Sums of Series – Quick Reminder

$$\sum_{r=1}^{n} 1 = n$$
$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} kf(r) = k\sum_{r=1}^{n} f(r)$$
$$\sum_{r=1}^{n} \big(f(r) + g(r)\big) = \sum_{r=1}^{n} f(r) + \sum_{r=1}^{n} g(r)$$
$$\sum_{r=m}^{n} f(r) = \sum_{r=1}^{n} f(r) - \sum_{r=1}^{m-1} f(r)$$

e.g. Find $\sum_{r=1}^{6}(5r-2)$

$$\begin{aligned}
&= 5\textstyle\sum_{r=1}^{6}(r) - 2\sum_{r=1}^{6}(1) \\
&= 5\left(\tfrac{1}{2}(6)(7)\right) - 2(6) \\
&= 105 - 12 \\
&= 93
\end{aligned}$$

e.g. $f(k) = \sum_{r=1}^{k}(2r+1)$

a) Show that $f(k) = k(k+p)$ where p is to be determined.
b) Hence evaluate $\sum_{r=7}^{20}(2r+1)$

a) $$\begin{aligned}
f(k) &= \textstyle\sum_{r=1}^{k}(2r+1) \\
&= 2\textstyle\sum_{r=1}^{k}(r) + \sum_{r=1}^{k}(1) \\
&= 2\left(\tfrac{1}{2}(k)(k+1)\right) + k \\
&= k(k+1) + k \\
&= k\big((k+1)+1\big) \\
&= k(k+2) \quad \text{i.e. } p = 2
\end{aligned}$$

b) $$\begin{aligned}
\textstyle\sum_{r=7}^{20}(2r+1) &= f(20) - f(6) \\
&= (20)(22) - (6)(8) \\
&= 440 - 48 \\
&= 392
\end{aligned}$$