HEIGHT AND DISTANCE 10

The mathematician does not study mathematics because it is useful.
He studies it because he delights in it and he delights in it because it is beautiful.

- Henri Poincare

10.1 Introduction

In previous chapter, we have studied about trigonometric ratios and techniques of solving right angled triangles. We shall now see how these techniques are used to solve problems regarding heights and distances in life around us. Trigonometry is one of the most ancient subjects studied by scholars all over the world. Note that in practice only some distances can be measured but not all. For instance, height of a hill (distance between its foot and summit), width of a river, distance between two celestial objects can not be measured by a measure tape. So, method of trigonometric ratios is very useful in measuring such distances. When dealing with heights or depths, we have to measure two kinds of angles (upward and downward from our eye-level). We describe these two kinds of angles more precisely as follows.

10.2 Angle of Elevation and Angle of Depression

Horizontal Ray : A ray parallel to the surface of the earth emerging from the eye of an observer is called a horizontal ray.

Ray of Vision : The ray from the eye of an observer towards the object is called the ray of vision or ray of sight.

Angle of Elevation : If the object under observation is above an observer, but not directly above the observer, then the angle formed by the horizontal ray and the ray of sight in a vertical plane is called the angle of elevation. Here horizontal ray, observer and object are in the same vertical plane.

In figure 10.1, the object P under observation is at a higher level than the observer O but not directly above O. Let $\overrightarrow{OM}$ be the horizontal ray in the vertical plane containing O and P. Then the union of the ray of vision $\overrightarrow{OP}$ and horizontal ray $\overrightarrow{OX}$ is $\angle$POM. If $m\angle POM = e$, then e is called the measure of the angle of elevation $\angle$POM, of the object P at the point of observation O.

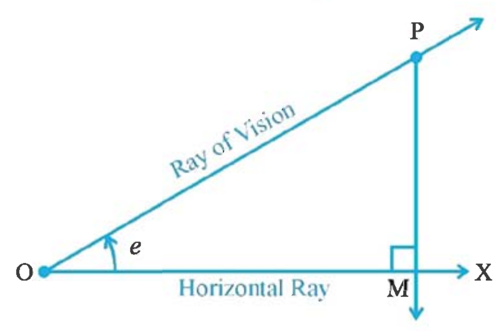

Figure 10.1

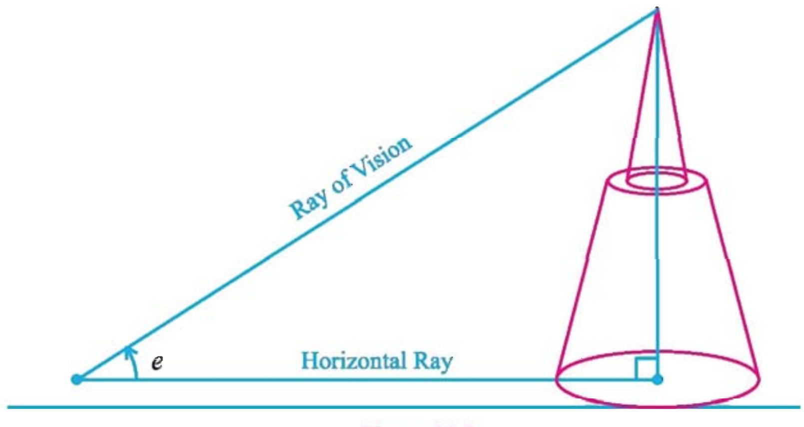

Figure 10.2

Angle of Depression : If the object under observation is at a lower level than an observer but not directly under the observer, then the angle formed by the horizontal ray and the ray of sight is called the angle of depression. Here horizontal ray, observer and the object are in the same vertical plane.

In figure 10.3, the object under observation is at a lower level than the observer O but not directly under O. Let $\overrightarrow{ON}$ be the horizontal ray in the vertical plane containing O and Q. Then the union of the ray of vision $\overrightarrow{OQ}$ and horizontal ray $\overrightarrow{ON}$ is $\angle$NOQ.

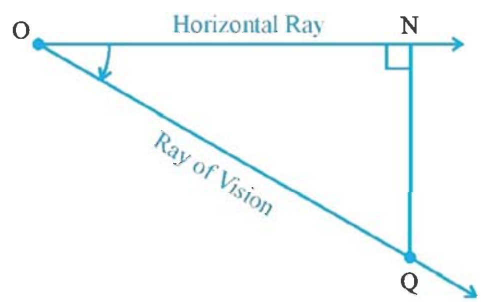

Figure 10.3

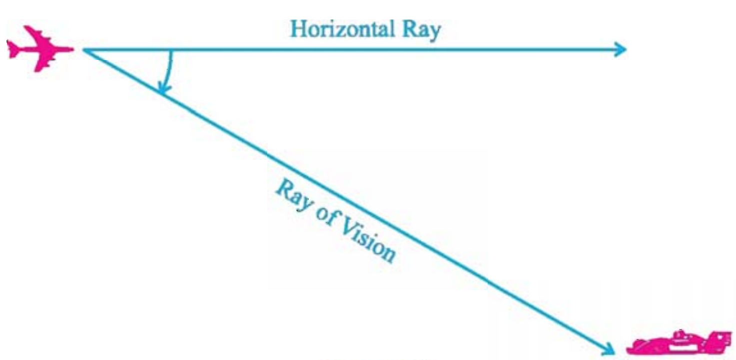

Figure 10.4

10.3 Solution of a Right Triangle

If the measure of any one side and any other element of a right angled triangle are given, then the solution of the right angled triangle can be obtained.

