2 = 5$
 - (2) x and y are each positive integers.
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

113. If a and b are nonzero integers, is a^b an integer?

(1) b^a is negative.

(2) a^b is negative.

(A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked

(B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked

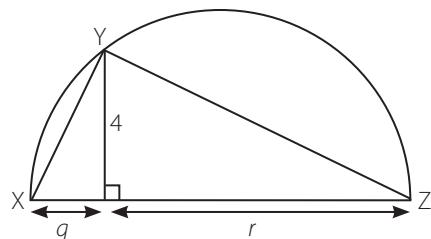
(C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient

(D) EACH statement ALONE is sufficient to answer the question asked

(E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



114. For nonnegative integers x and y , what is the remainder when x is divided by y ?
- (1) $\frac{x}{y} = 13.8$
- (2) The numbers x and y have a combined total of less than 5 digits.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



115. If arc XYZ above is a semicircle, what is its length?

- (1) $q = 2$
- (2) $r = 8$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



116. If y is an odd integer and the product of x and y equals 222, what is the value of x ?
- (1) x is a prime number.
- (2) y is a 3 digit number.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

117. If x and y are positive integers, is $\frac{x}{y}$ an integer?

- (1) Every factor of y is also a factor of x .
 - (2) Every factor of x is also a factor of y .
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



118. The infinite sequence $a_1, a_2, \dots, a_n, \dots$ is such that $a_1 = x, a_2 = y, a_3 = z, a_4 = 3$ and $a_n = a_{n-4}$ for $n > 4$. What is the sum of the first 98 terms of the sequence?

- (1) $x = 5$
- (2) $y + z = 2$
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

119. If x is a positive integer less than 10, is $14,743 + x$ prime?

- (1) $\frac{x}{2}$ is odd.
(2) $x^2 = 36$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked,
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



120. Steve works at an apple orchard and is paid by the bushel for the apples he harvests each day. If Steve harvests 42 bushels or less per day, he is paid y dollars per bushel. If Steve harvests more than 42 bushels per day, he receives y dollars per bushel for the first 42 bushels and is paid 1.5 times that amount for each additional bushel. How many bushels of apples did Steve harvest yesterday?
- (1) Yesterday, Steve was paid \$180 for the apples that he harvested.
- (2) Today, Steve harvested 10 more bushels of apples than yesterday and was paid a total \$240.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

121. If $yz \neq 0$, is $\frac{x-y+z}{2z} < \frac{x}{2z} - \frac{y}{2z} - \frac{x}{y}$?

- (1) $\frac{x}{y} < -\frac{1}{2}$
(2) $xy < 0$

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



122. In the first hour of a bake sale, students sold either chocolate chip cookies, which sold for \$1.30, or brownies, which sold for \$1.50. What was the ratio of chocolate chip cookies sold to brownies sold during that hour?
- (1) The average price for the items sold during that hour was \$1.42.
- (2) The total price for all items sold during that hour was \$14.20.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

123. Set T is a finite set of positive consecutive multiples of 14. How many of these integers are also multiples of 21?
- (1) Set T consists of 30 integers.
- (2) The smallest integer in set T is a multiple of 21.
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed



124. If r , s , and t are positive integers and $rst = 343$, what is the value of t ?

- (1) $r < s < t$
 - (2) $rs = 7$
-
- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
 - (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
 - (D) EACH statement ALONE is sufficient to answer the question asked
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

125. If n and a are positive integers, what is the units digit of $n^{4a+2} - n^{8a}$?

- (1) $n = 3$
- (2) a is odd.

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked
- (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked
- (C) BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient
- (D) EACH statement ALONE is sufficient to answer the question asked
- (E) Statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed

HOMEWORK

Challenge Questions



HOMEWORK



SOLUTIONS

18. C

Question Type: *What Is the Value?* This question asks how many cars will fit in the parking lot after it is made larger.

Given information from the question stem: The number of cars in each row will be decreased from 40 to 30. However, there will be fewer total rows in the lot as well. In order to answer the question “How many total cars will fit in the entire lot?” you will need to know how many rows times how many cars per row. You already know how many cars per row (30), so this question will hinge on whether you can determine the number of total rows in the new lot.

Statement 1: The new configuration will have 3 fewer rows than the current lot does. However there is no indication of how many rows were originally there. This is not sufficient. Therefore choices A and D are eliminated.

Statement 2: Before the change there are 10 rows of cars. Yet this does not tell you how many rows there will be after the change. Remember: Answer choice B is that “Statement 2 ALONE is sufficient.” This problem is a good opportunity to practice looking at Statement 2 alone and forgetting what you learned from Statement 1. Since Statement 2 alone is also not sufficient, choice B is eliminated.

Together: The statements allow for the calculation that there are 7 rows after the changes to the parking lot. 7 rows @ 30 cars per row = 210 cars. This is sufficient and the answer is C.

19. E

Question Type: Yes/No. This question asks whether William is taller than Jane.

Given information from the question stem: The question asks whether William is taller than Jane. If you can show that William is DEFINITELY taller than Jane or that William is DEFINITELY NOT taller than Jane, the statement (or statements) is sufficient.

Statement 1: You can write this as $W > A$ (William taller than Anna). This is not sufficient because we do not know anything yet about Jane. Choices A and D are eliminated.

Statement 2: You can write this as $A < J$ (Anna is not as tall as Jane). This is not sufficient because it provides no information about William. Choice B is eliminated.

Together: Taken together, you can write the statements as:

$$W > A, \text{ and } J > A$$

Note that it's important to get the inequalities pointed in the same direction (Statement 2 was given as $A < J$, but that also means that $J > A$). This helps you to notice that both William and Jane are taller than Anna, but that is all you can conclude. You still have no way to establish the relationship between William and Jane. Because you do not have enough information even taken together, the answer is E.

20. B

Question Type: Yes/No. This type asks whether $\frac{11x}{23}$ is less than $\frac{7x}{13}$.

Given information from the question stem: You can think of the question stem as asking whether $\frac{11x}{23}$ is less than $\frac{7x}{13}$. $\frac{11}{23}$ is slightly less than $\frac{1}{2}$ and $\frac{7}{13}$ is slightly more than $\frac{1}{2}$. Since both numbers are multiplied by the same variable, x , it would appear that the answer will be "yes" $\frac{11x}{23}$ is less than $\frac{7x}{13}$, at least if x is a positive integer. However, if x is a negative number or a fraction it may not be the case.

Statement 1: "x is an integer." With this information you know that x is not a fraction. However, x could still be negative. If x is negative then $\frac{7x}{13}$ will be "more negative" (further to the left on the number line) and therefore a smaller number than $\frac{11x}{23}$. This would give the answer of "no." You can also get a "yes" from Statement 1 if x is a positive integer. For this reason Statement 1 is not sufficient. Eliminate choices A and D.

Statement 2: "x is positive." This eliminates the possibility of x being negative (and also eliminates the possibility of x being 0, which would also have given a "no" answer to the question). However, Statement 2 does allow for x to be a non-integer—a fraction or decimal. Does it matter if x is a non-integer? It does not. Even if x is $\frac{1}{2}$, $\frac{7x}{13}$ will still be a larger number than $\frac{11x}{23}$. The number property at work in this question is "positive/negative" and the fact that x cannot be negative and cannot be zero is enough to ensure that the answer to the question is always "yes" $\frac{11x}{23}$ is smaller than $\frac{7x}{13}$. This statement is sufficient and the answer is B.

The important takeaways from this question: When a problem deals with the combination of inequalities and variables and/or when a statement specifically defines a variable as positive (>0) or negative (<0), be certain to check positive/negative number properties as part of your analysis.

**21. E**

Question Type: *What Is the Value?* This question asks how old Fred will be in y years.

Given information from the question stem: This question is asking for a specific number for the age that Fred will be y years from now. You can represent that as $F + y$.

Statement 1: Doris is 12 years older than Fred can be written " $F + 12 = D$." This is clearly not sufficient to determine Fred's age now or in the future. Eliminate choices A and D.

Statement 2: The sum of the ages of Doris and Fred is y years can be written " $F + D = y$." This statement only consists of three variables without any actual numbers, so it cannot give us a specific age for Doris or for Fred. This is not sufficient, so eliminate choice B.

Together: If you take both statements together you can use the second equation " $F+D = y$ " and substitute " $F+12$ " for D . This yields the equation " $F + F + 12 = y$." Without knowing either what y is or what F is this equation cannot be solved, as it is one equation with two variables. The answer is E.

22. B

Question Type: *What Is the Value?* This question asks for the value of the quadratic $4x^2 - 3x + 1$.

Given information from the question stem: This question asks for a specific value for the quadratic " $4x^2 - 3x + 1$." You might be tempted to factor this equation right away, but it is best to look at the statements first. You may need to factor but the question may be just as valuable in its current form.

Statement 2: For this question Statement 2 is the clearer of the two statements, so we can evaluate this one first. Statement 2: Using conceptual understanding you know that if you have values for each variable in an equation or inequality that you will be able to arrive at a single value and that this information will be sufficient. In this case the only variable in the quadratic in the question stem is "x" and we have a value for x given by Statement 2. There is no need to insert x into the quadratic; Statement 2 is automatically sufficient. The answer will either be B or D.

Statement 1: Statement 1 provides two values for x. Since either " $(x - 1) = 0$ " or " $(x+2) = 0$," x must either equal 1 or -2. At this point you have more than one value for x and this statement is "not sufficient" because it will yield more than one solution: If $x = 1$, the result of the quadratic will be 2; if $x = -2$, the result of the quadratic will be 23. The answer is B.

23. E

Question Type: *What Is the Value?* This question asks for the final score of a game between two teams, X and Y.

Given information from the question stem: This question is seeking a specific value for the final score in game between team X and team Y.

Statement 1: This statement tells you that the first-half score for $X + 14 =$ the Second Half Score for X. This statement is clearly not sufficient as it does not give you a score for either team X or Team Y. Eliminate choices A and D.

Statement 2: This statement tells you that during the first half the two teams scored equal points but that during the second half team y scored 4 more points. This statement also does not give you any fixed value for the score of X or of Y. Also not sufficient: Eliminate choice B.

Together: Even taken together these two statements do not yield enough information. Neither statement gives you an actual number. Playing Devil's Advocate, you could simply choose 10 as the number of points each team scored in the first half. Then add 14 points to X's first-half score to get the second-half score according to Statement 1, so X would score $10+24 = 34$ points. Y scored 4 more than X (Statement 2) so that would mean 38 points for Y. But there is no reason why "10" should be the first-half score any more than "20" or "100." This question allows for an infinite variety of final scores. The information is not sufficient. The answer is E.

24. C

Type of Question: Yes/No. This question asks whether the value of b (symbol) c is greater than 10.

Given information from the question stem: This question asks you to determine "yes" or "no" is b symbol C is greater than 10. Now you are told that the symbol is an "operation" and you might think that this means addition, subtraction, multiplication, and division only. But it does not. The symbol can really require you to take any mathematical action. It could have you add a million and then subtract 2 million. So do not make any assumptions here.

Statement 1: This statement tells you what the symbol represents. It says that for any two numbers "x symbol y" you square each of the numbers and then add them. This tells you what action the symbol requires. However, unless you know what the numbers are that you are squaring, you cannot get a definite answer. For example, if x and y each equal 1, then answer will be "no" because



$1^2 + 1^2 = 2$. But if x and y are each large numbers like 10, then the answer will be "yes" because $10^2 + 10^2 = 200$. Because you don't know the value of b and c , this statement is not sufficient.

Statement 2: This statement gives you values for b and c . It may seem that this statement is sufficient because each standard operation—multiplication, division, addition, subtraction—will yield a result of less than 10. However, as mentioned above, the symbol does not necessarily represent one of these four operations and Statement 1 makes this possibility very clear. Since you do not know what to do with the values for b and c , you cannot determine what b symbol c will equal. This statement is not sufficient.

Together: Taken together the two statements provide you with the values for b and c as well as the action that you should perform with these numbers. You take $2^2 + 1^2$ which equals 3, so the answer to the question is "no" b symbol c is not larger than 10. Together the statements are sufficient. The answer is C.

25. B

Question Type: *What Is the Value?* This question asks for the measure of angle ADC.

Given information from the question stem: This problem asks you for a specific number for the measure of angle ADC. The question stem indicates that this figure is a parallelogram. This information is valuable because there are properties of parallelograms that are very useful in determining the measure of an angle. The most important property for this problem is that opposite angles have equal measures. So angles BAD and BCD are equal, as are angles ABC and ADC. If you know the measure of any of the four angles then you can determine the measure of the other three.

Statement 1: This statement does not give the measure of any of the angles. This statement is clearly not sufficient on its own. Answers A and D are eliminated.

Statement 2: This statement gives you the measure of angle BCD. This angle measures 70 degrees. That means that angle BAD also measures 70 degrees. Since the angles of a parallelogram measure 360 in total that leaves 220 degrees for the remaining two angles. Since these angles are equal that means that angle ADC (as well as angle ABC) equals 110 degrees. Since this statement gives you a single measure for angle ADC it is sufficient. Answer Choice B is correct.

26. B

Question Type: Yes/No. This question asks whether the area of circle X is greater than the area of circle Y.

Given information from the question stem: This question asks you to determine whether circular region x has a larger area than does circular region y.

Statement 2 seems easier to evaluate so you can start there.

Statement 2: This statement gives you the relationship between the radius of circle x and that of circle y. Circle x has the larger radius. Since the formula for the area of a circle is πr^2 and π is a constant, it is true that a larger radius definitely means a larger area of the circle. This statement is sufficient. The answer is either B or D.

Statement 1: This statement compares the circumference of circles x and y with that of circle z. The statement can be rewritten as "circumference x > circumference z" and "circumference y > circumference z." This does not allow for a comparison of circles x and y; it simply indicates that each is larger than circle z. This information is not sufficient. The correct answer is B.

27. D

Question Type: *What Is the Value?* This question asks you for a specific value for the side of a triangle—side JL.

Given information from the question stem/diagram: The figure drawn above the question indicates that this is a 45 – 45 – 90, or isosceles right triangle. The ratio of the sides of a 45–45–90 is $x:x:x\sqrt{2}$. Note also that the specific angles are labeled, making JK the hypotenuse, and sides JL and KL the shorter sides.

Statement 1: This statement gives the measure of the hypotenuse $JK = 2\sqrt{2}$. With this information you can determine that both JL and KL will equal 2. This is sufficient. The answer is either A or D.

Statement 2: This statement gives the measure of side $KL = 2$. Side JL will have the same measure since this is an isosceles triangle. This statement is sufficient. The answer is D.

**28. E**

Question Type: Yes/No This question asks you to determine which of two shipping options is less expensive. This can be treated like a “yes/no” (Is Option A cheaper?) question since there are only two options. You may not even need to know the exact cost of each method as long as you can compare the two.

Given information from the question stem: Company A’s charge is based on the distance of the delivery and Company B’s charge is based on the weight of the package.

Statement 1: Statement 1 provides the scale at which the charges are applied. Company A charges a fixed amount plus .01 per mile and Company B charges a fixed amount plus \$8.50 per pound. This statement provides necessary information to calculate the charges but does not provide the distance nor the weight, so this statement is not sufficient. The answer will be B, C, or E.

Statement 2: This statement provides the distance that the package will travel. However it does not provide any way of calculating or even comparing the charges for companies A and B. This statement is not sufficient. Eliminate choice B.

Together: Taken together the statements give an amount that Company A will charge for the package. Statement 1 provides the scale and Statement 2 the distance. However, the weight of the package remains unknown so that the charge from Company B cannot be calculated nor compared to Company A’s charge. These statements are not sufficient. The answer is E.

29. D

Question Type: *What Is the Value?* This question asks for a specific number indicating the length—in words—of the letter.

Statement 1: This statement can be translated into the following equation: “ $(t - 2)80 = t(60)$ ” where t is the number of minutes it takes to type the letter at a rate of 60 words per minute. This equation then simplifies to “ $80t - 160 = 60t$.” Subtract $60t$ from both sides and add 160 to each, and the equation becomes “ $20t = 160$ ” and “ $t = 8$.” Therefore $8 \cdot 60$ gives a total of 480 words. *Of course the equation did not have to be solved. As soon as you got $(t - 2)80 = 60t$ you should see that this is a linear equation with just one variable and it can be solved for a single value.* This is sufficient. The answer is either A or D.

Statement 2: This statement can be translated into an equation using the formula $w = rt$ (work = rate • time). The work is $\frac{1}{2}$ of the letter. The rate is 40 words per minute and the time is 6 minutes. The equation is “ $\frac{1}{2}w = 40 \cdot 6$ ” or “ $\frac{1}{2}w = 240$ ” and “ $w = 480$.” This statement is sufficient as well; the answer is D.

30. D

Question Type: Yes/No This is a yes/no question asking you to determine whether the positive square root of a is an integer.

Statement 1: This statement tells you that “ b ” is an integer and that “ $a = b^4$.” This means that the “square root of a ” would equal “ b^2 .” This tells you that “square root of a ” must be an integer since multiplying an integer times itself always results in another integer. If “ b ” were, for example, 3, then “ a ” would be 81 and “the square root of a ” would equal 9. There is no need to plug in numbers for this statement; it is sufficient. If b is an integer then so is b^2 . The answer will be either A or D.

Statement 2: “ $a = 81$ ”; the positive square root of “ a ” would then be 9. This is an integer. This statement is sufficient. The correct answer is D.

31. C

Question Type: *What Is the Value?* This question asks you for a specific value for the price of the longer (28 inch) necklace.

Given information from the question stem: Necklaces come in two lengths and there is no “volume discount” since all necklaces of the same length sell for the same price regardless of number purchased.

Statement 1: Translated this becomes “ $x + y = \$68$ ” where x is 18 inch and y is 28 inch. This is two variables and just one equation. There is no way to distribute the \$68 total. This is not sufficient. Eliminate choices A and D.

Statement 2: Translated this becomes “ $2x + y = \$96$.” This is just like Statement 1 in that you have two variables and only one equation. This, too, is not sufficient. Eliminate choice B.

Together: Taken together the statements give you two linear equations with the same two variables. As long as these equations are distinct this will be sufficient. To ensure you come to an answer, you can use the Elimination Method for multiple variables, subtracting Statement 1’s equations from Statement 2’s:

$$2x + y = 96$$

$$-x - y = -68$$

$x = 28$, and you can plug that in to the first equation to solve for y . If $28 + y = 68$, then $y = 40$. These statements together allow you to solve for y , the price in question, so the correct answer is C.

**32. A**

Question Type: Yes/No This question asks: "Is $x > y$?"

Statement 1: $x = y + 2$; this statement clearly indicates that x is larger than y . In words, this says " x is two greater than y ." This is sufficient. The correct answer is either A or D.

