



Commonly used T-tests

T-test for one sample:

For example

T-test for two related samples:

For example:

For example:

T-test for two independent samples:

For Example:

Commonly used Z-tests

For example:

Z-test's for different purposes:

z-test for single proportion:

z-test for difference of proportions:

For example:

z-test for single mean:

z-test for single variance:

z-test for testing equality of variance:

Flowchart

Commonly used T-tests

- **Student's t-test** is a **method** of **testing hypotheses** about the **mean of a small sample** drawn
 - from a **normally distributed** population
 - when the population **standard deviation** is unknown.



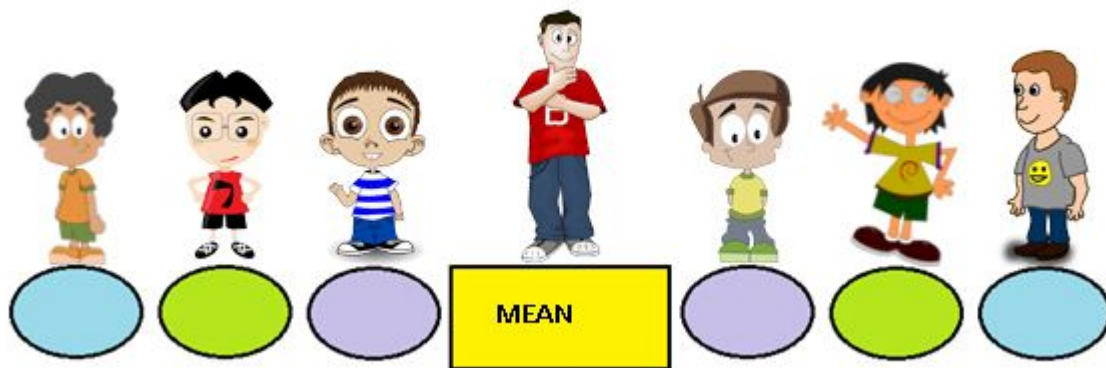
- In any **statistical hypothesis testing** situation, if the test statistic follows a **Student's t-test distribution** under **null hypothesis**, it is a **t-test**.



- **Most frequently used t-tests are:**
 - For comparison of mean in single sample.
 - Two samples related.
 - Two samples unrelated tests.
-

- In **one-sample location test**, it is tested whether or not the mean of the population has a value as specified in a **null hypothesis**.

- In **two independent sample location test**, equality of **means** of **two populations** is tested.



- To compare the **mean delta**(difference between two related samples) against **hypothesized value of zero** in a null hypothesis.
- It is also known as **paired t-test** or **repeated-measures t-test**.
- For a **binary variable** (such as cure, relapse, hypertension, diabetes, etc.,) which is either **yes** or **no** for a subject.

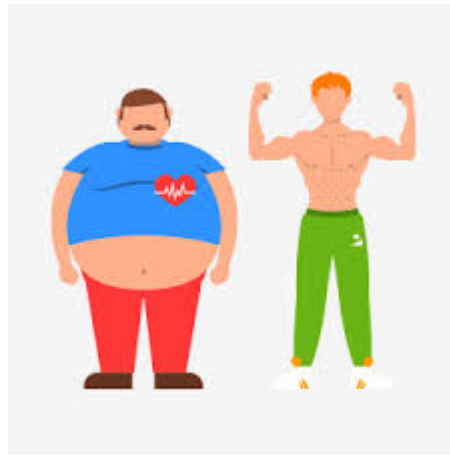


- If we take **1** for **yes** and **0** for **no** and consider this as a **score** attached to each **study subject**.
- Then the **sample proportion (p)** and the **sample mean** would be the **same**.
- Therefore, the **approach of t-test** for mean can be used for **proportion** as well.

T-test for one sample:

For example

- In a random sample of 30 **hypertensive males**, the observed mean **body mass index (BMI)** is 27.0 kg/m² and the **standard deviation** is 4.0.



- Also, suppose it is known that the mean **BMI in non-hypertensive males** is 25 kg/m².
- If the question is to **know whether or not** these 30 observations could have **come from a population** with a **mean** of 25 kg/m².
- To determine this, **one sample t-test** is used with the **null hypothesis** H_0 : Mean = 25, against alternate hypothesis of H_1 : Mean \neq 25.
- Since the **standard deviation** of the **hypothesized population** is not known.
 - Therefore, **t-test** would be **appropriate**.
 - Otherwise, **Z-test** would have been used.