Suppose In ΔABC, $m\angle$ABC = 90, $m\angle$ACB = 30 and AC = 20 m.

Here, $m\angle ACB + m\angle BAC = 90$

$\therefore \quad 30 + m\angle BAC = 90$

$\therefore \quad m\angle BAC = 60$

Now, $sin30 = \frac{AB}{AC}$

$\therefore \quad sin30 = \frac{AB}{20}$

$\therefore \quad \frac{1}{2} \times 20 = AB$

$\therefore \quad AB = 10\ m$

$tan30 = \frac{AB}{BC}$

$\therefore \quad tan30 = \frac{10}{BC}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{10}{BC}$

$\therefore \quad BC = 10 \times \sqrt{3}$

$= 10 \times 1.73$

$BC = 17.3\ m$

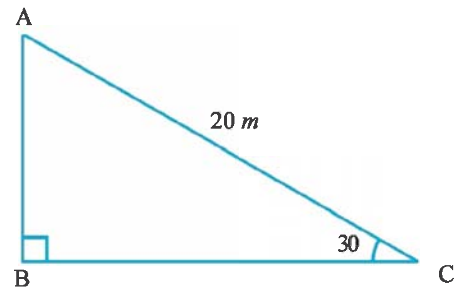

Figure 10.5

Note : In solving these examples, we shall take the values of $\sqrt{3}$ as 1.73, and $\sqrt{2}$ as 1.41.

Example 1 : A tower stands vertically on the ground. From a point on the ground which is 100 *m* away from the foot of the tower, the angle of elevation of the top of the tower is found to have measure 60. Find the height of the tower.

Solution : Suppose $\overline{AB}$ respresents the tower. O is the point 100 *m* away from the tower, OB is the distance of the point from the tower and $\angle AOB$ is the angle of elevation.

Then, OB = 100 *m* and $m\angle BOA = 60$.

In ΔAOB, $tan60 = \frac{AB}{OB}$

$\therefore \quad \sqrt{3} = \frac{AB}{100}$

$\therefore \quad AB = 100 \times \sqrt{3}$

$\therefore \quad AB = 100 \times 1.73$

$= 173\ m$

$\therefore$ The height of the tower is 173 *m*.

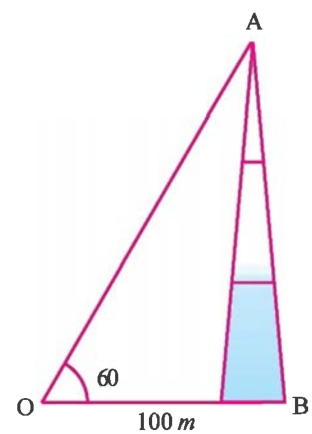

Figure 10.6

Example 2 : As observed from a fixed point on the bank of a river, the angle of elevation of a temple on the opposite bank has measure 30. If the height of the temple is 20 *m*, find the width of the river.

Solution : Here $\overline{AB}$ is the temple on the opposite bank of the river and C is the point of observation on the other bank of the river. So $\overline{BC}$ is the width of the river.

Then, AB = 20 *m* and $m\angle ACB = 30$.

In ΔABC, $tan30 = \frac{AB}{BC}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{20}{BC}$

$\therefore \quad BC = 20 \times \sqrt{3}$

$\therefore \quad BC = 20 \times 1.73$

$= 34.6\ m$

$\therefore$ Thus, the width of the river is 34.6 *m*.

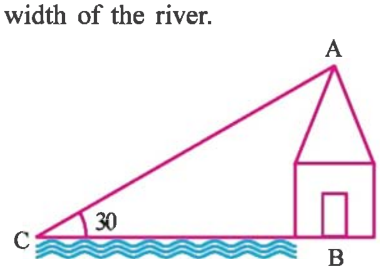

Figure 10.7

Example 3 : An observer 1.5 *m* tall is 28.5 *m* away from a tower. The angle of elevation of the top of the tower from her eyes has measure 45. What is the height of the tower ?

Solution : Here, $\overline{AD}$ is the tower having height h and $\overline{EC}$ be the observer of height 1.5 *m* at a distance of 28.5 *m* from the tower $\overline{AD}$.

Then, BC = DE = 28.5 *m* and

BD = CE = 1.5 *m*, $m\angle ACB = 45$.

In ΔABC, $tan 45 = \frac{AB}{BC}$

$\therefore \quad 1 = \frac{AB}{28.5}$

$\therefore \quad AB = 28.5$

Now, $h = AB + BD = 28.5 + 1.5 = 30$

$\therefore \quad h = 30\ m$

Hence, height of the tower is 30 *m*.

