

1. Which of the following is the time complexity of Binary search in a sorted array of size  $(n)$ ?

→

$$T(n) = T\left(\frac{n}{2}\right) + c$$

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + c$$

$$T(n) = [T\left(\frac{n}{4}\right) + c] + c$$

$$T(n) = T\left(\frac{n}{4}\right) + 2c$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + c$$

$$T(n) = T\left(\frac{n}{8}\right) + 3c$$

$$\therefore T(n) = T\left(\frac{n}{2^k}\right) + kc$$

Let, consider,  $n = 2^k$   
 $\log n = k$

$$\therefore T(n) = T(1) + \log n \cdot c$$

$$\therefore \boxed{O(\log n)}$$

2. What is the space complexity of the iterative version of Binary search.

→ ~~space complexity of~~  $O(1)$   
because no extra space is required.

3. For  $n = 10 \rightarrow 10^2 = 100$

For  $n = 50 \rightarrow 50^2 = 2500$

for  $n = 100 \rightarrow 100^2 = 10000$

for  $n = 500 \rightarrow 500^2 = 250000$

$\therefore$  for  $n = 500$ , the largest value of  $n^2$  exists.

So, the answer is  $\boxed{500}$ .

4. function solve (array)

for i from 1 to length (array)  
for j from 1 to length (array)  
print (array[i] + array[j])

loop runs  
'n' time  
as the length  
of 'n'

loop runs 'n' times  
as length is 'n'

Takes constant  
time  $O(1)$ .

∴ Nested loop ∴ Time Complexity  $O(n^2)$ .

for input array space Required  $O(n)$ .

for loop variable space Required  $O(1)$ .

∴ space Complexity  $O(n)$ .

5. function solve (n)

Sum = 0

for i from 1 to n

'i' takes  
constant space  
 $O(1)$

Loop runs 1

to n

∴  $O(n)$ .

Sum = Sum + i

return Sum.

'sum' takes constant  
space  $O(1)$

Take constant  
time  $O(1)$

∴ space Complexity  $O(1)$

∴ Time Complexity =  $O(n)$



6.

function solve (array)

for i from 1 to length (array)

if array[i] % 2 == 0  
 print (arr[i])

else  
 print (arr[i] \* 2)

Loop Runs 'n' times  
 $\therefore O(n)$ , as length is n.

Take Constant time  $\therefore O(1)$

Take space  $O(n)$ .  
 i, arr, Constant Space,  $O(1)$

Take Constant time  $\therefore O(1)$

$\therefore$  space complexity  $O(n)$ .

Time complexity  $O(n)$ .

7.

function solve (base, exponent)

$O(1)$  if exponent == 0  
 return 1

Recursive Call  
 $T(n) = T(n/2) + c$   
 $O(\log n)$

half = power (base, floor (exponent / 2))  
 return half \* half

else  
 $O(1)$  return half \* half \* base

$\therefore$  space complexity =  $O(\log n)$

=  $O(\log(\text{exponent}))$

Space complexity  $O(\log(\text{exponent}))$ .

8. function sumCalculator (n)

$O(1)$   $\leftarrow$  sum = 0  
 i = 1

while i < n

sum = sum + i  
 i = i \* 2  
 return sum

for each iteration the loop doubles the value.

$\therefore$  Time complexity =  $O(\log n)$

No extra space Required.

$\therefore$  space complexity =  $O(1)$

$i = 1, 2, 4, 8$   
 $2^0, 2^1, 2^2, 2^3$   
 $2^k = n$   
 $k = \log n$

9.

function solve(n)

$$O(1) \leftarrow \left[ \begin{array}{l} \text{if } n == 0 \\ \text{return } 1 \end{array} \right] \rightarrow O(1) \text{ space}$$

result = 1

For each loop  
runs 1 to n  
 $\therefore O(n)$ .

$$\left[ \begin{array}{l} \text{for } i \text{ from } 1 \text{ to } n \\ \text{result} = \text{result} + i \end{array} \right] \rightarrow O(1) \text{ space.}$$

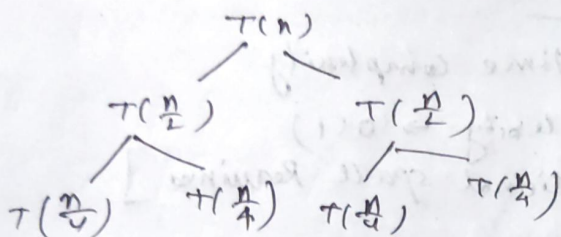
return result

 $\therefore$  Time Complexity =  $O(n)$ 

No extra space.

