The article “Inference for high-dimensional sparse econometric models” by Belloni, Chernozhukov and Hansen (2011) analyses the process of modelling econometric phenomena. The key aspect of the economic problems that this model tackles is the sparsity in the data. This implies that only a small portion of the data yields impactful outcomes but it is not possible to identify with ease which are these relevant factors. However, exact sparsity cannot be realistically assumed and as such, the model they analyse is an approximately sparse one, instead. Therefore, while all regressors are potentially non-zero values, only a certain number of unknown variables are necessary to approximate the regression with an acceptable degree of accuracy. As such, the purpose of this paper and the question it aims to solve is: What are the estimation methods for HDS models and how can they be improved?

The first condition for the method they propose is the ASM condition, which defines a parametric vector equivalent to our real target function and which will undergo all the necessary procedures in its stead as a semi-parametric model. In this scenario, l1 norm (the penalty for the Lasso estimator) may be introduced for an estimation of the model as a solution to the optimization as it removes the regressions that do not fulfil its condition. In addition to that, a Post-Lasso estimator is created after the penalty is issued, a simple quantile regression that further thins the list of regressors. With the SE condition that prevents the penalty to be approximated to infinite value, in the general linear models it is possible to create their first theorem of the process of this method. As such, it is possible to construct confidence sets for the coefficient as long as the intervals are efficiently bound. Their second theorem further proves the efficiency of the method as it is possible to bound post-Lasso to Lasso’s rate of convergence. It’s important to note here that Post-Lasso has smaller shrinkage bias and as such even if the first Lasso estimator or Iterated Lasso may fail to determine the right set of coefficients, Post-Lasso may be able to do it properly.

For linear instrumental variable models, the condition ASIV allows for a third theorem based on it that states that the IV estimator for Lasso or Post-Lasso is efficient in the semi-parametric models as long as the optimal instruments fulfil the necessary conditions. First of all, a rate condition that may be adjusted (though the adjusted version may not guarantee semi-parametric efficiency) and, secondly, that the instruments are smooth enough to be approximated by only a small number of series terms. Furthermore, another theory constructs two confidence regions in which the chance of including a false point that belongs to neither of those is nearly zero as long as its sufficiently distant to an identified point in either of the regions and the instruments with which the model has been built are not too weak. It’s called an Inverse Lasso as it collects all the possible values of the coefficient whereas the Lasso regression assigns a value of cero to those coefficients when there is a structural disturbance.

Additionally, a “double-Post-Lasso” method is introduced by applying Lasso methods to two related equations at the same time to increase the chances of successfully recovering the variables that approximate the target function. This results in a much more robust procedure for computational experiments that can work with weaker regularity conditions that its predecessors couldn’t overcome. As such, as long as the ASTE condition holds, this method achieves semi-parametric efficiency.

To sum it up, that the results of this article pose a great advance in knowledge in terms of the methods that are used to analyse the HDS models with a new approach using the Lasso estimator and penalty to identify weak signals among a vast group of regressors. Additionally, their method proves to be more efficient than the classic two-stage-least-squares (2SLS) and, in general, performs better than Fuller’s estimator in terms of the estimator risk and with a generally smaller median bias. A couple ideas in order to further advance on this question, more models could be analysed and compared to the Lasso estimator and the application of the Lasso estimator or an adjusted version of it to a series of observations across a time series where scores are dependent on previous and even future ones.

It is, however, necessary to reiterate as there is no complete uniformity in real life, their models might still be affected by it even if the errors and irrelevant information is minimized. This also means that the conditions in which their theories are built upon may not be fulfilled and the pivotal nature of the estimates in relation to the defined variables may be compromised, as such, the approximation would be a weak one with irrelevant results and therefore invalid. But alas, this is the risk all models face when trying to explain reality.