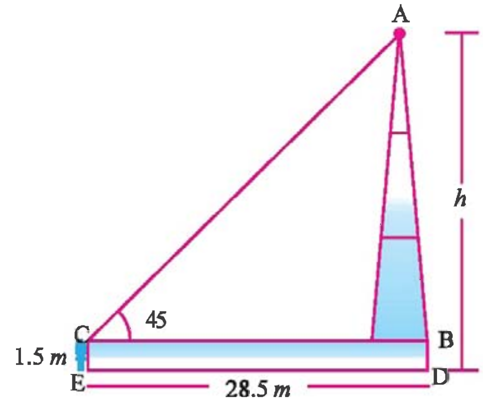

Figure 10.8

Example 4 : A Palm tree breaks due to storm and its upper end touches the ground and makes an angle of measure 30 with the ground. If the top of the tree touches the ground 15 *m* away from the bottom, find the height of the tree.

Solution : Here, $\overline{AC}$ is the tree broken at point B such that broken part $\overline{CB}$ takes the position $\overline{BD}$ and touches the ground at D.

Then, AD = 15 *m* and $m\angle ADB = 30$.

In ΔDAB, we have $tan 30 = \frac{AB}{AD}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{AB}{15}$

$\therefore \quad AB = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$

Now, $cos 30 = \frac{AD}{BD}$

$\therefore \quad \frac{\sqrt{3}}{2} = \frac{15}{BD}$

$\therefore \quad BD = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$

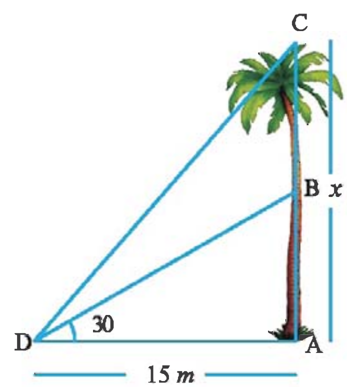

Figure 10.9

So, the height of the tree AC = AB + BC

$= AB + BD$

$= 5\sqrt{3} + 10\sqrt{3}$

$= 15\sqrt{3}$

$= 15(1.73) = 25.95\ m$

Hence the height of the tree is 25.95 *m*.

Example 5 : The angle of elevation of the top of a tower as observed from the foot of a temple has measure 60. The angle of elevation of the top of the temple as observed from the foot of the tower has measure 30. If the temple is 50 *m* high, find the height of the tower.

Solution : Here $\overline{CD}$ is the tower and $\overline{AB}$ is the temple Their feet are the points B and C respectively. $m\angle ACB = 30$ and $m\angle CBD = 60$, Also AB = 50. Let BC = y and CD = x.

In ΔABC, $tan30 = \frac{AB}{BC}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{50}{y}$

$\therefore \quad y = 50\sqrt{3}$

Now in ΔDBC,

$tan60 = \frac{DC}{BC}$

$\sqrt{3} = \frac{x}{y}$

$\therefore \quad x = \sqrt{3} \times y = \sqrt{3} \times 50\sqrt{3}$

$\therefore \quad x = 50 \times 3 = 150\ m$

$\therefore$ The height of the tower is 150 m.

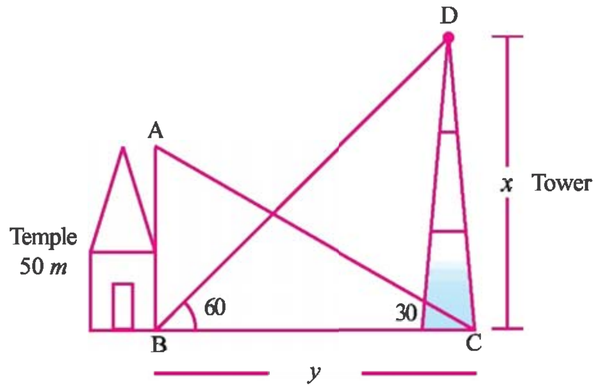

Figure 10.10

Example 6 : As the angle of elevation of the sun increases from 30 to 60, the length of the shadow of a building gets reduced by 10 m. Find the height of the building.

Solution : Here $\overline{AB}$ is the building and $\overline{BD}$ is its shadow when angle of elevation of the sun has measure 30 and $\overline{BC}$ is its shadow when angle of elevation of the sun has measure 60.

Then, $m\angle ADB = 30$, $m\angle ACB = 60$, $DC = 10\ m$

Let $AB = h$, $BC = x$, then $BD = BC + CD$

$\therefore \quad BD = x + 10$

In ΔABC, $tan60 = \frac{AB}{BC}$

$\therefore \quad \sqrt{3} = \frac{h}{x}$

$\therefore \quad h = \sqrt{3}x$ **(i)**

In ΔABD, $tan30 = \frac{AB}{BD}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{h}{x+10}$

$\therefore \quad x + 10 = \sqrt{3}h$

$\therefore \quad x + 10 = \sqrt{3}(\sqrt{3}x)$ **by (i)**

$\therefore \quad x + 10 = 3x$

$\therefore \quad 2x = 10$

$\therefore \quad x = 5$

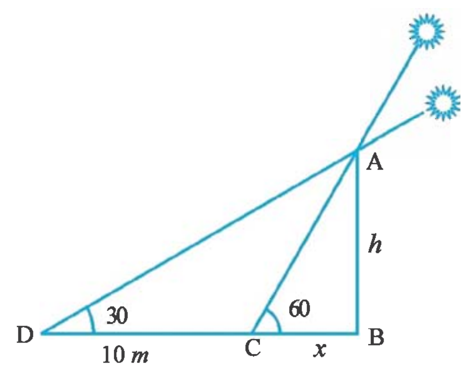

Figure 10.11

Now, by (i) $h = \sqrt{3}x$

$\therefore \quad h = \sqrt{3} \times 5$

$\therefore \quad h = 5(1.73)$

$\therefore \quad h = 8.65\ m$

Hence, the height of the building is 8.65 m.