Statement 2: $\frac{x}{2} = y - 1$; this statement can be investigated using the Play Devil's Advocate technique. Inserting a value for x —for example, $x = 2$ —will yield a value for y . If $x = 2$, then y also equals 2, since $\frac{2}{2} = 2 - 1$. Since x and y are equal then the answer is "no," x is not greater than y . Now you would be looking for values that give you a "yes," values that make $x > y$ and that work with Statement 2. Inserting a larger value for x , for example " $x = 6$ ", will give you a value of " $y = 4$ " since $\frac{6}{2} = 4 - 1$. In this case $6 > 4$, and therefore $x > y$, yielding an answer of "yes." Since this statement allows for values of x and y that give you answers of "yes" and of "no," this statement is not consistent and is therefore not sufficient. The correct answer is A.

33. A

Question Type: Yes/No. "is the average of a and b less than 40?"

Given information from the question stem: You're dealing with averages. One helpful tool to use when asked about the average of a group of numbers is to use the formula "Average * number = total." In this case if the average is 40 and there are two numbers (a and b), then the question is really asking is $\frac{A+B}{2} < 40$ or is " $A + B < 80$."

Statement 1: if the average of $3a$ and $3b$ is 117 this means that " $3a + 3b = 234$." If you divide by sides by 3 then you get " $a + b = 78$." This is sufficient to answer the question. "Yes, the average of $a + b < 40$." This statement is sufficient. The correct answer is either A or D.

Note: It would not be necessary to actually determine the value for " $a + b$." Once you know that you can divide both sides of the equation " $3a + 3b = 234$ " by 3 and get a value for " $a + b$ " then you do not need to actually make the calculation. Either the average will be less than 40 or it will not be, but in either case you will have a consistent answer.

Statement 2: This statement does not provide any fixed values for a or b and so it cannot be determined if the average of $a + b$ will be less than 40. This statement is not sufficient. The answer is A.

34. E

Question Type: *What Is the Value?* This question asks you for a specific number indicating the amount of money that Penny earned during week W.

Given information from the question stem: Penny is paid hourly and works part time, and has two different employers, Company X and Company Y. Therefore, her total weekly pay will be (Hours Worked at X)(Pay Rate at X) + (Hours Worked at Y)(Pay Rate at Y).

Statement 1: This statement provides the hourly rates for her work for the two companies, X and Y. However it does not provide a total number of hours worked for either company. This is not sufficient. Answers A and D are eliminated.

Statement 2: Penny worked 60 hours total during week W. This means that she worked 60 hours for X and Y combined. However, nothing is indicated as to the rate that she was paid for this work. This statement is not sufficient and answer B is eliminated.

Together: Taken together, you have the total number of hours worked and the hourly rate for each of the two companies. Yet, it is not possible to know how to divide the 60 hours between X and Y. Since she is paid a different amount by each of the two companies it is necessary to know precisely how many hours she worked for each. Therefore, the statements together are still not sufficient. The correct answer is E.

35. A

Question Type: Yes/No This question asks whether x is a positive number.

Statement 1: “ $-5x - 3 > -2x$ ” can be simplified by adding 3 to both sides to yield “ $-5x > -2x + 3$ ” and then adding $2x$ to both sides to yield “ $-3x > 3$. * Then, to isolate x, divide both sides by 3 and $-x > 1$. To change the sign of x, multiply both sides by a negative 1 and the statement becomes “ $x < -1$. * This algebraic manipulation has answered the question directly. “No, x is not positive. x is a negative number.” This statement is sufficient. The answer is either A or D.

Statement 2: x^2 is positive allows for x to be either positive or negative. Since the answer is not consistent this statement is not sufficient. The correct answer is A.

One important takeaway here: Even though Statement 1 provided the answer “no,” it was indeed sufficient. A definite “no” means you have sufficient information to arrive at one exact answer. So while your instincts might tell you “No → Eliminate,” remember that “no” means “sufficient.”

**36. A**

Question Type: *What Is the Value?* This question asks you to provide a specific number of paintings in the museum.

Statement 1: 4 new paintings will increase the number of paintings in the museum by 10%. This can be written as “ $4 = 0.1x$ ” with x equaling the total number of paintings. Clearly this will be sufficient to provide a specific answer to the question and there is no need to actually solve this equation. This statement is sufficient. The answer will be either A or D.

Statement 2: The ratio of impressionist to non-impressionist paintings is 3:2. This information does not allow you to arrive at a specific number since you do not know either the number of impressionist or the number of non-impressionist paintings. Not sufficient. The answer is A.

37. B

Question Type: *What Is the Value?* This question asks for a specific number of positive integers that are factors of n .

Given information from the question stem: n itself is an integer.

Statement 1: “ n is the product of a prime and a non-prime positive integer.” From the Data Sufficiency toolkit, you can use “conceptual understanding” on this statement and see that it allows for too many possibilities for you to get a single answer, or you can “Play Devil’s Advocate” and use a couple of small numbers to prove to yourself that this statement is not sufficient. For example, you can simply choose 3 as the prime number and 4 as the nonprime positive integer. The result, 12, has six factors: 1, 2, 3, 4, 6, 12. However, if you choose 3 as the prime and 1,000,000,000 as the non-prime numbers then the result will clearly have many more than six factors. This statement yields more than one answer and is therefore not sufficient. The answer is B, C, or E.

Statement 2: “ n and 20 are each divisible by the same number of positive integers.” This statement is clearly sufficient. It is saying that when you determine the number of positive factors of 20 then you have the answer as n has the same number of factors. It is not necessary for you to determine the number of positive integers that 20 is divisible by; you know that it will be just one number and that is sufficient. The answer is B.

38. E

Question Type: *What Is the Value?* This problem asks you to find the remainder when k is divided by j.

Given information from the question stem: j and k are integers.

Statement 1: This problem offers a great opportunity to pick numbers/Play Devil's Advocate." The easiest numbers to pick are likely prime numbers, since you know that by definition all primes have the same number of factors (two). 5 divided by 3 leaves a remainder of 2, so now you want to find an answer that gives you something other than 2. You want to Play Devil's Advocate to arrive at an alternative answer to show that the statement is not sufficient. Typically the best way to do that is to try numbers with different properties, and 2—the only even prime—gives you just that opportunity. 3 divided by 2 leaves a remainder of 1, and since we've found two different answers (2 and 1) to this specific-value question, we can prove that this statement is not sufficient.

Statement 2: Notice this: The numbers we chose for Statement 1 are also possibilities given Statement 2. So we can use the same numbers to show that Statement 2 alone is not sufficient, and that even taken together the statements are not sufficient. Accordingly, the correct answer is E.

39. C

Question Type: *What Is the Value?* This question asks you for the number of units that line A produced in 2006.

Given information from the question stem: Production on line A increased by 5% from 2006 to 2007 and production on line B increased by 10%.

Statement 1: This statement can be written as "A + B = 100,000 units." This information is not sufficient since you do not know how many of the 100,000 units were produced by line A and how many by line B. Not sufficient. Eliminate answers A and D.

Statement 2: This statement tells you the combined number of units in 2007. This can be written as $1.05A + 1.10B = 107,500$. Similar to the above, this statement is not sufficient alone as you have two variables and one equation. Eliminate choice B as well.

Together: Now you have two distinct linear equations with two variables and you can solve. There is no need to actually solve for A since you are certain that you can. Together the statements are sufficient and the answer is C.

**40. E**

Question Type: Yes/No This question asks whether the average sale for month J was at least 20 percent higher than for month M. Mathematically, that's: $J > 1.2M$?

Given information from the question stem: The average sale for month M was "d" dollars.

Statement 1: Total revenue from sales in month M was \$3,500. This statement provides no information as to the number of sales so there is no way to determine the size of the average sale. This statement is not sufficient alone. Eliminate choices A and D.

Statement 2: Total revenue from sales in month J was \$6,000. This statement is very similar to Statement 1 and is not sufficient for the same reason. Eliminate choice B.

Together: Even together these two statements do not provide you the average sale for either month J or month M. These two statements are not sufficient together. The correct answer is E.

41. C

Question Type: Yes/No The question asks whether xy is divisible by 9. Another way to say this is "Is xy a multiple of 9?" and, further breaking down the question, "Does xy have prime factors that include $3 \cdot 3$?" By manipulating the question using conceptual understanding you are often in a stronger position to evaluate the answer choices. Now you know that if the information can guarantee that 3^2 is a factor of xy then that information is sufficient.

Given information from the question stem: x and y are positive integers.

Statement 1: The product xy is divisible by 6. This tells you that 3 and 2 are factors of xy . However this statement is not sufficient alone. Conceptual understanding will tell you that if 3 is a factor of xy then it is possible that 9 is also a factor of xy . However it is equally possible that 9 is not a factor of xy . If you are in doubt you can use numbers and Play Devil's Advocate. xy could equal 6 or 12, both of which are divisible by 6 as the statement requires and are not multiples of 9. This would yield an answer of "no." xy could also equal 36, which is a multiple of 6 and also of 9. This would yield the answer of "yes." Since yes and no answers are both possible this statement is not consistent and is therefore not sufficient. Eliminate choices A and D.

Statement 2: x and y are perfect squares. Conceptually this is clearly not

sufficient. While each of the numbers must be a perfect square this statement does not guarantee that the result will even be a multiple of 3, much less of 9. Eliminate choice B.

Together: Statement 1 tells you that xy must include the prime factors of 2 and 3 (in order to be a multiple of 6). Since Statement 2 requires x and y to be perfect squares the only way to have a 2 and a 3 as prime factors of x and y is to have 2^2 and 3^2 as factors of x and y . In fact, the smallest values for x and y are 4 and 9. So the answer to the question "Is xy divisible by 9?" is "yes." Together the statements yield one consistent answer and the correct answer is C.

42. D

Question Type: *What Is the Value?* This question asks for a specific number of executives.

Given information from the question stem: Bonuses were paid at \$125 for executives and \$75 to non-executives. There are 100 non-executive employees.

Statement 1: Total employees = 120. This statement is very clearly sufficient. It is as simple as $120 - 100 = 20$ (executives). Sufficient. The correct answer is either A or D.

Statement 2: This statement gives you a total for the amount paid in bonuses. The total of \$10,000 is divided between executives and non-executives at the rates given in the question stem. This information is sufficient. You can create the following equation: " $75(100) + 125E = \$10,000$." If you solve the equation E will yield the number of executives. However there is no need to solve. Since it is very clear that you will get just one answer this statement is also sufficient. The correct answer is D.

43. C

Question Type: *What Is the value?* This question asks for the value of $y - x$.

Given information from the question stem: $x = y^2$. While it may not be important for this question, you should train yourself to notice things like "x cannot be negative" here, as y^2 has to be ≥ 0 .

Statement 1: $x = 4$. This is not sufficient because even taken with the fact above it still results in two answers. Substitute x in the equation from the facts and you get $4 = y^2$. So y can equal 2 or -2. These two values for y will give you different answers for " $y - x$ " and are therefore not specific enough. You need a single value in order to be sufficient and you do not have one. The answer will either be B, C, or E.



Statement 2: $x + y = 2$. This is not sufficient even if taken together with the information from the question stem. A quick example using Play Devil's Advocate might be the simplest way to show that this is not sufficient. " $x = 1$ and $y = 1$ " is compatible with the given information and with Statement 2. However, so is one of our examples from statement 1 " $x = 4$ and $y = -2$." After all $4 + -2 = 2$. These different values for x and y will result in different answers to " $y - x = ?$ " This statement is also not sufficient. The answer is either C or E.

Together: Taken together the statements are sufficient. Only one set of values will work with the two statements and the fact. " $x = 4$ and $y = -2$." The correct answer is C. One important takeaway from this question: Do not assume that a variable is positive! When people answer this question incorrectly it's usually because they only considered 2 as a value (and not -2) for y when they saw Statement 1. When a statement gives you a solution for y^2 , remember that y could be either positive or negative. "Forgetting about negatives" is one of the easiest (and most common) mistakes to make on Data Sufficiency problems.

44. C

Question Type: *What Is the Value?* This question asks for the specific number of show dogs.

Statement 1: Exactly 3 of the dogs won prizes of at least \$500. This statement is not sufficient alone. There is no way for you to determine the total number of show dogs. Eliminate choices A and D.

Statement 2: 40% of the dogs have not won a prize of \$500 or more. This statement is also not sufficient alone. As in Statement 1, there is no way to determine the total number of dogs. Eliminate choice B.

Together: One of the best ways to solve a GMAT problem is to find a way to state the same thing in two different ways, allowing you to create an equation. You know from Statement 1 that 3 dogs have won prizes of \$500 or more and you know from Statement 2 that 60% of the dogs have won a prize of \$500 or more. So that means "3 dogs = 60% of the total dogs" or " $3 = .6d$ " She has 5 show dogs total. This information is sufficient and the correct answer is C.

45. D

Question Type: *What Is the Value?* This question asks for a specific number for the base memory capacity of the computer.

Given information from the question stem: Each M-type unit will increase base capacity by 3 megabytes.

Statement 1: 2 M-types units will increase the computer's base memory by 300%. From the fact we know that this is a total addition of 6 megabytes. So "6 = 3x," where x is the current total base memory of the computer in megabytes. "x = 2." This statement is sufficient, so the answer is either A or D.

Statement 2: The ratio of total memory capacity after adding 2 M-types units to the total memory capacity after adding 1 M-type unit is "1.6: 1." Therefore 6 (which is 2M-types units) + x (the current base memory) = 1.6 (3 + x). Therefore "6 + x = 4.8 + 1.6x." You can see that you have just one variable and you will be able to solve for x, so there is no need to do so. This is sufficient as well. The correct answer is D.

46. D

Question Type: Yes/No This question asks whether $a^2 > 3a - b^4$.

Statement 1: $3a - b^4 = -5$. This statement may not appear sufficient at first. You cannot manipulate it algebraically to mirror the question stem as might be the first impulse. However, the important thing is that " $3a - b^4$ equals a negative number." Remember that a^2 cannot be less than zero, as a squared number cannot be negative. The answer, then, is "yes," as you can absolutely conclude that a^2 will be greater than -5. The correct answer is either A or D.

Statement 2: $a > 5$ and $b > 0$. Again, this statement may not appear to be sufficient. It does not give specific values for a or b. However, if you "Just Do It" and plug in the numbers then you will see that it is sufficient. A good strategy for these "greater than" statements is to use the actual numbers given. Although the statement says $a > 5$ and $b > 0$ you can just use 5 and 0 in the inequality. The inequality becomes " $25 > 15 - 0$?" The answer is clearly "yes": $25 > 15$. And as you increase both "a" and "b" the result becomes a stronger "yes." For example, if "a = 6 and b = 2" the inequality is " $36 > 18 - 16$?" Conceptually it looks like this, so long as a is greater than 3 then a^2 will be greater than 3a. Whatever number "b" is can only take away from the 3a it cannot add to it. In fact, Statement 2 gives more information than strictly necessary; " $a > 3$ " would be sufficient. The correct answer is D.

**47. D**

Question Type: Yes/No This question asks whether x is even.

Given information from the question stem: x is a positive integer.

Statement 1: $5x$ is even. If the product of multiplication is even then at least one of the numbers being multiplied must be even. 5 is odd so x must be even. This is sufficient. The answer is either A or D. Keep in mind here the importance of leveraging the “ x is an integer” portion of the question stem. Had that not been in place, a number like $\frac{2}{5}$ would have also worked here, as $5(\frac{2}{5}) = 2$, an even integer. But the question limits you to only using integers, so you are allowed to employ even/odd number property concepts here.

Statement 2: $x - 5$ is odd. In order to get an odd result from the addition or subtraction of integers you need to have one odd and one even number. Since 5 is odd x must be even. This is sufficient as well. The correct answer is D.

48. E

Question Type: Yes/No Is $x^2 > x^3$?

Given information from the question stem: While no new information is explicitly provided, the question structure—exponents and inequalities—should already get your mind thinking in terms of positive/negative and integer/non-integer number properties. This problem will almost certainly test your ability to consider different types of numbers (negatives, fractions).

Statement 1: x^3 is positive. Given that x^3 is positive, x must be positive as well. However, x could still be a non-integer between 0 and 1. For example, if $x = \frac{1}{2}$ then $x^2 = \frac{1}{4}$ and $x^3 = \frac{1}{8}$. In this case $x^2 > x^3$ so that would be a “yes.” If x is a positive integer like 2 then $x^3 > x^2$, which provides the answer “no.” Since this statement allows for both a “yes” and a “no” it is not sufficient. Eliminate choices A and D.

Statement 2: x does not equal 1. Given the possibility of fractions between 1 and 0, this statement is clearly not sufficient on its own. Each of the values that was used in the analysis of Statement 1 works with this statement as well. This is also not sufficient. Eliminate choice B.

Together: The same values work for both statements, giving you a “yes” and a “no” even with both statements together. If x is between 0 and 1 the answer is yes. If x is greater than 1 the answer is no. The correct answer choice is E.

49. B

Question Type: *What Is the Value?* What percent of males in a certain poll voted for coffee Y?

Given information from the question stem: A total of 1,600 people voted. Each voted for X or Y. No one voted for both. And because you're dealing with people/votes, the numbers for X and Y must be integers.

Statement 1: 40% of females voted for brand X. This statement is not sufficient to tell you what percent of males voted for brand X. Not sufficient. The answer will either be B, C, or E.

Statement 2: 65% of the males voted for X. This statement might appear to be insufficient as it does not even talk about brand Y. However, since you know that there are only two brands, X and Y, the percent of males who chose X + the percent who chose Y must equal 100%. So if 65% chose X, the other 35% voted for Y. And since the question asks for the percent, not the actual number of males, 35% is sufficient. The correct answer is B.

50. C

Question Type: *What Is the Value?* What is the month when Jeff received the highest bill: June, July, or August?

Given information from the question stem: The average for the 3 months was \$75. It is a good idea to just multiply this right away to get the total amount for the 3 months = \$225 total.

Statement 1: The lowest bill was received in July. This means that the July bill must be lower than \$75. However, the highest bill could still be for June or August. Since this is not specific it is not sufficient. Eliminate choices A and D.

Statement 2: The total for July and August was \$160. This gives you the exact amount for June. $\$225 - \$160 = \$65$ for June. Be careful not to include the information from Statement 1 at this point. You do not know whether July or August is the highest bill. This statement alone is not sufficient. Eliminate choice B.

Together: Taken together you know that the June bill is \$65 and that the July bill is for less than this amount. Since the total amount of all three has to be \$225, and June and July account for less than \$130, the August bill must account for at least \$95, and therefore be the highest total. This is sufficient. The answer is C.

**51. E**

Question Type: Yes/No What is the value of two-digit integer x ?

Given information from the question stem: x is a two-digit integer.

For this question you can start with Statement 2 since that statement is more obviously not sufficient.

