Master in High Performance Computing

Numerical Solution of PDEs using the Finite Element Method

Triangulation, DoFHandler, FiniteElement

Luca Heltai <luca.heltai@sissa.it>

International School for Advanced Studies (www.sissa.it)
Mathematical Analysis, Modeling, and Applications (math.sissa.it)
Master in High Performance Computing (www.mhpc.it)
SISSA mathLab (mathlab.sissa.it)

sissa mathLab innovating with mathematics

The Abdus Salam International Centre for Theoretical Physics

SISSA

Aims for this module

- Gain familiarity with three core classes
  - **Triangulation**
  - **DoFHandler**
  - **FiniteElement**
- Create and interrogate meshes
- Create and interrogate sparsity patterns

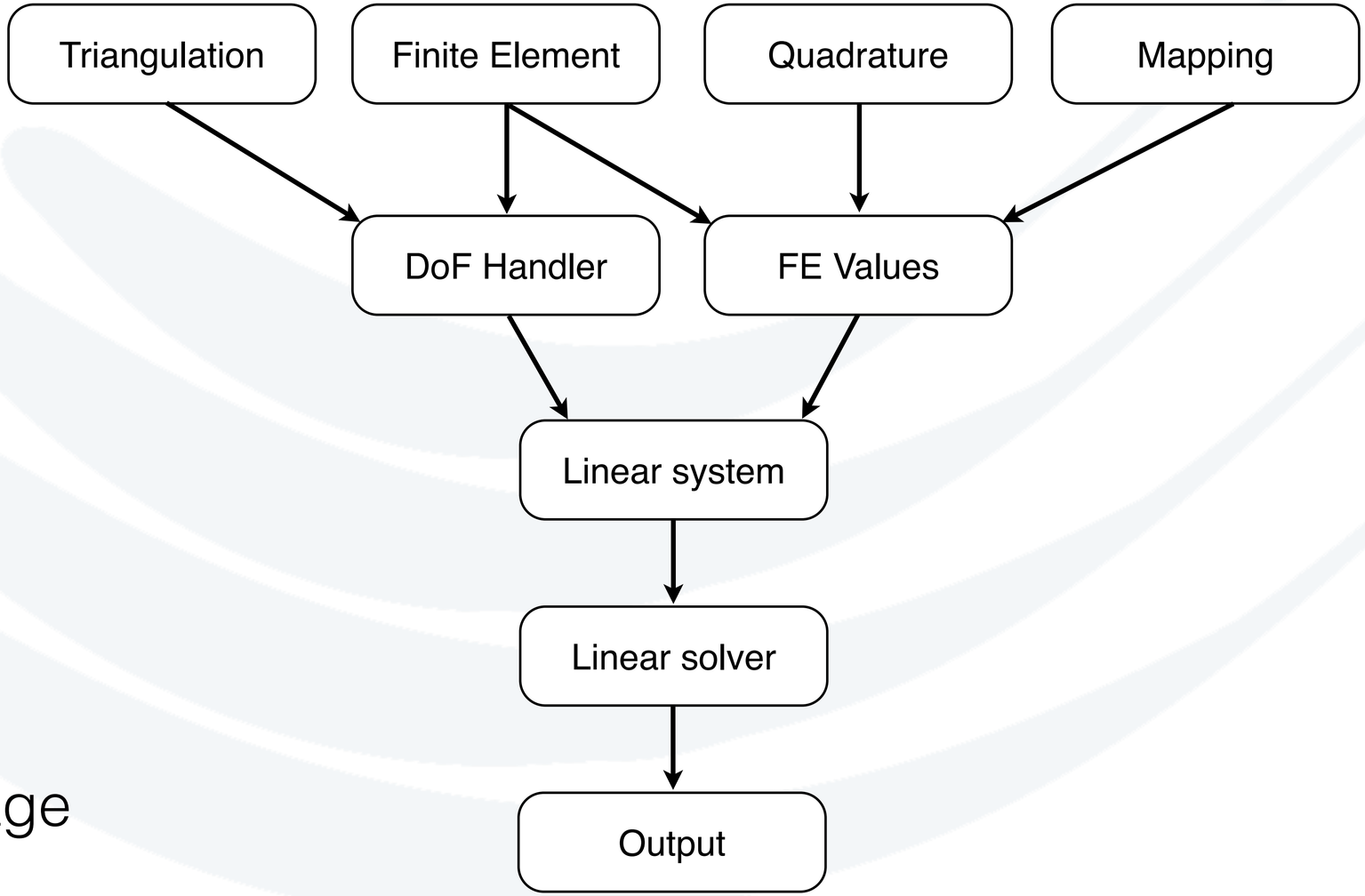
Reference material

- Main page
  https://dealii.org/current/doxygen/deal.II/index.html
- Tutorials
  - Step-1
    https://dealii.org/current/doxygen/deal.II/step_1.html
  - Step-49
    https://dealii.org/current/doxygen/deal.II/step_49.html
  - Step-2
    https://dealii.org/current/doxygen/deal.II/step_2.html