Another Method : $m\angle ACB = 60 = m\angle ADC + m\angle DAC = 30 + m\angle DAC$

(Interior Opposite Angles)

$\therefore \quad m\angle DAC = 30$

$\therefore \quad AC = CD = 10$

Now, $sin60 = \frac{AB}{AC}$

$\therefore \quad AB = AC\ sin60$

$= (10)\frac{\sqrt{3}}{2} = 5\sqrt{3} = 5(1.73) = 8.65\ m$

Example 7 : A man is standing on the top of a building 60 m high. He observes that the angle of depression of the top and the bottom of a tower has measure 30 and 60 respectively. Find the height of the tower.

Solution : Let $\overline{AB}$ be the building and $\overline{CD}$ be the tower.

Let CD = h

Let $\overline{CE}$ be the perpendicular from C to $\overline{AB}$. The angles of depression of the top C and the bottom D of the tower $\overline{CD}$ have measures 30 and 60 respectively from A.

Then, $m\angle ACE = 30$ and $m\angle ADB = 60$

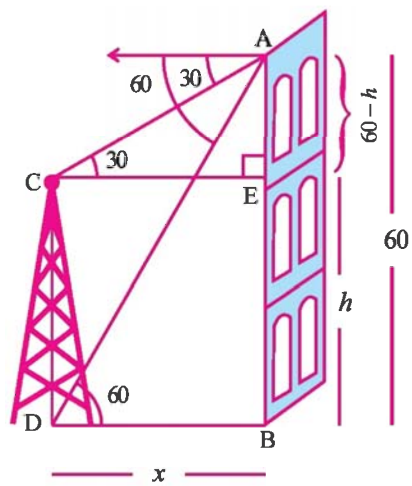

Figure 10.12

Let BD = CE = x

Here, AB = 60, CD = EB = h

$$AB = AE + EB$$

$\therefore$ $60 = AE + h$

$\therefore$ $AE = 60 - h$

In ΔAEC, $tan30 = \frac{AE}{CE}$

$\therefore$ $\frac{1}{\sqrt{3}} = \frac{60 - h}{x}$

$\therefore$ $x = (60 - h)\sqrt{3}$ **(i)**

In ΔABD, $tan60 = \frac{AB}{BD}$

$\therefore$ $\sqrt{3} = \frac{60}{x}$

$\therefore$ $x = \frac{60}{\sqrt{3}}$ **(ii)**

From (i) and (ii) we have,

$$(60 - h)\sqrt{3} = \frac{60}{\sqrt{3}}$$

$\therefore$ $3(60 - h) = 60$

$\therefore$ $60 - h = 20$

$\therefore$ $h = 40\ m$

Hence, the height of the tower is 40 m.

Example 8 : The angle of elevation of a jet plane from a point on the ground has measure 60. After a flight of 30 seconds. The angle of elevation has measure 30. If the jet plane is flying at a constant height of $4500\sqrt{3}$ m, find the speed of the jet plane.

Solution : Let O be the point of observation, C and D be the two positions of the jet plane. The angles of elevation of the jet plane in two positions C and D from the point O have measures 60 and 30 respectively. A and B are feet of perpendiculars from C and D to the ground.

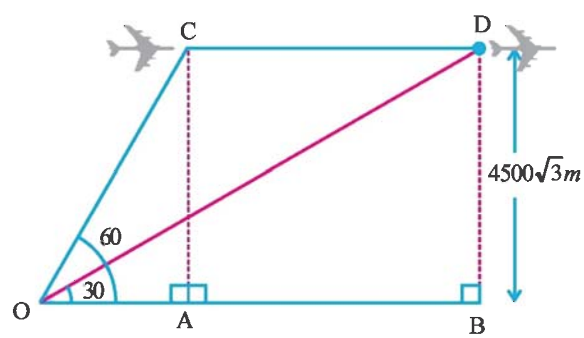

Figure 10.13

$m\angle COB = 60$, $m\angle DOB = 30$ and

$BD = AC = 4500\sqrt{3}\ m$

In ΔOAC, $tan60 = \frac{AC}{OA}$

$\therefore \quad \sqrt{3} = \frac{4500\sqrt{3}}{OA}$

$\therefore \quad OA = 4500\ m$

In ΔOBD, $tan 30 = \frac{BD}{OB}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{4500\sqrt{3}}{OB}$

$\therefore \quad OB = 4500 \times 3 = 13500\ m$

Now, $CD = AB = OB - OA$

$\therefore \quad CD = 13500 - 4500$

$\therefore \quad CD = 9000\ m$

Thus, the jet plane travels 9000 m in 30 sec.

Hence speed $= \frac{9000}{30} = 300\ m/sec = \frac{300 \times 60 \times 60}{1000}\ km/hr$

$\therefore$ Speed of the jet plane = 1080 km/hr

Example 9 : A straight highway leads to the foot of a tower. A man standing on the top of the tower observes a car at an angle of depression with measure 30. The car is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car has measure 60. Find the further time taken by the car to reach the foot of the tower.