Statement 2: x is divisible by 9. In other words, x is a multiple of 9. However, there are many two-digit multiples of 9, such as 18, 27, 36, etc. This is clearly not sufficient. Eliminate choices B and D.

Statement 1: The product of the two digits is 14. What two-digit integers have a product of 14? 27 has a product of 14, as does 72. Because there is more than one value of x this statement is also not sufficient. Eliminate choice A.

Together: Taken together the two values used in Statement 1 still work here. Both 27 and 72 are multiples of 9, and both multiply to 14. Taken together the statements are not sufficient. The correct answer is E.

52. D

Question Type: *What Is the Value?* This question asks for the specific value of $x - y$.

Given information from the question stem: $x + y = 6$.

Statement 1: $x^2 - y^2 = 12$. You should recognize this as the difference of squares: $x^2 - y^2 = (x + y)(x - y)$. Factoring this will make it fit the question and the information that you already have. You can rewrite this to say " $(x + y)(x - y) = 12$." Given that you know that " $x + y = 6$ " then it must be the case that " $x - y = 2$." This statement is sufficient. The answer is either A or D.

Statement 2: $2y + x = 8$. From this statement and from the equation in the question stem, you can determine that $y = 2$. Either the substitution or elimination method (for solving multivariable equations) will work, but for this purpose let's use elimination. If you take the equation $2y + x = 8$ and subtract the equation $x + y = 6$, all that is left is $y = 2$. If you plug this number back into the equation in the fact, then $x + 2 = 6$. Therefore $x = 4$ and $x - y = 2$. This statement agrees with Statement 1 and is also sufficient. The correct answer is D.

53. A

Question Type: *What Is the Value?* This question asks for the specific number of steps between the first and second floor.

Given information from the question stem: The two floors are 9 vertical feet apart.

Statement 1: Each step is $\frac{3}{4}$ foot high. This statement allows you to create the equation " $9 = \frac{3}{4}x$ " where x is the number of steps. This is sufficient to tell you the number of steps needed to climb 9 vertical feet. The correct answer is A or D.

Statement 2: Each step is 1 foot wide. This statement is less helpful since you do not know the overall width of the entire staircase; you only know from the fact that it is 9 feet tall. There is no way to determine the number of steps from this information. This statement is not sufficient. The correct answer is A. And notice the trap here: In classic "Why Are You Here?" form, Statement 2 is designed to make you think you might need it. Since each stair has a height and a width, you might feel better knowing all dimensions. But you do not need the width; whether this is a long, processional staircase or a ladder, it needs to extend 9 vertical feet.

54. C

Question Type: Yes/No Is $2a - b + c > a - b - 2c$? It will help to simplify this question through algebraic manipulation before moving forward. If you add b to both sides, you can streamline the question to "Is $2a + c > a - 2c$?" If you then subtract a from both sides and subtract c from both sides, you get "Is $a > -3c$?" This is important because now the statements are much easier to analyze.

Given information from the question stem: a , b , and c are integers.

Statement 1: a is positive. This is not sufficient given that " c " could still be a negative number and $-3c$ could be larger than a . Eliminate choices A and D.

Statement 2: c is positive. This statement indicates that the right side of the inequality is negative since c is positive. However, a could also be a negative number, and potentially more negative than c and therefore smaller. If a is a positive number it will clearly be larger than $-3c$. So the inequality might be true or not true. Therefore the statement is not sufficient. Eliminate choice B.

Together: Taken together you know that the left side of the inequality will be positive and the right side will be negative. Any positive number is larger than any negative number, so " $a > -3c$." Alternatively, you could add $3c$ to both sides to make the question "Is $a + 3c > 0$?" Clearly if a and c are both positive, this sum will be positive. Together the statements are sufficient. The correct answer is C.

**55. B**

Question Type: *What Is the Value?* This question asks for the dollar expenditure on hops.

Given information from the question stem: $H + Y + M = \$100,000$ (H = amount spent on hops, Y = amount spent on yeast, M = amount spent on malt).

Statement 1: $Y = 1.2M$. This would be sufficient if the equation in the facts only included these two variables. But with three variables in the above equation, there is no way to distribute the \$100,000. This statement is not sufficient. Eliminate choices A and D.

Statement 2: $Y + M = H$. This statement seems at first glance to be like Statement 1. However, it contains some hidden information. You're solving for H , and this statement allows you to plug in " H " for " $Y + M$ " in the equation $Y + M + H = 100,000$. If you replace $Y + M$ with H , you have $H + H = 100,000$, so H must be 50,000. Note here the opportunity to Think Like the Testmaker. C is too easy an answer, as it adds a third equation to the three-variable set-up. The testmaker is begging you to pick C, so if you initially went for that answer you should ask yourself whether you can Leverage Assets, and here if you spend a bit more time manipulating Statement 2 you should see that it is sufficient. Because Statement 2 is sufficient, the correct answer is B.

56. A

Question Type: *What Is the Value?* This statement asks for a specific value for the fraction $\frac{x^5y^4z^2}{z^2y^4x^2}$.

Given information from the question stem: The product xyz does not equal zero. This means that none of these three is equal to zero. At this point, it's also helpful to streamline the given fraction. Since the same variables appear in both the numerator and denominator, you can "combine like terms" by factoring out terms with common bases:

x: The numerator includes x^5 and the denominator includes x^2 . You can divide out the x^2 term to yield x^3 in the numerator.

y: Both the numerator and denominator include y^4 , so this term divides out entirely.

z: Both the numerator and denominator include z^2 , so this term divides out entirely.

So what you're left with is the question "What is x^3 ?"—a much easier question to answer!

Statement 1: $x = 1$. This is clearly sufficient given the work that you have done on the question stem. If $x = 1$ the entire question = 1. This is sufficient and the answer is either A or D.

Statement 2: $y = 1$. This is not actually helpful, given that both the numerator and the denominator have y^4 and this can therefore be eliminated. It literally does not matter what the value of y is. This statement is not sufficient. The correct answer is A.

57. D

Question Type: *What Is the Value?* This question asks for a specific value for the number of liters (x) of concentrated cleaner in the solution.

Given information from the question stem: 5 liters of solution = x liters of concentrated cleaner + y liters of water. Succinctly: $x + y = 5$.

Statement 1: $\frac{x}{y} = \frac{1}{8}$. This statement means that the ratio of $x:y = 1:8$. That means that $\frac{1}{9}$ of the solution is concentrated cleaner. So $\frac{1}{9} \cdot 5 = x$. So $x = \frac{5}{9}$ of a liter. This is sufficient. The answer is either A or D.

Statement 2: $\frac{x}{5} = \frac{1}{9}$. You may see immediately that this is sufficient, as it is one linear equation with one variable—and that variable, x , is what you're asked to solve for. To see the math, you can cross-multiply to get $9x = 5$. And if you divide both sides by 9 you get $x = \frac{5}{9}$ of a liter. This is also sufficient. The correct answer is D.

58. E

Question Type: *What Is the Value?* This question asks for the length of the shortest of three pieces of rope (s).

Given information from the question stem: The three pieces of rope are each different lengths. (Let s = shortest, m = middle, and L = longest.)

Statement 1: $m + L = 12$. This statement is not sufficient because you do not have the total length of rope and cannot say what the length of the shortest piece is exactly. Not sufficient. Eliminate choices A and D.

Statement 2: $s + m = 11$. This statement is not sufficient because, although you have a total that includes the shorter piece, you have one equation with two variables and no way to say what the specific length of the shorter piece is. Eliminate choice B.

Together: Together you have two equations, one from each statement. They are: $m + L = 12$ and $s + m = 11$. This allows you to determine that the longest piece



is 1 foot longer than the shortest piece so that $L - s = 1$. However, you are still not able to fix an exact value to s , m , or L . The longest piece is longer than 6 feet and the shortest piece is shorter than 5.5 feet, but since the values do not have to be integers and since each of the three pieces can vary there is no way to get the specific values. These statements are not sufficient. The correct answer is E.

59. B

Question Type: Yes/No Is the number $3(a + b) + c$ divisible by 3?

Given information: a , b and c are integers. You can Use Conceptual Knowledge here. Given that each of these numbers is an integer, $3(a + b)$ will be a multiple of 3 regardless of the values of a and b .

Therefore you are simply focused on the question “Is c a multiple of 3?” If c is a multiple of 3 then the whole thing is divisible by 3. If c is not a multiple of 3 then the whole thing is not divisible by 3.

Statement 1: $a + b$ is not divisible by 3. As mentioned above the value of $a + b$ does not matter. So this statement is not sufficient alone. This is the advantage of doing your work early. When you truly understand the question you can work through the statements very efficiently. This statement is not sufficient. The correct answer is B, C, or E.

Statement 2: c is divisible by 3. This is an exact answer to the question that you developed by manipulating the question stem. If c is a multiple of 3 and since we know from algebra that $3(a + b)$ will also be a multiple of 3 then we also know that adding two multiples of 3 gives a result that is a multiple of 3. Try it. Make c = any multiple of 3. Then make a and b any integers at all. $3(a + b)$ will also be a multiple of 3 and “ $3(a + b) + c$ = a multiple of 3.” The answer is always “yes.” It is consistent and sufficient. The correct answer is B.

60. A

Question Type: *What Is the Value?* This question asks for a specific value for $x - y$.

Statement 1: $x - y = y - x$. This statement can be simplified to “ $2x = 2y$ ” and finally to “ $x = y$.” You can then finish the Algebraic Manipulation by subtracting y from both sides and you get “ $x - y = 0$ ” This clearly gives you a specific value for “ $x - y$.” This statement is sufficient. The answer is either A or D.

Statement 2: $x - y = x^2 - y^2$. The left side of the equation is the difference of squares so we can rewrite this as “ $x - y = (x + y)(x - y)$.” This statement leaves you with two possibilities. In order for this to be true, either “ $x - y = 0$ ” so that both sides of the equation = 0, or $x + y = 1$ so that both sides of the equation =

$x - y$. These two different possibilities allow infinite values for " $x - y$ " since if " $x + y = 1$ " " $x - y$ " could equal anything. This statement is not sufficient. The correct answer is A.

61. B

Question Type: Yes/No Is n an integer less than 5? Notice that the question requires two components to provide the answer "yes": n must be an integer and n must be less than 5.

Statement 1: $5n$ is a positive integer. This statement is not sufficient. If you Play Devil's Advocate you can get both integers and non-integers for n . For example, n could equal 1, which is an integer less than 5. Or n could = $\frac{1}{5}$ and $5n = 1$. This would give you a "no" since $\frac{1}{5}$ is not an integer.

You could also manipulate the "algebra" on this one. If $5n =$ positive integer, then $n = \frac{\text{(positive integer)}}{5}$. That allows n to equal something like $\frac{5}{5}$ (which is 1) or $\frac{1}{5}$ (which is not an integer).

This statement is not sufficient. Eliminate choices A and D.

Statement 2: $\frac{n}{5}$ is a positive integer. In order for $\frac{n}{5}$ to be a positive integer, n must be at least equal to 5 and must be a multiple of 5. For example, $\frac{5}{5} = 1$ and $\frac{10}{5} = 2$, etc. So this statement is sufficient because the answer is "no." n is an integer but is not less than 5.

Again, you could also perform the "algebra" by saying that $\frac{n}{5} =$ (positive integer). This means that $n = 5$ (positive integer). And since the lowest positive integer is 1, then n at a minimum is 5(1). Accordingly, n cannot be less than 5.

Because you can answer definitively "no" to this question then this statement is sufficient and the answer is B.

62. E

Question Type: *What Is the Value?* What percentage of the combined total of groups A and B is comprised of men?

Given information from the question stem: Every man is either a member of Group A or Group B or both. So in Venn Diagram terms, the "Neither" category is empty, but there may very well be men who are in "Both" groups. 20% of Group A is men and 65% of Group B is men.

Statement 1: Group A contains 50 people. So the number of men in group A is 50(20%). So 10 men are in Group A. This is not sufficient since you do not know the number of men in Group B. Eliminate choices A and D.



Statement 2: Group B contains 100 people. The number of men in Group B is 100(65%) or 65 men. This is not sufficient since it does not tell you about group A. Eliminate choice B.

Together: It may appear that these two statements are sufficient when taken together. There are 10 men from Group A and 65 from Group B for a total of 75 men, right? Not necessarily. It is possible for some of the people in groups A and B to be in both groups. It is possible, for example that all 10 men in Group A are also in Group B. So instead of 75 men total there could be as few as 65. This means that you cannot get one specific value for the percentage of men. This is not sufficient. So the answer is E.

63. A

Question Type: Yes/No This question asks whether $b > a$.

Given information from the question stem: a and b are consecutive negative integers.

On this question you can start with Statement 2 first since that statement is easier to evaluate.

Statement 2: b is an odd number. It should not matter if b is odd when determining if " $b > a$." You can quickly Play Devil's Advocate with a couple of consecutive negative integers. b could = -5 and a could equal -6. In this case $b > a$. Of course b could also equal -5 and a could equal -4, in which case b is not greater than a . Since you can get both a "yes" and a "no" for this statement it is not sufficient. Eliminate choices B and D (since you did Statement 2 first.)

Statement 1: $a + 1$ and $b - 1$ are consecutive negative integers. You already know from the facts that a and b are consecutive negative integers. Now when you add one to " a " and subtract one from " b " you also have consecutive negative integers. Playing Devil's Advocate is a good strategy here to take the abstraction of variables and make it concrete with numbers. You can use the numbers from above: Let $b = -5$ and $a = -6$. Adding 1 to a and subtracting 1 from b changes it to $b = -6$ and $a = -5$, meaning that these are still consecutive negative numbers. So b can be greater (less negative) than a .

Can the opposite be true? Can a be larger than b ? Try using the reverse numbers. Make $a = -5$ and $b = -6$. Now when you add one to a and subtract one from b the result is $a = -4$ and $b = -7$, so this will not work with Statement 2. a cannot be larger than b ; b must be the larger number. The statement is sufficient. The correct answer is A.

64. D

Question Type: Yes/No The question asks: "Is $x^2 = xy$?"

For this question it is easier to begin with Statement 2.

Statement 2: $x = y$. If $x = y$ then x^2 will automatically equal xy . So this statement is sufficient. The answer is either B or D.

Statement 1: $x^2 + y^2 = -(x^2 - y^2)$. Generally with algebra you want to eliminate parentheses, so in this case you'll want to multiply each term in $(x^2 - y^2)$ by -1 to distribute that negative sign; the statement becomes $x^2 + y^2 = -x^2 + y^2$. If you then manipulate this statement algebraically by adding x^2 to both sides and by subtracting y^2 from both sides you get $2x^2 = 0$. For this to be true x must 0. And if $x = 0$, then $0^2 = 0(y)$ by definition—both sides of the equation would equal 0. This statement is also sufficient and the correct answer is D.

65. B

Question Type: Yes/No This question asks: "Is the tenths digit of the decimal equal to 4?"

Given information from the question stem: $2.5 < x < 4.5$.

Statement 1: $x + 0.006 < 4.5$. If you subtract 0.006 from both sides you get: $x < 4.494$. This statement is not very helpful in narrowing down the value for x . You already know from the fact that $2.5 < x < 4.5$ and now you can say that $2.5 < x < 4.494$. This leaves many possible values for x and many possibilities for the tenths digit of that decimal. This statement is not sufficient. Eliminate choices A and D.

Statement 2: $x + 0.06 > 4.5$. Subtracting 0.06 from both sides makes the statement $x > 4.44$ combined with the fact above you get $4.44 < x < 4.5$. Since x must be greater than 4.44 and less than 4.5, the tenths digit of x will definitely be a 4. This statement is sufficient to answer the question with a consistent "yes." The correct answer is B.

66. D

Question Type: What Is the Value? This question asks for a specific value for b .

Given information from the question stem or diagram: The marks on the number line shown are equally spaced, and a and b represent specific marks where a is 3 marks above 0 and b is 7 marks above 0. Therefore, a is $\frac{3}{7}$ as far away from 0 as b is, so you can represent that mathematically as $a = \frac{3}{7}b$.



Statement 1: $a = \frac{1}{4}$. If the numbers are evenly spaced then you can simply take the fact that you discerned above that $a = (\frac{3}{7})b$ and replace a with $\frac{1}{4}$. So $\frac{1}{4} = \frac{3}{7}b$. This is sufficient to yield a specific value for b . The answer is either A or D.

Statement 2: $b - a = \frac{1}{3}$. Again, you can lean on the information you derived from the question stem and diagram. If $a = (\frac{3}{7})b$, then $b - a = \frac{1}{3}$ can be rewritten as $b - (\frac{3}{7})b = \frac{1}{3}$, or $(\frac{4}{7})b = \frac{1}{3}$. This allows you to solve for b . Like Statement 1, this is sufficient to get a specific value for b . The correct answer is D.

67. E

Question Type: Yes/No Did more than $\frac{2}{5}$ of the ice cream made in July contain over 20% milk?

Given information from the question stem: The factory makes two flavors of ice cream: chocolate and caramel. Each flavor comes in over 20% milk and under 20% milk. This means that there are four total possibilities: chocolate over 20%, chocolate under 20%, caramel over 20%, and caramel under 20%. You are looking for the combined fraction of the ice cream that is over 20% milk.

Statement 1: 80% of the caramel ice cream contained over 20% milk. This statement is not sufficient because you know nothing about the chocolate ice cream. Eliminate choices A and D.

Statement 2: 1,000 gallons of chocolate ice cream contained over 20% milk. This statement does not tell you how many gallons of chocolate ice cream were made as whole. It is not as useful as statement 1 and is certainly not sufficient alone. Eliminate choice B.

Together: You know that $\frac{4}{5}$ of the caramel ice cream is over 20% milk, but you do not know what the proportion is for the chocolate ice cream. The number "1,000 gallons" from Statement 2 is not particularly useful in this situation. Even with both statements together there is not enough information. The correct answer is E.

68. E

Question Type: Yes/No "Is y greater than 0?" Or, in other words, "Is y positive?"

Given information from the question stem: x does not equal 0.

Statement 1: $xy = 24$. In order to get a positive result from multiplication you either need two positive or two negative numbers. It is possible for both x and y to be positive (giving you a "yes") and for both to be negative (giving you a "no").

This statement allows both yes and no and is not consistent and not sufficient. The answer will be choice B, C, or E.

Statement 2: $x - y = 5$. This can be restated as " $x = y + 5$." This is not sufficient alone since y could be a negative number. For example, $x = -5$ and $y = -10$. This statement allows x to be positive as well as negative so it is not sufficient. Eliminate choice B.

Together: Taken together the statements tell you that " x is 5 greater than y " and that " $xy = 24$." The question is, can you Play Devil's Advocate and find both positive and negative values for y that work with both statements. First x and y can be positive. Let $x = 8$ and $y = 3$. This satisfies both statements and yields a "yes, y is positive." Simply making $x = -3$ and $y = -8$ also works with both statements and gives you a "no, y is not positive." Even taken together these statements are not sufficient. The correct answer is E.