First and **BIGGEST** tip

- Program **defensively**
  - Program and test in **debug** mode
    - Additional compiler warnings
    - Add assertions
  - Perform studies in **release** mode

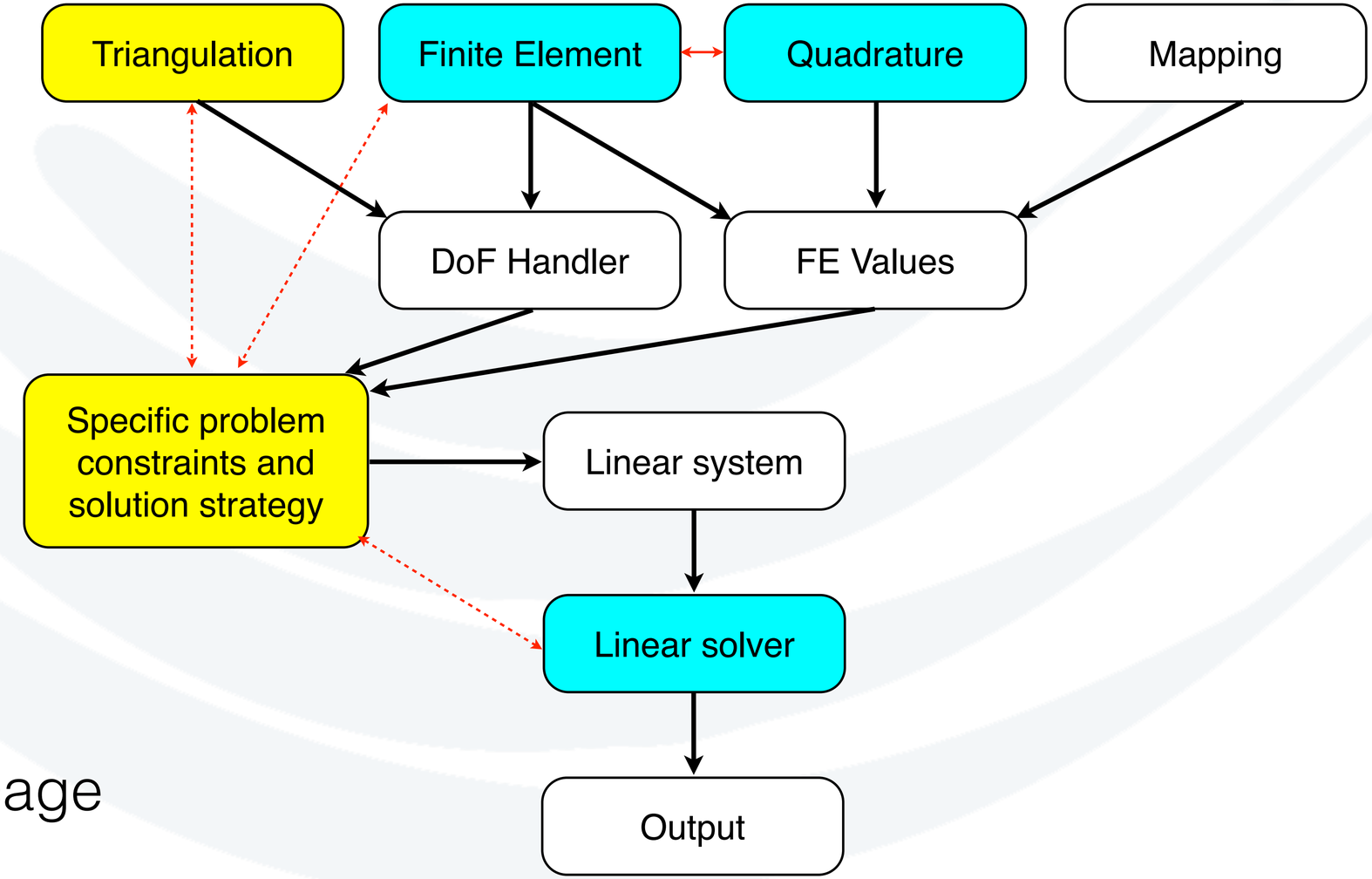
Structure of a prototypical FE problem



Main page

https://dealii.org/current/doxygen/deal.II/index.html

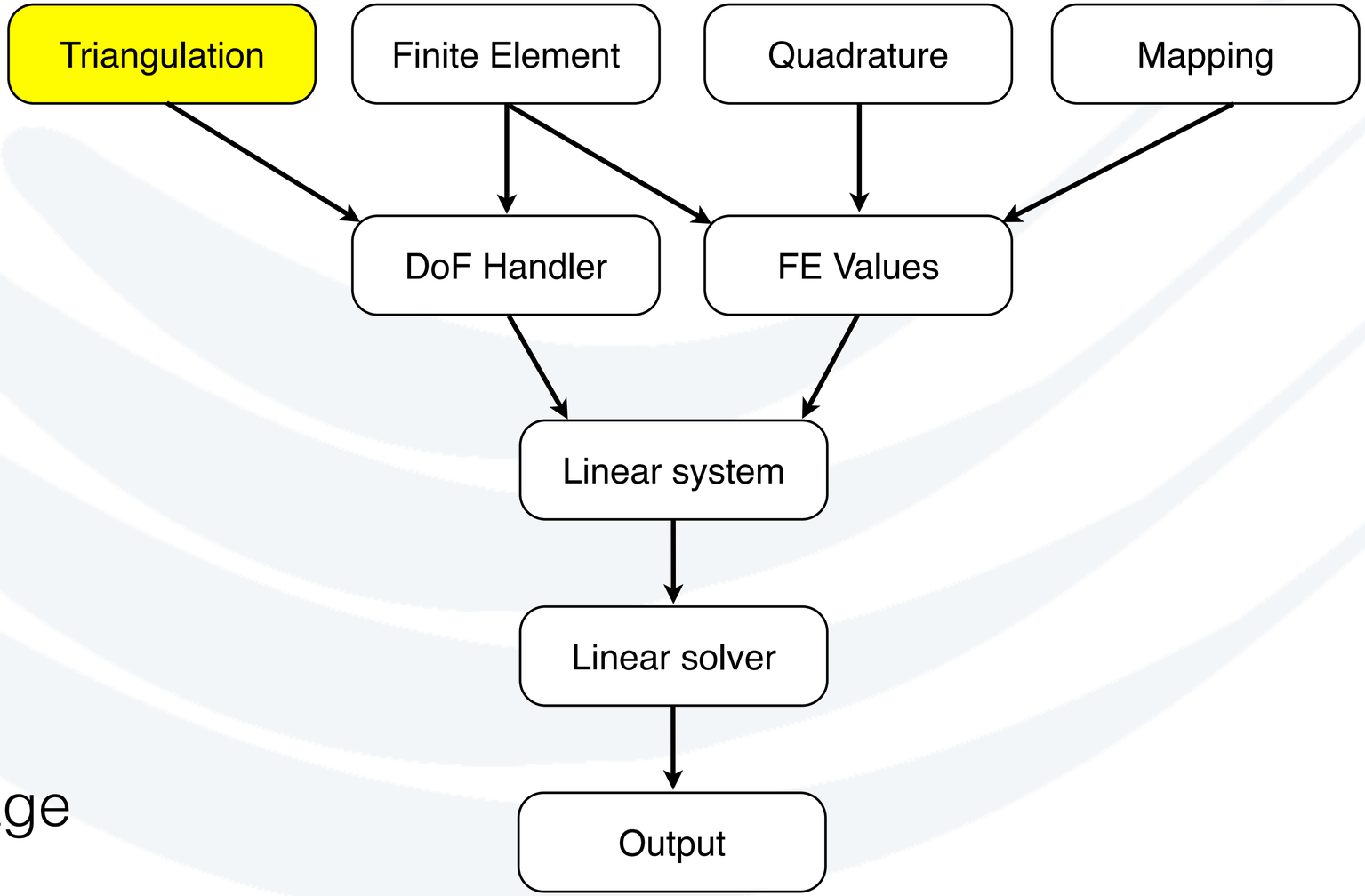
Structure of a prototypical FE problem



Main page

