Using logs to solve exponential equations

The crucial thing to remember is that you must manipulate the equation first so that you are taking the log of a single exponential expression. This is because $\log(a+b) \neq \log a + \log b$.

To undo p^x we use $\log_{\mathrm{p}} x$.

e.g. solve $5 \times 7^{x+1} - 3 = 8$ giving your answer in exact form

$7^{x+1} = \frac{11}{5}$
$x + 1 = \log_7\left(\frac{11}{5}\right)$
$x = \log_7\left(\frac{11}{5}\right) - 1$

There are several equivalent answers
e.g. $x = \log_7(11) - \log_7(5) - 1$ or $x = \log_7\left(\frac{11}{35}\right)$

e.g. solve $2^{2x+1} - 11 \times 2^x + 5 = 0$ giving your answers to 2dp where appropriate

$2 \times (2^x)^2 - 11 \times (2^x) + 5 = 0$

Use the index laws to rewrite as a quadratic:
$2^{2x+1} = 2^1 \times 2^{2x} = 2 \times (2^x)^2$

Let $y = 2^x$. Then:

$2y^2 - 11y + 5 = 0$
$(2y-1)(y-5) = 0$
$y = \frac{1}{2}$ or $y = 5$

Convert back to an equation in x

$2^x = \frac{1}{2}$ or $2^x = 5$
$x = \log_2\left(\frac{1}{2}\right)$ or $x = \log_2(5)$
$x = -1$ or $x = \log_2(5)$

By default you should give exact answers, simplifying any logs which give a simple rational answer, unless the question specifies a required accuracy. However, there is no harm in giving both exact and rounded.

($x = -1$ or 2.32 (2dp))

NB It is quite normal to have solutions to the quadratic in y which are ≤ 0 and therefore give you a "math error" when you try to take a log. Don't panic! There is simply no corresponding x solution.

e.g. $3^{2x+2} + 17 \times 3^x - 2 = 0$
Let $y = 3^x$. Then $9y^2 + 17y - 2 = 0$
$(9y-1)(y+2) = 0$
$y = \frac{1}{9}$ or $y = -2$
$3^x = \frac{1}{9}$ or $3^x = -2$
We may discard the second equation as 3^x can never be negative, so the only solution is $x = -2$