1

FG[X]HI
ABCDE

ABCDE in in the front row can be arranged in 5!;
G (the boy who misbehaves) can be either to the right of the coach (X, which is fixed in the middle) or to the left, so 2 options, and other 3 boys (F, H, I) in the back row can be arranged in 3! ways;

Total: 5!*2*3!=1,440.

Answer: D.

2

There are 4 single digit prime numbers: 2, 3, 5 and 7. Hence, last two digits (tens and units) can take 4*4=16 different values: 22, 23, ..., 77.

So, in each hundred there are 16 such numbers. In 17 hundreds there will be 17*16=272 such numbers, but 4 out of them will be more than 1670, namely: 1672, 1673, 1675 and 1677. Which means that there are 272-4=268 numbers between 0 and 1670 which have a prime tens digit and a prime units digit.

Answer: A.

OR

We have 10 digits (0 - 9) and 4 of them are prime (2, 3, 5, 7). So (4/10)th of consecutive numbers will have prime digits in their unit's place.
Similarly, if we want prime digits in ten's place, again (4/10)th of 100 consecutive numbers will have prime digits in ten's place.

So in 100 consecutive numbers, (4/10)*(4/10) = 16/100 will have prime digits in both unit's and ten's places.

Number of numbers with both units and tens digit prime in the first 1600 = (16/100) * 1600 = 256
Number of numbers with both units and tens digit prime in leftover 70 numbers = (3/4) * 16 = 12

Required number of numbers = 256 + 12 = 268

3

Selecting 5 contestants for tribe A: $C_{10}^{5}=252$. The rest 5 contestants will automatically form tribe B.

Answer: C.

Notice that if we were asked about different ways of splitting 10 people into 2 equal groups of 5 then the answer would be 252/2=126, since we wouldn't have group #1 and group #2 in that case.

4

Direct approach:

Since there should be at least one manager and at least one non-manager among team of 3, then there should be either 1 manager and 2 non-managers OR 2 managers and 1 non-manager: $C_3^1*C_8^2+C_3^2*C_8^1=84+24=108$.

Reverse approach:

Total # of teams of 3 possible is $C_{11}^3=165$;
# of teams with only managers or only non-managers is: $C_3^3+C_8^3=1+56=57$;

# of teams of 3 with *at least* one manager or *at least* one non-manager is: 165-57=108.

Answer: B.

5

Total # of permutation of 5 distinct letters will be 5!=120;

Glue A and B together, consider it to be one unit: {AB}{C}{D}{E} --> # of permutation of these 4 units will be 4!=24, A and B within its unit also can be arranged in 2 ways : {AB} or {BA}, so total # of ways to arrange A, B, C, D, and E so that A and B to be together will be 4!*2=48;

The same for A and D: total # of ways to arrange A, B, C, D, and E so that A and D to be together will be 4!*2=48;

Now, the above 48+48=96 cases will contain the arrangements when A is adjacent to both B and D, so we should subtract this cases to get rid of the double counting. The # of case when A is adjacent to both B and D will be: consider {BAD} {C}{E} --> # of permutation of these 3 units will be 3!, {BAD} also can be arranged in 2 ways: {BAD} or {DAB}, so total # of ways to arrange A, B, C, D, and E so that A is adjacent to both B and D will be 3!*2=12;

The # of arrangements when A is adjacent to neither B nor D will be total-(48+48-12)=120-84=36.

Answer: D.

6

As A and E must be among 3 letters than the third letter must be out of B, C and D. 3C1=3 ways to choose which one it'll be. Now, 3 different letters can be arranged in 3!=6 ways, so final answer is 3*6=18.

Answer: D.

We are asked about the # of **arrangements** of 3 letters: {ABE} is a different arrangement from {EBA}, so for every **group** of 3 letters (for every **selection** of 3 letters) there will be 3 different arrangements possible and as there are total of 3 groups (3 selections) possible then there will be total of 3*6=18 arrangements.

