

1

The car ends **within a half mile** of the sign indicating 2 1/2 miles means that the car will end in **one mile interval, between the signs indicating 2 and 3 miles.**

Now, it doesn't matter where the car starts or what distance it travels, the probability will be $P = (\text{favorable outcome}) / (\text{total \# of outcomes}) = 1/3$ (as the car starts at random point and travels some distance afterwards we can consider its end point as the point where he randomly appeared, so the probability that the car appeared within 1 mile interval out of total 3 miles will be 1/3).

Answer: C.

2

Standard approach:

$(\text{any but Joey})(\text{Joey})(\text{any}) + (\text{any but Joey})(\text{any but Joey})(\text{Joey}) = 11/12 * 1/11 * 1 + 11/12 * 10/11 * 1/10 = 2/12.$

Answer: E.

Another approach:

Actually even OE has one more step than necessary: since there are two slots for Joey from 12 possible than the probability is simply 2/12.

Consider this line 12 members in a row. Now, what is the probability that Joey is 1st in that row? 1/12. What is the probability that he's 2nd? Again 1/12. What is the probability that he's 12th? What is the probability that he's second or third? $1/12 + 1/12 = 2/12$. What is the probability that he's in last 6? 6/12...

Answer: E.

3

OR probability:

If Events A and B are independent, the probability that Event A OR Event B occurs is equal to the probability that Event A occurs plus the probability that Event B occurs minus the probability that both Events A and B

occur: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$

This is basically the same as 2 overlapping sets formula:

$\{\text{total \# of items in groups A or B}\} = \{\text{\# of items in group A}\} + \{\text{\# of items in group B}\} - \{\text{\# of items in A and B}\}.$

Note that if event are mutually exclusive then $P(A \text{ and } B) = 0$ and the formula simplifies to: $P(A \text{ or } B) = P(A) + P(B).$

Also note that when we say "A or B occurs" we include three possibilities:

A occurs and B does not occur;

B occurs and A does not occur;

Both A and B occur.

AND probability:

When two events are independent, the probability of both occurring is the product of the probabilities of the individual

events: $P(A \text{ and } B) = P(A) * P(B).$

This is basically the same as **Principle of Multiplication**: if one event can occur in m ways and a second can occur independently of the first in n ways, then the two events can occur in mn ways.

BACK TO THE ORIGINAL QUESTION:

Two dice are tossed once. The probability of getting an even number at the first die or a total of 8 is

- A. 1/36
- B. 3/36
- C. 11/36
- D. 20/36
- E. 23/36

1. The probability of getting an even number at the first die is 1/2 (as the probability of even = the probability of odd = 1/2);
2. The probability of getting a total of 8 is $5/6^2$, as there are 5 different favorable scenarios: (2,6), (6,2), (3,5), (5,3) and (4,4);
3. The probability of getting an even number at the first die AND a total of 8 is $3/6^2$ (from above case);

Hence, The probability of getting an even number at the first die OR a total of 8 is $1/2 + 5/36 - 3/36 = 20/36$.

Answer: D.

4

A coin is tossed 7 times. Find the probability of getting more heads than tails in all 7 tosses?

- A. $1/2$
- B. $63/128$
- C. $4/7$
- D. $61/256$
- E. $63/64$

Assuming the coin is fair - $P(H)=P(T)=1/2$

We can do as proposed by the explanation in your initial post:

Total outcomes: 2^7

Favorable outcomes:

4 heads --> combination of HHHHTTT --> $7!/(4!*3!)=35$ (# of permutation of 7 letters out of which 4 H's and 3 T's are identical);
5 heads --> combination of HHHHHTT --> $7!/(5!*2!)=21$;
6 heads --> combination of HHHHHHT --> $7!/(6!*1!)=7$;
7 heads --> combination of HHHHHHH --> 1;

$P(H>T) = \text{Favorable outcomes} / \text{Total outcomes} = (35+21+7+1)/2^7 = 1/2$.

BUT: there is MUCH simpler and elegant way to solve this question. Since the probability of getting either heads or tails is equal ($1/2$) and a tie in 7 (odd) tosses is not possible then the probability of getting more heads than tails = to the probability of getting more tails than heads = $1/2$. How else? Does the probability favor any of tails or heads? (The distribution of the probabilities is symmetrical: $P(H=7)=P(T=7)$, $P(H=5)=P(T=5)$, ... also $P(H>4)=P(T>4)$)

Answer: A.

If it were: A fair coin is tossed 8 times. Find the probability of getting more heads than tails in all 8 tosses?

Now, almost the same here: as 8 is even then a tie is possible but again as distribution is symmetrical

then $P(H>T) = \frac{1 - P(H=T)}{2} = P(T>H)$ (so we just subtract the probability of a tie and then divide the given value by 2 as $P(H>T)=P(H<T)$). As $P(H=T) = \frac{8!}{4!*4!} = 70$ (# of permutation of 8 letters HHHHTTTT, out of which 4 H's and 4 T's are identical) then $P(H>T) = \frac{1 - P(H=T)}{2} = \frac{1 - \frac{70}{256}}{2} = \frac{93}{256}$. You can check this in following way: total # of outcomes = $2^8=256$, out of which in 70 cases there will be a tie, in 93 cases $H>T$ and also in 93 cases $T>H$ --> $70+93+93=256$.

5

Percent of students who are 25 years old or older is $0.4*48 + 0.2*52 = 30$, so percent of people who are less than 25 years old is $100-30=70$.

Answer: B.

This can also be done with an approximation method. Since percent of men and women are almost the same (48% and 52%) then percent of students who are 25 years old or older will be close to the average of 40% and 20%, so close to 30%, which makes percent of people who are less than 25 years old close to 70%.

Answer: B.