1. A professor has 15 students and during lecture will (uniformly) at random choose a student to answer a question. The professor asks 8 questions during the lecture. What is the probability no student will have to answer more than one question?
   Figure out chances of unique students being chosen for each question:
   $\frac{15}{15} \cdot \frac{14}{15} \cdots \frac{8}{15}$
   $= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{15^8}$
   $= 0.101$

2. An integer from the range 00000 - 99999 is generated uniformly at random. We are interested only in even integers that start with 2 odd digits where all **digits are unique**. If we randomly generate 8 of these numbers in succession, what is the probability we get exactly 5 numbers that meet our criteria?
   Total space of possible integers: 10^5

Integers using specified format can be made using the following format:
3 digits $\rightarrow 5 \cdot 4 \cdot 5 = 100$
4 digits $\rightarrow 5 \cdot 4 \cdot 7 \cdot 5 = 700$
5 digits $\rightarrow 5 \cdot 4 \cdot 7 \cdot 6 \cdot 5 = 4200$
Chance a randomly generated number is formatted this way: $\frac{5000}{10^5} = 0.05$
Using the binomial probability (is or isn't one of our numbers):
Answer $\rightarrow nCr(8, 5)(0.05)^5(1 - 0.05)^3 \approx 0.000015$

3. You roll 3 six-sided, fair dice. Let A be the event that at least 2 dice show 4 or above. Let B be the event that all 3 dice show the same value. Are A and B independent?

The probability one dice roll will return a value ≥ 4 is 0.5.

Probability that at least two dice are 4 and above:
$nCr(3, 2)(0.5)^3 + nCr(3, 3)(0.5)^3 = 0.5$

Probability that all dice have the same value:
$6 \times \frac{1}{6} \times \frac{1}{6} = \frac{6}{216} = \frac{1}{36}$

Probability that at least two dice are 4 and above AND they all have the same value:
$\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{72}$

$P(A) \times P(B) = \frac{1}{36} \times \frac{1}{2} = \frac{1}{72}$

Since these two results are the same, these events are independent.

4. In poker, a flush is any 5-card hand where all the cards of the same suit. For this problem we will not distinguish between an ordinary flush and special flushes (like straight and royal flushes), meaning we will call any hand that has all 5 cards from the same suit a flush. Poker-player Paul loves a flush. What is the expected number of hands of poker he has to play to get a flush. (We assume each hand is dealt from a new deck containing of randomly ordered cards).
   $4 \times \frac{nCr(13,5)}{nCr(52,5)}$ is the probability of a flush given any 5 cards drawn from the deck (4 suits, 13 cards per suit, 5 cards drawn from deck).
   $= Expectednumberofhands\$ \sim \frac{1}{P(A)} = \frac{1}{0.00198079231693} \approx 504$ hands of poker.

5. A basketball team has a superstar. When their superstar plays, they win 70% of the time. When their superstar does not play they win 50% of the time. Entering a 5 game stretch, the superstar had been recovering from an injury and said the chance they would play the next 5 games was 75%. You go on a trip to the jungle (no internet access). When you return you find out the team won 4 of the 5 games. What is the probability the superstar played those 5 games? You may assume the superstar doesn't get injured during those games (either they play all or none of the 5).
   Superstar Win: 0.7
   No Superstar Win: 0.5
   Binomial probability
   $\rightarrow$wins with superstar: $nCr(5,4)(0.7)^4(0.3)^1 = 0.36015$
   $\rightarrow$wins without superstar: $nCr(5,4)(0.5)^4(0.5)^1 = 0.15625$

Total chance of 4 wins including consideration for p(superstar playing):
$\rightarrow (0.75)(0.36015) + (0.25)(0.15625) = 0.309175$

Chance that superstar was playing given total chance of 4 wins
$\frac{p(win,superstar)(p(superstar))}{p(4wins)}$
$= \frac{0.36015 \cdot 0.75}{0.3092}$
≈ 0.8736