T-test for two related samples:

- Two samples can be regarded as related in a **pre-** and **post-design** (self-pairing).
- It can also be **in two groups** where the subjects have been matched on a third factor a **known confounder** (artificial pairing).
- In a **pre** and **post-design**, each subject is used as **his** or **her** own control.

For example:

- An **investigator** wants to assess effect of an intervention in **reducing systolic blood pressure (SBP)** in a pre- and post-design.



- Here, for each patient, there would be **two observations of SBP**, that is, before and after.



- Here, instead of individual observations, **difference between pairs of observations** would be of interest.
- The problem reduces to **one-sample situation** where the null hypothesis would be to test the **mean difference in SBP equal to zero**
- It is used to **test against** the alternate hypothesis of **mean SBP being not equal to zero**.
- The underlying assumption for **using paired t-test** is that under the null hypothesis the population of difference in **normally distributed**.
- This can be judged using the **sample values**.
- Using the **mean difference** and the **standard error** of the mean difference, **95% confidence interval** can be computed.



- The other situation of the two sample being related is the **two-group matched design**.

For example:

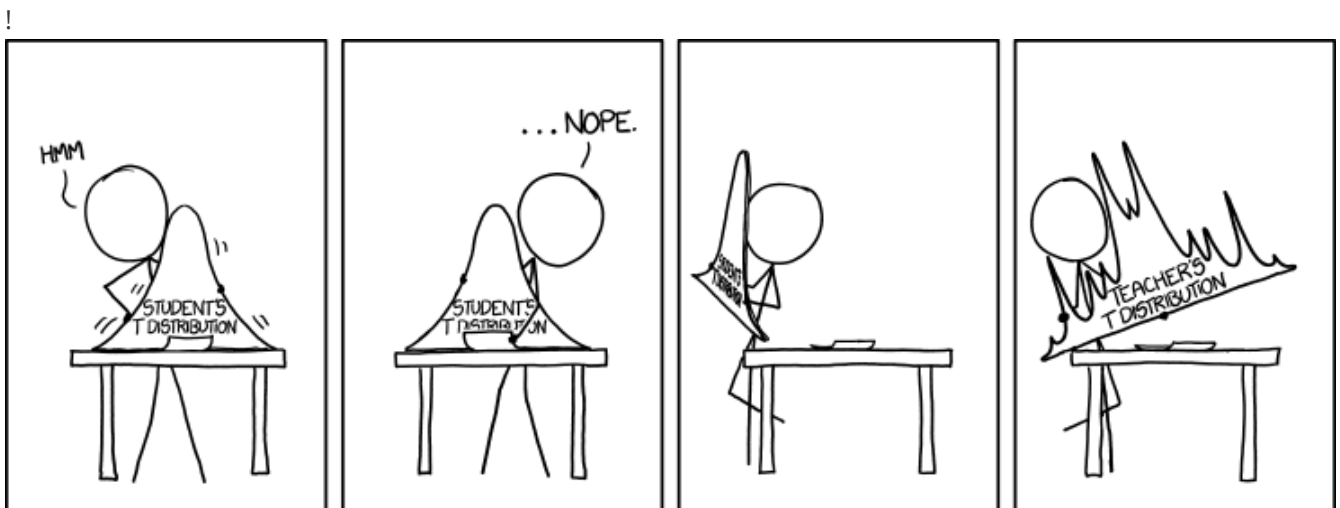
- In a **case-control study** to assess association between **smoking** and **hypertension**.
- Both **hypertensive** and **non-hypertensive** are matched on some third factor, say **obesity**, in a pair-wise manner.



- Same approach of **paired analysis** would be used.
- In this situation, **cases and controls** are different subjects. However, they are related by the factor.

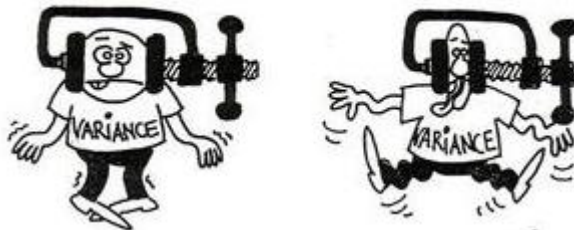
T-test for two independent samples:

- To test the null hypothesis that the means of **two populations are equal**.
- **Student's t-test** is used provided the **variances** of the **two populations are equal**.



- The **two samples** are assumed to be **random sample**.