https://dealii.org/current/doxygen/deal.II/index.html

Structure of a prototypical FE problem

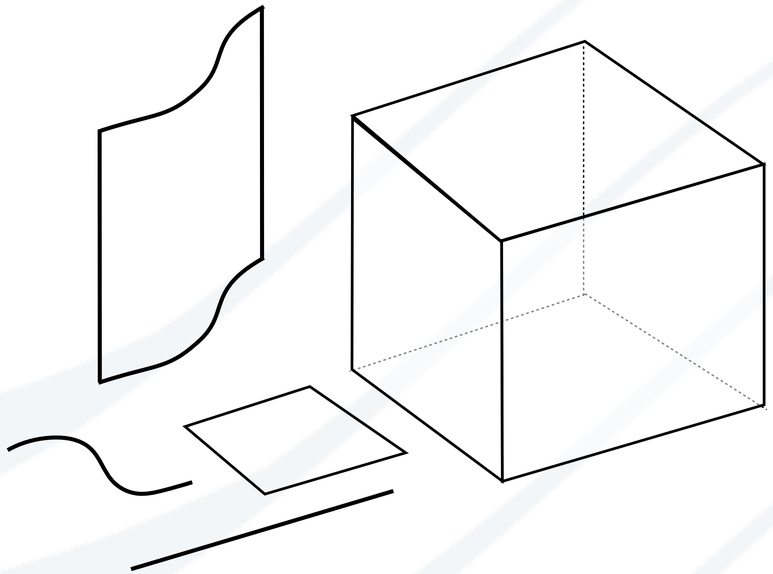


Main page

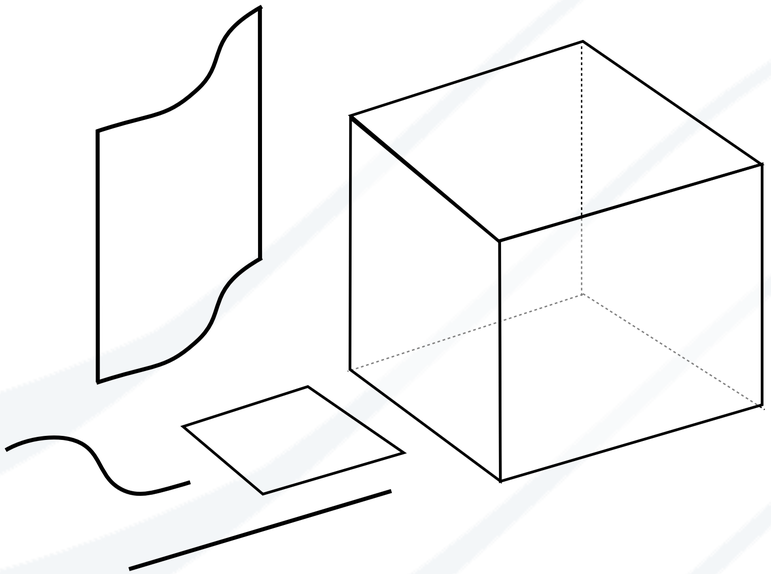
https://dealii.org/current/doxygen/deal.II/index.html

Interaction with geometry: the Triangulation class

- Describes problem geometry
  - Support for lines, quad, hex elements
  - Conceptually even higher order!
  - Structured/unstructured meshes
  - Co-dimension 1 or 2 case