69. A

Question Type: Yes/No This question asks: "Is a less than b ?"

Given information from the question stem: $\frac{a}{3} = \frac{4}{b}$. If you cross-multiply this becomes $ab = 12$.

Statement 1: b is greater than or equal to 4. If this is the case then both a and b must be positive and a must be less than or equal to 3. This means that "yes, a is less than b ." This statement is sufficient. The correct answer is either A or D.

Statement 2: b is less than or equal to 5. This statement is not as helpful as Statement 1. First, you do not know if a and b are positive or negative. a could be -3 and b could be -4, which does work with the facts above and would yield a "no, a is not less than b ." Even if a and b are positive then a could equal 3 and b equal 4 in which case a would still be larger than b . Of course, b can be larger than a as well if $b = 4$ and $a = 3$. So Statement 2 allows for both a "yes" and a "no" and is not consistent and not sufficient. The correct answer is A.

70. C

Question Type: Yes/No. The question asks: "Does $x = 2$?"

Given information in the question stem or diagram: No important information is given in the question stem.

Statement 1: $x^2 - 3x + 2 = 0$. In this case it is best for you to "Just Do It" and see what values you get for x . Factoring the quadratic yields $(x - 2)(x - 1) = 0$. So $x = 2$ or $x = 1$. This gives you both a "yes" and a "no" answer to the question so it is



not sufficient. Eliminate choices A and D.

Statement 2: $x^2 - x - 2 = 0$. Factoring this quadratic gives you $(x - 2)(x + 1) = 0$. So x either equals 2 or -1. This statement is also not sufficient alone since those values give a "no" and a "yes." Eliminate choice B.

Together: When taking these statements together you need to see if there is only one value that is allowed by BOTH of the statements. $x = 2$ is the only value that both statements allow so the correct answer is C. Note: This is only hard if you make a mistake with the factoring or get confused about what constitutes sufficiency when considering the two statements together. This would be trickier if one statement gave two values other than 2. Then that statement would be sufficient as the answer would be a definitive NO, even if there were two possibilities from the statement. As it stands, neither alone is sufficient and the answer is C.

71. B

Question Type: Yes/No This question asks: "Is $y < 2.4$?"

Statement 1: $y < 2.5$. This is not sufficient because y could still be greater than 2.4 but less than 2.5, such as 2.49, and the answer would be "no." However y could of course be equal to 1, and then the answer would be "yes." This statement is not sufficient so you can eliminate answers A and D.

Statement 2: $y < 2.3$. If y is less than 2.3 then it must always be less than 2.4, so this statement will always give a "yes" answer to the question. The correct answer is thus B. Note: While not particularly difficult, this question is here to make sure people understand the importance of limits in this type of inequality question. Statement 1 sets a limit that creates a "maybe" answer but Statement 2 sets a limit that will create a definitive "yes" to the given question.

72. D

Question Type: *What Is the Value?* This question asks for a specific value for the price per microwave (P).

Given information in the question stem or diagram: The total price of M equally priced microwaves is \$18,000 so $MP = \$18,000$, where P is the price per microwave. Note: To get a definite value for P , you simply need some way to find the number of total microwaves or another unique equation with only M and P in it.

Statement 1: This statement allows you to create the equation: $M(P + 10) = 18,000 + 5,000$, which simplifies to $MP + 10M = \$23,000$. This equation, when combined with the equation in the facts, allows you to solve for M and P because you have

two unique equations with two unknowns. Since $MP = 18,000$ you can substitute directly for MP into the equation above to see that $18,000 + 10M = 23,000$ and $M = 50$. This means $P = 360$. This is sufficient so the correct answer is either A or D. Note: This is much more difficult to conceptualize than it is to consider algebraically. The key is to make sure that the equation given in the statement is indeed unique from the one given in the question stem. When doing algebra on this type of question, you do not need to solve completely, but make sure that you can “see the finish line” and get exactly one value for P .

Statement 2: This statement allows you to create the equation: $M(P - 9) = 18,000 - .25(18,000)$, which simplifies to $MP - 9M = 13,500$. Again since you can substitute so easily for MP from the question stem, there is no need to do the rest of the math. Once you know that you will get just one value for M , which allows you to solve for P , then you know that the information is sufficient. The correct answer is D.

73. E

Question Type: *What Is the Value?* The question asks for the price of the most expensive bicycle.

Given information in the question stem or diagram: Total number of bicycles = 6; average price per bicycle = \$1,000. This means that the total amount sold by the store was \$6,000.

Statement 1: The median price was \$1,000. This means that the mean equals the median. However, this does not allow you to find the price of the most expensive bicycle. Depending on the prices of the other five bicycles, the most expensive one can have a wide range of prices. For instance, there could have been three \$800 bikes and three \$1,200 bikes that sold, and the most expensive would be \$1,200. Or there could have been one \$1,300 bike, four \$1,000 bikes, and one \$700 bike. In each case you have met the conditions in the statement and the question stem but gotten multiple values. Not sufficient, so eliminate choices A and D.

Statement 2: The range of prices was \$600. This information allows you to narrow down the range of values for the most expensive bicycle but does not allow you to establish this price. It is clearly not sufficient by itself so you can eliminate choice B.

Together: It is much more difficult to make the choice between C and E on this question. With all the information in the question stem and the statements, you know that the six bicycles have a median and an average of \$1,000 and a range of \$600. Given this, many people will assume that the most expensive bike must be \$1,300 and the least \$700. For example, if the price of the most expensive is



\$1,300 that makes the price of the least expensive \$700 and, as shown in the example previously, the middle four could all be \$1,000. However, you could also make the most expensive be \$1,400. Make the lowest price one equal to \$800 (range has to be \$600) and the remaining four could be \$800, \$1,000, \$1,000, and \$1,000. This means that multiple values will still work for the highest priced bicycle and so the information not sufficient. The correct answer is E. Note: This problem is about avoiding assumptions. People assume that because the mean and the median are the same, that it must be an evenly spaced set. This is not true as shown above. If you are dealing with an evenly spaced set, the mean always equals the median, but if the mean equals the median in a set, this does not prove it is evenly spaced. Remember to always prove E on a question like this by Playing Devil's Advocate and doing your best to find multiple possibilities for the highest priced item.

74. A

Question Type: *What Is the Value?* This question asks for the value of $j - k$.

Given information in the question stem or diagram: There is no important information given in the question stem. Note: When you are asked for a difference versus an actual variable, you typically need less information than you think.

For this question you should start with Statement 2 because it is easier and clearly insufficient.

Statement 2: $n = 3$. n is not even part of the question so this statement is clearly not sufficient but creates the classic "Why Are You Here?" scenario. You need to consider very carefully why this statement might be important when considering Statement 1. Maybe you will need it or maybe you won't! Eliminate choices B and D.

Statement 1: $2j + 4n = 2k + 4n - 6$. This statement also appears to be insufficient but you should do some manipulation to make sure that you cannot isolate $j - k$. Without Statement 2 you have no value for n and it appears that you need it in this equation. However, a careful look at the equation shows that the same element " $+ 4n$ " is on both sides and will cancel out. You can simply subtract $4n$ from both sides and n disappears leaving you with $2j - 2k = -6$. Now you can simplify by dividing both sides by 2 to see that $j - k = -3$. While you do not know the values of either j or k , you do know the difference and this is sufficient alone. The correct answer is A. Note: This is classic example of the "Why Are You Here?—Temptation" construct. Remember: Before you ever pick answer C in a case like this, make absolutely sure that you really need the information in the other statement.

75. E.

Question Type: *What Is the Value?* This question asks for the value of x .

Given information in the question stem or diagram: $20x + 30y = 280$. This should be simplified before you move to the statements. Divide all terms by 10 to see that this equation is really $2x + 3y = 28$. Note: On the GMAT, you should always simplify algebraically whenever you are given information that is not in simplified form.

Statement 1: $4x = 56 - 6y$. This statement appears to give you a second linear equation, which would be sufficient to solve for x when combined with the given information. However, before you do anything with this equation you should simplify it as you did with the question. If you add $6y$ to both sides of the equation and divide by 2, you see that this equation is really the same as the given equation: $2x + 3y = 28$. These are not two distinct linear equations so this information is not sufficient. Eliminate choices A and D. Note: Be ready for this set-up in Data Sufficiency algebra questions: the “disguised” second equation that is really the same as the first.

Statement 2: $y^2 = 16$ or $y = 4$ or -4 . The equation gives two different values for y , but it is important to glance at the original equation to make sure that the different values for y will actually result in different values for x . In this case they do as the original equation is a linear equation. Since this information gives you two different values for x it is not sufficient.

Together: Given that the equation from Statement 1 is the same as in the given information, the analysis of the two statements together is the same as for Statement 2. Statement 1 is no help and since Statement 2 is not sufficient alone, the answer is E.

76. D.

Question Type: Yes/No. This question asks: “Is y^2 divisible by 4.”

Given information in the question stem or diagram: y is an integer.

Statement 1: y is even. With a conceptual understanding of factors, you know that if y is an even number then y^2 will have to be a multiple of 4. If you want to quickly test this you can use some small even numbers. For example, if $y = 2$, then $y^2 = 4$; yes, that is divisible by 4. If $y = 4$, then $y^2 = 16$; this is also divisible by 4. Even 0, which is an even number, will work. $0^2 = 0$; and yes, 0 is divisible by 4. This confirms what we already knew from our conceptual understanding: an even number when squared is a multiple of 4. The answer is either A or D.



Statement 2: y^3 is divisible by 4. For some, this may not seem sufficient at first glance. y^3 might be divisible by 4 while y^2 is not. For example, y could be the cube root of 4. However, if you leverage properly the information in the question stem (always so important!) then you know that "y is an integer." So y cannot be the cube root of 4; it must be an integer. Once you have established that y must be an integer then the only way for y^3 to be divisible by 4 is for y to be an even number. In other words, it gives you the same information that you already found sufficient in Statement 1. Thus, the correct answer is D.

Note: This is another good example of "When one statement is easy, the other is hard/counterintuitive." The first statement is relatively simple so you should be extra careful analyzing the second statement. As is often the case, the key with Statement 2 is to make sure you properly leverage the given information.

77. B.

Question Type: Yes/ No. The question asks: "Is $xy < 8$?"

Statement 1: $x < 2$ and $y < 4$. At first glance this might appear sufficient. When you multiply a positive number smaller than 2 by a positive number smaller than 4, the result is smaller than 8; however, you must consider negative numbers. x could equal -3, for example, and y could be -5. Together their product would equal 15. So this statement is not sufficient and you can eliminate choices A and D. Note: This statement is all about avoiding assumptions. Do not assume that x and y are positive as that is not given in the question stem!

Statement 2: $0 < x < \frac{1}{2}$ and $y^2 < 225$. This means that x is a positive number between 0 and $\frac{1}{2}$ and $15 > y > -15$. When you multiply a positive number that is smaller than $\frac{1}{2}$ by a number that is between 15 and -15, the result must be smaller than 8. Even if you took the values of $\frac{1}{2}$ and 15 the product is 7.5. Unlike in Statement 1, it is okay when y is negative because x must be positive, so the product would be negative and less than 8. With this information, you know that xy must be smaller than 8 and this statement is sufficient. The answer is B.

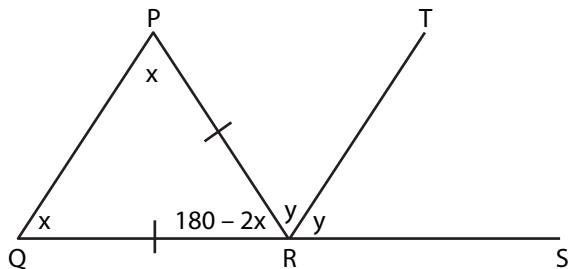
78. B

Question Type: Yes/No. The question asks: "Are lines TR and PQ parallel?"

Given information in the question stem or diagram: QRS is a straight line and QR = PR. If you leverage the information in the question stem, you can add several facts: Shape QPR is a triangle, and if sides QR and PR are equal, then it is an isosceles triangle with angle QPR and angle PQR each the same measure. If you call those angles each x , then the third angle – angle QRP will measure $180 - x$.

Statement 1: Length PQ = Length PR. This tells you that not only is the triangle QPR isosceles, it is equilateral, with all angles 60 degrees. So now you know just about everything you need to know about the triangle, but keep in mind that you know nothing about line TR. This statement is not sufficient, so eliminate choices A and D.

Statement 2: Line TR bisects angle PRS. Here is where leveraging the information in the question stem—and labeling your diagram—will pay off. You already know that line QRS is a straight line, meaning that the three angles that meet at point R (angle QRP, angle PRT, and angle TRS) will have to sum to 180. And you know that angles PQR and QPR are equal, which you've labeled as x in the diagram. If line TR bisects angle PRS, then the two angles PRT and TRS are equal. You can call them each y , and then your diagram should now look like:



And since $180 - 2x + 2y$ has to equal 180 (for line QRS to be a straight line), then $2x$ has to equal $2y$, and $x = y$, so angle TRS equals PQR, meaning that lines PQ and TR are, indeed, parallel. Statement 2 is sufficient, so the answer is B.

Note this important takeaway from this problem: Geometry-based DS questions are quite often candidates for the Leverage Assets strategy. Be certain to label diagrams with all known information, as Geometry has a lot of potential for seemingly small pieces of information to leverage into sufficiency.

**79. A**

Question Type: *What Is the Value?* This question asks for number of bolts that contain BOTH cotton and wool.

Given information in the question stem or diagram: 20 bolts of fabric total; each is 100% cotton, or 100% wool, or a mixture of BOTH cotton and wool. This means that neither = 0 and you can set up your Venn diagram equation before you go to the statements: Total = Cotton (C) + Wool (W) – Both (B) or $20 = C + W - B$. Remember that C and W represent the total sets not those with "Only Wool" or "Only Cotton."

Statement 1: Of the 20 bolts, 18 contain some wool and 14 contain some cotton. Plugging this information into the equation above you see that this statement is sufficient. $20 = 18 + 14 - B$. $B = 12$. Or logically you know that there are 20 total bolts but $18 + 14 = 32$. This means the difference must be the number of bolts that are double counted (those with both). The answer is either A or D.

Statement 2: Of the 20 bolts 6 are 100% wool. As in any Venn Diagram problem, when you are given "only" information such as this, you must use a diagram or think logically (the equation above is for total set information, not "only" information). If "Wool Only" = 6 then you know that the total of cotton bolts (including those with both) is 14. However there is no way to determine the value of "Both" from this information so it is not sufficient. The correct answer is A. Note: Almost all Venn Diagram problems hinge on reading carefully and distinguishing between "Total Set" information and "Only" information. In this problem, it is the difference between bolts containing Wool or Cotton (which includes both) and bolts containing 100% Wool or Cotton or Only Wool or Cotton (which does not include both).

80. D

Question Type: Yes/No. This question asks: "Is $x < 3$?"

Given information in the question stem or diagram: x , y , and z are the lengths of three sides of a triangle. Note: Even before going to the statements you should recognize that the third side rule of triangles will almost certainly come into play: The third side (i.e., any side) of a triangle is always between the sum and the difference of the other two sides.

Statement 1: $z = y + 3$ OR $z - y = 3$. This statement does not give you the value of any of the sides and does not allow you to determine the actual value of side x . However, using the third side rule you know that x must be greater than the difference between z and y , which is given as 3 in this statement. Therefore x

must be greater than 3 and you can answer this question with a definitive “no” from this information. The information is sufficient so the correct answer is either A and D. Note: People tend to underleverage this piece of information because it does not lock down a value for x (which is not necessary) and because people have forgotten the third side rule.

Statement 2: $y = 3$ and $z = 6$. This gives very similar information to statement 1 and also allows you to apply the third side rule. Since the explicit values are given you can see that x must be greater than the difference between x and y (which is again 3). Statement 2 is also sufficient and the answer is thus D. Note: The answer could never be C on this question. While this is not a common construct (and thus not mentioned in the lesson portion of this book), it is one that you will see occasionally. The first piece of information ($z = y + 3$) is automatically known from the second, so putting them together could never help you! It is either answer A, B, D, or E whenever one statement is known from the other, so C should be eliminated automatically here.

81. E

Question Type: Yes/No. The question asks: “Is x an even number?”

Given information in the question stem or diagram: x is a positive integer.

Statement 1: If x is divided by 3, the remainder is 2. This statement can be easily addressed by considering numbers and Playing Devil’s Advocate. First attempt to get an even number from Statement 1: The number 8 gives you a remainder of 2 when divided by 3. This is an even number so you have a “yes” answer. Next choose a number exactly 3 higher or lower than 8: 11 is an odd number and also gives a remainder of 2 when divided by 3. So 11 works but gives you a “no” answer. This statement is not sufficient, so eliminate choices A and D.

Statement 2: If x is divided by 5, the remainder is 2. This statement can be treated in the same way as Statement 1. First get a “yes” answer. If $x = 12$, then $\frac{12}{5} = 2$ remainder 2. Next choose a number exactly 5 above or below 12. If $x = 7$, then $\frac{7}{5} = 1$ remainder 2. So this works as well and gives you a “no” answer. This statement is also not sufficient, so eliminate choice B.

Together: Taken together you know that x must be 2 greater than a multiple of both 5 and 3. So x must yield a remainder of 2 when divided by 15. So 17 would work. $\frac{17}{5} = 3$ remainder 2, and $\frac{17}{3} = 5$ remainder 2. 17 is an odd number so you have a “no” answer. Now add 15 to 17 to get 32. 32 is an even number and will give you a “yes” answer. $\frac{32}{5} = 6$ remainder 2 and $\frac{32}{3} = 10$ remainder 2. So 32 satisfies both statements as well. Note: Number picking is of course not



required to answer this question. With a good conceptual understanding of the number line, and quotient/remainder problems in particular, you can see quite clearly that the answer will be E in this problem. As is often the case, Arithmetic problems in Data Sufficiency can be quickly and reliably answered if you have a deep conceptual understanding of the subject matter.

82. C

Question Type: *What Is the Value?* This question asks for the surface area before the changes were made.

Given information in the question stem or diagram: The surface of the rectangular field was changed so that one dimension was reduced by 10 feet and the other dimension was increased by 20 feet.

Statement 1: After the changes, the surface area was 2,500 square feet. This statement is not sufficient because it does not allow you to determine the dimensions of the field either before or after the change. For example, the new dimensions might be $25 \cdot 100$, but which number was reduced by 10 and which was increased by 20? And $25 \cdot 100$ is just one example. Many other dimensions are possible for a 2,500 square foot field so this statement is clearly not sufficient. Eliminate choices A and D.