 $\therefore$  Space complexity =  $O(1)$ .

10.


 $\therefore$  Time Complexity  
=  $O(\log n)$ 
 $\therefore$  Space complexity  
=  $O(1)$ 

11.

function solve(n)

$$O(1) \leftarrow \left[ \begin{array}{l} \text{if } n \leq 1 \\ \text{return } 1 \end{array} \right]$$

sum = 0

loop runs  
'n' times  
 $\therefore O(n)$

$$\left[ \begin{array}{l} \text{for } i \text{ from } 1 \text{ to } n \\ \text{sum} = \text{sum} + \text{solve}(i-1) \end{array} \right]$$

return sum.

 $\therefore$  Time complexity =  $O(n)$  $\therefore$  Space complexity =  $O(n)$ 

12.

function solve(array, k)

 $O(k) \leftarrow$  for i from 1 to k
 $O(n) \leftarrow$  for j from 1 to length(array)  
print(array[j]).

 $\therefore$  Nested loop  $\therefore$  Time complexity =  $O(n+k)$ 
Space complexity =  $O(1)$



13.  $i < \text{array}(\text{length})$

$j = i+1, j < \text{array}(\text{length})$

$i = 0$ , loop runs  $n-1$

$i = 1$ , " "  $n-2$

$i = n-2$  loop runs  $1$

$i = n-1$  loop runs  $0$

$$\begin{aligned} T(n) &= (n-1) + (n-2) + \dots + 1 \\ &= \frac{n(n-1)}{2} \end{aligned}$$

$O(n^2) \rightarrow$  Time complexity

space complexity  $\rightarrow O(1)$

[ $\therefore$  NO additional space required]

14.

15. Time complexity :  $O(n^2)$   
Space complexity :  $O(1)$ .

16. function solve (array)

result = 0

$O(n) \leftarrow$  [for i from 1 to length(array)  
result = result + array[i]

$O(n) \times$  [for i from 1 to length(array)  
result = result - array[i]

return result.

∴ Time complexity =  $O(n)$ .

∴ Space complexity =  $O(1)$

[∵ NO additional space require].

17.  $T(n) = 2T(n-1) + O(1)/c$

$T(n-1) = 2T(n-2) + c$

$T(n) = 2[2T(n-2) + c] + c$

$T(n) = 4T(n-2) + 3c = 2^2 T(n-2) + (2^2 - 1)c$

$T(n-2) = 2T(n-3) + c$

$T(n) = 4[2T(n-3) + c] + 3c$

$= 8T(n-3) + 7c$

$= 2^3 T(n-3) + (2^3 - 1)c$

$= 2^K T(n-2^K) + (2^K - 1)c$

$n - K = 0$

$n = K$

$T(n) = 2^n T(1) + 2^n$

∴ Time complexity =  $O(2^n)$ .

∴ Space complexity =  $O(n)$



18. Time Complexity =  $O(m_1 + m_2 + m_1)$   
 Space Complexity =  $O(m_1 + m_2)$ .

19. Time complexity =  $O(n^3)$   
 Space Complexity =  $O(1)$ .

20. Time Complexity =  $O(2^n \times n)$   
 Space Complexity =  $O(n)$ .

21. Time complexity =  $O(n)$   
 Space Complexity =  $O(n)$ .

22. Time Complexity =  $O(n^2)$   
 Space complexity =  $O(1)$

23. Time Complexity =  $O(n^2)$   
 Space complexity =  $O(1)$ .

24. Time Complexity =  $O(n)$   
 Space Complexity =  $O(1)$

25. Time complexity =  $O(n \times m)$   
 Space complexity =  $O(n \times m)$ .

26. Time complexity =  $O(n)$   
 Space complexity =  $O(1)$

~~27.~~

27.

26.

26.

function solve (array)

count = 0

$O(n)$   $\leftarrow$  for  $i$  from 1 to length (array)

if array [ $i$ ] > 0

count = count + 1

return count.

$\therefore TC = O(n)$   $\therefore SC = O(1)$ .

27.

function solve (array)

result []

Take  $O(n)$ .  $\leftarrow$  [for  $i$  from 1 to length (array)  
if not result.contains (array [ $i$ ])

$O(1)$   $\leftarrow$  result.append (array [ $i$ ])

return result.

$\therefore TC = O(n)$

$\therefore SC = O(n) \rightarrow$  as result array stores  $n$  unique elements.

28.

$TC = O(\log n)$

$SC = O(1)$

29.

$1 \rightarrow n$   
 $i \rightarrow 1$

$\frac{n(n+1)}{2} = O(n^2)$ .

$TC = O(n^2)$   $SC = O(1)$