

G



D

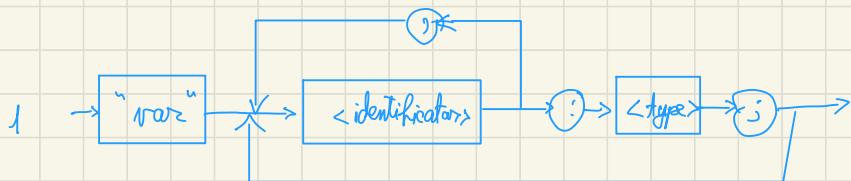


D

1. 1.1 . (suma)  
 (suma, produs, dif)      lista de param

-23      constante  
 + \$A10

1.2 var      a,b,c : intreg;  
 s : string;  
 vector : array[1..20] of boolean;



2. () , [ ], { }, 0 ..

1) BNF

<lista\_par> ::= (<l-ids>)

<l-ids> ::= | <l\_id> | <l\_id> ,

EBNF

<lista\_par> ::= ("( " | " ) " | "[ " | " ] " | "{" | " } " )"

BNF

2) <intreg-zeceinal-hexadecimal> ::= <numn> <numm>

<numn> ::= + | - | E

<numm> ::= <nz2> | \$ <nz2>

<nz2> ::= <cifra> | <cifra> <nz2>

<nz2> ::= <cifrah> | <cifrah> <nz2>

EbNF

instrucción ::= [ "+ " "-" ] ( espacio | espacio { } | " \$" espacio { } espacio { } )

3. a) Terminali : BEGIN, end., ., ;, ID, :=, +, -, (), then

no terminali : <program>, <lista-instr>, <instr>, <atributos>, <instr-if>, <expr>, <variables>

b) begin

if(ab) then

ab = a+c

end.

begin

ab = acc;

abc = a

end.

3.2

a) elem. lexicales : var, integer, begin, end

operadores : :=, +

separadores : ;, , , , ,

constante : f, 11, 5, 3

id : f, a1, a2, a3

struct sintactice : program, lista, expresión, atributo, ...

b)

<PROG> ::= VAR <VARS> : integer ; begin <instr> end.

<VARS> ::= ID, <VARS> ID

<INSTR> ::= <atrib> | <atrib> <INSTR>

<atrib> ::= ID := <expr>

<expr> ::= const | ID + <expr>

c) var  $x, y$ : integer ;

begin

$x := 15$ ;

$y := x + 3$

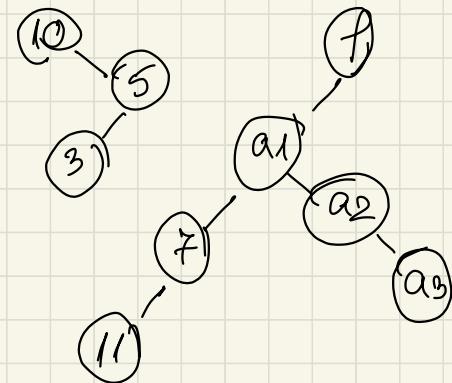
end.

Atom	Cod atom
id	0
CONST	1
var	2
:	3
;	4
begin	5
end	6
,	7
+	8
integer	9
:=	10
.	11

FIFO	Cod atom	Cod TS
	2	-
	0	-
	7	-
	0	-
	7	-
	0	-
	3	-
	9	-
	4	-
	5	-
	0	-
	10	-
	1	-
	4	-
	0	-
	10	-
	1	-
	4	-
	...	-

Tabel ord. lexic.

Simbol	Cod TS
"	11
?	7
a1	4
a2	2
a3	3
f	1



1.1.

$$A: L_A = \{ uw \mid w \in \{0,1\}^* \} \cup \{0\}$$

$$B: L_B = \{ -uv \mid w \in \{0,1\}^* \} \cup L_A$$

$$C: L_C = \{ xy.z \mid y \in \{0,1\}^*, z \in \{0,1\}^* \} \cup L_A$$

$$D: L_D = \{ xy \mid x \in \{1,0\}, y \in \{0,1\}^* \} \cup \{0\}$$

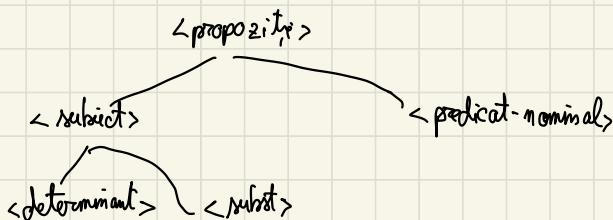
$$E: L_E = \{ -xy \mid x \in \{1,0\}, y \in \{0,1\}^* \} \cup L_A$$

$$F: L_F = \{ xy.z \mid x \in \{1,0\}, y \in \{0,1\}^*, z \in \{0,1\}^* \} \cup L_A \cup \{0.z \mid z \in \{0,1\}^* \}$$

2.1 a)  $L(G) = \{ab, b\}$ 

P:  $A \rightarrow aB \quad (1)$  derivare:  $A \xrightarrow{(1)} aB \xrightarrow{(2)} ab$   
 $A \rightarrow B \quad (2)$   $A \xrightarrow{(2)} B \xrightarrow{(3)} b$   
 $B \rightarrow b \quad (3)$

b) arbore de derivare



$L(G) = \{ 0 \text{ fumăte este derivabilă}, 0 \text{ fet. este cont.}, \text{ orice fet. este deriv.}, \text{ orice fumăt este cont} \}$

2.2

a) ab, ac

$$A \rightarrow ab$$

$$A \rightarrow aC$$

$$B \rightarrow b$$

$$C \rightarrow c$$

$$b) A \rightarrow aB$$

$$B \rightarrow bC$$

$$C \rightarrow c$$

3.1

 $L_E$ :

BNF :

$$\langle E \rangle := \langle s \rangle \langle \text{cifra} \rangle \langle \text{múltiplo} \rangle \mid \langle s \rangle \langle \text{cifra} \rangle$$

$$\langle s \rangle ::= + \mid - \mid \epsilon$$

$$\langle \text{cifra\_n} \rangle ::= 1 \dots 9$$

$$\langle \text{cifra} \rangle ::= 0 \mid \langle \text{cifra\_n} \rangle$$

$$\langle \text{múltiplo} \rangle ::= \langle \text{cifra} \rangle \langle \text{múltiplo} \rangle \mid \langle \text{cifra} \rangle$$

EBNF :

$$E ::= [ "+ " \mid "- " ] ( 1 | 2 | \dots | 9 ) \{ 0 \mid 1 \mid 9 \} \{ 0 \mid " 0 " \}$$

 $L_A$ 

BNF :

$$\langle A \rangle ::= \langle \text{cifra\_n} \rangle \langle m \rangle \mid \langle \text{cifra} \rangle$$

$$\langle \text{cifra\_n} \rangle ::= 1 \dots 9$$

$$\langle \text{cifra} \rangle ::= 0 \mid \langle \text{cifra\_n} \rangle$$

$$\langle m \rangle ::= \langle \text{cifra} \rangle \langle m \rangle \mid \langle \text{cifra} \rangle$$

