

# Dealing with different exponential functions in the same equation

## The idea

- Take log of both sides – your choice of which base to use, unless it is a “show that” question
- Use the log laws to change powers inside the log to multiples outside the log
- Expand brackets and gather  $x$  terms
- Divide through

## Examples

1. Solve  $3^{x+1} = 7^{2x-1}$

I have chosen to take log base 3

$$\begin{aligned}x + 1 &= \log_3(7^{2x-1}) \\x + 1 &= (2x - 1) \log_3(7) \\x + 1 &= (2 \log_3(7))x - \log_3(7) \\1 + \log_3(7) &= (2 \log_3(7))x - x \\1 + \log_3(7) &= (2 \log_3(7) - 1)x \\x &= \frac{1 + \log_3(7)}{2 \log_3(7) - 1}\end{aligned}$$

2. Solve  $5^{x-1} = 2^{9-x}$

This time I have chosen to take log base 10 (which is the default base)

$$\begin{aligned}\log(5^{(x-1)}) &= \log(2^{(9-x)}) \\(x - 1) \log(5) &= (9 - x) \log(2) \\x \log(5) - \log(5) &= 9 \log(2) - x \log(2) \\(\log(5) + \log(2))x &= (\log(5) + 9 \log(2)) \\x &= \frac{\log(5) + 9 \log(2)}{\log(5) + \log(2)} = \frac{\log(2560)}{\log(10)}\end{aligned}$$