- When this assumption of **equality of variance is not fulfilled**, the form of the test used is a **modified t-test**.
- These tests are also known as **two-sample independent t-tests** with equal variance or unequal variance, respectively.



- The only difference in the **two statistical tests** lies in the denominator, that is, in determining the **pooled variance**.
- Prior to choosing **t-test for equal** or **unequal variance**, very often a test of variance is carried out to compare the two variances.
- It is **recommended** that this should be **avoided**.
- Using a **modified t-test** even in a situation when the **variances are equal**, has **high power**.
- Therefore, to compare the means in the two unrelated groups, using a **modified t-test is sufficient**.
- When there are **more than two groups**, use of **multiple t-test**(for each pair of groups) is **incorrect** because it may give **false-positive result**.
- Hence, in such situations, **one-way analysis of variance (ANOVA)**, followed by correction in P value for multiple comparisons (post-hoc ANOVA), if required, is used.



- It is used to **test the equality** of more than two means as the **null hypothesis**
- It ensures that the total **P value** of all the pair-wise **does not exceed 0.05**.

For Example:

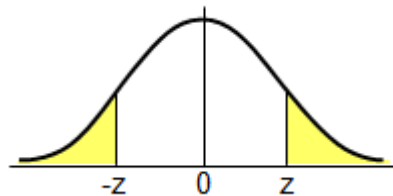
- Do men spend the **same amount of money** on clothing as women?
- We can't reasonably ask the **entire population of men** and **women** how much they spend.
- So we'll draw a **sample of men** and **women**.
- These **samples are independent** because they **don't overlap**: everybody is either man or woman, never both.



- If the average amount spent is precisely equal for **all men and women**.
- We'll probably still see **slightly different means** between our samples.
- However, very different sample means suggest that the **population means weren't equal** after all.
- A **t test** tells us if a **sample difference is big enough** to draw this conclusion.

Commonly used Z-tests

- It is a **statistical test** where **normal distribution** is applied.



- It is basically used for **dealing with problems** relating to **large samples when $n \geq 30$** (n = sample size).

For example:

- Suppose we want to test if both **tea** and **coffee** are **equally popular** in a particular city.



- We can take a sample of size say **500 from the city** out of which suppose **280 are tea drinkers**.
- To **test the hypothesis**, we can use **Z-test**.

Z-test's for different purposes:

z-test for single proportion:

- It is used to **test a hypothesis** on a **specific value** of the **population proportion**.
- Statistically speaking, we **test the null hypothesis** $H_0: P = P_0$ against the alternative hypothesis $H_a: P > P_0$
 - Where **P** is the **population proportion**
 - **P_0** is a **specific value of the population proportion** we would like to test for acceptance.



The example on **tea drinkers** explained above requires this test.

- In that example, **$P_0 = 0.5$** .
- Notice that in this particular example, proportion refers to the **proportion of tea drinkers**.

z-test for difference of proportions:

- It is used to **test the hypothesis** that **two populations** have the **same proportion**.

For example:

- Suppose one is interested to test if there is any significant difference in the **habit of tea drinking** between **male** and **female citizens** of a city.



- In such a situation, **z-test** for **difference of proportions** can be applied.
- One would have to obtain **two independent samples** from the city, **one from males** and the **other from females**.
- Determine the **proportion of tea drinkers** in each sample in order to perform this test.



z-test for single mean:

- It is used to test a hypothesis on a **specific value** of the **population mean**.
- Statistically speaking, we test the null hypothesis $H_0: \mu = \mu_0$ against the alternative hypothesis $H_a: \mu > \mu_0$ or $\mu < \mu_0$
 - Where μ is the **population mean**
 - μ_0 is a **specific value of the population** that we would like to test for acceptance.
- Unlike the **t-test** for **single mean**, this test is used if $n \geq 30$ and population **standard deviation** is known.

z-test for single variance:

- It is used to test a hypothesis on a **specific value** of the **population variance**.
- Statistically speaking, we test the null hypothesis $H_0: \sigma = \sigma_0$ against $H_a: \sigma > \sigma_0$ or $\sigma < \sigma_0$
 - Where σ is the **population mean**.
 - σ_0 is a specific value of the **population variance** that we would like to test for acceptance.
- In other words, this test enables us to test if the given sample has been drawn from a population with **specific variance σ_0** .
- Unlike the **chi square test** for **single variance**, this test is used if $n \geq 30$.

z-test for testing equality of variance:

- It is used to test the hypothesis of equality of **two population variances** when the **sample size** of each sample is **30 or larger**.



Flowchart