- Grid creation
  - Built-in basic grid generation and manipulation tools
  - Can read in grids



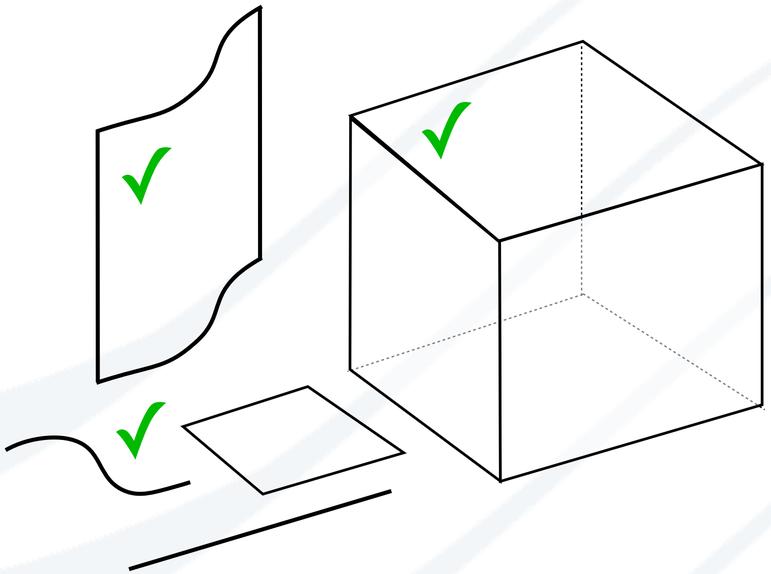
Interaction with geometry: the Triangulation class

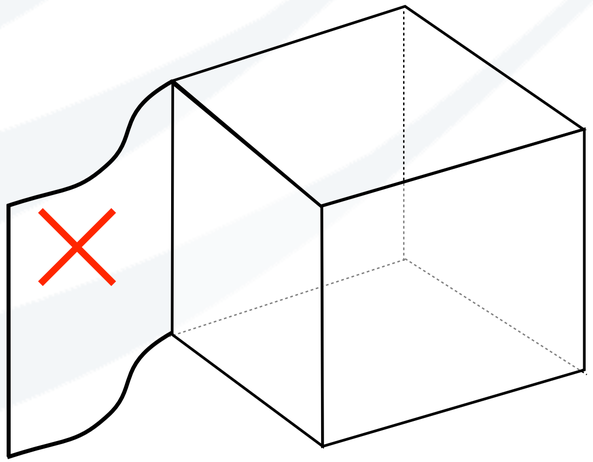


- Assign helper ID's
  - Materials
  - Boundaries
  - Manifolds
- Allows storage of custom data-structure attached to each cell/face
- Cells know about neighbour cells
  - Useful for DG methods