4.1 a)

$$L \in \{ab, aabb, aaabbb, \dots\} = \{a^m b^m \mid m \in \mathbb{N}\}$$

b)  $G = (N, \Sigma, S, P)$

$$N = \{A\}$$

$$\Sigma = \{a, b\}$$

$$S = A$$

$$P: A \rightarrow ab$$

$$A \rightarrow aAb$$

5.1  $L = \{x^m y^m \mid m \in \mathbb{N}\}$

$$G: S \rightarrow E \quad (1)$$

$$S \rightarrow xSy \quad (2)$$

I  $L \subseteq L(G)$  ?

$$m=0, x^0 y^0 = x^0 y^0 = \epsilon \quad \left\{ \begin{array}{l} \Rightarrow x^0 y^0 \in L(G) \\ S \stackrel{(1)}{\Rightarrow} \epsilon \end{array} \right.$$

$$m > 0 \quad S \stackrel{(2)}{\Rightarrow} xSy \stackrel{(2)}{\Rightarrow} x^2 Sy^2 \stackrel{\dots}{\Rightarrow} \dots \Rightarrow x^m Sy^m \stackrel{(1)}{\Rightarrow} x^m y^m \Rightarrow x^m y^m \in L(G)$$

II  $P(k)$  După exact  $k$  deriva  $\Rightarrow x^k Sy^k$  sau  $x^{k-1} y^{k-1} \in L(G), \in L \quad (k \in \mathbb{N}^*)$

$$P(1): S \stackrel{(1)}{\Rightarrow} \epsilon = x^{t-1} y^{t-1} \quad \left\{ \begin{array}{l} \Rightarrow P(1) \in T \\ \stackrel{(2)}{\Rightarrow} xSy \rightarrow xy \end{array} \right.$$

$$\left. \begin{array}{l} \Rightarrow \\ L \subseteq L(G) \end{array} \right.$$

$P_p$  că  $P(k)$  este T. Dacă că  $P(k+1) \in T$

$$P(k) \cap T : S \stackrel{k \text{ deriva}}{\Rightarrow} x^k Sy^k \stackrel{(1)}{\Rightarrow} x^k y^k \quad \left\{ \begin{array}{l} \Rightarrow \\ \stackrel{(2)}{\Rightarrow} x^{k+1} Sy^{k+1} \end{array} \right. \quad \left\{ \begin{array}{l} P(k+1) \in T \\ x^{k+1} y^{k+1} \text{ nu putem continua} \end{array} \right.$$

$$\text{Dacă } (k) \text{ și } (k+1) \Rightarrow L = L(G)$$

$$5.2 \quad L = \{a^{2n}bc \mid n \in \mathbb{N}\}$$

$$\therefore L = \{a^{2m+1} \mid m \in \mathbb{N}\}$$

$$S \rightarrow bc$$

$$S \rightarrow aaS$$

# Seminar 3

18 - 10.2023

1.1 a b

$p \quad g \quad p \quad o$   
 $g \quad p \quad p \quad 1$

AFD

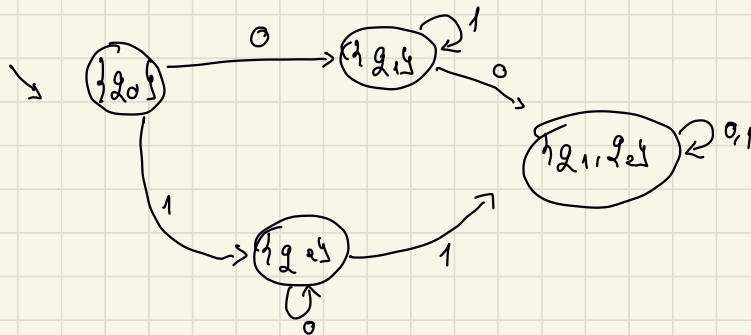
0 1

$g_0 \quad g_1 \quad g_2 \quad 0$   
 $g_1 \quad g_2 \quad g_0 \quad g_1$   
 $g_2 \quad g_0 \quad g_1 \quad g_2$

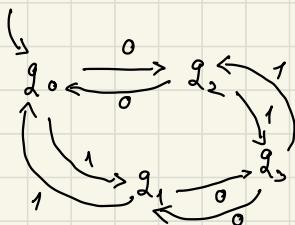
AFN

0 1

$g_{0,1} \quad g_{1,1} \quad g_{2,1} \quad 0$   
 $g_{1,1} \quad g_{2,1} \quad g_{0,1} \quad 0$   
 $g_{2,1} \quad g_{0,1} \quad g_{1,1} \quad 1$   
 $g_{0,1} \quad g_{1,1} \quad g_{2,1} \quad 1$



2.



a)  $1010 \in L(M)$ ?

$$(g_0, 1010) \xrightarrow{(g_1, 010)} (g_1, 010) \xrightarrow{(g_2, 10)} (g_2, 0) \xrightarrow{(g_3, 0)} (g_0, \epsilon) \left. \begin{array}{l} \\ \\ g_0 \in T \end{array} \right\} \Rightarrow 1010 \in L(M)$$

$$d(g_0, 1) = g_1$$

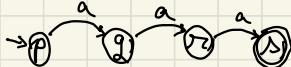
$1100 \in L(M)$ ?

$$(g_0, 1100) \xrightarrow{(g_1, 100)} (g_1, 100) \xrightarrow{(g_0, 00)} (g_0, 00) \xrightarrow{(g_1, 0)} (g_0, \epsilon) \left. \begin{array}{l} \\ \\ g_0 \in T \end{array} \right\} \Rightarrow 1100 \in L(M)$$

b) 1011

$$(g_0, 1011) \vdash (g_1, 011) \vdash (g_2, 11) \vdash (g_3, 1) \vdash (g_4, \varepsilon) \left( \begin{array}{l} \Rightarrow 1011 \notin L(M) \\ g_3 \text{ is } F \end{array} \right)$$

$$3. \text{ a) } L = \{aaa\}, \Sigma = \{a\}$$



$$b) L = \{w_1 a a a w_2 \mid w_1, w_2 \in [a, b]^*\}$$



$$c) \rightarrow \overset{0,1}{\textcircled{q}_1} \xrightarrow{0} \overset{0}{\textcircled{q}_2} \xrightarrow{0} \overset{0,1}{\textcircled{q}_3}$$

d) 

```

graph LR
    q0((q0)) -- a --> q1((q1))
    q1 -- a --> q2((q2))
    q2 -- a --> q3((q3))
    q3 -- a --> q4((q4))
    q4 -- a --> q5((q5))
    q5 -- a --> q6((q6))
    q0 -- b --> q1
    q1 -- b --> q3
    q3 -- b --> q4
    q4 -- b --> q6
    q5 -- a --> q6
    q5 -- c --> q6
    q6 -- c --> q5

```

c)  $\rightarrow$

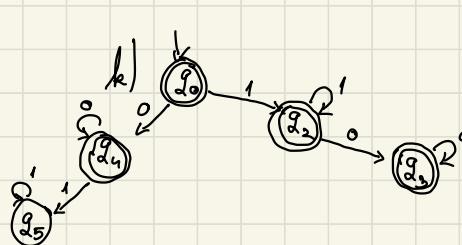
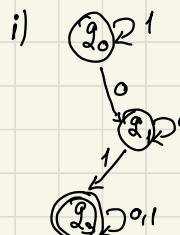
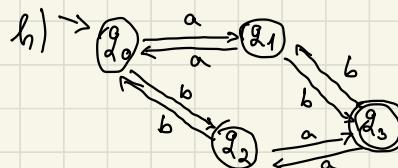
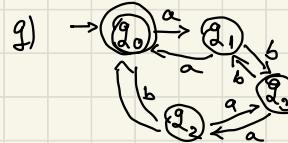
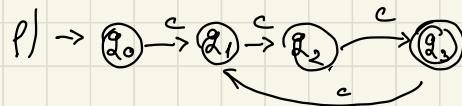
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graph LR
    q0((q0)) -- "a|bc" --> q1((q1))
    q0 -- "a" --> q2((q2))
    q1 -- "b" --> q2
    q1 -- "c" --> q3((q3))
    q2 -- "a|bc" --> q3
    q2 -- "b" --> q4((q4))
    q3 -- "c" --> q4
    q4 -- "a, b|bc" --> q4
  
```