Solution : Let $\overline{AB}$ be the tower and height of the tower AB = h m. At C the angle of depression of the car has measure 30 and six seconds later it reaches D where the angle of depression is 60.

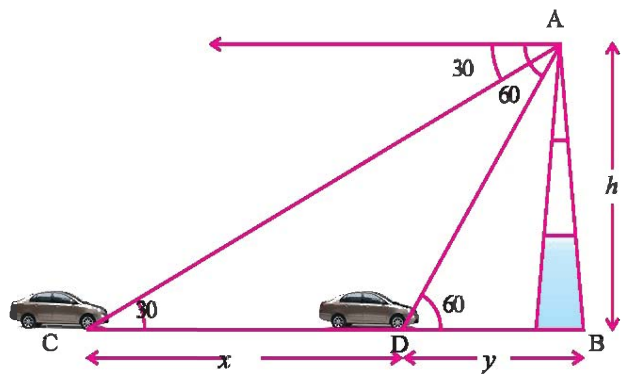

Figure 10.14

Let $CD = x$, $DB = y$

Here, $AB = h\ m$, $m\angle ACB = 30$ and $m\angle ADB = 60$.

In ΔACB, $tan 30 = \frac{AB}{CB}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{h}{x + y}$

$\therefore \quad x + y = \sqrt{3}h$ **(i)**

In ΔABD, $tan 60 = \frac{AB}{BD}$

$\therefore \quad \sqrt{3} = \frac{h}{y}$

$\therefore \quad h = \sqrt{3}y$ **(ii)**

From (i) and (ii), we have

$x + y = \sqrt{3}(\sqrt{3}y)$

$\therefore \quad x + y = 3y$

$\therefore \quad x = 2y$

The car has uniform speed. Suppose the car travels distance at v metere / sec

It travels $x = 2y$ in six seconds

$\therefore$ It travels distance y = BD in 3 seconds.

Hence, further time taken by the car to reach the foot of the tower is 3 seconds.

Example 10 : A 1.3 m tall girl spots a ballon moving with the wind in horizontal line at a constant height of 91.3 m from the ground. The angle of elevation of the balloon from the eyes of the girl at an instant has measure 60. After some time, the angle of elevation is reduced in measure to 30. Find the distance travelled by the balloon during the interval.

Solution : Let A and P be the positions of the balloon when its angles of elevation from the eyes of the girl at O have measures 60 and 30 respectively.

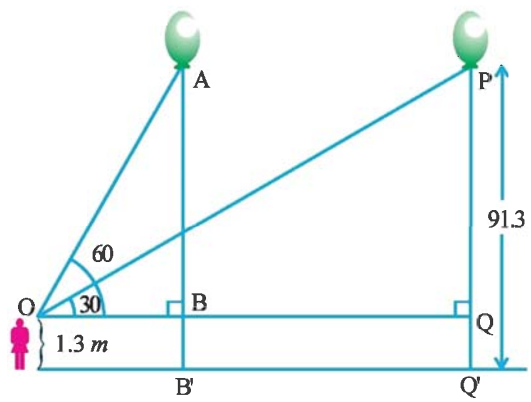

Figure 10.15

Here, AB' = PQ' = 91.3 and

BB' = QQ' = 1.3

$\therefore$ PQ = PQ' − QQ'

= 91.3 − 1.3 = 90 m

$\therefore$ PQ = AB = 90 m

In ΔABO, $\tan 60 = \frac{AB}{OB}$

$\therefore \quad \sqrt{3} = \frac{90}{OB}$

$\therefore \quad OB = \frac{90}{\sqrt{3}} = \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 30 \times \sqrt{3}$ **(i)**

In ΔPQO, $\tan 30 = \frac{PQ}{OQ}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{90}{OQ}$

$\therefore \quad OQ = 90\sqrt{3}$ **(ii)**

$\therefore$ The distance travelled by the balloon = BQ = OQ − OB

$\therefore \quad BQ = 90\sqrt{3} - 30\sqrt{3}$

$= 60\sqrt{3}$

$= 60 \times 1.73$

$= 103.8$ m

Hence, the distance travelled by the balloon is 103.8 m.

Example 11 : As observed from the top of a building 60 m above the surface of a lake, the angle of elevation of a kite flying in the sky has measure 30 and the angle of depression of the image of the kite in the lake has measure 60. Find the height of the kite above the surface of the lake.

Solution : Let $\overline{BE}$ be the surface of the lake and $\overline{AB}$ be the building. Let F be the reflection of kite C. Horizontal line $\overline{AD}$ intersect $\overline{CE}$ in D.

$m\angle CAD = 30$, $m\angle FAD = 60$

AB = 60 m. Let CE = h, BE = l.

Then CD = $h - 60$ and DF = $h + 60$

In ΔADC, $tan30 = \frac{CD}{AD}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{h-60}{l}$

$\therefore \quad l = \sqrt{3}(h - 60)$ **(i)**

In ΔADF, $tan60 = \frac{DF}{AD}$

$\therefore \quad \sqrt{3} = \frac{h+60}{l}$

$\therefore \quad \sqrt{3}l = h + 60$ **(ii)**

From (i) and (ii)

$\sqrt{3}[\sqrt{3}(h - 60)] = h + 60$

$3(h - 60) = h + 60$

$3h - 180 = h + 60$

$2h = 240$

$h = 120\ m$

Hence, the height of the kite above the surface of the lake is 120 m.

