1

Couple of things before solving:

If exponentiation is indicated by stacked symbols, the rule is to work from the top down, thus:
$a^{m^n}=a^{(m^n)}$ and not $(a^m)^n$, which on the other hand equals to a^{mn}.

So:
$(a^m)^n=a^{mn}$;

$a^{m^n}=a^{(m^n)}$ and not $(a^m)^n$.

Back to the original question:
Let's replace # by @ as # looks like the symbol "not equal to" and it might confuse someone.

Given: $x@n=x^{(x@(n-1))}$ and $x@1=x$:
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$x@2=x^{(x@1)}=x^x$, as $x@1=x$;
$x@3=x^{(x@2)}=x^{(x^x)}=x^{x^x}$;
$x@4=x^{(x@3)}=x^{(x^{x^x})}=x^{x^{x^x}}$;
...
Basically n in x@n represents the # of stacked x-es.

A. $(3@2)@2=(3^3)@2=(27)@2=27^{27}=3^{81}$

B. $3@(1@3)=3@(1^{1^1})=3@1=3$

C. $(2@3)@2=(2^{2^2})@2=16@2=16^{16}=2^{64}$

D. $2@(2@3)=2@(2^{2^2})=2@16$ this will be huge number
2^2^2^2^2^2^2^2^2^2^2^2^2^2^2^2=2^2^2^2^2^2^2^2^2^2^2^2^2^2^4=2^2^2^2^2^2^2^2^2^2^2^2^16=.... ;

E. $(2@2)@3=(2^2)@3=4@3=4^{4^4}=4^{256}=2^{512}$

Option D (2^2^2^2^2^2^2^2^2^2^2^2^2^2^2^2) will be much bigger number than numbers from other answer choices.

Answer: D.

2

For which of the following functions is f(a+b)= f(a)+f(b) for all positive numbers a and b?

A. $f(x)=x^2$
B. $f(x)=x+1$
C. $f(x)=\sqrt{x}$
D. $f(x)=\frac{2}{x}$
E. $f(x)=-3x$

A. $f(a+b)=(a+b)^2=a^2+2ab+b^2\neq f(a)+f(b)=a^2+b^2$

B. $f(a+b)=(a+b)+1\neq f(a)+f(b)=a+1+b+1$

C. $f(a+b)=\sqrt{a+b}\neq f(a)+f(b)=\sqrt{a}+\sqrt{b}$.

D. $f(a+b)=\frac{2}{a+b}\neq f(a)+f(b)=\frac{2}{a}+\frac{2}{b}$.

E. $f(a+b)=-3(a+b)=-3a-3b=f(a)+f(b)=-3a-3b$. Correct.

Answer: E.

OR, as f(a+b)= f(a)+f(b) must be true for all positive numbers a and b, then you can randomly pick particular values of a and b and check for them:

For example: $a=2$ and $b=3$

A. $f(a+b)=f(5)=5^2=25\neq f(a)+f(b)=f(2)+f(3)=2^2+3^2=13$

B. $f(a+b)=f(5)=5+1=6\neq f(a)+f(b)=f(2)+f(3)=(2+1)+(3+1)=7$

C. $f(a+b)=f(5)=\sqrt{5}\neq f(a)+f(b)=f(2)+f(3)=\sqrt{2}+\sqrt{3}$

D. $f(a+b)=f(5)=\frac{2}{5}\neq f(a)+f(b)=f(2)+f(3)=\frac{2}{2}+\frac{2}{3}=\frac{5}{3}$

E. $f(a+b)=f(5)=-3*(5)=-15=f(a)+f(b)=f(2)+f(3)=-3*(2)-3*(3)=-15$. Correct.

It might happen that for some choices of a and b other options may be "correct" as well. If this happens just pick some other numbers and check again **these "correct" options only**.

3

If the function Q is defined by the formula Q = 5w/(4x(z^2)), by what factor will Q be multiplied if w is quadrupled, x is doubled, and z is tripled?
A. 1/9
B. 2/9
C. 4/9
D. 3/9
E. 2/27

Given: $Q=\frac{5w}{4x*z^2}$.

Now, quadruple w, so make it $4w$; double x so make it $2x$; triple z and substitute these values instead of x, y, and z in the original equation:

. Thus Q is multiplied by $\frac{2}{9}$.

Answer: B.

Else plug-in values for x, y, and z. Let $x=y=z=1$ --> $Q=\frac{5w}{4x*z^2}=\frac{5}{4}$.

$4w=4$, $2x=2$ and $3z=3$ --> $\frac{5*4}{4*2*3^2}=\frac{4}{18}*\frac{5}{4}=\frac{2}{9}*\frac{5}{4}$. Thus Q is multiplied by $\frac{2}{9}$.

Answer: B.

4

The question should read:

$A_x(y)$ is an operation that adds 1 to y and then multiplies the result by x. If x = 2/3, then $A_x(A_x(A_x(A_x(A_x(x)))))$ is between

(A) 0 and ½
(B) ½ and 1
(C) 1 and 1½
(D) 1½ and 2
(E) 2 and 2½

According to the stem:
$A_x(x)=(x+1)x=x^2+x$;
$A_x(A_x(x))=A_x(x^2+x)=(x^2+x+1)*x=x^3+x^2+x$;
$A_x(A_x(A_x(x)))=A_x(x^3+x^2+x)=(x^3+x^2+x+1)*x=x^4+x^3+x^2+x$;
...

We can see the pattern now, so $A_x(A_x(A_x(A_x(A_x(x)))))=x^6+x^5+x^4+x^3+x^2+x$.

For $x=\frac{2}{3}$ we'll get: $(\frac{2}{3})^6+(\frac{2}{3})^5+(\frac{2}{3})^4+(\frac{2}{3})^3+(\frac{2}{3})^2+(\frac{2}{3})$.

So, we have the sum of the 6 terms of the geometric progression with the first term equal to $\frac{2}{3}$ and the common ratio also equal to $\frac{2}{3}$.

Now, the sum of **infinite geometric progression** with **common ratio** $|r|<1$, is $sum=\frac{b}{1-r}$, where b is the first term. So, if we had infinite geometric progression instead of just 6 terms then its sum would be $Sum=\frac{\frac{2}{3}}{1-\frac{2}{3}}=2$. Which means that the sum of this sequence will never exceed 2, also as we have big enough number of terms (6) then the sum will be very close to 2, so we can safely choose answer choice D.

Answer: D.

One can also use direct formula.
We have geometric progression with $b=\frac{2}{3}$, $r=\frac{2}{3}$ and $n=6$;

$S_n=\frac{b(1-r^n)}{(1-r)}$ --> $S_6=\frac{\frac{2}{3}(1-(\frac{2}{3})^6)}{(1-\frac{2}{3})}=2*(1-(\frac{2}{3})^6)$. Since $(\frac{2}{3})^6$ is very small number then $1-(\frac{2}{3})^6$ will be less than 1 but very close to it, hence $2*(1-(\frac{2}{3})^6)$ will be less than 2 but very close to it.

Answer: D.