```

graph LR
    f((f)) --> q0((q0))
    q0 -- a --> q1((q1))
    q0 -- b --> q2((q2))
    q1 -- a --> q2
    q2 -- "a/b" --> q2

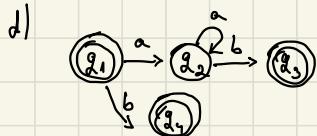
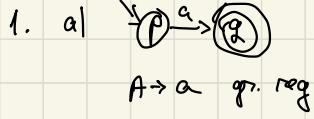
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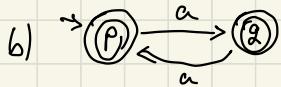
1.2

# Seminar 4

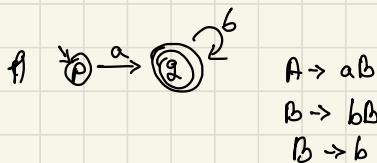
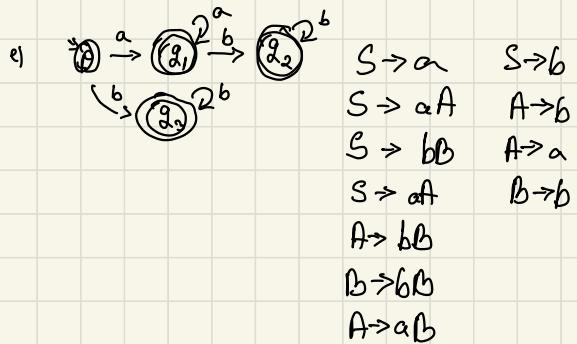
25. 10. 2023



$$\begin{aligned} S &\rightarrow \epsilon \\ S &\rightarrow b \\ S &\rightarrow aA \\ A &\rightarrow aA \end{aligned}$$



$$\begin{aligned} A &\rightarrow a \\ A &\rightarrow \epsilon \\ A &\rightarrow ab \\ A &\rightarrow aB \end{aligned}$$



*Nachricht din care iei singură prima secvență meteterminată*

2.

$$\begin{aligned} AF \rightarrow G &: \quad O_N^{\leftarrow} \\ \rightarrow \odot &: \quad S \rightarrow \epsilon \\ O_A^{\leftarrow} \rightarrow \odot &. \quad A \rightarrow c \\ O_A^{\leftarrow} \rightarrow O_B &: \quad A \rightarrow cb \\ \rightarrow \odot &_{SA} \end{aligned}$$

3. a)

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow ab \\ B &\rightarrow aS \\ B &\rightarrow a \end{aligned}$$

b)

$$\begin{aligned} S &\rightarrow \epsilon \\ S &\rightarrow aF \\ A &\rightarrow aB \\ B &\rightarrow aC \\ B &\rightarrow a \\ C &\rightarrow aF \end{aligned}$$

c)

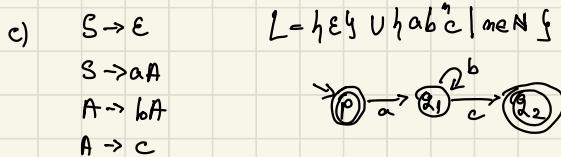
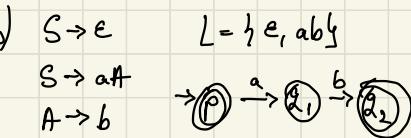
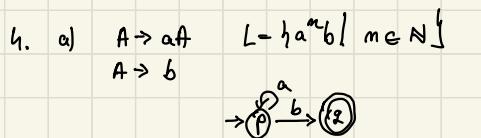
$$\begin{aligned} A &\rightarrow aF \mid aB \\ B &\rightarrow bb \mid b \end{aligned}$$

d)

$$\begin{aligned} S &\rightarrow 1A \\ S &\rightarrow 0 \\ A &\rightarrow 0A \\ A &\rightarrow 1A \\ A &\rightarrow 1 \\ A &\rightarrow 0 \\ S &\rightarrow 1 \end{aligned}$$

e)

$$\begin{aligned} S &\rightarrow a1 \dots 1z \\ S &\rightarrow \dots \end{aligned}$$



$G \rightarrow AF$

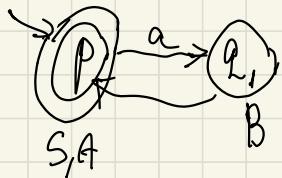
$N \Theta + 10$

$S \rightarrow \epsilon$

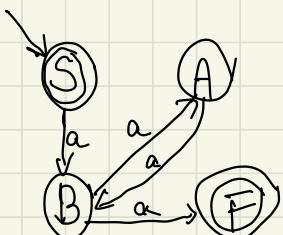
$A \rightarrow c$      $A \xrightarrow{a} \textcircled{Q}$

$A \rightarrow cB$

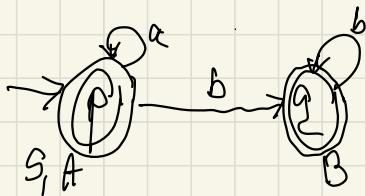
5. a)  $L = \{a^{2m} \mid m \in \mathbb{N}\}$



$S \rightarrow \epsilon$   
 $S \rightarrow aB$   
 $B \rightarrow aA$   
 $B \rightarrow a$   
 $A \rightarrow aB$



b)  $L = \{ a^m b^n \mid m, n \in \mathbb{N} \}$



$$S \rightarrow E$$

$$S \rightarrow aA$$

$$A \rightarrow a$$

$$A \rightarrow bB$$

$$B \rightarrow b$$

$$S \rightarrow a$$

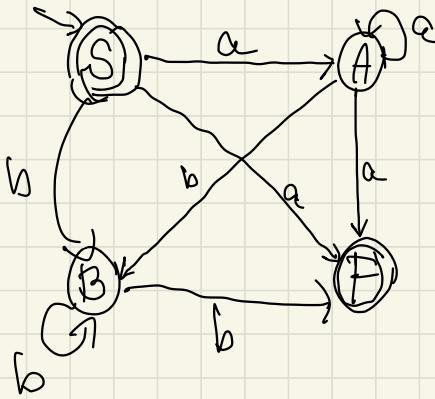
$$A \rightarrow aA$$

$$S \rightarrow bB$$

$$S \rightarrow b$$

$$A \rightarrow b$$

$$B \rightarrow bB$$



un limbaj e regulat dacă se poate face AF

1.1 lema de pompări pt. lb. regulare  $\rightarrow$  ciclu

$L$  - lb. reg. at.  $\exists p \in N^*$  unic pt. lb.  $\forall k$  mare a.c.  $|w|_{kp} = p$ ,  $\exists$  descompunere  $w = xyz$ ,  $0 < |y| \leq p$  a.s.  $\forall i \in N$   $xy^iz \in L$