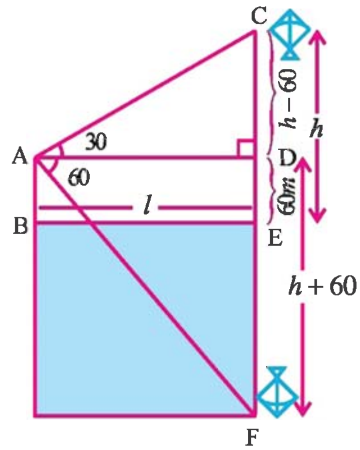

Figure 10.16

Example 12 : A flag-staff of height h stands on the top of a school building. If the angles of elevation of the top and bottom of the flag-staff have measures α and β are respectively from a point on the ground, prove that the height of the building is $\frac{htan\beta}{tan\alpha - tan\beta}$.

Solution : Let $\overline{AB}$ be the flag-staff, $\overline{BC}$ be the school building and D be the point of observation.

Now, AB = h. Let BC = H and CD = d. $m\angle ADC = \alpha$ and $m\angle BDC = \beta$.

In ΔADC, $tan\alpha = \frac{h+H}{d}$

$d = \frac{h+H}{tan\,\alpha}$ **(i)**

In ΔBDC, $tan\beta = \frac{H}{d}$

$d = \frac{H}{tan\,\beta}$ **(ii)**

From (i) and (ii)

$$\frac{h+H}{tan\,\alpha} = \frac{H}{tan\,\beta}$$

$\therefore \quad htan\beta + Htan\beta = Htan\alpha$

$\therefore \quad htan\beta = Htan\alpha - Htan\beta$

$\therefore \quad H(tan\alpha - tan\beta) = htan\beta$

$\therefore \quad H = \frac{htan\beta}{tan\alpha - tan\beta}$

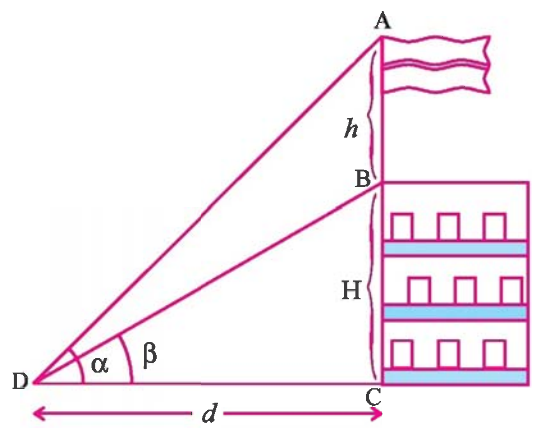

Figure 10.17

Example 13 : A ladder rests against a wall at an angle having measure α with the ground. Its foot is pulled away from the wall by a m keeping ladder on the ground. By doing this, its upper end on the wall slides down by b m. Now the ladder makes an angle of measure β with the ground. Then prove that $\frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

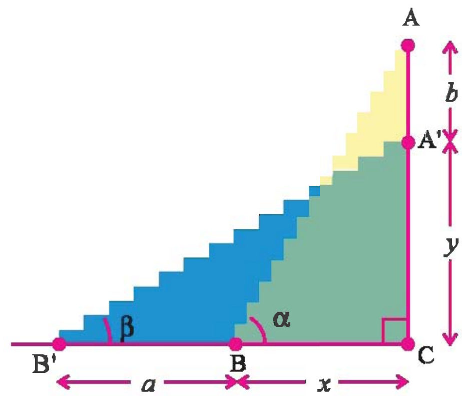

Figure 10.18

Solution : Let $\overline{AB}$ be the ladder when its top A is on the wall and bottom B is on the ground such that $m\angle ABC = \alpha$. Now the ladder is pulled away from the wall through a distance a, so that its top A slides and takes position A' and bottom B slides and takes position B', such that $m\angle A'B'C = \beta$. Let BC = x, A'C = y.

Now, AA' = b and B'B = a.

In ΔABC, $\sin\alpha = \frac{AC}{AB}$, $\cos\alpha = \frac{BC}{AB}$

$\therefore \quad \sin\alpha = \frac{b+y}{AB}$, $\cos\alpha = \frac{x}{AB}$ **(i)**

In $\Delta A'B'C$, $\sin\beta = \frac{A'C}{A'B'}$, $\cos\beta = \frac{B'C}{A'B'}$

$\therefore \quad \sin\beta = \frac{y}{A'B'}$, $\cos\beta = \frac{a+x}{A'B'}$

Now, AB = A'B'

$\therefore \quad \sin\beta = \frac{y}{AB}$, $\cos\beta = \frac{a+x}{AB}$ **(ii)**

From (i) and (ii)

$$\sin\alpha - \sin\beta = \frac{b+y}{AB} - \frac{y}{AB} \text{ and } \cos\beta - \cos\alpha = \frac{a+x}{AB} - \frac{x}{AB}$$

$$\therefore \quad \sin\alpha - \sin\beta = \frac{b}{AB} \text{ and } \cos\beta - \cos\alpha = \frac{a}{AB}$$

$$\therefore \quad \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta} = \frac{\frac{a}{AB}}{\frac{b}{AB}}$$

$$\therefore \quad \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta} = \frac{a}{b}$$

Example 14 : A jet plane is at a vertical height of h. The angles of depression of two tanks on the horizontal ground are found to have measures α and β $(\alpha > \beta)$ Prove that the distance between the tanks is $\frac{h(\tan\alpha - \tan\beta)}{\tan\alpha \cdot \tan\beta}$.