Generally:
The words **"Permutation"** and **"Arrangement"** are synonymous and can be used interchangeably.
The words **"Combination"** and **"Selection"** are synonymous and can be used interchangeably.

7

Find the number of ways in which 4 letters may be selected from the word "Examination"?
A. 66
B. 70
C. 136
D. 330
E. 4264

We have, 11 letters: {A, A, E, I, I, O, M, N, N, T, X};
Out of them there are 3 pairs: {A, A}, {I, I} and {N, N}.
So, # of distinct letters is 8: {A, E, I, I, O, M, N, N, T, X}

As pointed out, there are 3 different cases possible.

{abcd} - all 4 letters are distinct: $C^4_8=70$;

{aabc} - two letters are alike and other two are distinct: $C^1_3*C^2_7=63$ (C^1_3 is a # of ways to choose which two letters will be alike from 3 pairs and C^2_7 # of ways to choose other two distinct letters from 7 letters which are left);

{aabb} - two letters are alike and other two letters are also alike: $C^2_3=3$ (C^3_3 is a # of ways of choosing two pairs of alike letter from 3 such pairs);

Total=70+63+3=136.

Answer: C.

So, as you can see you can not just pick 4 letters with C^4_{11} and then divide it by some factorial as there are 3 *different* cases possible and each has its own factorial correction.

8

Notice that each digit can appear more than once in a code.

Since there should be 4 letters in a code (X-X-X-X) and each letter can take 5 values (A, B, C, D, E) then total # of combinations of the letters only is 5*5*5*5=5^4.

Now, we are told that the first and last digit must be a letter digit, so number digit can take any of the three slots between the letters: X-X-X-X, so 3 positions and the digit itself can take 3 values (1, 2, 3).

So, total # of codes is 5^4*3*3=5,625.

Answer: E.

9

In the Land of Oz only one or two-letter words are used. The local language has 66 different letters. The parliament decided to forbid the use of the seventh letter. How many words have the people of Oz lost because of the prohibition?

A. 65
B. 66
C. 67
D. 131
E. 132

The answer to the question is indeed E. The problem with above solutions is that they do not consider words like AA, BB, ...

The number of 1 letter words (X) that can be made from 66 letters is 66;
The number of 2 letter words (XX) that can be made from 66 letters is 66*66, since each X can take 66 values.
Total: 66+66*66.

Similarly:
The number of 1 letter words (X) that can be made from 65 letters is 65;
The number of 2 letter words (XX) that can be made from 66 letters is 65*65, since each X can take 65 values.
Total: 65+65*65.

The difference is (66+66*66)-(65+65*65)=132.

Answer: E.

10

Notice that we are told that no digit in these numbers could be zero. So, we have only 9 digits to use for **XYZ**, where **Z** is an even number.

If the first digit (X) is 7 or 9 (*2 values*) then the third digit (Z) can take all *4 values* (2, 4, 6, or 8) and the second digit (Y) can take *7 values* (9 minus two digits we already used). So, for this case we have 2*4*7=56 numbers;

If the first digit (X) is 8 (*1 value*) then the third digit (Z) can take only *3 values* (2, 4, or 6,) and the second digit (Y) can take *7 values* (9 minus two digits we already used). So, for this case we have 1*3*7=21 numbers;

Total: 56+21=77 numbers.

Answer: E.

11

We are told that "*if* team A wins one of the prizes, team B wins also one of the prizes". Consider following cases:

A wins one of the prizes, then B must also win one of the prizes, and in this case we can have 4 triplets: {ABC}, {ABD}, {ABE}, {ABF}. Each triplet can be arranged in 3!=6 ways. Hence in the case when A wins one of the prizes 4*6=24 arrangements are possible.

A does NOT win one of the prizes, then three winners must be from other 5 teams. 3 winners out of 5 (B, C, D, E, F) teams can be chosen in $C_5^3=10$ ways and each case (for example {CDE}) can be arranged in 3!=6 ways, hence in the case when A does NOT win one of the prizes 10*6=60 arrangements are possible.

Total = 24+60 = 84.

Answer: D.