Statement 2: The length and width of the field were equal after the changes. Don't be tempted to bring information from Statement 1! You do not yet have that information. Statement 2 alone is not sufficient because just knowing that the dimensions are equal does not tell you what those dimensions are. They could be $30 \cdot 30$ or $40 \cdot 40$, etc. Eliminate choice B.

Together: Taken together you know that the area after the changes is 2,500, which means that after the changes the length and width are each 50 since they must be equal. One of these—either the length or width—used to be 10 longer and one used to be 20 shorter. It is not possible to determine which was longer and which was shorter, but it does not matter. It is because the sides are equal after the changes that you can solve this problem. So the dimensions before the change are $60 \cdot 30$ or 1,800 square feet. Together the statements give you a specific number and are sufficient. The answer is C.

Note: This problem is designed to make you carry information between statements. Always be careful about polluting your mind from one of the statements, and make sure that you are only using the information from the question stem and the statement you are assessing.

83. D

Question Type: *What Is the Value?* This question asks for the ratio of revenue from sales of x to revenues from sales of y. So “revenue x: revenue y = ?”

Given information in the question stem or diagram: Item x sells for 10% less than item y.

Statement 2 is very clear on this problem so you should begin there.

Statement 2: The store’s revenue from sales of X = \$6,000 and the revenue from sales of y = \$8,000. This statement is clearly sufficient. The ratio is simply 6,000:8,000 or 3:4. The answer is either B or D. Note: This is a very good example of the construct “When one statement is very easy, the other will be hard and counterintuitive.” Everyone knows this is sufficient so there will be something tricky about Statement 1. Slow down and make sure you are not over- or under-leveraging the given information in Statement 1.

Statement 1: The store sells 20% more of item y than item x. This statement, when combined with the given information, is sufficient. 20% more of item y is sold at 10% higher prices. Revenue = (Unit Price)(Number of Units). Conceptual understanding will allow you to stop at this point and realize that although you do not have the actual number of units or the actual price per unit, you can calculate the ratio of revenue from x to that of revenue from y. This is sufficient and the answer is D. Note: Most people under-leverage this information because it does not give any hard number values. Using your hint from the other statement to dig deeper here, you should eventually realize that the percentage data is sufficient.

84. A

Question Type: *What Is the Value?* This question asks for the percent change in profits of company x from the second to third year.

Statement 1: The increase in profits was the same from year 1 to 2 as from year 1 to 3. This is a tricky statement that most people under-leverage. At first glance, it appears insufficient but a careful analysis shows that this information proves that there was no change from year 2 to year 3. The only way that the change could be the same from year 1 to the year 2 as from year 1 to year 3 is with a 0% change from year 2 to year 3. This statement requires that you read very carefully and leverage the information given. The answer is A or D.

Statement 2: The profits in year 1 were \$13,800 and in year 3 were \$15,900. This indicates that the increase in profits from year 1 to 3 was \$2,100 and the



percentage gain from year 1 to year 3 could be calculated. But it is not possible to know how much of this increase occurred between year 1 and 2 versus year 2 and 3 so it is not sufficient. The correct answer is then A. Note: People are apt to make two mistakes on this question. First, they may think that the hard numbers from Statement 2 are required with Statement 1 and pick choice C. This would be a classic “Why Are You here?—Temptation” set-up. They give you something that you do NOT need in the second statement but that is hard to say no to. Also, many people will pick choice E because they do not read the first statement carefully and realize that this information guarantees a 0% increase in that time period.

85. E

Question Type: *What Is the Value?* This question asks for the lateral area of the pyramid-shaped box. The lateral area represents the area of the faces of the polygon added together.

Given information in the question stem or diagram: The box is pyramid shaped and constructed with four lateral faces and an open bottom.

Statement 1: The base is a polygon with all sides of equal length and the perimeter of the base is 1 meter. This statement does not provide enough information to determine the area of the faces of the pyramid. While you know that each side of the polygon at the base is .25 meters, you have no idea about the height of the pyramid and thus the height of each lateral face. This is not sufficient so you can eliminate choices A and D.

Statement 2: The lateral faces are isosceles triangles that have the same size and shape. This statement is clearly insufficient by itself, as you are given no hard numbers. The question, then, is whether this statement helps you when combined with Statement 1. The answer is C or E.

Together: Taken together, the information still does not give you any idea about the height of the pyramid. If Statement 2 said that the lateral faces were equilateral triangles, then the answer would be C. However, this pyramid could be 3 meters high, or 10 meters high, greatly changing the area of the faces. Note: This question is a good example of the “prove E” concept. If you are going to choose answer E, as you should in this problem, do so because you can identify the missing information, not just because the problem seems difficult. Here you are still missing the height of the pyramid so the correct answer is indeed E.

86. D

Question Type: Yes/No. The question asks: "Is y greater than or equal to 9?"

Given information in the question stem or diagram: x and y are integers and $x + y = 23$. Note: When specific information like this is given in the question stem, make sure you carefully leverage it with each statement.

Statement 1: $x - 6 < 9$. Adding 6 to both sides you see that $x < 15$. Since the question asks about y , substitute from the equation into the inequality. If $x + y = 23$, then $x = 23 - y$ and, after substituting into the inequality, $23 - y < 15$. Move y from the left to right and 15 from the right to left to see that $y > 8$. Since y must be an integer, this proves that y must be at least 9 and this information is sufficient. You could also do this logically, but algebraic manipulation leaves no doubt in your mind! This statement is sufficient so the answer is A or D. Note: In any problem that mixes equations and inequalities, do not forget that you can substitute from the equation into the inequality.

Statement 2: $x^3 = 2744$. It is tempting in this statement to try to find the cube root of 2,744 to get an exact value for x . However, since doing this will give you one value for x and since you can then subtract that value from 23 and get an exact value for y , this must be sufficient. Whatever value you find for y will either be "yes, greater than/ equal to 9" or "no, less than 9." There is no need to do the math here and that is certainly the trick in this statement. The correct answer is D. Note: This second statement is testing exclusively if you understand the concept of sufficiency and how it applies to each of the two question types. If you are missing this type of question (or spending lots of extra time on it!) review the rules/drills relating to sufficiency at the beginning of the book.

87. C

Question Type: *What Is the Value?* This question asks for the number of pigs on the farm.

Given information in the question stem or diagram: Pigs (P) + Cows (C) + Horses (H) = 60. $P + C + H = 60$.

Statement 1: $H:C = 2:9$. This statement is not sufficient because it does not give an exact number of cows and horses, so it is not possible for you to determine the number of pigs. However, you should check to make sure that multiple cases are possible given a total of 60 animals. For instance, you could have 2 horses, 9 cows, and 49 pigs; or it could be 4 horses, 18 cows, and 38 pigs. Not sufficient, so you can eliminate answers A and D. Note: if the total was, for instance, 20 animals, then this statement would be sufficient because it would be impossible



for there to be more than 2 horses and 9 cows (or total would be over 20); this would leave 9 pigs and a definite answer to the question.

Statement 2: $C > 36$. This statement also does not give you specific numbers for any of the animals, so it is clearly insufficient and the answer is C or E. However, you should already be considering the “Why Are You Here?” strategy and wondering how this information might matter when combined with the first statement.

Together: Taken together you know that $C > 36$ and $H:C = 2:9$. The hidden fact on many ratio questions is that the number of animals (or children or photocopiers) must be an integer. There is no such thing as $\frac{1}{3}$ of a horse. This means that you have more information here than you might think. From the two statements together, you know that C must be a multiple of 9 and must be greater than 36. The next multiple of 9 is 45. If there are then 45 cows there are 10 horses because of the ratio in Statement 1. That leaves a total 5 pigs since $P + C + H = 60$. What about 54 cows? That is a multiple of 9. This is not possible, because 54 cows would mean 12 horses, and that is over 60 animals. Together the two statements are sufficient and there are 5 pigs on the farm. The correct answer is C. Note: People miss this because they under-leverage the information given in these statements and do not take the hint given in the second statement.

88. E

Question Type: *What Is the Value?* This question asks for the value of “ $x + y$.”

Given information in the question stem or diagram: No important information is given in the question stem.

On this question you should begin with Statement 2 as that statement is easier to evaluate.

Statement 2: $3x + 2y = 24$. Since there is no way to algebraically isolate $x + y$ from this equation, it is not sufficient. However, if this equation had been $2x + 2y = 24$ you could have divided by 2 to find $x + y$. However, since x and y have different coefficients, you cannot do that. This statement is not sufficient, so eliminate answers B and D.

Statement 1: Clearly, this statement requires substantial algebraic manipulation before you can evaluate its sufficiency. Begin by simplifying the left side. You should see that the numerator $4x^2 - 4y^2$ fits the Difference of Squares format and can be written as $(2x - 2y)(2x + 2y)$. This works because the coefficients are perfect squares as well as the variables. So now the equation is $\frac{(2x - 2y)(2x + 2y)}{2x + 2y} = 2x - 2y$. Simplified further it becomes $2x - 2y = 2x - 2y$. This

statement is not sufficient as there is an infinite set of solutions for x and y (i.e., it is true for any values of x and y —a useless equation!) Eliminate choice A.

Together: Taken together the statements are still not sufficient because Statement 1 does not contribute anything when looking back at Statement 2. The correct answer is thus E. Note: This question is designed for people to pick answer C, as they believe you have two equations and two unknowns. However, by Playing Devil's Advocate and manipulating the first statement properly, you can see that the first statement is a meaningless equation that doesn't contribute any information to the problem.

89. E

Question Type: *What Is the Value?* This question asks for the value of the two-digit number x .

Given information in the question stem or diagram: x is a two-digit number.

Statement 1: The sum of the two digits is 4. There are only four two-digit numbers that have digits that total 4. They are 13, 22, 31, and 40. Since that allows for more than one value for x , this statement is not sufficient. Eliminate choices A and D.

Statement 2: The difference between the two digits is 2. This means that for the tens digit (T) and the units digit (U), either: $T - U = 2$, or $U - T = 2$. Many people confuse this statement and think that the tens digit must be larger, such as 64, where $T - U = 2$. However, 46 would also be acceptable since the difference between the digits is 2. Clearly this statement is not sufficient alone as there are many two-digit numbers where $T - U = 2$ or $U - T = 2$. Eliminate choice B.

Together: When taking the statements together it is best to start with the more limiting statement. Statement 1 only allows four values: 13, 22, 31, and 40. How many of these values are compatible with Statement 2? Two of them: 31 is $T - U = 2$; and 13 is $U - T = 2$. They each have a difference of 2. Since there are still two possible values for the two-digit number x , the correct answer is E. Note: This is a classic C vs. E problem. Almost everyone gets it down to choice C or E, but many people forget to differentiate between 13 and 31. (They either miss one in their list of possibilities for the first statement or assume that it must be 31 for the reasons discussed above.) Remember to do your best to exhaust all possibilities before picking answer C.

**90. E**

Question Type: Yes/No. This question asks: "Is the square root of $(7ab)$ an integer?"

Given information in the question stem or diagram: No important information is given in the question stem.

Statement 1: $a = 7$. This statement would change the question to: "Is the square root of $7 \cdot 7 \cdot b$ an integer?" This statement is not sufficient because while you know that 7^2 will still be an integer when the square root is taken, you do not know whether b is a perfect square. Not sufficient. Eliminate answers A and D.

Statement 2: $b =$ an integer raised to the third power. This is also not sufficient. It says nothing about a . In order for the square root of $7ab$ to be an integer you need either a or b to have 7 as a factor and for the other variable to be a perfect square. This statement does not guarantee either of these. Eliminate choice B.

Together: Statement 1 brings the necessary 7 and takes care of that part. All that you need is for b to be a perfect square and then you would be sure that this square root is an integer. However, Statement 2 tells you that b is a perfect cube, not a perfect square. Many people will erroneously pick choice C at this point, thinking that this information proves that it could NEVER be an integer (which would give you a definitive "no" and be sufficient). However, there are many perfect cubes (1, 64, etc.) that are also perfect squares. Therefore together this information could result in either an integer or a non-integer and the answer is E. Note: This is another great example of the importance of Playing Devil's Advocate. Your gut reaction is that the statements together prove that it could never be an integer, but you need to consider all other ways it COULD be an integer. Considering the number 1 for the value of b is probably the easiest way to show that there is at least one way it could be an integer.

91. C

Question Type: Yes/No. The question asks: "Is $xy > 24?$ "

Given information in the question stem or diagram: There is no important information given in the question stem.

Statement 1: $y - 2 < x$. This statement is clearly not sufficient since it does not give you any fixed value for x or y . x could be any number that is greater than " $y - 2$," easily making this product less than or greater than 24. Eliminate choices A and D.

Statement 2: $2y > x + 8$. Like Statement 1, this statement does not fix values for x or y but simply states them in relation to each other. This cannot be sufficient. Eliminate choice B.

Together: It is very easy to eliminate each of these statements individually so it is a classic C vs. E question. Inequality questions in Data Sufficiency form are one of the most difficult question types on the GMAT. The common mistake that most students make is to pick numbers and try to reason out a solution rather than to apply algebraic manipulation. Using your understanding of combining inequalities, it is possible to isolate x and y and learn more about them individually. First let's eliminate x and isolate y :

Step 1: Rewrite the inequalities to line up variables:

$$y - x < 2$$

$$2y - x > 8$$

Step 2: Multiply top inequality by -1 to get the signs pointing the same way and then combine to eliminate x :

$$-y + x > -2$$

$$2y - x > 8$$

$$y > 6$$

Repeat step 2 to eliminate y by multiplying the top inequality by -2 to get the signs pointing the same way and then combine:

$$-2y + 2x > -4$$

$$2y - x > 8$$

$$x > 4$$

If $y > 6$ and $x > 4$, then you know that the product of xy must be greater than 24 and the answer to the question is C. Attempting to do this with number picking is both time consuming and ineffective. Also, and importantly, you must do the necessary work to find the limit. If you did this work and found that $x > 3$ and $y > 6$, then the answer would be E, as xy might or might not be greater than 24. Many students make the mistake of assuming that if you have two inequalities with two variables, then you can definitely answer the question. That is only true when dealing with equations, not inequalities.

**92. E**

Question Type: Yes/No. This question stem asks: "Is ab a prime number?"

Given information in the question stem or diagram: There is no important information given in the question stem.

Statement 1: a is a prime number. This statement is not sufficient but it is tricky. Using conceptual knowledge, many people will assume that the product ab could never be prime. When you multiply any prime number with another positive integer, it could never be prime unless that integer is 1. However, since b could be 1 or any fraction, this statement is not sufficient, and you can eliminate choices A and D. For instance, if a is 7 and b is 1 or $\frac{2}{7}$ then the answer is "yes." However if b is any positive integer other than 1, then the answer is "no." Note: This question is about avoiding assumptions! Do not assume that b is a whole number, and don't forget about the very important number 1.

Statement 2: b is not a prime number. If b is not prime then b could easily be a fraction or equal to 1. In either case it is very possible for ab to be a prime number and it is very easy to show that it might not be. Not sufficient. Eliminate answer B.

Together: Taken together you know that a is prime and b is not. But this leaves you in the same dilemma as with Statement 1 alone. Certainly a and b can multiply to equal a non-prime number such as "a = 2 and b = 6." 12 is not a prime number. However, if b = 1 then whatever prime number a equals will remain and ab will be that prime number. As in the first statement, the key in this problem is to avoid assumptions and remember that nothing limits b to an integer and nothing excludes the value of 1. Answer is E.

93. D

Question Type: *What Is the Value?* This question asks for the least number of tiles required to cover a floor.

Given information in question stem or diagram: Rectangular floor is 4 meters wide; floor is to be covered with non-overlapping square tiles; tiles are 0.25 meters on each side.

Statement 1: L = 3W. This means that the length = 12 meters and the area is 48 square meters. This statement is sufficient. The number of tiles can be determined since you have the area of the tiles and the area of the floor. The answer is A or D.

Statement 2: The area = 48 square meters. This gives the same information as in Statement 1 so the answer must be D or E. (This is the "no news is good news"

scenario referenced in the lesson. Answer choices A, B, and C are not possible if the statements give equivalent information.) Since each statement is sufficient alone the answer is D.

Note: This problem is only difficult because people will forget to leverage the information in the question stem. If you look at both statements and forget that you know the width, then you will certainly pick answer C, as then you would need both statements. However, since you know the width, each one gives the necessary information.

94. A

Question Type: Yes/No. This question asks: "Is $x > 3$?"

Given information in question stem or diagram: No important information is given in the question stem.

Statement 1: The first step in this statement is to translate the wording into the following equation: $x + x^2 = 12$. Since this is a quadratic equation, you should set everything equal to zero so that $x^2 + x - 12 = 0$. Factoring this, you see that $(x + 4)(x - 3) = 0$ and x would be -4 or 3. The difficulty in this statement is that many people assume that this information is not sufficient because there are two values, one negative and one positive. However, remember that to prove sufficiency in a yes or no question requires only a definitive answer, not one value. Since each of these values (-4 and 3) gives a "no" answer to the question, this statement is sufficient. The answer is either A or D.

Statement 2: $x^2 > 9$. If $x^2 > 9$ then either $x > 3$, which gives you a "yes" answer, or $x < 3$, which gives you a "no" answer. For example x could be -5 (which when squared is > 9) or 5 (which when squared is also > 9). This statement is thus not sufficient, and the correct answer is A.

Note: This question is created to prey on two common mistakes, one relating to Data Sufficiency itself and one relating to algebra: 1.) People (even those who have done lots of data sufficiency) tend to forget to look for the "no" answer in Yes/No questions and they often make mistakes about what is really required for sufficiency on Yes/No questions. 2.) People forget about the negative possibilities when dealing with squared variables in inequalities.

95. D

Question Type: *What Is the Value?* This question asks for the specific value of positive integer a .

Given information in the question stem or diagram: There is a lot of information



to leverage from this question stem. a , b , and c are distinct positive integers; $a < b < c$; and the square root of $abc = c$. You should first manipulate the last one algebraically by squaring both sides to see that $abc = c^2$. Divide both sides by c (you can do this because you know that c cannot be 0 from the question stem) and the equation becomes $ab = c$. So before you even go to the statements you know that $ab = c$ and all of the variables are different positive integers.

Statement 1: $c = 8$. Combined with what you learned from the question stem, this means that $ab = 8$. Since a and b are distinct positive integers and $a < b$, the only possibility is $a = 2$ and $b = 4$. You might consider $a = 1$ and $b = 8$ but since the integers must be distinct, you cannot have $b = 8$ since $c = 8$. This is sufficient but you will only see that if you properly leverage every piece of information given in the question stem. Remember: When you are given even a small piece of information in the question stem it is usually very important. The correct answer is A or D.