Interaction with geometry: the Triangulation class

- Can enforce topologies
  - Manifolds on boundary
  - Internal manifolds
- Disadvantage
  - Cannot mix triangulation types
  - e.g. Volumetric body with extended manifold surface

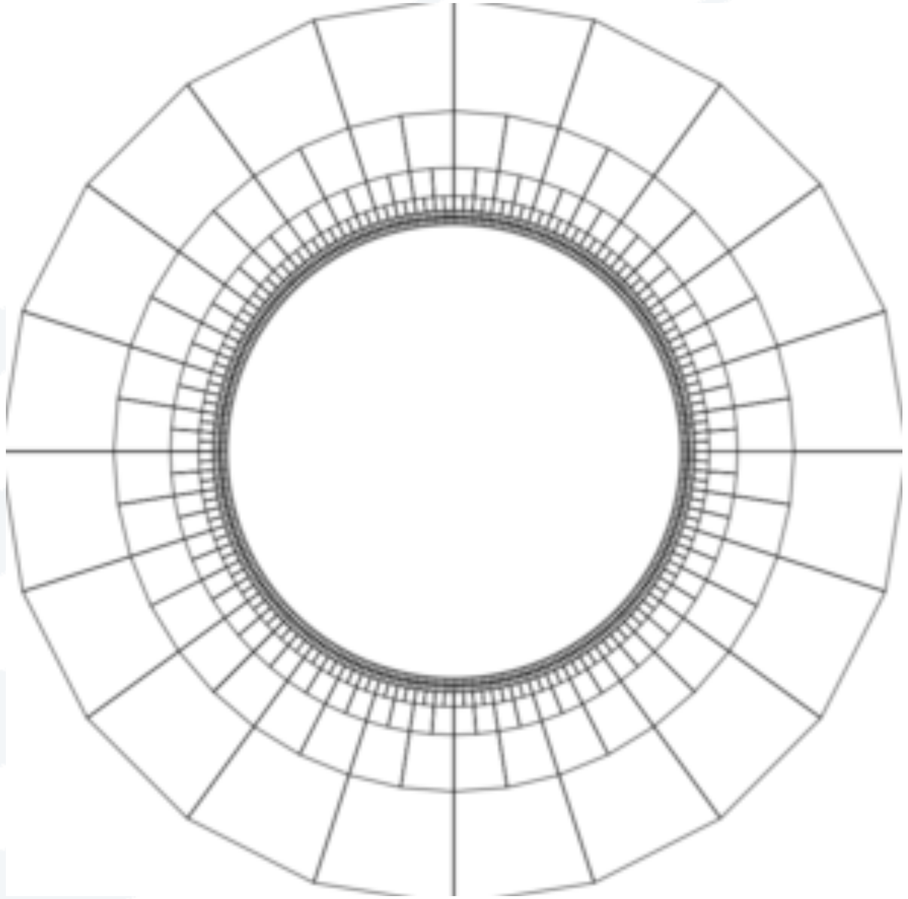




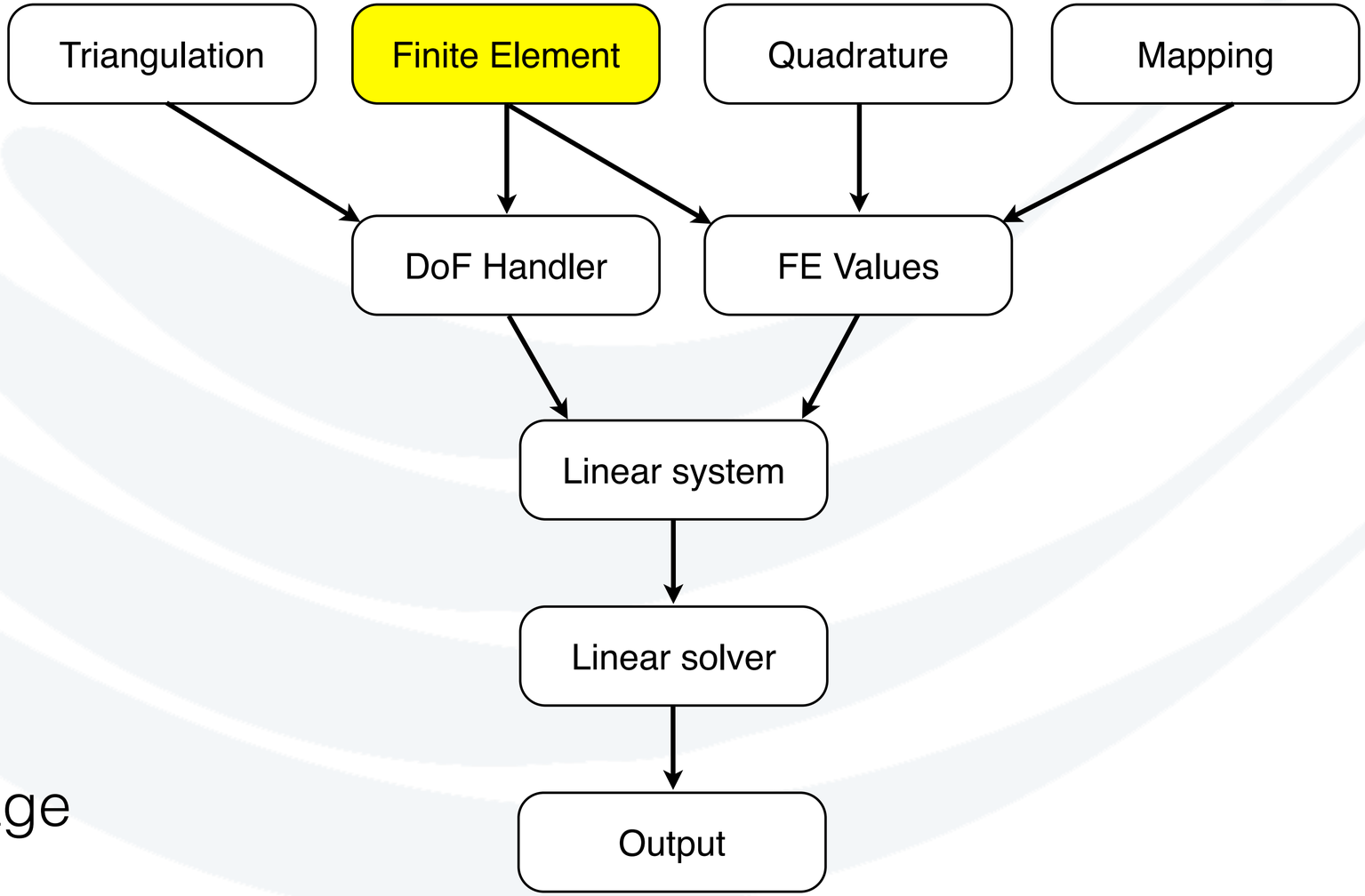
Interaction with geometry: the Triangulation class

- Demonstration: Step-1, step-49
  https://www.dealii.org/current/doxygen/deal.II/step_1.html
  https://www.dealii.org/current/doxygen/deal.II/step_49.html
  http://www.math.colostate.edu/~bangerth/videos.676.5.html
  http://www.math.colostate.edu/~bangerth/videos.676.6.html

- Key points
  - deal.II headers
  - Creating a triangulation
  - Boundary topology
  - Traversing a triangulation
