Trig Differentiation

First Principles

SUPER CRUCIAL NOTE

Whenever calculus is involved, you may assume that you are working in radians unless specifically told otherwise (which NEVER happens!)

To differentiate $\sin x$ and $\cos x$ we use both the addition formulae and small angle approximations.

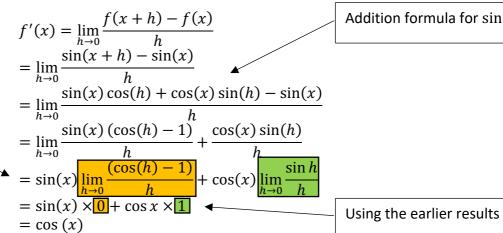
First note that for small h, $\cos(h) \approx 1 - \frac{h^2}{2}$ and $\sin(h) \approx h$ so...

$$\lim_{h \to 0} \frac{(\cos(h) - 1)}{h} = \lim_{h \to 0} \frac{\left(1 - \frac{h^2}{2} - 1\right)}{h} = \lim_{h \to 0} \frac{-h}{2} = 0$$

$$\lim_{h \to 0} \frac{\sin h}{h} = \lim_{h \to 0} \frac{h}{h} = 1$$

Now if $f(x) = \sin x$ then:

Because we treat xas fixed, cos(x) and $\sin(x)$ are constant and can be brought out to the front of the limit



Using the earlier results

A similar approach will prove that if $f(x) = \cos(x)$ then $f'(x) = -\sin(x)$

The Results

$$\frac{d}{dx}(\sin(kx)) = k\cos(kx)$$

$$\frac{d}{dx}(\cos(kx)) = -k\sin(kx)$$

Example

Find the stationary points on $y = 3x - \cos(3x)$ for $0 \le x \le 2\pi$

$$\frac{d}{dx}(\cos(3x)) = -3\sin(3x)$$

Stationary points occur when $\frac{dy}{dx} = 0$:

 $\frac{dy}{dx} = 3 + 3\sin(3x)$

Then don't forget the double negative!

$$3 + 3\sin(3x) = 0$$

$$\sin(3x) = -1$$

$$3x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$$

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

The coordinates are $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, $\left(\frac{7\pi}{6}, \frac{21\pi}{6}\right)$, $\left(\frac{11\pi}{6}, \frac{33\pi}{6}\right)$