$A \rightarrow B \rightarrow TB \rightarrow TA$  (reducere la absurd)

1. a) nu este regulat  $L = \{a^n b^{2n} \mid n \in N^*\}$

$\forall p \in N^*$ ,  $\exists w \in L$ ,  $w = a^p b^{2p}$ ,  $|w| = 4p \geq p$

$\forall w = xyz$ ,  $0 < |y| \leq p$

aaa ... aaa ... aaabb... bbb... bbb = w

cas I :  $x = \text{aaa ... a}$

$$\begin{array}{l} y = \text{restul} \\ z = b \dots bbb \end{array}$$

cas II :  $x = \text{cc e în fact}$

$$\begin{array}{l} y = bb \dots bb \\ z = b \dots bbb \end{array}$$

cas III :  $x = \text{aa ... a}$

$$\begin{array}{l} y = aa \dots a \\ z = \text{restul} \end{array}$$

cas I :  $x = a^k$ ,  $0 \leq k \leq p$

$$\begin{array}{l} y = a^{p-k} b^j, 0 \leq k < p, 0 \leq j \leq p \\ z = b^{2p-j} \end{array}$$

$$\text{pentru } i=2, xy^2z = a^k (a^{p-k} b^j)^2 b^{2p-j} = a^k a^{p-k} b^j a^{p-k} b^j b^{2p-j}$$

$$= a^{p+k} a^{p-k} b^{2p}, j > 0, p-k > 0 \Rightarrow \notin L(1)$$

cas II :  $x = a^p b^j$

$$\begin{array}{l} y = b^k \\ z = b^{2p-k-j} \end{array}, \quad 0 \leq j < 2p, 0 < k \leq p$$

$$i=2, xy^2z = a^p b^j b^k b^{2p-k-j}$$

$$= a^p b^{dp+k} \quad \left\{ \begin{array}{l} \Rightarrow \notin L(2) \\ k > 0 \end{array} \right.$$

caz III analog (3)

(1,2,3)  $\Rightarrow$  lema de pompare nu are loc  $\Rightarrow L$  nu e regular

b)  $L = \{a^k \mid k \text{ prim}\}$

$\nexists p \in \mathbb{N}^*, \exists w \in L, |w| \geq p, w = a^k, k \text{ prim}$   
 $k \geq p$

$w = xyz, 0 < |y| \leq p$

$$x = a^j, j \geq 0, j < k$$

$$y = a^l, 0 < l \leq p$$

$$z = a^{k-l-j}$$

$$\text{fii } i = k+1, xy^iz = a^j a^l a^{k-j-l} = a^{k(l+1)}$$

$k \text{ prim}, k \geq 2, l+1 > 0 \Rightarrow k(l+1) \text{ nu e prim}$

$\Rightarrow \nsubseteq L \Rightarrow$  lema de pompare nu are loc  $\Rightarrow L$  nu e regular.

d)  $L = \{a^{2^m} \mid m \in \mathbb{N}\}$

$\nexists p \in \mathbb{N}^*, \exists w \in L, |w| \geq p, w = a^{2^p}, |w| = 2^p \geq p$

$w = xyz, 0 < |y| \leq p$

$$x = a^m, 0 \leq m < 2^p$$

$$y = a^{\frac{p}{2^p-m-p}} \quad 0 < n \leq p$$

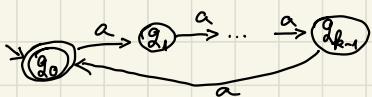
$$z = a^{2^{p+m}-m-n}$$

$$i = 2, xy^iz = \dots = a^{2^{p+m}}$$

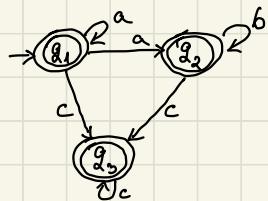
$$2^{p+m} > 2^p \quad 2^{p+i} = 2 \cdot 2^p = 2^{p+2^p} \Rightarrow 2^{p+m} < 2^{p+1}$$

$$m \leq p \Rightarrow \nsubseteq L \dots$$

e)  $L = \{a^{km} \mid m \in \mathbb{N}\}$



f)  $L = \{a^m b^n c^p \mid m, n, p \in \mathbb{N}\}$



Prop. de inclusiune din curs:  $L_1, L_2, L_1 \cap L_2 \dots$

2. a) limbajul de la 1b) nu e regular.

$L_1 = \{a^k \mid k \text{ mesuri } f\} \Rightarrow L_1 \text{ nu e reg.}$   
 $\overline{L_1} \text{ e 1b}$

b)  $p=2$

Fie  $w = aaaa$ ,  $|w| = 4 > 2$

$w = xyz$ ,  $x = a$

$y = a$ ,  $0 < |y| = 1 \leq 2 = p$

$z = aa$

$\forall i \in \mathbb{N}^*, xy^i z = a a^{2i} a = a^{2(i+1)} \in L \forall i \in \mathbb{N}^*$

1.2.1 a)  $L = \{a^m \mid m \in \mathbb{N}^*\}$



g. reg :  $A \rightarrow a$

$A \rightarrow aB$

$B \rightarrow a$

$B \rightarrow aB$

g. nonreg.  $S \rightarrow a$

$S \rightarrow Sa$

$$b) \quad L = \{a^m b^{2n} \mid m \in \mathbb{N}^*\} \quad S \rightarrow ab^2 \\ S \rightarrow aSbb$$

1.3.1

$$a) \quad A \rightarrow aB$$

b)  $A \rightarrow B$

$$B \rightarrow b$$

$$A \rightarrow bB$$

$$\beta \rightarrow bA$$

$$B \rightarrow aB$$

$$\beta \rightarrow \alpha$$

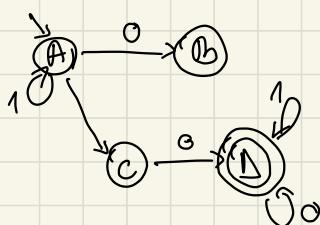
1. a)  $01110111 \rightarrow (1^*01)^* (11+0)^*$

b)  $11100111 \quad (1^*0)^* + (0^*1) \quad X$

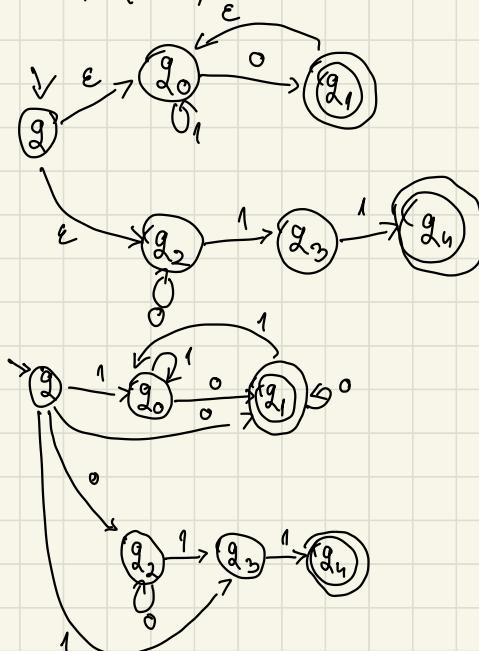
c)  $011100101 \quad \downarrow \quad 01^*01^* (11^*0^*)^*$