Statement 2: The average of a , b , and c is $\frac{14}{3}$. This means that the total of $a + b + c = 14$. This statement is even trickier than the last but requires a similar leveraging of all available information. It may seem at first glance that there are many possibilities for the values of a , b , and c . However, the only way that $ab = c$ and $a + b + c = 14$ is for $a = 2$, $b = 4$, and $c = 8$. There is no other way to have three distinct numbers add up to 14 and have $ab = c$. This statement is also sufficient and the correct answer is D. This question provides an excellent example of a phenomenon you will see often in Data Sufficiency: When a lot of information is given in the question stem, statements are usually sufficient with much less information than you might first think.

96. C

Question Type: *What Is the Value?* This question asks for the value of integer x .

Given information in the question stem or diagram: You are told that x is an integer, so consider how that might be important with each statement.

Statement 2 is much easier to consider on this problem, so you can start there.

Statement 2: $x = 3$ or 4 . Clearly this is not one specific value for x so it is not sufficient. Eliminate choices B and D. Since this statement is quite easy, consider how it might be important in relation to Statement 1.

Statement 1: $\frac{1}{5} < \frac{1}{1+x} < \frac{1}{2}$. This can be restated using only the denominators: In order to have a smaller fraction (with the same numerator) you need a larger denominator so $5 > 1+x > 2$. Therefore x has to be < 4 and x has to be > 1 . The

only integers between 4 and 1 are 3 and 2. So x could be 2 or 3. Thus Statement 1 is also not sufficient. Eliminate choice A.

Together: When taken two statements that each give specific numbers, the key is the overlap. The only numbers that are acceptable when taking the statements together are those that satisfy the conditions in each of the two statements. In this case the only number that each statement allows is 3. Therefore $x = 3$ and the correct answer is C. Note: Before you pick answer C on this problem, you should carefully make sure that you couldn't do it from each of the statements alone.

97. B

Type of question: *What Is the Value?* The question asks for “the speed of car y (beyond point p) as a percent of the speed of car z.” Note: This is the same as asking for Distance/Time of car y as a percent of the Distance/Time of car z.

Given information in the question stem or diagram: The roads that car y and z are traveling on are parallel until point P. At point P the cars are even and then car y makes a turn of x degrees and continues to keep up with car z. The key information that you need to leverage from this given information is that the times will be the same for each car.

Statement 1: “The speed of car z = 50 mph.” This statement does not give any information about the speed of car y, nor about the extra distance that car y must travel. This statement is not sufficient on its own. Eliminate choices A and D, and note that this is a classic “Why Are You Here?” set-up. This statement is clearly insufficient on its own so you will want to consider whether it is important in relation to Statement 2.

Statement 2: “The measure of angle $x = 45$ degrees.” This statement may not appear sufficient until you realize that you do not actually need to know the speeds of cars y and z. Because the cars are keeping up with each other, the time that each car travels must be equal. That means that the increased percentage of distance that car y has traveled will be the same as the increased percentage of the rate of car y compared to car z. In other words, determining the ratio of the distance of y to the distance of z is enough to answer this question because the times are necessarily the same. While you do not need to do it, the actual mathematical solution is below. Also, note that this is a classic example of the “Why Are You Here?—Temptation” set-up that is used so often on harder problems. They have tempted you with a statement (the speed of car z) that is not necessary to determine sufficiency. Statement 2 is sufficient by itself.

The road that car y travels on after point P forms a triangle with the road that y



was traveling on (that road is parallel to the road z is traveling on). This triangle is a right triangle with the right angle formed by the dotted line. Statement 2 gives the measure of x at 45 degrees. That means you have a 45–45–90 triangle with the distance that y has traveled as the hypotenuse and the distance z has traveled as one of the shorter sides. The hypotenuse is $\sqrt{2}$ bigger than the shorter sides, so car y is $\sqrt{2}$ faster than car z. As noted above, this is sufficient and the correct answer is B.

98. A

Question Type: *What Is the Value?* This question asks you for the specific value of $\frac{b-c}{b}$.

Given information in the question stem or diagram: a, b, and c are integers, and $\frac{a}{b-c} = 1$. Before going to the statements it might be helpful to restate that equation. If you multiply both sides by $b - c$, the equation becomes $a = b - c$. Looking back to the question you can now phrase the question as “What is $\frac{a}{b}$?”. Remember that often the important work in Data Sufficiency questions relates to manipulating the information in the question stem, not just in the statements.

Statement 1: “ $\frac{a}{b} = \frac{3}{5}$.” The work that you did on the question has paid off. This statement exactly answers the question: $\frac{a}{b} = \frac{3}{5}$. This is sufficient, and the answer is either A or D.

Statement 2: “a and b have no common factors greater than one.” This means that a and b are what is called “co-prime.” For example, 8 and 9 are “co-prime.” Neither has to be a prime number but in relation to each other they have no common factors greater than 1. This statement is not sufficient, as there are many co-prime numbers. Since this is clearly insufficient by itself, it is a “Why Are You Here?” set-up and you should wonder why testmakers are delivering this statement. Is it important? It would be important if you were trying to find the value of a or b individually, but since the question only asks for a ratio, the individual values of a and b are not important and the answer is A.

99. A

Question Type: *What Is the Value?* This question asks for the specific value of $x - y$.

Given information in the question stem or diagram: No important information is given in the question. Note that you only need to find the difference between the two variables. This will usually require less information than solving for the individual variables.

Statement 1: This is a perfect example of using algebraic manipulation to find the value of $x - y$. By digging deep into your algebra toolkit and by using your

understanding of the common algebraic equations, you should manipulate accordingly: $(x + y)^2 = 4xy$ can be rephrased as $x^2 + 2xy + y^2 = 4xy$. Subtract $4xy$ from both sides to create the quadratic equation $x^2 - 2xy + y^2 = 0$. This can be factored to $(x - y)(x - y) = 0$. Therefore $x - y = 0$. This statement is sufficient, and the answer is either A or D.

Statement 2: $x^2 - y^2 = 0$. This statement again requires proper algebraic manipulation and recognition of the difference of squares: $x^2 - y^2 = 0$ is the same as $(x + y)(x - y) = 0$. However, it is not clear whether $x + y = 0$ or $x - y = 0$ so this statement is not sufficient. The correct answer is A. Note: This problem demonstrates how important it is to have a high level of fluency in algebra, and with the common algebraic equations in particular. There is no way to prove sufficiency here without this fluency.

100. B

Question type: Yes/No. The question asks: "Is the product $(a)(a + 1)(a + 2)$ divisible by 48?" You can think ahead of time that in order to be divisible by 48 this product must have the prime factors of 48: $2^4 \cdot 3$. So the question is really: Does this product contain at least four 2s and one 3?

Given information in the question stem or diagram: "a is a positive integer." Since a is a positive integer the rule that "in any three consecutive integers one of those integers will be a multiple of 3" applies. That means that before you even go to the statements you know that the product $(a)(a + 1)(a + 2)$ will be a multiple of 3. The question then can be simplified even more from above because you know that the factor of 3 will be present. The simplified question is: Does this product have 2^4 as a factor? It is essential that you always leverage all given information in the question before moving to statements. Also note that this problem (as is true for most arithmetic problems) is best solved with your conceptual understanding of factors and divisibility. While you could prove sufficiency/insufficiency with number picking, it would be cumbersome and risky in this example.

Statement 1: a is even. If a is an even number, it means that a will contain at least one 2 as a factor. It also means that $a + 2$ will be even and that one of those two even numbers will be a multiple of 4. For example, if $x = 2$ then $(x + 2) = 4$. This means that you have at least 2^3 as a factor. However, this statement is not sufficient as it only guarantees three 2s in the product and not the required four 2s. Eliminate choices A and D.

Statement 2: If $4a$ is divisible by 32 then "a" must be divisible by 8. If a contains three 2's as factors then this information is sufficient as you know that $(a + 2)$ will



have to contain at least one 2 as well. This statement is sufficient to prove that the product will contain $2^4 \cdot 3$ and the correct answer is B.

101. A

Question Type: *What Is the Value?* This question asks for the area of the rectangle PQRS.

Given information in the question stem or diagram: The diagram is given and importantly one side is defined with PS = 4, so both PS and QR = 4. To find the area you need to somehow determine either RS or PQ from the information in the statements. The question is really: What is RS or PQ?

Statement 1: QTR is an equilateral triangle. If this is true, then with QR as the base, the height of the equilateral triangle is equivalent to the side that you are trying to determine (PQ or RS). Since the base of QTR is 4 you can determine the necessary height with your knowledge of 30–60–90 triangles or simply with your knowledge that if you know one thing about an equilateral triangle, then you know everything! While you do not need to calculate it, the height would be $2\sqrt{3}$ as it is the long leg in a 30–60–90 triangle formed by $\frac{1}{2}$ the base of QTR (2), the hypotenuse QT (4) and the height ($2\sqrt{3}$) which is equivalent to PQ or RS. Statement 1 is sufficient, so the answer is A or D.

Statement 2: In this difficult statement, the testmakers are playing with a common trick. They have polluted your brain with the first statement and want you to assume that if “segments PT and TS have equal lengths,” then QTR must again be equilateral. However, this statement does nothing to help you determine the length of the sides (QP and RS) because it only proves that QTR is isosceles. There is no limit put on the lengths of PQ and RS (because you do not know the angle of TQR and TRQ) with this statement, so it is not sufficient. Remember: One of the keys to success in Data Sufficiency is to consciously avoid assumptions, but that can be hard when you are set up so nicely to make assumptions with the other statement. Statement 2 is not sufficient, so the correct answer is A.

102. A

Question Type: Yes/No. The question asks: “Is $\frac{1}{x} + \frac{1}{y} = 16$?”

Given information in the question stem or diagram: xy does not equal 0. This is important only because x and y cannot be 0, as they are denominators. Importantly, you should manipulate the equation in question. As you learned with the Algebra Toolkit, in almost any equation involving fractions, the first step to simplification is to remove the denominators. Here that is done by multiplying the equation by xy to create the new question: Is $y + x = 16xy$?

Statement 1: If you have done the proper manipulation as shown above, it is clear that this statement is exactly the same as the changed question and thus sufficient. The answer is A or D. Note: Do not forget that on many harder Data Sufficiency questions it is easier to change the question to match the statements than vice versa. This provides an excellent example of this phenomenon and shows the importance of proper algebraic simplification in Data Sufficiency questions.

Statement 2: If you take this information ($x = y$) and substitute it into the question above you get: Is $2x = 16x^2$? Or: Is $2y = 16y^2$? These questions are impossible to answer, as you do not know anything about the value of x or y. Statement 2 is not sufficient so the answer is A.

Note: If you picked answer C on this question you have fallen for the “C Trap”! Most test-takers immediately see that this question can be answered with both statements together, as substituting x for y turns Statement 1 into a single variable equation, allowing you to solve for one variable, which we know from Statement 2 is exactly the same as the other. But in situations in which C seems “too easy,” one should take the more-comprehensive statement and try to exhaust it, as the GMAT is apt to reward you for being able to obtain more value from each resource. Statement 1, as shown earlier, is actually the same as the question, but this is only apparent after the proper manipulation shown above.

103. E

Question Type: *What Is the Value?* This is a Venn Diagram question and the question asks: “How many students take both French and Spanish?”

Given information in the question stem or diagram: In any Venn Diagram problem you should draw out the diagram and remember that Total = Set 1 + Set 2 – Both + Neither. It is important to always consider “Neither” in these types of DS questions and to note the difference between total set information and “only” information (for instance, the difference between people who take “only French” and “French”).

Statement 1: French = 100 and Spanish = 125. This can be written to say that French + Spanish – Both + Neither = Total or $100 + 125 - \text{Both} + \text{Neither} = \text{Total}$. This statement is not sufficient alone because you do not know how many total students there are, nor how many students take neither French nor Spanish. Not sufficient. Eliminate choices A and D.

Statement 2: Total students = 200. This is not sufficient alone since there is no way to allocate the 200 students among the categories. Eliminate choice B.



Together: Many students will think that the answer is C because they have forgotten to consider “Neither.” You do not know how many students take neither French nor Spanish, so there is no way to determine “Both.” Without this information you only know that at least 25 students take BOTH (that is if Neither = 0) and as many as 100 might take BOTH if the neither category was 75. This is not sufficient and the correct answer is E. Note: In any choice between C and E such as this you must learn to Play Devil’s Advocate and consider anything you might be missing. In this example (as with many Venn problems) that is making sure you remember to consider the value of “Neither.”

104. B

Question Type: Yes/No. The question asks: “Is x positive?”

Given information in the question stem or diagram: No important information is given in the question stem.

Statement 1: $x^2 = 9x$. Many students will think that $x = 9$ after dividing both sides by x and thus believe that this statement is sufficient. However, it is important to consider all of the possibilities, and you are not allowed to divide by x if it is 0. Given this, the best way to approach this question is to subtract $9x$ from both sides to get $x^2 - 9x = 0$. Factor out x and see that: $x(x - 9) = 0$. So clearly x can equal 9 or 0. Because this is a yes or no question, you must check each value to get a “yes” or a “no.” Just because you have two values for x does not make the statement automatically insufficient. $x = 9$ gives you a “yes” since x is positive. $x = 0$ gives you a “no” since zero is not greater than zero. So this statement is not sufficient and the answer is either B, C, or E. Note: This is a common and tricky algebraic set-up in data sufficiency. Mathematically, whenever you have a squared variable you should set the equation equal to zero and factor.

Statement 2: $|x| = -x$. This statement appears more difficult than it really is. The key of course is to figure out what this equation tells you about x . Hopefully you recognize that x must be negative OR zero given this statement, but if not, you can try some numbers to prove it. Try a positive number for x such as 9. That gives you $|9| = -9$, which means “9 = -9.” This is not true so you can see that any positive numbers are excluded by this statement. Try a negative number: $x = -5$. Now $|-5| = -(-5)$, which means $5 = 5$. This is true so you can see that this statement allows for negative numbers. You will also want to try zero, but as in the last statement, many students forget about zero. $|0| = -0$. This is true ($0 = 0$) so negative numbers as well as 0 work for this statement. Taking these back to the question you see that any negative numbers give you a “no” answer. Zero

also gives you a “no” answer so there is a consistent “no” and this statement is sufficient. The answer is B.

Note: The difficulty of this question lies in both the tricky algebra and in the fact that you might pick answer C even if you do the algebra properly. Why? Because people forget that “no” is just as good as “yes” in the second statement and they also see that 0 nicely overlaps between the statements, making it tempting to think “C.” If the question was asking “What is the value of x ?” then the answer would be C. However, since this is a Yes/No question, the second statement gives you a definitive answer and is thus sufficient.

105. A

Question Type: *What Is the Value?* This question asks for the average of the terms in set J.

Given information in the question stem or diagram: No important information is given in the question stem.

Statement 1: The sum of any three terms in the set is 21. This is a very difficult statement to consider. Clearly this WILL be true if all the terms are 7, but you need to make sure there are not any other possibilities. Will it work with any other sets, such as “6, 7, 8, 6, 7, 8”? Adding certain numbers in this set will give you 21, but not “any three terms.” The only ways to ensure that any three randomly chosen terms will sum to 21 are either to have only three terms (say, $6 + 7 + 8$) or to have all terms in the set equal 7. In either case, the average must be 7, so this is sufficient information. The answer is A or D.

Statement 2: Set J consists of 12 terms. This statement is not sufficient on its own since it does not give any values for the terms in the set. However, as is the case with any clearly insufficient statement such as this, you must consider whether it is important with the other statement. This statement was clearly designed to make choice C a more attractive option, but you do not need to know anything about the number of terms for the first statement to be sufficient. The correct answer is A.

106. B

Question Type: Yes/No. The question asks whether line M runs through point $(6,6)$.

Given information in the question stem or diagram: Line M is tangent to a circle; the circle is centered on point $(3,4)$.

Statement 1: Line M runs through point $(-8,6)$. This statement is not sufficient because there is no information about the size of the circle or the direction that



line M is running. Simply having a single point on line M will not be sufficient to answer the question. Eliminate choices A and D.

Statement 2: Line M is tangent to the circle at point (3,6). Many people will immediately combine this with Statement 1 in their minds and think that with both statements you know that M is the straight line $y = 6$, in which case line M would definitely run through point (6,6). However, before you jump to this stage, remember that you only get to combine statements when neither statement is sufficient alone. Before you pick choice C in a question like this, it is always essential that you take a closer look at each statement individually.

The definition of tangent is very important for Statement 2 and it is what differentiates Statement 2 from Statement 1. A line that is tangent to a circle touches that circle in only one place and is perpendicular to that circle at that point. Given that the center of the circle is at (3,4) and the point of tangency is at (3,6), you can conclude that line M is perpendicular to the line $x = 3$. Therefore line M must be the line $y = 6$, and it does pass through (6,6). This statement is sufficient alone and the answer is B. Note: This is another great example of the “C trap” set-up in which testmakers give you two statements that fit together nicely but are not both required to answer the question. To combat this set-up, use the reward system from the lesson and always double-check each statement alone before picking answer C.

107. A

Question Type: *What Is the Value?* The question asks you for the remainder when x is divided by 10.

Given information in the question stem or diagram: x is a positive integer. When x is divided by 7 the quotient is q and the remainder is 1. Number picking is an important and valid strategy in quotient/remainder questions, so you might even want to consider possibilities for x before moving to the statements. If the remainder is 1 when x is divided by 7, then x could be 1, 8, 15, 22, 29, 36, 43, etc., yielding different quotients for q .

Statement 1: When x is divided by 5 the quotient is q and the remainder is 1. In the question stem, you learned that when x is divided by 7 the quotient is also q and the remainder is also 1. Most students understand that if you divide x by two different divisors and get the same remainder, x must equal the sum of the remainder and a multiple of the divisors’ LCM. For instance, if you divide a number by 5 and 7 and get a remainder of 1, then that can happen at 35 + 1; 70 + 1; 105 + 1; etc.

However, if you read carefully you see that the quotient is the same in both operations, a puzzling result. Clearly if you divide 36 by 7 it will give you a different quotient than when you divide 36 by 5. However, students will overlook this important fact (that q is the same in both cases) and believe that there are multiple possibilities for x and thus multiple possibilities for the remainder when you divide x by 10. The only way you can have the same quotient and the same remainder when you divide by two different numbers is if you are dividing a smaller number by a bigger number—in this case 1. Remember that when a smaller integer is divided by a larger integer, the quotient is always 0 and the remainder is the dividend itself. Here when 1 is divided by 7 the quotient is 0 and the remainder is 1, and when 1 is divided by 5 the quotient is also 0 and the remainder is 1. Statement 1 is sufficient, as x must be 1 and the remainder when x is divided by 10 is also 1. The answer is either A or D.

