1

If the OA is B then it should be:

Sequence S consists of 24 nonzero integers. If each term in S after the second is the product of the previous two terms, how many terms in S are negative?

(1) The third term in S is **positive** --> either all terms are positive, so zero negatives or +--... and not all terms are positive, so more than zero negatives. Not sufficient.

(2) The fourth term in S is **negative** --> again two cases:

--+--+--+...
OR
-+--+--+-...

The same here: in both cases there is a repeated pattern of three terms in which 2 are negative and 1 positive (--+ or -+-) so in both cases out of 24 terms 2/3 will be negative, so there will be 16 negative terms. Sufficient.

Answer: B.

2

An infinite sequence of positive integers is called a perfect sequence. If each term in the sequence is a perfect number, that is, if each term can be expressed as the sum of its divisors, excluding itself. For example, 6 is a perfect number, as its divisors, 1, 2, and 3, sum to 6. Is the infinite sequence S a perfect sequence?

(1) Exactly one term in S is a prime number --> primes have exactly two divisors 1 and itself, hence no prime is a perfect number, which means that S is not a perfect sequence. Sufficient.

(2) In sequence S, each term after the first in S has exactly 3 divisors --> a number to have exactly 3 divisors must be square of a prime, for example 3^2=9 has 3 divisors: 1, 3, and 9 (1, p, and p^2). No, such number is a perfect number: 1+3 cannot equal to 9, (1+p cannot equal to p^2 for integer p), which means that S is not a perfect sequence. Sufficient.

Answer: D.

3

A set of nonnegative integers consists of {x, x + 7, 2x, y, y + 5}. The numbers of this set have four distinct values. What is its average (arithmetic mean)?

Notice couple of things:
1. We are told that all numbers in the set are **integers**;
2. We are told that all those integers are **non-negative**;
3. We are told that the set contains four distinct values out of five (so two integers out of 5 are alke and other three are distinct).

(1) x ≠ 5. Clearly insufficient.

(2) 4y + 12 = 6(y + 2) --> 4y+12=6y+12 --> 2y=0 --> y=0. So, we have that our set is {x, x + 7, 2x, 0, 5}. Since we know that two integers out of 5 are alike then:

1. x, x+7 or 2x is 0.
If x=0, then 2x=0 too, so in this case we'll have three alike integers x, 2x, and y. Thus this case is out.
If x+7=0, then x=-7 and we know that all integers must be non-negative so this case is out too.

2. x, x+7 or 2x is 5
If x=5, then x+7=12 and 2x=10. This scenario is OK. The set in this case would be: {5, 12, 10, 0, 5}
If x+7=5, then x=-2 and we know that the integers in the set must be non-negative. Thus this case is out.
If 2x=5, then x=5/2 and we know that the numbers in the set must be integers. Thus this case is out.

3. Two out of x, x+7 and 2x are alike.
x=2x is not possible, since in this case x=2x=0 and in this case we'll have three alike integers x, 2x, and y.
x=x+7 has no solution.
2x=x+7 --> x=7. This scenario is OK. The set in this case would be: {7, 14, 14, 0, 5}.

Two cases are possible. Not sufficient.

(1)+(2) Since from (1) x ≠ 5, then from (2) x=7. So, the set is {7, 14, 14, 0, 5}. Sufficient.

Answer: C.

4

In the sequence of positive numbers x_1, x_2, x_3, ..., what is the value of x_1?

(1) $x_i=\frac{x_{(i-1)}}{2}$ for all integers $i>1$ --> we have the general formula connecting two consecutive terms (basically we have geometric progression with common ratio 1/2), but without the value of any term this info is insufficient to find x_1.

(2) $x_5=\frac{x_4}{x_4+1}$ --> we have the relationship between x_5 and x_4, also insufficient to find x_1 (we cannot extrapolate the relationship between x_5 and x_4 to all consecutive terms in the sequence).

(1)+(2) From (1) $x_5=\frac{x_4}{2}$ --> $\frac{x_4}{2}=\frac{x_4}{x_4+1}$ --> $x_4=1$ --> $x_4=1=x_1*(\frac{1}{2})^3$ --> $x_1=8$. Sufficient.

Answer: C.