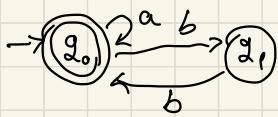
d)  $1000011 \quad [10^* + 11]^* (0^*1)^*$

2. a)  $(01+1)^*00 \quad (0+1)^*$



b)  $(1^*0)^* + 0^*11$





BUNĂ

$$\begin{cases} q_1 = q_1 a + q_2 b + \varepsilon \\ q_2 = q_1 b \end{cases}$$

$$q_1 = q_1 a + q_1 b^2 + \varepsilon$$

$$q_1 = q_1 (a+b^2) + \varepsilon \Rightarrow q_1 = \varepsilon (a+b^2)^* = (a+b^2)^* - \text{expr. mg.}$$

$$\begin{cases} q_1 = q_1 a + q_2 b + \varepsilon \\ q_2 = q_1 b \\ q_3 = q_1 c + q_2 a \\ \quad = (a+b^2)^* + c \end{cases}$$

$$q_1 + q_3 = q_1 a + q_1 b^2$$

$$\begin{cases} i=1 \\ j \\ m=2 \end{cases}$$

$$1 \quad \{a^{2n} \mid n \in \mathbb{N}\}$$

$$S \rightarrow E$$

$$S \rightarrow aA$$

$$A \rightarrow aB$$

$$B \rightarrow aA$$

$$A \rightarrow a$$

Gr regular

tip 3



$$S \xrightarrow{\alpha} aS$$

$$S \rightarrow E$$

Gr Index

tip 2

$$S \rightarrow E$$

$$S \rightarrow aA$$

$$aA \rightarrow aaaa$$

$$A \rightarrow a$$

Gr meindag  
tip 1

2.

$$1) \quad L = \{wuw \mid w \in \{a,b\}^*\}$$

Tip 0

$$S \rightarrow aSA$$

$$S \rightarrow bSB$$

$$S \rightarrow M$$

$$Ma \rightarrow Ma$$

$$Mb \rightarrow Mb$$

$$bB \rightarrow bB$$

$$aA \rightarrow Aa$$

$$aA \rightarrow Ab$$

$$aB \rightarrow Ba$$

$$M \rightarrow E$$

$$S \rightarrow M$$

$$S \rightarrow aSCSA \quad | \quad bSCSB$$

$$C \rightarrow aC$$

$$C \rightarrow bC \quad C \rightarrow E$$

$$C \rightarrow a$$

$$C \rightarrow b$$

$$Ma \rightarrow Ma$$

$$Mb \rightarrow Mb$$

$$bB \rightarrow bB$$

$$aA \rightarrow Aa$$

$$bA \rightarrow Ab$$

$$aB \rightarrow Ba$$

$$M \rightarrow E$$

$$3) L = \{a^m b^n c^m \mid m \in \mathbb{N}^*\}$$

$S \rightarrow a SBC$

$S \rightarrow a BC$

$aB \rightarrow ab$

$bB \rightarrow bb$

$C \rightarrow c$

$CB \rightarrow BC$

$$4) L = \{a^m b^n c^m \mid m \in \mathbb{N}^*\}$$

$S \rightarrow aSBC\Delta$

$S \rightarrow aBC\Delta$

$aBC \rightarrow abc$

$\Delta \rightarrow d$

$CB \rightarrow B C$  tip 1

$BC \rightarrow C\Delta$

$B\Delta \rightarrow Bd$

$bB \rightarrow bb$

$bC \rightarrow bc$

$cC \rightarrow cc$

$c\Delta \rightarrow cd$

$d\Delta \rightarrow dd$

$$5) L = \{a^{2^m} \mid m \in \mathbb{N}\}$$

$S \rightarrow L aR$

$L \rightarrow E$

$R \rightarrow E$  tip 3

$L \rightarrow LA$

$Aa \rightarrow aaA$

$AR \rightarrow R$

6. mu e ic

7. ic

8. ic

9. ic

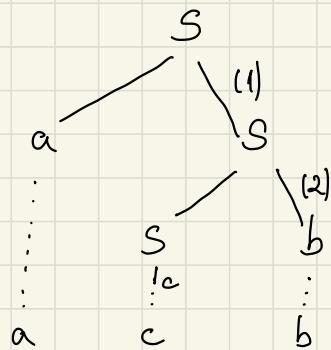
10. ic

11

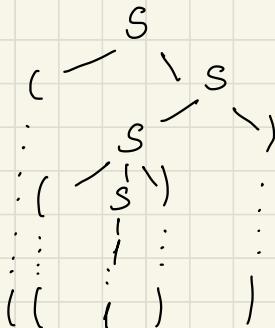
2. Gv. ambigua - dacă și arborei (...)

b) Pozai tel

a)  $S \rightarrow aS$   
 $S \rightarrow Sb$   
 $S \rightarrow c$



c)  $S \rightarrow (S)$   
 $S \rightarrow S)$   
 $S \rightarrow (S)$   
 $S \rightarrow 1$



```

graph TD
    S1[S] --> S2[S]
    S1 --> P1["(1)"]
    S2 --> a1[a]
    S2 --> Dots1[...]
    S2 --> a2[a]
    P1 --> c1[c]
    P1 --> Dots2[...]
    c1 --> b1[b]
    c1 --> Dots3[...]

```

*E ambiigua*

rez:  $S \rightarrow aD$   
 $S \rightarrow Sb$   
 $S \rightarrow c$   
 $D \rightarrow ab$   
 $D \rightarrow c$

S-a obț. același rez din 2 arb. dif.  $\Rightarrow$  e ambiguă

$$S \rightarrow S'$$

$S \rightarrow S$ )

$$S \rightarrow 1$$

$$S^1 \rightarrow S$$

$$S' \rightarrow I$$

# Seminar 8

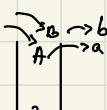
22. 11. 2023

1. e)  $L = \{ w \in \Sigma^* \mid \text{wreha}_b b^* \text{, w tilde ist inv lui w} \}$

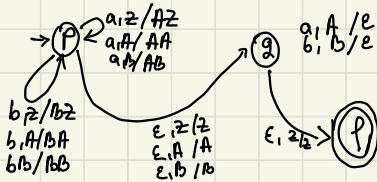
dimbyj regular similar:  $L' = \{ w \mid \text{wreha}_b b^* \}$

$\rightarrow \text{PDA } 2^{ab}$

ex: abbbba



APD Göt. istina wie



	aff. stura	a	b	$\epsilon$	
P	$z$	$(p, Az)$	$(p, Bz)$	$(g, z)$	0
	$A$	$(p, AA)$	$(p, BA)$	$(g, A)$	
	$B$	$(p, BA)$	$(p, BB)$	$(g, B)$	
g	$z$			$(p, \epsilon)$	0
	$A$	$(g, \epsilon)$			
	$B$		$(g, \epsilon)$		
p	$z$	-	-	-	1
	$A$	-	-	-	
	$B$	-	-	-	

$(p, abbbba, z) \vdash (p, bbbbba, Az)$   
 $(p, bbbba, BAz) \vdash$   
 $(p, bba, BBAz) \vdash$   
 $(g, ba, BAZ) \vdash$   
 $(g, a, AZ) \vdash$   
 $(g, \epsilon, z) \vdash$   
 $(g, \epsilon, \epsilon) \vdash$   
banda de s-a golt  
intreaga gola stura  
 $\Rightarrow abbbba \in L_\epsilon(\mu)$