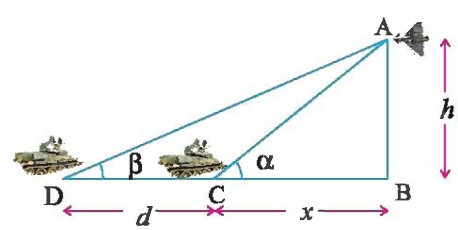

Figure 10.19

Solution : Let A be the jet plane, C and D are two tanks.

Here AB = h, BC = x and CD = d.

$m\angle ACB = \alpha$ and $m\angle ADB = \beta$

In ΔABC, $tan\alpha = \frac{AB}{BC}$

$\therefore \quad tan\alpha = \frac{h}{x}$

$\therefore \quad x = \frac{h}{tan\,\alpha}$ **(i)**

In ΔABD, $tan\beta = \frac{AB}{BD}$

$\therefore \quad tan\beta = \frac{h}{x+d}$

$\therefore \quad x + d = \frac{h}{tan\,\beta}$ **(ii)**

From (i) and (ii)

$$\frac{h}{tan\,\alpha} + d = \frac{h}{tan\,\beta}$$

$$\therefore \quad d = \frac{h}{tan\,\beta} - \frac{h}{tan\,\alpha}$$

$$\therefore \quad d = \frac{h(tan\alpha - tan\beta)}{tan\alpha \cdot tan\beta}$$

Hence, the distance between the tanks is $\frac{h(tan\alpha - tan\beta)}{tan\alpha \cdot tan\beta}$.

EXERCISE 10

1. A pole stands vertically on the ground. If the angle of elevation of the top of the pole from a point 90 *m* away from the pole has measure 30, find the height of the pole.
2. A string of a kite is 100 *m* long and it makes an angle of measure 60 with the horizontal. Find the height of the kite, assuming that there is no slack in the string.
3. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of pole is 10 *m* and the angle made by the rope with ground level has measure 30. Calculate the distance covered by the artist in climbing to the top of the pole.
4. A tree breaks due to a storm and the broken part bends such that the top of the tree touches the ground making an angle having measure 30 with the ground. The distance from the foot of the tree to the point where the top touches the ground is 30 *m*. Find the height of the tree.
5. An electrician has to repair an electric fault on the pole of height 5 *m*. He needs to reach a point 2 *m* below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use, which when inclined at an angle of measure 60 to the horizontal would enable him to reach the required position.
6. As observed from a fixed point on the bank of a river, the angle of elevation of a temple on the opposite bank has measure 30. If the height of the temple is 20 *m*, find the width of the river.
7. As observed from the top of a hill 200 *m* high, the angles of depression of two vehicles situated on the same side of the hill are found to have measure 30 and 60 respectively. Find the distance between the two vehicles.
8. A person standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank has measure 60. When he retreats 20 *m* from the bank, he finds the angle to have measure 30. Find the height of the tree and the breadth of the river.

9. The shadow of a tower is 27 m, when the angle of elevation of the sun has measure 30. When the angle of elevation of the sun has measure 60, find the length of the shadow of the tower.

10. From a point at the height 100 m above the sea level, the angles of depression of a ship in the sea is found to have measure 30. After some time the angle of depression of the ship has measure 45. Find the distance travelled by the ship during that time interval.

11. From the top of a 300 m high light-house, the angles of depression of the top and foot of a tower have measure 30 and 60. Find the height of the tower.

12. As observed from a point 60 m above a lake, the angle of elevation of an advertising ballon has measure 30 and from the same point the angle of depression of the image of the ballon in the lake has measure 60. Calculate the height of the balloon above the lake.

13. Watching from a window 40 m high of a multistoreyed building, the angle of elevation of the top of a tower is found to have measure 45. The angle of elevation of the top of the same tower from the bottom of the building is found to have measure 60. Find the height of the tower.

14. Two pillars of equal height stand on either side of a road, which is 100 m wide. The angles of elevation of the top of the pillars have measure 60 and 30 at a point on the road between the pillars. Find the position of the point from the nearest end of a pillars and the height of pillars.

15. The angles of elevation of the top of a tower from two points at distance a and b metres from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$ metres.

16. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change its measure from 30 to 45, how soon after this, will the car reach the tower ?

17. If the angle of elevation of a cloud from a point h metres above a lake has measure α and the angle of depression of its reflection in the lake has measure β, prove that the height of the cloud is $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha} m$.

18. From the top of a building $\overline{AB}$, 60 m high, the angles of depression of the top and bottom at a vertical lamp post $\overline{CD}$ are observed to have measure 30 and 60 respectively. Find,
    (1) the horizontal distance between building and lamp post.
    (2) the height of the lamp post.
    (3) the difference between the heights of the building and the lamp post.