Statement 2: x is less than 50. This is clearly not sufficient by itself. This statement is here for people who miss that the quotient is the same and think that x could be any (multiple of 35) + 1. By knowing that x is less than 50 it would seem to lock in the value at 36. However, from the discussion above it is clear that this would be incorrect. This is a very clever “C trap” because it makes the person choosing answer C feel like they have understood the problem while the answer is really A. The only way to get this right is by knowing this tricky and easily overlooked fact in Arithmetic and by reading the information closely (leveraging that q is the same when dividing by two different numbers—a puzzling result).

108. B

Question Type: *What Is the Value?* This question asks you for the number of families that have a dog.

Given information in the question stem or diagram: There are a total of 60 families in the neighborhood. 38 of these families have a cat, or $C = 38$. Remember that you have been given the total number that have cats (not only cats) and you have been told nothing about how many of these total families have neither a cat nor a dog.

Statement 1: This statement tells you that 28 families have only a cat. Since you know that there are 38 total families with a cat, this means that 10 families must have both. At this point, if you forget about neither, you might think that this is sufficient because the number of families with dogs would be $60 - 28$ (those with only a cat), or 32. However, you do not know how many families have neither a cat nor a dog so perhaps fewer than 32 families have a dog. This is not sufficient, so eliminate choices A and D.



Statement 2: In this statement you are told that "Both = Neither." This information does not appear to be sufficient at first glance, but a closer look at the venn diagram formula for two set problems shows that it is. Total Families = # of Families with a Cat (C) + # of Families with a Dog (D) – Families with Both (B) + Families with Neither (N) or with variables: $T = C + D - B + N$ Combining this with the facts we know that $T = 60$. $C = 38$ and $B = N$, meaning that if you substitute "B" for "N" (they are equal) you get $38 + D - B + B = 60$. Simplify and $38 + D = 60$. $D = 22$. This statement is sufficient. The correct answer is B.

Note: This is a very difficult example of the common "C trap" scenario. Together, you can clearly find the value for the number of families with dogs. Only with a very careful analysis of each statement alone will you realize that Statement 2 is sufficient. The fact that the two statements make it pretty easy to solve gives you a hint to look VERY carefully at each statement.

109. A

Question Type: Yes/No. The question asks whether $\frac{5x-2}{3} - \frac{5x-1}{4} > 0$.

Given information in the question stem or diagram: x does not equal 0, and you are given a question in inequality form that can be easily simplified. Before you move to the statements, you should simplify the question. Multiply both sides of the inequality by 12 (LCM of 3 and 4) to see that the question is really: Is $20x - 8 - (15x - 3) > 0$? With a bit more simplification (being VERY careful with parentheses and negative/positive signs) the question becomes: Is $20x - 8 - 15x + 3 > 0$? After combining like terms, it's: Is $5x > 5$? As it turns out this question is really asking: Is $x > 1$?

Statement 1: If you have properly simplified the question, it is clear that this statement exactly answers the question and is thus sufficient. If you have not, it will be both difficult and tedious to prove with number picking. Remember: With Data Sufficiency questions involving inequalities, manipulating algebraically is almost always the best way to prove sufficiency. Often this involves manipulating the question, not just the statements. Since this is sufficient, the answer is A or D.

Statement 2: $x = |x|$. This indicates that x is a positive number (since you are told in the facts that x does not equal 0). However, this does not mean that x is larger than 1 so you have a "maybe" answer. (Numbers between 0 and 1 would give a "no" answer while all others would give a "yes" answer.) This statement is not sufficient. The correct answer is A. Note: This problem is very hard if you do not manipulate the question, but relatively easy if you do!

110. D

Type of Question: *What Is the Value?* This question asks for the greatest common factor of x and y (in other words, the largest integer that will divide into both numbers x and y).

Given information in the question stem or diagram: x and y are positive integers. Also note that any factor question such as this generally relies on your conceptual understanding of the underlying Arithmetic, not on manipulation or number picking.

Statement 1: When x is divided by y the remainder is 1. Conceptually, this means that any factor of y other than 1 cannot divide into x , because it, too, will have a remainder of 1 when divided into x . Imagine as an example if x were 31 and y were 30. None of the factors of y greater than 1 (2, 15, 3, 10, 5, 6) can divide into 31; they must all necessarily yield a remainder of 1 as well. So if $y = 10$ and $x = 31$ there will be the same result. Therefore, if there is a remainder of 1 when x is divided by y then these numbers are “co-prime,” meaning that the only factor they can share is 1. Statement 1 is sufficient as the greatest common factor (GCF) must be 1. This would also be true if x is smaller than y , as x would have to be 1 and it would share only the factor of 1 with any other number. This statement is sufficient. The answer is A or D.

Statement 2: $x^2 - 2xy + y^2 = 1$. This requires that you first do some algebraic manipulation. As with any of the common algebraic equations, if they are given to you in one form you should typically convert it to the other form. Here $x^2 - 2xy + y^2 = 1$ is the same as $(x - y)^2 = 1$. You can determine from this equation that positive integers x and y must be consecutive numbers on the number line. For this equation to be true, the difference between x and y must be 1. For instance x could be 2 and y could be 3, or x could be 3 and y could be 2. Regardless of which it is, the greatest common factor of any consecutive integers on the number line is always 1 for the same reason as discussed in Statement 1: The factors of any positive integer x (when x is greater than 1, which is a factor of all numbers) cannot divide into a number directly adjacent to x on the number line. Therefore, this statement is also sufficient and the answer is D.

Note: Like many arithmetic DS questions, this one requires a deep understanding of the underlying math. Data Sufficiency construct thinking will not help you on this problem, as it is more about the math than the Data Sufficiency question type.

**111. C**

Question Type: Yes/No. The question asks: "Is $(y - 10)^2 > (x+10)^2$?"

Given information in the question stem or diagram: While no information is given in the question stem, it is essential that you change/simplify the question. As in most difficult inequality problems, algebraic manipulation, not number picking, is the key to success. In this very difficult example, most of the algebraic manipulation has to be done with the question stem itself. To simplify the question, you should first expand the two common algebraic equations so the question looks like this: Is $y^2 - 20y + 100 > x^2 + 20x + 100$?

Remove 100 from each side to get the question: Is $y^2 - 20y > x^2 + 20x$?

The next step is difficult and not obvious. By moving x^2 from the right side to the left side in the question and moving $20y$ from the left side to the right side, you create the difference of squares on the left and an easily factored expression on the right: Is $y^2 - x^2 > 20y + 20x$?

Factor each side to change the question to: Is $(y + x)(y - x) > 20(y + x)$?

At this point many students will incorrectly divide each side by $(y + x)$ to cancel that expression. But remember that you cannot divide by variables in an inequality unless you know the signs of the variables involved. At this point you have simplified the question as much as possible, and you should look at each statement with your new question in front of you: Is $(y + x)(y - x) > 20(y + x)$?

Statement 1: " $-y > x + 5$ " or, after rearranging, " $y + x < -5$." Given the "new" question that you have just created above, this is a very important piece of information, as it says that $y + x$ is negative. If you know the sign of $y + x$ you can then again simplify the question: Is $(y + x)(y - x) > 20(y + x)$?

Divide both sides by $(y + x)$ and flip the sign because you know from Statement 1 that $y + x$ is negative. Using Statement 1, then, the question becomes: Is $y - x < 20$?

While you cannot answer this question yet, Statement 1 has helped get you closer by allowing more simplification of the question. Statement 1 is not sufficient. Eliminate choices A and D.

Statement 2: " $x > y$." This is clearly insufficient by itself. (With simple number picking or conceptual thinking you can see that you can get both a "yes" and a "no" from the original question.) This is not sufficient. Eliminate choice B.

Together: When you take the two statements together you know that the question is really: Is $y - x < 20$? AND you know that $x > y$. That second statement means that $y - x$ must be a negative number (whenever you subtract a larger number from a smaller number you get a negative result) and you are sure that $y - x$ will always be less than 20. The statements are sufficient together, and the answer is C. Note: This type of inequality question in data sufficiency form is a very difficult one for students. Ninety percent of the time, these questions require that you use algebraic manipulation to prove sufficiency, and this problem is no exception.

112. C

Question Type: *What Is the Value?* This question asks you for the value of y .

Given information in the question stem or diagram: No important information is given in the question stem.

Statement 1: $x^2 - y^2 = 5$. This is not sufficient because x and y can be many pairs of real numbers. For instance x could be $\sqrt{15}$ and y could be $\sqrt{10}$, or they could be negative integers such as $x = -3$ and $y = -2$, or positive integers such as $x = 3$ and $y = 2$. Not sufficient. Eliminate choices A and D. Note: If you assume that x and y are positive integers (which many people do!), then this would appear to be sufficient.

Statement 2: x and y are each positive integers. This is clearly insufficient, as y could be any of the infinite set of positive integers. Eliminate choice B but note that is a perfect example of the "Why Are You Here?" construct.

Together: Taken together, you should consider why Statement 2 (which is clearly insufficient by itself) might be important when analyzing Statement 1. In other words, consider what hints are being given by the second statement. If x and y are each positive integers, then x must be greater than y , as the difference is positive 5. Furthermore, there is only one set of perfect squares that can yield a difference of 5 and that is $(3)^2 - (2)^2$. As you move up the number line, the difference between any set of perfect squares increases to infinity, so there is only one place on the number line where the difference is 5. As a result, x must be 3 and y must be 2. If you take both statements together, they are sufficient. The correct answer is C.

113. E

Question Type: Yes/No. The question asks: "Is a^b an integer?"

Given information in the question stem or diagram: a and b are nonzero integers. Note: The fact that they cannot be zero is likely important so make sure you leverage that as you look at each statement.



Statement 1: “ b^a is negative.” This statement tells us that b is negative and that a is odd. (If a were even the result could not be negative.) Taking this information back to the question stem, you will find that it is not sufficient. For almost all values of a^b , where b is negative, the result is a non-integer, as a negative exponent means that you are creating a fraction. But if the denominator of that fraction is 1 or -1 (which is allowed by this statement) then a^b is either 1 or -1, and that means that a^b is an integer. Statement 1 is therefore not sufficient, so eliminate choices A and D. This is a good illustration for how important it is to Play Devil’s Advocate when number picking. It is easy to show that a^b can be a non-integer, but you must consider all possibilities to see that there is an exception.

Statement 2: “ a^b is negative.” This isn’t particularly helpful, as it tells us only that b must be odd (positive or negative) and that a is negative. If you go back to the question stem you see that if b is negative, then you can have a fraction, but if b is positive then we’re guaranteed an integer.

Together: Even taken together, the possibility still exists for a^b to be -1, particularly if a were -1 and b were -1. This results in an integer. All other possibilities, such as $a = -3$ and $b = -1$, which results in $-\frac{1}{3}$, yield nonintegers, but that one lone catch of -1 makes statements together insufficient, leading to answer choice E. Note: This problem is all about Playing Devil’s Advocate. If you are forced to number pick or think conceptually, make sure you consider numbers like 1,-1, 0, and fractions to find those tricky exceptions.

114. C

Question Type: *What Is the Value?* This question asks for the value of the remainder when x is divided by y .

Given information in the question stem or diagram: x and y are integers; x and y are not negative. Note: Anytime you are told that a variable is “non-negative” you should always remember that it could be zero as well as positive when analyzing the statements (although it does not end up mattering in this example!).

Start with Statement 2, the easier statement.

Statement 2: “The numbers x and y have a combined total of fewer than 5 digits.” This is clearly insufficient on its own, as it allows for thousands of combinations of x and y . Eliminate choices B and D.

Statement 1: $\frac{x}{y} = 13.8$. This question, with Statement 1, tests your understanding of the relationship between a remainder and the decimal representation of a

quotient. Perhaps the easiest way to recognize what information is conveyed by the decimal about the remainder is to express 13.8 as a mixed fraction: $13\frac{4}{5}$. Still, this does not guarantee that our remainder is 4. We know only that the remainder is a multiple of 4.

Next, express the mixed fraction $13\frac{4}{5}$ as an improper (top-heavy) fraction: $\frac{69}{5}$. If $\frac{x}{y} = \frac{69}{5}$, then perhaps $x=69$ and $y=5$, or perhaps $x=138$ and $y=10$. In other words, $\frac{x}{y} = \frac{69}{5} = \frac{138}{10} = \frac{207}{15}$, etc. If $x=69$ and $y=5$, the remainder in question is 4. But if $x=138$ and $y=10$, the remainder in question is 8. Statement 1 alone is not sufficient. Eliminate answer A.

Together: What we learned from Statement 1 is that x is a multiple of 69 and y is the corresponding multiple of 5. For example, x and y could be the second positive multiples of 69 and 5, respectively: 138 and 10. Of course, from Statement 2, we know that 138 and 10 have too many total digits to be the values of x and y . All bigger possible multiples of 69 and 5 would certainly have five or more total digits, as well. Thus, with both statements combined, we know that $x=69$ and $y=5$, and that the remainder in question is 4. The correct answer is C.

Note: There are several ways to deal with remainder questions on the GMAT, and the specific form of the question should dictate what approach you take. If you are dealing with a decimal representation on a remainder question, converting the decimal to a mixed fraction, and then to an improper fraction, is a useful approach. Note also that many remainder questions on the problem-solving format ask not for a precise value of a dividend, divisor, or remainder, but for a possible value of some piece (e.g., “Which of the following *could* be the value of the remainder?”). From the above explanation, you can see that these “could be” questions hinge on divisibility issues, or what the piece in question is a multiple of.

115. D

Question Type: What Is the Value? This question asks for the length of the arc XYZ. This is really asking you for the diameter ($q+r$), as that will allow you to calculate the circumference.

Given information in the question stem or diagram: You are told that arc XYZ is a semicircle and given all the geometric information in the diagram itself. You know that angle XYZ is measures 90 degrees; anytime a triangle is inscribed in a circle, and one side of the inscribed triangle is a diameter of the circle, that diameter is the hypotenuse of a right triangle.

Before you even begin work on this one, recognize this: Choice C is much too easy an answer. If you know the diameter of a semicircle (which Statements 1



and 2 together hand you on a silver platter), you can easily find the length of that semicircle (half the circumference). Don't take the bait on choice C—or at least recognize that there's a very, very high likelihood that the answer will be something other than C, so it's worth your time to do some math here.

Recognize also that statements 1 and 2 both give you the same kind of information: One segment of the diameter and the side of a smaller right triangle within the larger right triangle. So if one statement is sufficient, the other one will likely be sufficient too.

Statement 1: This tells us that $q = 2$. And since we also have that height of 4, we can solve for line XY. But remember: Data Sufficiency is about whether you CAN SOLVE the problem, not about THE ANSWER itself. So save yourself the messy Pythagorean work and just call line XY "known." It's an ugly number (well, not that ugly—it's the square root of 20), but you don't need the exact number. Note now that that line XY is also part of one other Pythagorean calculation, for the large triangle (XYZ). That side squared plus side YZ squared will give us $(2 + r)^2$. And our goal is to solve for r (because $r + 2$ is the diameter, which leads us to the circumference). So let's think about other calculations we know for r. The right-hand side smaller triangle also includes r and side YZ. On that one, $r^2 + 4^2 = (YZ)^2$. So we have two equations with r:

$$(\text{KNOWN})^2 + (YZ)^2 = (2 + r)^2$$

$$(YZ)^2 = r^2 + 4^2$$

Here, notice that because $(YZ)^2$ is in two equations, we can substitute for it and get everything in terms of r. Substituting the second equation into the first, you have:

$$(\text{KNOWN})^2 + r^2 + 4^2 = (2 + r)^2$$

And remember: The numbers themselves don't really matter—just the fact that we'll get numbers. So on the left-hand side:

$$r^2 + (\text{a known number}) = 4 + 4r + r^2$$

The r^2 terms will subtract out, and now you have a linear equation with just 4r. You can prove here that you will get a value for r, so Statement 1 is sufficient. Also, these three triangles are all similar to each other and you can come to the same solution by leveraging that fact (and is much faster IF you see it).

Statement 2: Since, as we've determined, each statement essentially gives you

the same amount and type of information, Statement 2 is sufficient, also. You do not want to go through the same process in the other direction; just know you can do it. The answer is D, but the takeaways may be more important:

1. When the math looks messy in a specific-value problem, know that you don't need to actually worry about official numbers.
2. When choice C looks too good to be true, it probably is.
3. Geometry is about leveraging assets, so it's quite possible for geometry based statements to be "sneaky sufficient." Make sure that you leverage all your assets in geometry-based Data Sufficiency.

116. A

Question Type: *What Is the Value?* This question asks you for the value of x .

Given information in the question stem or diagram: y is an odd integer, and $xy = 222$.

Statement 1: x is a prime number. When combined with the information from the question stem, this is sufficient. If xy is even number and if y is an odd integer, this means that x must be even (or a non-integer, but "x is prime" rules that out). Since x is a prime number, it can only be 2, the only even prime number. This gives you one specific value for x and is sufficient. The answer is either A or D.

Statement 2: y is a 3 digit number. The mistake that students make here is that they carry information from Statement 1 to Statement 2. Your brain has been polluted by being told that x is a prime number in the first statement! You now assume that x is a positive integer but you don't know that from the question stem. If x were a positive integer (as all prime numbers are), then the only way for y to be an odd three-digit number and $xy = 222$ is for $x = 2$ and $y = 111$. However, you do not know that x is positive nor that it is an integer. $x = -2$ and $y = -111$ would work, as would $x = \frac{222}{999}$ and $y = 999$. Each of these variations fits both the information from the question stem and Statement 2. There is not just one value for x , so this statement is not sufficient, and the correct answer is A. Note: It is essential that you always consciously ask yourself whether you are carrying information from the other statement. Make sure you "reset" your brain every time you move to the next statement.

117. A

Question Type: Yes/No. The question asks: "Is $\frac{x}{y}$ an integer?"

Given information in the question stem or diagram: x and y are positive integers.



Statement 1: Every factor of y is also a factor of x . This means that whatever factors are in the denominator, those same factors are also in the numerator. So if $y = 30$ (with factors of $2 \cdot 3 \cdot 5$) then x will have at least these factors. In other words, x will be a multiple of 30. If x has all of the factors of y then $\frac{x}{y}$ will always be an integer. This statement is sufficient, and the answer is either A or D.

Statement 2: Every factor of x is also a factor of y . This statement may seem to be identical to Statement 1, but it is not! All of the factors of x will also be present in y , but y could contain other factors. For instance, x could be 30 and y could be 60, meeting all the conditions in this statement. Or they could each be equal to 30. This statement allows for $\frac{x}{y}$ to be the integer 1 but also many non-integers, so it is not sufficient and the correct answer is A. Note: In any question like this, where the statements appear to be the same, you should be highly suspicious of choice D as the correct answer!

118. E

Question Type: *What Is the Value?* This question asks for the sum of the first 98 terms of the sequence.