Norim

$G \vdash C \rightarrow AP\Delta$

$S \rightarrow \epsilon$

$S \rightarrow aSa$

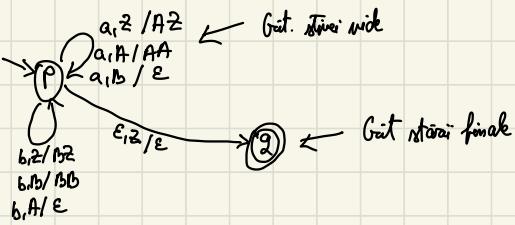
$S \rightarrow bSb$

AP $\Delta$  Gs. störn mit

		a	b	$\epsilon$
	S			( $g_1, \epsilon$ )
g				( $g_1, aSa$ )
	a	( $g_1, \epsilon$ )		
	b	( $g_1, \epsilon$ )		

$$\rho) L = \{ w \mid w \in \{a, b\}^+ \text{, } m_{\rho}(w) = m_{\rho_b}(w) \}$$

AP $\Delta$



$S \rightarrow \epsilon$

$S \rightarrow aSb$

$S \rightarrow bSa$

$S \rightarrow SS$

		a	b	$\epsilon$
	S			( $g_1, \epsilon$ )
g				( $g_1, aSb$ )
	a	( $g_1, \epsilon$ )		
	b	( $g_1, \epsilon$ )		

$$a) L = \{a^m b^{2m} \mid m \geq 0\}$$

$$S \rightarrow aSbb$$

$$S \rightarrow \epsilon$$

	a	b	$\epsilon$
S			$(g_1, aSbb)$ $(g_1, \epsilon)$
$g_2$	$a$	$(g_2, \epsilon)$	
b		$(g_2, \epsilon)$	



$$c) L = \{a^m b^m \mid m \geq m \geq 0\}$$

$$S \rightarrow \epsilon$$

$$S \rightarrow aS$$

$$S \rightarrow aSb$$

	a	b	$\epsilon$
$g_2$	S		$(g_1, aS)$ $(g_1, aSb)$ $(g_1, \epsilon)$
a	$(g_2, \epsilon)$		
b		$(g_2, \epsilon)$	

$$g) L = \{a^{2m} b^{2m} \mid m \geq 0\}$$

$$S \rightarrow aaSbb$$

$$S \rightarrow \epsilon$$

$$S \rightarrow aS$$

$$S \rightarrow Sb$$

$$S \rightarrow \epsilon$$

	a	b	$\epsilon$
$g_2$	S		$(g_1, aS)$ $(g_1, Sb)$ $(g_1, \epsilon)$
a	$(g_2, \epsilon)$		
b		$(g_2, \epsilon)$	

$$d) L = \{a^m b^m \mid m \geq m \geq 0\}$$

$$S \rightarrow \epsilon$$

$$S \rightarrow Sb$$

$$S \rightarrow aSb$$

$$h) L = \{a^m b^n \mid m \geq 0\} \cup \{b^m a^n \mid n \geq 0\}$$

	a	b	$\epsilon$
$g_2$	S		$(g_1, A)$ $(g_1, B)$
A			$(g_1, \epsilon)$ $(g_1, aAb)$
B			$(g_1, \epsilon)$ $(g_1, bBa)$
a	$(g_2, \epsilon)$		
b		$(g_2, \epsilon)$	

$$S \rightarrow A \mid B$$

$$A \rightarrow aAb$$

$$B \rightarrow bBa$$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

Seminar 9

29.11.2023

Analiza sintactică descendătoare LL(1)

ex. suplimentar

Tabelul de analiză LL(1)

$$S \rightarrow aSb \quad (1)$$

$$S \rightarrow bSa \quad (2)$$

$$S \rightarrow SS \quad (3)$$

$$S \rightarrow \epsilon \quad (4)$$

conflict  
⇒ q. nu e  
de tip LL(1)

	a	b	\$
S	(aSb) <sub>1</sub> , (aSb) <sub>2</sub> , (SS) <sub>1</sub>		
a	pop	err	err
b	err	pop	err
\$	err	err	acc

	First <sub>1</sub>	FOLLOW <sub>1</sub>
S	a, b, ε	\$, a, b

2.  $S \rightarrow aS$

$$S \rightarrow aSbS$$

$$S \rightarrow i$$

conflict  
⇒ q. nu e de tip  
LL(1) \*

	First <sub>1</sub>	FOLLOW <sub>1</sub>
S	a, i	\$, b

\*

	a	b	i	\$
S	(a <i>i</i> ) <sub>1</sub>			
a	pop			
b		pop		
i			pop	
\$				pop

Nu e LL(1) - deoarece putem realiza o  
față la stânga.

$$\boxed{S \rightarrow aS}$$

$$\boxed{S \rightarrow aSbS}$$

$$S \rightarrow aSP \quad (1)$$

$$P \rightarrow \epsilon \quad (2)$$

$$P \rightarrow bS \quad (3)$$

$$S \rightarrow i \quad (4)$$

$$S \rightarrow aSP \quad (1)$$

$$P \rightarrow \epsilon \quad (2)$$

$$P \rightarrow bS \quad (3)$$

$$S \rightarrow i \quad (4)$$

	First <sub>1</sub>	FOLLOW <sub>1</sub>
S	a, i	\$, b
P	b, ε	\$, b

	a	b	i	\$
S	(a <i>SP</i> ) <sub>1</sub>		(i, <i>SP</i> ) <sub>1</sub>	
P		(b, <i>SP</i> ) <sub>1</sub> , (b, <i>SP</i> ) <sub>2</sub>		(ε, <i>SP</i> ) <sub>1</sub>
a	pop			
b		pop		
i			pop	
\$				acc

$aaibis \in L(G) ?$

$(aaibis, \$, \epsilon) \xrightarrow{\text{push 1}}$

st. de in.      st. de      now  
biere      bieren

$(aaibis, asp\$, 1) \xrightarrow{\text{pop}}$

$(aibis, sp\$, 1) \xrightarrow{\text{push}}$

$(aibis, aspp\$, 1) \xrightarrow{\text{pop}}$

$(ibis, spp\$, 1) \xrightarrow{\text{push}}$

$(ibis, ip\$, 1) \xrightarrow{\text{pop}}$

$(bis, pp\$, 1) \xrightarrow{\text{push}}$

$(bis, bsp\$, 1) \xrightarrow{\text{pop}}$

$(i\$, sp\$, 1) \xrightarrow{\text{push}}$

$(i\$, ip\$, 1) \xrightarrow{\text{pop}}$

$(\$, p\$, 1) \xrightarrow{\text{push}}$

$(\$, p\$, 1) \xrightarrow{\text{acc}} \Rightarrow aaibis \in L(G)$  n/p sprawdzone: 114342

aibb

$(aibb, \$, \epsilon) \xrightarrow{\text{push 1}}$

$(aibb, asp\$, 1) \xrightarrow{\text{pop}}$

$(ibb, sp\$, 1) \xrightarrow{\text{push}}$

$(ibb, ip\$, 1) \xrightarrow{\text{pop}}$

$(bb, p\$, 1) \xrightarrow{\text{pop}} \Rightarrow aibb \notin L(G)$

5.  $S \rightarrow bLe \quad (1)$

$S \rightarrow i \quad (2)$

$L \rightarrow S \quad (3)$

$L \rightarrow SpL \quad (4)$

	First <sub>1</sub>	Follower <sub>1</sub>
$S$	$b, i$	$e, p, \$$
$L$	$b, i$	$e$

Étapes Dernières

	$b$	$e$	$i$	$p$	$\$$
$S$	$(bLe, 1)$		$(i, 2)$		
$L$	$(S, 3)$		$(S, 3)$		
$b$	$pop$				
$e$		$pop$			
$i$			$pop$		
$p$				$pop$	
$\$$					$acc$