19. A bridge across a valley is h metres long. There is a temple in the valley directly below the bridge. The angles of depression of the top of the temple from the two ends of the bridge have measures α and β. Prove that the height of the bridge above the top of the temple is $\dfrac{h(\tan\alpha \cdot \tan\beta)}{\tan\alpha + \tan\beta} m$.

20. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On walking 192 metres towards the tower, the tangent of the angle is found to be $\frac{3}{4}$. Find the height of the tower.

21. A statue 1.46 m tall, stands on the top of a pedestal. From the point on the ground the angle of elevation of the top of the statue has measure 60 and from the same point, the angle of elevation of the top of the pedestal has measure 45. Find the height of the pedestal.

22. Select a proper option (a), (b), (c) or (d) from given options and write in the box given on the right so that the statement becomes correct :

(1) On walking ...... metres on a hill making an angle of measure 30 with the ground, one can reach the height of 'a' metres from the ground. ☐

(a) $\frac{\sqrt{3}}{2}a$ (b) $\frac{2a}{\sqrt{3}}$ (c) $2a$ (d) $\frac{a}{2}$

(2) The angle of elevation of the top of the tower from a point P on the ground has measure 45. The distance of the tower from the point P is a and height of the tower is b. Then, ... ☐

(a) $a > b$ (b) $a < b$ (c) $a = b$ (d) $a = 2b$

(3) A 3 m long ladder leans on the wall such that its lower end remains 1.5 m away from the base of the wall. Then, the ladder makes an angle of measure ...... with the ground. ☐

(a) 30 (b) 45 (c) 60 (d) 20

(4) A tower is $50\sqrt{3}$ m high. The angle of elevation of its top from a point 50 m away from its foot has measure ...... ☐

(a) 45 (b) 60 (c) 30 (d) 15

(5) If the ratio of the height of a tower and the length of its shadow is $1 : \sqrt{3}$, then the angle of elevation of the sun has measure ...... ☐

(a) 30 (b) 45 (c) 60 (d) 75

(6) If the angles of elevation of a tower from two points distance a and b ($a > b$) from its foot on the same side of the tower have measure 30 and 60, then the height of the tower is ...... ☐

(a) $\sqrt{a+b}$ (b) $\sqrt{ab}$ (c) $\sqrt{a-b}$ (d) $\sqrt{\frac{a}{b}}$

(7) The tops of two poles of height 18 m and 12 m are connected by a wire. If the wire makes an angle of measure 30 with horizontal, then the length of the wire is ...... ☐

(a) 12 m (b) 10 m (c) 8 m (d) 4 m

(8) The angle of elevation of the top of the building A from the base of building B has measure 50. The angle of elevation of the top of the building B from the base of building A has measure 70. Then, ... ☐

(a) building A is taller than building B.

(b) Building B is taller than building A.

(c) Building A and building B are equally tall.

(d) The relation about the heights of A and B cannot be determined.

(9) If the angle of elevation of the top of a tower of a distance 400 m from its foot has measure 30, then the height of the tower is ...... ☐

(a) $200\sqrt{2}$ (b) $\frac{400}{\sqrt{3}}$ (c) $200\sqrt{3}$ (d) $\frac{400}{\sqrt{2}}$

(10) The angle of depression of a ship from the top of a tower 30 m height has measure 60. Then, the distance of the ship from the base of the tower is ...... ☐

(a) 10 (b) 30 (c) $10\sqrt{3}$ (d) $30\sqrt{3}$

(11) When the length of the shadow of the pole is equal to the height of the pole, then the angle of elevation of the source of light has measure ...... $\square$

(a) 45 (b) 30 (c) 60 (d) 75

(12) From the top of a building h metre high, the angle of depression of an object on the ground has measure θ. The distance (in metres) of the object from the foot of the building is ...... $\square$

(a) $h\,sin\theta$ (b) $h\,tan\theta$ (c) $h\,cot\theta$ (d) $h\,cos\theta$

(13) As observed from the top of the light house the angle of depression of the two ships P and Q anchored in the sea to the same side are found to have measure 35 and 50 respectively. Then from the light house.... $\square$

(a) P and Q are at equal distance.

(b) The distance of Q is more then P.

(c) The distance of P is more than Q.

(d) The relation about the distance of P and Q cannot be determined.

(14) Two poles are x metres apart and the height of one is double than that of the other. If from the mid-point of the line joining their feet, an observer finds the angle of elevation of their tops to be complementary, then the height of the shorter pole is ...... $\square$

(a) $\frac{x}{4}$ (b) $\frac{x}{\sqrt{2}}$ (c) $\sqrt{2}\,x$ (d) $\frac{x}{2\sqrt{2}}$

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Summary

In this chapter we have studied following points :

1. **Horizontal Ray :** A ray parallel to the surface of the earth emerging from the eye of the observer is called a horizontal ray.
2. **Ray of Vision :** The ray from the eye of an observer towards the object is called the ray of vision or ray of sight.
3. **Angle of Elevation :** If the object under observation is above an observer, but not directly above the observer, then the measure of the angle formed by the horizontal ray and the ray of sight is called the angle of elevation. Here horizontal ray, observer and object are in the same vertical plane.
4. **Angle of Depression :** If the object under observation is at a lower level than an observer but not directly under the observer, then the measure of the angle formed by the horizontal ray and the ray of sight is called the angle of depression.
5. The height of length of an object on the distance between two distant objects can be determined with the help of trigonometric ratios.

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