Given information in question stem or diagram: There is a lot of information to digest before moving to the statements. $a_1 = x$; $a_2 = y$; $a_3 = z$; $a_4 = 3$; $a_n = a_{n-4}$ for $n > 4$. This means that not only is $a_8 = a_4 = 3$, as each term is equal to the term four before it. This also means that $a_1 = a_5 = a_9$; $a_2 = a_6 = a_{10}$; $a_3 = a_7 = a_{11}$; etc. The sequence begins: $x, y, z, 3, x, y, z, 3$, and carries on infinitely in that pattern. Therefore, if you can determine the values of the first four terms in the sequence you can extrapolate from those in cycles of four. If you divide 98 by 4 you get 24 with a remainder of 2. This means that the sequence goes through 24 cycles of $x, y, z, 3$, through the 96th term, and then terms 97 and 98 are x and y , respectively. This is important because in order to find the sum of the first 98 terms you need not only the sum of the four-term pattern (which you can multiply by 24 to get the sum of the first 96 terms) but also the sum of the extra $x + y$ from the 97th and 98th terms. Leveraging all this, you can remove the sequence information from the question and realize that the question is really: What is $24(x + y + z + 3) + x + y$? Now it is just an algebra problem!

Statement 1: $x = 5$. This statement is clearly not sufficient, as there is no way to determine the value of y or z . Eliminate choices A and D.

Statement 2: $y + z = 2$. This statement is also clearly insufficient as, most conspicuously, it tells you nothing about x . But also notice that you need the individual value of y to fill in the algebra above, as you need $25y$ but only $24z$. The statement is not sufficient. Eliminate choice B.

Together: Taken together with the question stem, the two statements allow you to get the sum of the first four terms: $x + y + z + 3 = 10$. Thus the sum of the first 96 terms is $24*10$ or 240. However, you still need to be able to give a value for those last two numbers “ $x + y$.” You have the value of x from Statement 1, but Statement 2 combines two terms in the sequence and therefore does not give you a clear value for the variable y , which represents a_2 and therefore a_{98} . Even together these statements are not sufficient.

If the question had asked for sum of the first 96 or 97 terms, or the first 99 or 100 terms, you could have supplied that specific value. It is only because you cannot separate y and z —and because the 98th term falls directly between the two—that you cannot solve. The correct answer is thus E. Note: This is a very hard C vs. E choice. Almost everyone can get it down to those two choices, but it requires careful analysis to realize that the answer is E. As in so many DS questions, the key again is to simplify the question and all the given information before you move to the statements.

119. D

Question Type: Yes/No. This question asks: “Is $14,743 + x$ prime?”

Given information in the question stem or diagram: x is a positive integer and $x < 10$. That means that $x = 1, 2, \dots, 9$ and that $14743 + x$ will equal 14,744, 14,745, ..., 14,752. Also note: It is difficult to confirm that a large prime number is prime. For instance there is no good way to determine whether 1,000,001 is prime. However, it is very easy to show that 1,000,011 is NOT prime, because you know it is divisible by 3 (as the sum of its digits is divisible by 3). So your strategy here should really be to try to prove that the possible values are not prime, by finding factors of each possible value.

Statement 2 is easier because it gives you a specific value for x , so you should begin there.

Statement 2: $x^2 = 36$. Normally $x = 6$ or -6, but you have the fact that x is positive. So x must equal 6. This statement is sufficient because when you add 6 to 14,743 you will get a single value, and that number will either be a prime number or not; you do not really care which. You only need to know that the answer will be a consistent “yes” or a consistent “no” with only one number involved. If you are interested, 14,749 is not prime as it is clearly divisible by 7 so the answer is “no”! The answer is either B or D.

Statement 1: “ $\frac{x}{2}$ is odd.” With conceptual understanding you see that the only numbers less than 10 that work with this statement are even numbers (so that



they can still yield an integer when divided by 2) that are not multiples of 4 (so that the integer is not an even one). So $x = 2$ or $x = 6$. This means that you would be adding either 2 or 6 to 14,743. If you add 2 then you get 14,745, which is not prime since it ends in 5. If you add 6 you get 14,749, which is not a multiple of 2, 3, or 5. However it is a multiple of 7. A quick check of division shows that $14,749 = 7 \times 2,107$. Since neither of the numbers are prime, you have a consistent "no" and this statement is also sufficient. Again, you should note that the only plausible answer to this question will be "no" or the question would be unfair. 14,747, for instance, is prime but there is no way in two minutes you could EVER prove that, so the testmakers could not have x be 4! The correct answer is D.

120. E

Question Type: *What Is the Value?* This question asks for the specific number of bushels of apples harvested yesterday.

Given information in the question stem or diagram: There is a lot of confusing information given in the question stem. If Steve harvests 42 or fewer bushels he is paid y dollars per bushel; if he harvests more than 42 per day then he is paid y dollars per bushel for the first 42 and $1.5y$ for each additional bushel.

After reading through this complicated word problem, it is fairly clear that each statement alone could not be sufficient.

Statement 1: "Yesterday Steve was paid \$180 for the apples he harvested." This statement means that Steve might have only harvested one bushel if the rate of y was \$180 per bushel or he could have harvested 30 bushels if the rate of y was \$6 per bushel. This is clearly not sufficient. Eliminate choices A and D.

Statement 2: "Today Steve harvested yesterday's amount + 10 bushels and earned \$240." This statement alone only gives you information referencing the amount he harvested yesterday (which is unknown using only Statement 2). So, this is also not sufficient. This is a C vs. E question and the choice between those is very, very difficult.

Together: Using the two statements together, it appears that you can determine one value for y . If Statement 2 tells you that the additional 10 bushels added \$60 to his revenue, then the value of y appears to be \$6. Using this value of y , you can see that he would have harvested 30 bushels yesterday for \$180 and 40 bushels today for \$240. Most students will then pick answer C and assume the answer is 30 for the amount of bushels bought yesterday.

However, the question stem gave us two potential scenarios. Imagine if those

10 additional bushels that Steve harvested were already over the threshold of 42. Then the value of y would not be \$6 ($10y = 60$); it would be \$4 ($10(1.5)y = 60$), and y would be \$4 per bushel. Looking back at Statement 1 and using $y = \$4$ it is possible that Steve harvested 44 bushels yesterday. He would have earned \$4 per bushel for the first 42 (\$168) and then the increased rate of \$6 per bushel for the remaining 2, which would be \$12. Added together this gives the total value of \$180. Since there are two possible scenarios for the number of bushels harvested yesterday, the answer is E. This question is the ultimate test of Resource Management skills and your ability to Play Devil's Advocate. Did you realize that there were two possible scenarios given the stated information? Probably not, but for those who want to see them more clearly you can look below:

Scenario 1: If the total number of bushels harvested on both days were less than 42, then $y = \$6$.

$$\text{Yesterday: } (30 \text{ bushels})(\$6 \text{ per bushel}) = \$180$$

$$\text{Today: } (40 \text{ bushels})(\$6 \text{ per bushel}) = \$240$$

Answer to question = 30 bushels harvested yesterday.

Scenario 2: If the total number of bushels harvested on both days were more than 42, then $y = \$4$ and he would have earned \$6 per bushel for each one above 42.

$$\begin{aligned} \text{Yesterday: } & (42 \text{ bushels})(\$4 \text{ per bushel}) = \$168 \\ & + (2 \text{ bushels}) (\$6 \text{ per bushel}) = \$12 \\ & \text{Total} = \$180 \end{aligned}$$

$$\begin{aligned} \text{Today: } & (42 \text{ bushels})(\$4 \text{ per bushel}) = \$168 \\ & + (12 \text{ bushels})(\$6 \text{ per bushel}) = \$72 \\ & \text{Total} = \$240 \end{aligned}$$

Answer to question = 44 bushels harvested yesterday.

**121. A**

Question Type: Yes/No. This question asks: whether $\frac{(x-y+z)}{2z} < \frac{x}{2z} - \frac{y}{2z} - \frac{x}{y}$.

Given information in question stem or diagram: yz does not equal zero. This is only given to assure that no denominators are equal to 0. Before moving to the statements, it is a good idea to use algebraic manipulation to try to simplify this very difficult inequality question. The first move is suggested by the fact that three of the four fractions have a denominator of $2z$. While you might want to multiply both sides of the inequality by $2z$, you are not allowed to do that as you do not know the sign of the variable z . However, you can get all of the $2z$ denominator terms on one side by subtracting $\frac{x}{2z}$ from $\frac{x-y+z}{2z}$ and adding $\frac{y}{2z}$ to both sides. After you do that, you see that the new question is:

$$\text{Is } \frac{x-y+z}{2z} - \frac{x}{2z} + \frac{y}{2z} < -\frac{x}{y}?$$

Now you can combine the numerators on the left side (the x 's and y 's each cancel) and put over one denominator:

$$\text{Is } \frac{z}{2z} < -\frac{x}{y}?$$

This can be further simplified by canceling the z 's to show that the question is really:

Is $\frac{1}{2} < -\frac{x}{y}$? You can transform this so that you're not dealing with the negative sign next to the variables. Multiply both sides by -1 (remember to flip the sign when doing so!) to make the question much more manageable:

$$\text{Is } \frac{x}{y} < -\frac{1}{2}?$$

Now that you're down to only two variables and being asked about a simple x -to- y ratio, it's time to attack the statements.

Statement 1: $\frac{x}{y} < -\frac{1}{2}$. You have done all of the work already in manipulating the question, as this statement perfectly matches the rephrased question. This statement is clearly sufficient, and the answer is A or D. Note: If you did not manipulate the question algebraically, this statement would be very difficult to incorporate and evaluate in a reasonable period of time.

Statement 2: " $xy < 0$." This is very close to being sufficient but not does not give enough information to find the limit for $\frac{x}{y}$. This statement proves that $\frac{x}{y}$ must be negative, but it does not prove that it is less than $-\frac{1}{2}$. For instance, y could be 4 and x could be -1, and this would give a "no" answer. Or x could be 4 and y could be -1 and then it would be a "yes" answer. This statement is not sufficient. The answer is A.

122. D

Question Type: *What Is the Value?* This question asks for the ratio of chocolate chip cookies to brownies.

Given information in question stem or diagram: Chocolate chip cookies = \$1.30 each; brownies = \$1.50 each

Statement 1: The average price for the items sold was \$1.42. This information allows you to determine the ratio of cookies to brownies using the principles of the weighted average. The relative distance of the price of each item to the overall average price is the inverse of the relative weight of each item. Of course, you do not need to do this to prove sufficiency (you know you CAN do it) but here is the process: First find the distance for each of the prices to the average price. Cookie price to average price is a distance of 12 cents and brownie price to average price is a distance of 8 cents. So the distance ratio is CC:B = 12:8. Now simply invert the numbers and simplify to see the actual ratio: CC:B = 8:12 and CC:B = 2:3. Since you have the average price and the prices of each of the items you can judge the relative weight of the items and create a ratio. This statement is sufficient and the answer is either A or D.

Note: If the question had asked for the number of brownies or number of chocolate chip cookies, this statement would not have been sufficient.

Statement 2: "The total price for all of the items = \$14.20." This information allows you to create the equation $1.3C + 1.5B = 14.20$. This may seem like it is not sufficient since you have two variables with one equation. However, because C and B must be whole numbers, there is a limit put on the values of B and C that you must consider. The fact that the variables are only 20 cents apart and that the total value is a relatively small \$14.20 means that it is likely that there is only one value for C and B. To prove that this statement is sufficient by itself, consider the limits of the total number of treats that could have been bought. Try 11 items first (because the first statement gives 10 total items this would be a logical place to test). If you buy 11 of the cheaper item (cookies), you have spent too much money! 11 cookies would total $11 \cdot 1.30 = \$14.30$. This is too much; you know that the total is only \$14.20. Therefore the total number of items must be less than 11. Now try 9 items. If you buy 9 of the more expensive items (brownies), you have not spent enough money! 9 brownies = $9 \cdot \$1.50 = \13.50 , which is less than the \$14.20 you know was spent. Therefore, the total number must be greater than 9. The total number of items must = 10, and so you actually have two equations. The equation $1.3C + 1.5B = 14.20$ has embedded within it the equation $C + B = 10$. This statement is also sufficient, and the correct answer is D.



Note: This is another classic “C trap” question. Choice C is absurdly easy on this question, so it is unlikely to be the correct answer. Even if you note the C trap and avoid picking choice C it is still likely you will pick A. Most people will miss the unique equation in the second statement that allows you to actually solve for the variables. The integer constraint implicit in the context of the question makes all the difference and provides a reward for those who take care to Leverage Assets.

123. A

Question Type: *What Is the Value?* This question asks: “How many numbers in Set T are multiples of 21?”

Given information in the question stem or diagram: Set T is a finite set of positive consecutive multiples of 14. It might be a good idea to think about the relationship between multiples of 14 and 21 before moving on. Multiples of 14 are multiples of $2 \cdot 7$, and multiples of 21 are multiples of $3 \cdot 7$. In order to be both, a number needs to be a multiple of $6 \cdot 7$, or 42. Indeed 42 is the smallest positive number that is a multiple of both 14 and 21. What you need to realize before moving to the statements is that every third multiple of 14 is also a multiple of 21.

Statement 1: Set T consists of 30 integers. If every third multiple of 14 is a multiple of 21, then there would be 10 multiples of 21 in Set T. A quick glance at Statement 2 indicates that it may be important to know if the first member of the set is a multiple of 21, which would make this statement alone not sufficient. However, since it is every third member of Set T that is a multiple of 21, and since 30 is a multiple of 3, there will be exactly 10 sets of 3, and each of those will have one multiple of 21. It does not matter if the multiple of 21 is the first number in each group of 3 or the last. This is because 30 is an exact multiple of 3. If the set contained 31 integers, then it would be important to know if the multiple of 21 was first, but in this case you do not need that information. This statement is sufficient alone. The answer is either answer A or D.

Statement 2: The smallest integer in Set T is a multiple of 21. This statement is not sufficient alone since it does not indicate the number of members of the set. The correct answer is A. Note: This question is a very tricky example of the “Why Are You Here?—Temptation” construct. A good test-taker will very carefully analyze this second statement and prove whether it matters in relation to the first. In this case, it does not matter, so the answer is A, but in other cases it certainly will, and the answer would be C. The key here is that you take the time to make sure that the second statement does or does not matter.

124. D

Question Type: *What Is the Value?* This questions asks for the value of the integer t.

Given information in the question stem or diagram: r, s, and t are positive integers, and $rst = 343$.

Statement 1: “ $r < s < t$.” This statement not only puts the variables in ascending order but also subtly guarantees that the variables are all different numbers—no repeats. It would be a good idea at this point to get the prime factors of 343. $343 = 7^3$. The only factors of 343 are 1, 7, 49, and 343. Even before you get to Statement 1, you might recognize from the question stem that the only possible sets of values for r, s, and t are $1 \cdot 1 \cdot 343$, $1 \cdot 7 \cdot 49$, and $7 \cdot 7 \cdot 7$. With Statement 1 the only possible set is $1 \cdot 7 \cdot 49$, so t must be 49. The answer is A or D.

Statement 2: “ $rs = 7$.” This means that $7t = 343$. In order for rst to equal 343, t must = 49. This statement is clearly sufficient. The correct answer is D.

Note: This is a great example of another important construct discussed in the lesson portion of this book. Statement 2 is a very easy statement: T is clearly equal to 49, and this is quite obviously sufficient. Whenever you have one clearly sufficient statement, the other one will almost always be hard and counterintuitive! The first statement does not seem sufficient at first glance, but with a careful analysis of factors, you see that it is indeed sufficient. The easy 2nd statement is a hint to dig deeper in the first statement and leverage every piece of information that is available.

125. A

Question Type: *What Is the Value?* This question asks for the unit’s digit of $n^{4a+2} - n^{8a}$.

Given information in the question stem or diagram: n and a are positive integers.

Statement 1: In the Algebra lesson, you learned that all numbers have a repeating pattern of units digits when they are raised to certain powers. For instance, here is the progression for 3, which is given as the value for n in Statement 1:

Units Digit of $3^1 = 3$

Units Digit of $3^2 = 9$

Units Digit of $3^3 = 7$

Units Digit of $3^4 = 1$



Units Digit of $3^5 = 3$

Units Digit of $3^6 = 9$

Units Digit of $3^7 = 7$

Units Digit of $3^8 = 1$

As you can see, the pattern repeats every 4. This Data Sufficiency question is asking whether you can determine the units digit of $n^{(4a+2)} - n^{(8a)}$. If you can determine the units digit of each of those terms, then you can calculate the difference between their units digits. In Statement 1, you learn that n is 3, and it seems like you must also know something about a to answer the question. However, a closer look at the exponents shows that it does not matter what value a is. Remember that 3 raised to any multiple of 4 will always end in 1 (for instance, 3^{12} or 3^{24}), and 3 raised to any multiple of $4 + 2$ (for instance 3^{16} or 3^{26}) will always end in 9. Therefore, regardless of what a is, $n^{(4a+2)}$ will end in 9 and $n^{(8a)}$ will end in 1. Therefore you can answer the question, and the answer must be A or D. Note: This is only possible because the numbers before the exponent a are multiples of 4.

Statement 2: A quick look at this statement and it is clearly insufficient. You must know something about n to answer the question. Since most people do not think that Statement 1 is sufficient, this is an important “Why Are You Here?” statement. The good test-taker will at this point try a few odd and even values for a to see if it makes a difference. In doing that, you will quickly prove to yourself that you do not need to know anything about a in order for statement 1 to be sufficient. This is another classic example of “Why Are You Here?—Temptation,” and if you play the Data Sufficiency game properly—leveraging hints and deciding if information is really important—you can get this problem correct every time!



ANSWER KEY

LESSONS		HOMEWORK					
1. B	10. B.	18. C	36. A	54. C	72. D	90. E	108.B
2. C	11. E	19. E	37. B	55. B	73. E	91. C	109.A
3. D	12. C	20. B	38. E	56. A	74. A	92. E	110.D
4. A	13. D	21. E	39. C	57. D	75. E	93. D	111.C
5. E	14. A	22. B	40. E	58. E	76. D	94. A	112.C
6. E	15. A	23. E	41. C	59. B	77. B	95. D	113.E
7. C	16. A	24. C	42. D	60. A	78. B	96. C	114.C
8. A	17. E	25. B	43. C	61. B	79. A	97. B	115.D
9. C		26. B	44. C	62. E	80. D	98. A	116.A
		27. D	45. D	63. A	81. E	99. A	117.A
		28. E	46. D	64. D	82. C	100.B	118.E
		29. D	47. D	65. B	83. D	101.A	119.D
		30. D	48. E	66. D	84. A	102.A	120.E
		31. C	49. B	67. E	85. E	103.E	121.A
		32. A	50. C	68. E	86. D	104.B	122.D
		33. A	51. E	69. A	87. C	105.A	123.A
		34. E	52. D	70. C	88. E	106.B	124.D
		35. A	53. A	71. B	89. E	107.A	125.A