→ conflit : gr. nuc de tip LL1

$S \rightarrow bLe \quad (1)$

$S \rightarrow i \quad (2)$

$L \rightarrow SA \quad (3)$

$A \rightarrow E \quad (4)$

$A \rightarrow PL \quad (5)$

	First <sub>1</sub>	Follower <sub>1</sub>
$S$	$b, i$	$\$, p, e$
$L$	$b, i$	$e$
$A$	$p, e$	$e$

	$b$	$e$	$i$	$p$	$\$$
$S$	$(bLe, 1)$		$(i, 2)$		
$L$	$(SA, 3)$		$(SA, 3)$		
$A$		$(e, 4)$		$(PL, 5)$	
$b$	$pop$				
$e$		$pop$			
$i$			$pop$		
$p$				$pop$	
$\$$					$acc$

## Analiza sintactică descendente LR(0)

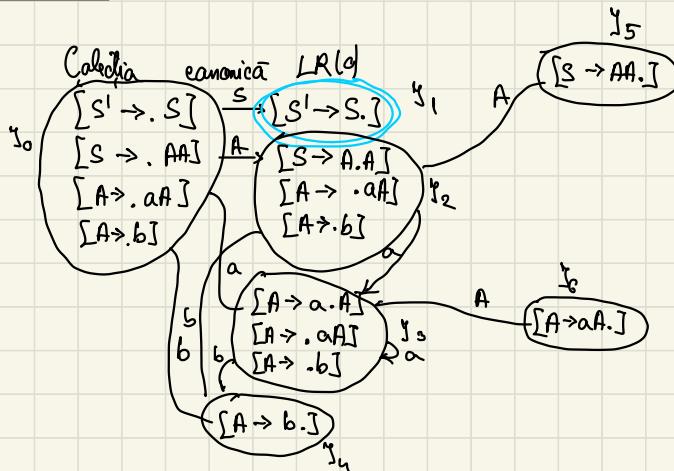
$$4.1 \quad S \rightarrow AA$$

$$A \rightarrow aA$$

$$A \rightarrow b$$

a) Verif. dacă este LR(0)

b) abab ∈ L(G)



Tabelul de analiză LR(0)

shift / accept / deplasare

	Astăuime	S	A	a	b
I <sub>0</sub>	shift	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>	I <sub>4</sub>
I <sub>1</sub>	accept				
I <sub>2</sub>	shift		I <sub>5</sub>	I <sub>3</sub>	I <sub>4</sub>
I <sub>3</sub>	shift		I <sub>0</sub>	I <sub>3</sub>	I <sub>4</sub>
I <sub>4</sub>	3				
I <sub>5</sub>	1				
I <sub>c</sub>	2				

$([\$0] \xrightarrow{\text{shift}} \text{abab\$}, \epsilon) \xrightarrow{\text{shift}}$   
 st. de luară st. de intrare nr. rag  
 prod

$(\$0a3, \text{bab\$}, \epsilon) \xrightarrow{\text{shift}}$   
 $(\$0a3b4, \text{ab\$}, \epsilon) \xrightarrow{\text{reduceze } S \rightarrow a} (\$0a3, \text{ab\$}, \epsilon)$   
 $(\$0a3A6, \text{ab\$}, 3) \xrightarrow{\text{reduceze } 2 \rightarrow a} (\$0a2a3, \text{ab\$}, 2)$   
 $(\$0A2a3, \text{ab\$}, 2) \xrightarrow{\text{shift}}$   
 $(\$0A2a3b4, \$, 2) \xrightarrow{\text{shift}}$   
 $(\$0A2a3b4, \$, 2) \xrightarrow{\text{reduceze } S \rightarrow b} (\$0A2a3b4, \$, 2)$   
 $(\$0A2a3b4, \$, 2) \xrightarrow{\text{reduceze } 2 \rightarrow a} (\$0A2a3b4, \$, 2)$

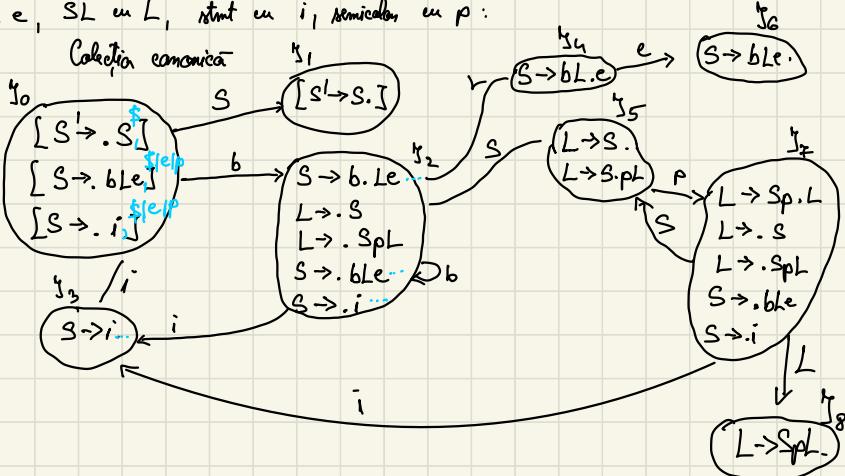
$(\$0A2A5, \$, 232) \xrightarrow{\text{reduce}} S \rightarrow AA$

$(\$0\$1, \$, 12323) \xrightarrow{\text{acc.}}$

$\Rightarrow abab \in L(G)$  și serial prod: 1, 2, 3, 2, 3

6. reden. begin cu b, end cu e, SL cu L, rest cu i, semicolon cu p:

- $S^1 \rightarrow .S$  (0)
- $S \rightarrow bLe$  (1)
- $S \rightarrow i$  (2)
- $L \rightarrow S$  (3)
- $L \rightarrow S_p L$  (4)



actuale	S	L	b	e	p	i
\$0 shift						\$1
\$1 accept						
\$2				\$5	\$4	\$2
\$3						\$3
\$4						\$6
\$5 shift, 3						\$7
\$6	1					
\$7 shift				\$5		
\$8						

Conflict  $\Rightarrow$  gramatica nu e de tip LR(0).

SLR

	First 1	Follow 1
S'	b, i	\$
S	b, i	p, \$, e
L	b, i	e

S	L	b	e	i	p	\$
i <sub>0</sub>	A <sub>1</sub>		A <sub>2</sub>		A <sub>3</sub>	
i <sub>1</sub>						acc
i <sub>2</sub>	A <sub>5</sub>	A <sub>4</sub>	A <sub>2</sub>		A <sub>3</sub>	
i <sub>3</sub>			r <sub>2</sub>		r <sub>2</sub>	r <sub>2</sub>
i <sub>4</sub>			A <sub>6</sub>			
i <sub>5</sub>			r <sub>3</sub>		A <sub>7</sub>	
i <sub>6</sub>			r <sub>1</sub>		r <sub>1</sub>	r <sub>1</sub>
i <sub>7</sub>	A <sub>5</sub>	A <sub>8</sub>	A <sub>2</sub>		A <sub>3</sub>	
i <sub>8</sub>			r <sub>4</sub>			

bipie  $\in L(G)$  ?

