

1

There are exactly 6 teams in league x. What was the total number of games played by the 6 teams last season?

(1) Each team in league x played each of the other teams at least once --> if each team played each of the other teams ONLY once then there was total of  $C_6^2 = 15$  games played (notice that each team played 5 games) but if each team played each of the other teams TWICE then there was total of  $2 * C_6^2 = 30$  games played (notice that each team played 10 games). Not sufficient.

(2) No team in league x played more than 7 games. Clearly insufficient.

(1)+(2) If each team played each of the other teams ONLY once then there was total of  $C_6^2 = 15$  games played (each of the 6 teams played 5 games), but if after each team played each of the other teams ONLY once, some two teams played between each other once more then the total number of games played was 16 (4 teams played 5 games and 2 teams played 6 games). Not sufficient.

Answer: E.

2

The integers n and t are positive and  $n > t > 1$ . How many different subgroups of t items can be formed from a group of n different items?

The question basically asks about the value of  $C_n^t = \frac{n!}{(n-t)! * t!}$ .

(1) The number of different subgroups of n - t different items that can be formed from a group of n different items is 680 --

$C_n^{n-t} = \frac{n!}{(n-(n-t))! * (n-t)!} = \frac{n!}{t! * (n-t)!} = 680$ , directly gives us the answer. Sufficient.

(2)  $nt = 51 \rightarrow 51 = 17 * 3 = 17 * 1$ , since  $n > t > 1$  then  $t=3$  and  $n=17 \rightarrow C_n^t = \frac{n!}{(n-t)! * t!} = \frac{17!}{14! * 3!} = 680$ . Sufficient.

Answer: D.

3

Question:  $C_x^3 * C_y^2 = ?$  So we need the values of x and y.

(1)  $C_{x+2}^3 = 56 \rightarrow$  there is only one x to satisfy this (meaning that there is only one number out of which we can make 56 different selections of 3, so  $x+2$  is some specific number which gives some specific x), but we know nothing about y. Not sufficient.

Just to illustrate how to find x:  $C_{x+2}^3 = 56 \rightarrow \frac{(x+2)!}{(x+2-3)! * 3!} = 56 \rightarrow \frac{(x+2)!}{(x-1)! * 3!} = 56 \rightarrow \frac{x(x+1)(x+2)}{3!} = 56 \rightarrow x(x+1)(x+2) = 6 * 7 * 8 \rightarrow x = 6$ .

(2)  $x = y + 1$ . Clearly insufficient.

(1)+(2) From (1) we know x, so from (2) we can get y. Sufficient.

Answer: C.

4

The point here is the following:

Suppose we are told that there are 10 ways to choose  $x$  people out of 5. What is  $x$ ?  $C_5^x = 10 \rightarrow \frac{5!}{x!(5-x)!} = 10 \dots$   
 $\rightarrow x!(5-x)! = 12 \rightarrow x = 3$  or  $x = 2$ . So we cannot determine single numerical value of  $x$ . Note that in some cases we'll be able to find  $x$ , as there will be only one solution for it, but generally when we are told that there are  $n$  ways to choose  $x$  out of  $m$ , there will be (in most cases) two solutions of  $x$  possible.

But if we are told that there are 10 ways to choose 2 out of  $x$ , then there will be only one value of  $x$  possible  $\rightarrow C_x^2 = 10 \dots$   
 $\rightarrow \frac{x!}{2!(x-2)!} = 10 \rightarrow \frac{x(x-1)}{2!} = 10 \rightarrow x(x-1) = 20 \rightarrow x = 5$ .

In our original question, statement (1) says that there are 126 ways to choose 5 out of  $x+2 \rightarrow$  there will be only one value possible for  $x+2$ , so we can find  $x$ . Sufficient.

Just to show how it can be done:  $C_{(x+2)}^5 = 126 \dots$   
 $\rightarrow (x-2)(x-1)x(x+1)(x+2) = 5! * 126 = 120 * 126 = (8 * 5 * 3) * (9 * 7 * 2) = 5 * 6 * 7 * 8 * 9 \dots$   
 $\rightarrow x = 7$ . Basically we have that the product of five consecutive integers  $(x-2)(x-1)x(x+1)(x+2)$ , equal to some number  $(5! * 126) \rightarrow$  only one such sequence is possible, hence even though we have the equation of 5th degree it will have **only one positive integer solution**.

Statement (2) says that there are 56 ways to choose 3 out of  $x+1 \rightarrow$  there will be only one value possible for  $x+1$ , so we can find  $x$ . Sufficient.

$C_{(x+1)}^3 = 56 \rightarrow (x-1)x(x+1) = 3! * 56 = 6 * 7 * 8 \rightarrow x = 7$ . Again we have that the product of three consecutive integers  $(x-1)x(x+1)$ , equal to some number  $(3! * 56) \rightarrow$  only one such sequence is possible, hence even though we have the equation of 3rd degree it will have **only one positive integer solution**.

5

Sammy has  $x$  flavors of candies with which to make goody bags for Franks birthday party. Sammy tosses out  $y$  flavors, because he doesn't like them. How many different 10-flavor bags can Sammy make from the remaining flavors? (It doesn't matter how many candies are in a bag, only how many flavors).

In order to calculate how many 10-flavor bags can Sammy make from the remaining  $(x-y)$  flavors, we should know the value of  $x-y$ . The answer would simply be  $C_{x-y}^{10}$ . For example if he has 11 flavors (if  $x-y=11$ ), then he can make  $C_{11}^{10} = 11$  different 10-flavor bags.

(1) If Sammy had thrown away 2 additional flavors of candy, he could have made exactly 3,003 different 10-flavor bags. We are told that  $C_n^{10} = 3,003$ , where  $n = (x-y)-2$ ; he can make 3,003 10-flavor bags out of  $n$  flavors. Now,  $n$  can take only one particular value, so we can find  $n$  (it really doesn't matter what is the value  $n$ , important is that we can find it), hence we can find the value of  $x-y$  ( $x-y=n+2$ ). Sufficient.

(2)  $x = y + 17 \rightarrow x-y=17$ . Directly gives us the value of  $x-y$ . Sufficient.

Answer: D.