  - Querying geometric information
  - Manipulating a triangulation
  - Aspects of grid refinement
  - Visualising a triangulation



Structure of a prototypical FE problem



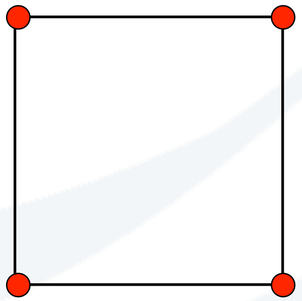
Main page

https://dealii.org/current/doxygen/deal.II/index.html

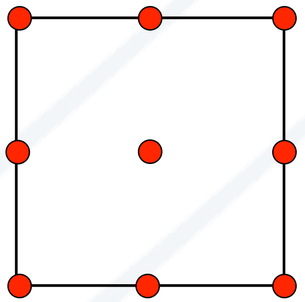
Assigning degrees-of-freedom: the FiniteElement classes

- Built in Finite Elements
  - Continuous
    - Piecewise Lagrange polynomials
  - Discontinuous
    - Monomials
    - Legendre polynomials
  - Vector-valued
    - Nedelec (H^{curl}, C/Dc)
    - Raviart-Thomas (H^{div}, C/Dc)
- A few more...
- Can develop finite elements from scratch
  - Specialisation for FE's derived by polynomial expansions
  - Enhanced/bubble elements