$(\$0, \text{ bipie}\$, \varepsilon) \xrightarrow{\text{left 2}}$   
 $(\$0b_1, \text{ pie}\$, \varepsilon) \xrightarrow{\text{left 3}}$   
 $(\$0b_2i_3, \text{ pie}\$, \varepsilon) \xrightarrow{\text{red 2}}$   
 $(\$0b_2SS, \text{ pie}\$, \varepsilon)$

## Analiza sintactică ascendenta LR(1)

4.1

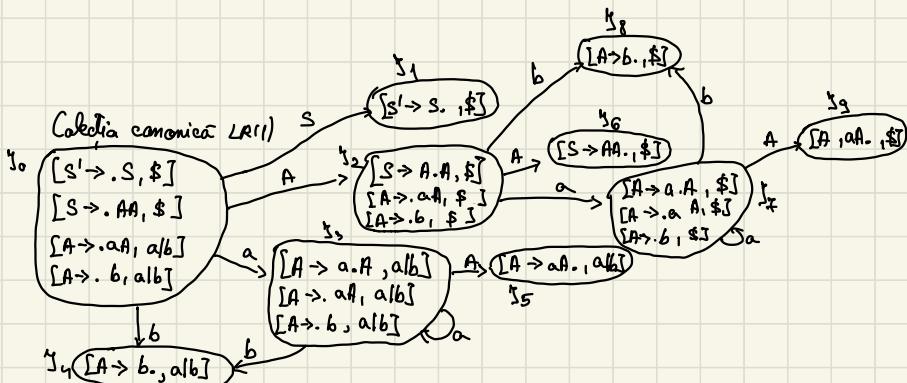
$$S' \rightarrow S \quad 0$$

$$S \rightarrow AA \quad 1$$

$$A \rightarrow aA \quad 2$$

$$A \rightarrow b \quad 3$$

Fază 1	
$S'$	a,b
$S$	a,b
$A$	a,b



## Tabelul de analiză LR(1)

	$S$	$A$	$a$	$b$	$\$$
$\Sigma_0$	$\Delta_1$	$\Delta_2$	$\Delta_3$	$\Delta_4$	
$\Sigma_1$					ace
$\Sigma_2$		$\Delta_6$	$\Delta_7$	$\Delta_8$	
$\Sigma_3$		$\Delta_5$	$\Delta_3$	$\Delta_4$	
$\Sigma_4$			$\Delta_3$	$\Delta_3$	
$\Sigma_5$			$\Delta_2$	$\Delta_2$	
$\Sigma_6$					$\Delta_1$
$\Sigma_7$	$\Delta_3$	$\Delta_2$	$\Delta_8$		
$\Sigma_8$					$\Delta_3$
$\Sigma_9$					$\Delta_2$
$\Sigma_{10}$					$\Delta_2$

$abab \in L(G)$

dim 0 cu a  $\Rightarrow 3$ , dim 3 cu A  $\Rightarrow 5$

$(\$0, abab\$, \epsilon) \xrightarrow{\Delta_3}$   
 $(\$0a3, bab\$, \epsilon) \xrightarrow{\Delta_4}$   
 $(\$0a3b4, ab\$, \epsilon) \xrightarrow{\Delta_3}$   
 $(\$0a3b4, ab\$, \epsilon) \xrightarrow{\Delta_2}$   
 $(\$0a3A5^*, ab\$, 3) \xrightarrow{\Delta_2}$   
 $(\$0A2, ab\$, 23) \xrightarrow{\Delta_7}$   
 $(\$0A2a7, b\$, 23) \xrightarrow{\Delta_8}$   
 $(\$0A2a7b8, \$, 23) \xrightarrow{\Delta_3}$   
 $(\$0A2a7A9, \$, 323) \xrightarrow{\Delta_2}$   
 $(\$0A2A6, \$, 2323) \xrightarrow{\Delta_1}$   
 $(\$0\$1, \$, 12323) \xrightarrow{\text{accept.}}$

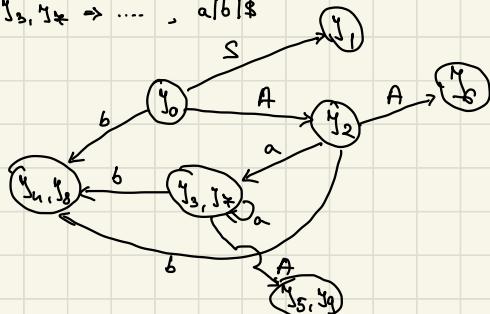
$\Rightarrow abab \in L(G)$  și reiau prod 1,2,3,2,3

$LALR = LR(1)$ , "compact"

$\gamma_1, \gamma_8 \Rightarrow [A \rightarrow b., a/b, \$]$

$\gamma_6, \gamma_9 \Rightarrow \dots, a/b/b/\$$

$\gamma_3, \gamma_4 \Rightarrow \dots, a/b/b/\$$



4.5

$E' \rightarrow E$

$E \rightarrow E + T$

$E \rightarrow T$

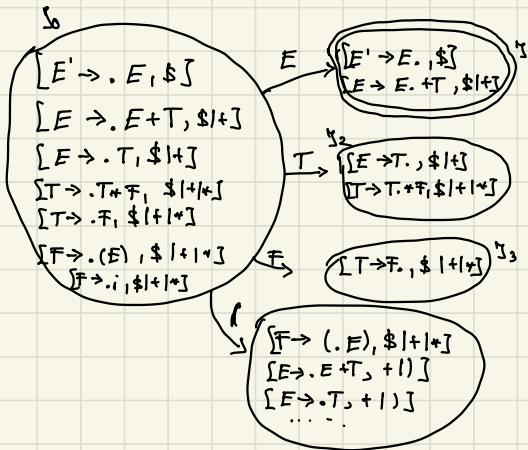
$T \rightarrow T * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow i$

	$T_{start}$
$E'$	i, (
$E$	i, (
$T$	i, (
$F$	i, (



Análisis ascendente:

Gramática de precedencia simple

$S \rightarrow aSSb$

$S \rightarrow c$

	$S$	$a$	$b$	$c$	$\$$
$S$	=	$\leq$	=	$\leq$	
$a$	=	$\leq$		$\leq$	
$b$		>	>	>	
$c$		>	>	>	
$\$$		$\leq$		$\leq$	

Reg. Wirth-Welz

$X = \cdot Y : A \rightarrow dXY\beta \in P$

$X \leftarrow \cdot Y : A \rightarrow dXB\beta \in P, B = \beta Y \beta^*$

$X \rightarrow \alpha : A \rightarrow dBY\beta \in P, B \Rightarrow^* \beta X, Y \Rightarrow^* \alpha$

$X \rightarrow \$ : S \Rightarrow^* dX$

$\$ \leftarrow X : S \Rightarrow^* Xd$

análisis ~ LR

Nx la Existencia