



AUTOMATA, FORMAL LANGUAGES AND LOGIC

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MODULE 5

Propositional Logic & First-Order Logic

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Syntax

- **Syntax** defines the sentences in the language.
- Can be thought of as a “grammar” of representation.
- Consider a stmt: **Buffalo is Black.**
- It can be represented as:
 - 1) $p = \text{Buffalo is black, or}$
 - 2) $\text{Black}(\text{Buffalo})$
 - 3) $\text{for all } x, \text{Buffalo}(x) \rightarrow \text{black}(x)$
means for all x , if x is buffalo, x is black

} Different
syntax

Logic In general

- Logics are formal languages for representing information such that conclusions can be drawn.

- Let

A= Postman delivers post from Monday to Friday

B= Today is Sunday.

C (conclusion)=Post is not delivered today.

Semantics

- **Semantics** OR "meaning" of sentences;
 - i.e., define **truth** of a sentence in a possible world.
 - Eg $2xy=3$ is **true** for whenever $x=3$, $y=1/2$ in a world but would be **false** for $x=3$, $y=2$ in a world
 - $X + Y = 4$ is **true** in a word where X is 2 and Y is 2. But **false** in a word where X is 1 and Y is 1.

Model

- The word “**Model**” is also used for “possible world”

Models are mathematical abstraction.

Think of a possible world having

x men and y women sitting at a table playing chess

The sentence $x+y = 4$ is true, **when** there are **4** people in total.

Model

If,
a sentence α is true in model M ,
we say, that M satisfies α OR
sometimes M is a model of α .

We use the notation $M(\underline{\alpha})$ to represent the set of all models of α .

Entailment

- **Entailment** means that one thing **follows from** another:

$$p \models q$$

means **p entails the sentence q**.

- In every model, in which **p is true**, **q is also true**.
- **The truth of q is contained in p**

$$\text{eg, } a + b = 9 \models 9 = a + b$$

Sentence $x = 0$ entails the sentence $xy = 0$.

Model

M is a model of a sentence α

If α is true in M and $M(\alpha)$ is the set of all models of α

$\alpha \models \beta$ means α entails the sentence β

i.e $\alpha \models \beta$

iff $M(\alpha)$ is a subset of $M(\beta)$.



THANK YOU

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