Sums of Polynomial Series – Up to Cubics

The Full List of (A-level Relevant) Formulae

$$\sum_{r=1}^{n} 1 = n$$

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

A couple of points:

- You need to remember the first two but the last two are in the formula booklet
- Interesting aid for the memory: $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$

e.g.

$$\begin{aligned}\sum_{r=1}^{5}(r+1)(r-3) &= \sum_{r=1}^{5}(r^2-2r-3)\\ &= \sum_{r=1}^{5} r^2 - 2\sum_{r=1}^{5} r - 3\sum_{r=1}^{5} 1\\ &= \frac{1}{6}(5)(6)(11) - (2)\left(\frac{1}{2}(5)(6)\right) - (3)(5)\\ &= 55 - 30 - 15\\ &= 10\end{aligned}$$

Multiply out before trying to use the summation formulae

e.g.

$$\begin{aligned}\sum_{r=6}^{10}(r^3-r^2) &= \sum_{r=6}^{10} r^3 - \sum_{r=6}^{10} r^2\\ &= \left(\sum_{r=1}^{10} r^3 - \sum_{r=1}^{10} r^2\right) - \left(\sum_{r=5}^{10} r^3 - \sum_{r=5}^{10} r^2\right)\\ &= \left(\frac{1}{4}(10)^2(11)^2 - \frac{1}{6}(10)(11)(21)\right) - \left(\frac{1}{4}(5)^2(6)^2 - \frac{1}{6}(5)(6)(11)\right)\\ &= (3025-385)-(225-55)\\ &= 2470\end{aligned}$$

We want to include the 6th term in our sum, so cancel everything up to the 5th term

p.50
Q6, 7, 8, 9, 12