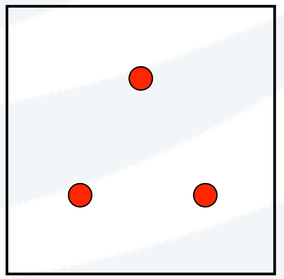
`FE_Q<2>(1)`



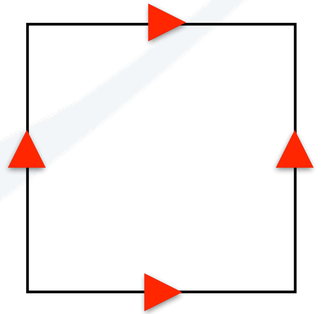
`FE_Q<2>(2)`



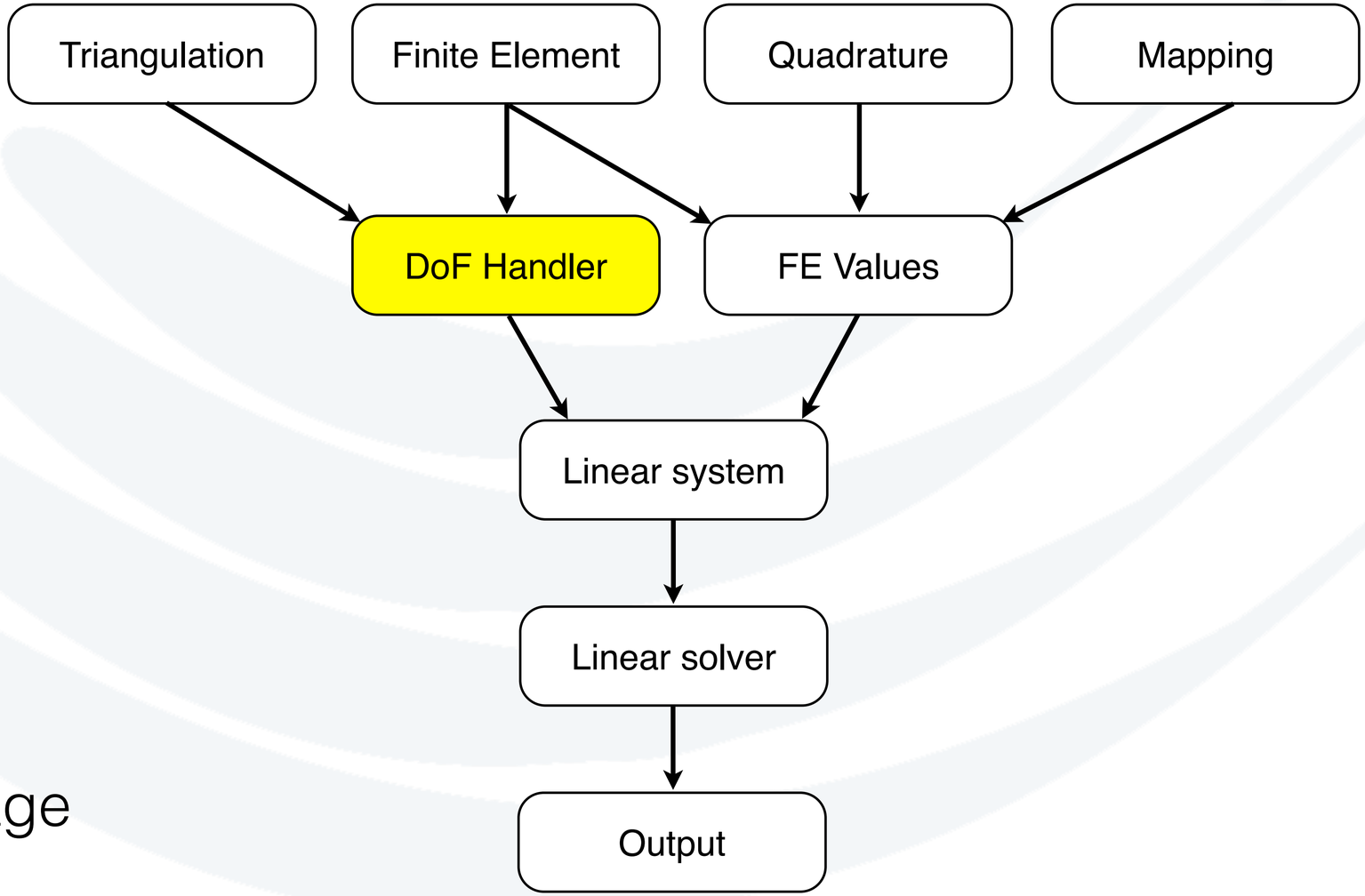
`FE_DGPMonomial<2>(1)`



`FE_Nedelec<2>(0)`



Structure of a prototypical FE problem

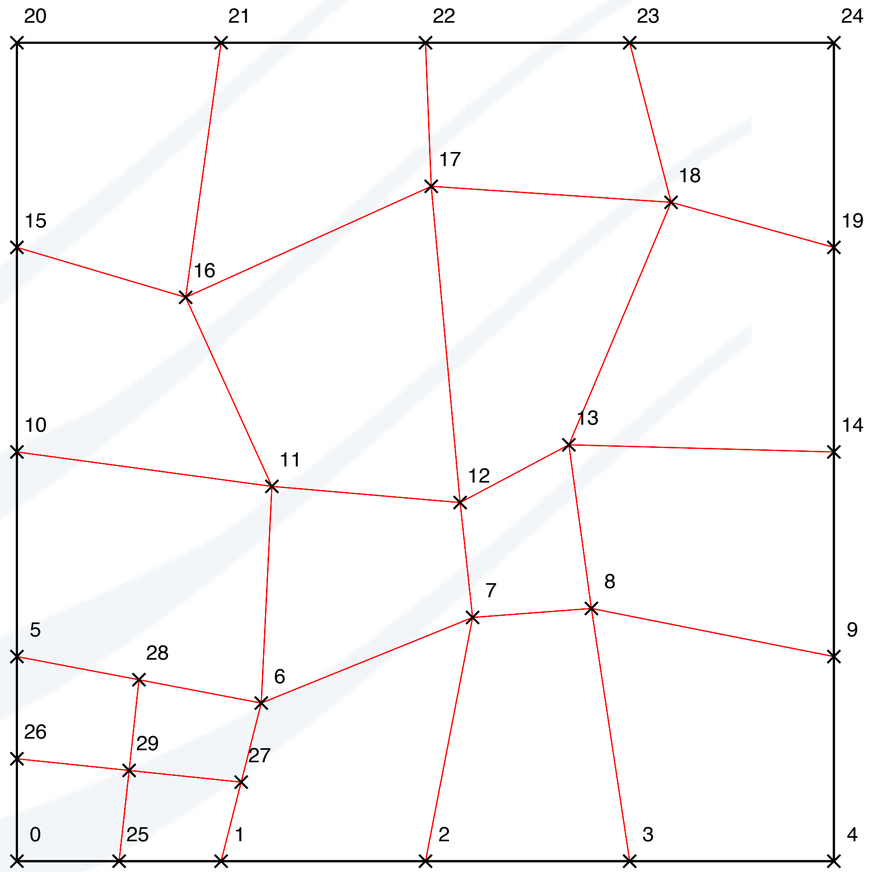


Main page

https://dealii.org/current/doxygen/deal.II/index.html

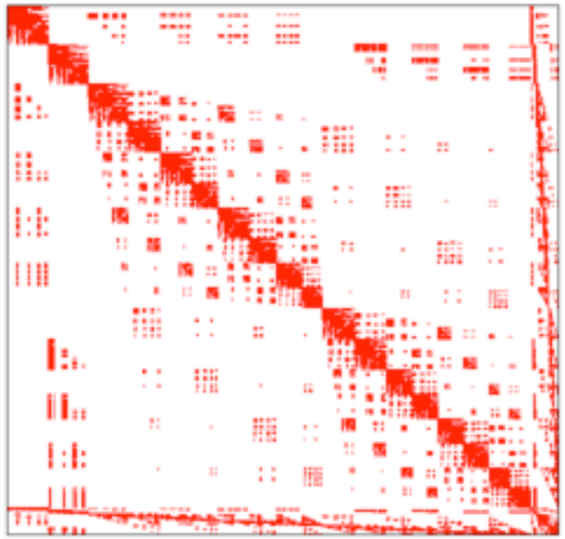
Assigning degrees-of-freedom: the DoFHandler class

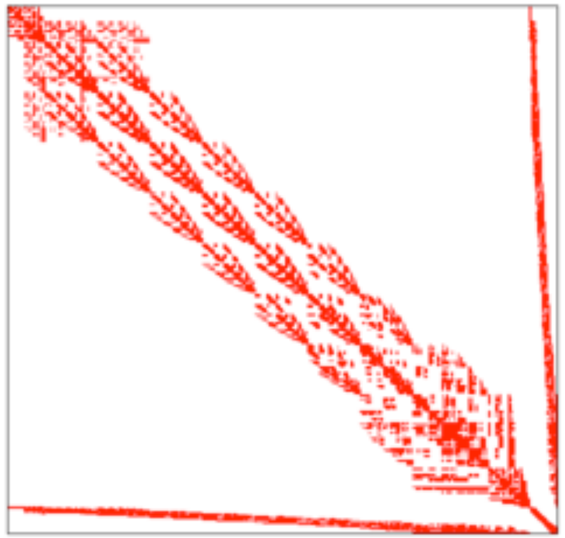
- DoFHandler assigns DoF's to grid
  - Important: separate from Triangulation!
- Unified way to access DoF's, regardless of FE used
  - e.g. Discontinuous elements: support points not necessarily at vertices
- Fast access and grid traversal
  - STL-type cell iterators
  - Access to faces, edges through these



Assigning degrees-of-freedom: the DoFRenumbering namespace

- Renumbering schemes
  - Cuthill McKee
  - King
  - Downwind
- Reduce bandwidth
- Collect like-components
- Induce block-structure
- Directional (fluid flow)
- MPI subdomain





Assigning degrees-of-freedom: the FiniteElement and DoFHandler classes

- Demonstration: Step-2
  https://www.dealii.org/current/doxygen/deal.II/step_2.html
  http://www.math.colostate.edu/~bangerth/videos.676.9.html

- Key points
  - Choosing a Finite Element
  - Distributing degrees-of-freedom on a mesh
  - Renumbering degrees-of-freedom
  - Visualising sparsity patterns