Skew Lines

Two lines are skew if they are not parallel, and do not intersect. They can only exist in space with more spatial dimensions than two.

Proving two lines are skew

$L_1 = \underline{r_1} + \lambda \underline{d_1}$
$L_2 = \underline{r_2} + \mu \underline{d_2}$

Part 1 - The direction vectors

If $\underline{d_1} = k * \underline{d_2}$ where $k \in \mathbb{N}$, then lines are parallel, and thus, not skew, to prove that the lines are skew, prove that no scale factor exists that can scale the first direction vector, $\underline{d_1}$ to become $\underline{d_2}$.

Part 2 - Intersection

The next thing to prove is that the lines do not intersect.
To do this, set up the following simultaneous equations:

$r_{x1} + \lambda d_{1x} = r_{2x} + \mu d_{2x}$
$r_{1y} + \lambda d_{1y} = r_{2y} + \mu d_{2y}$
$r_{1z} + \lambda d_{1z} = r_{2z} + \mu d_{2z}$

Using the first two equations to solve for λ and μ, substitute those values into the third equation. If the lines are skew, this should result in an invalid equation. Else, the lines intersect at the λ and μ values computed earlier.