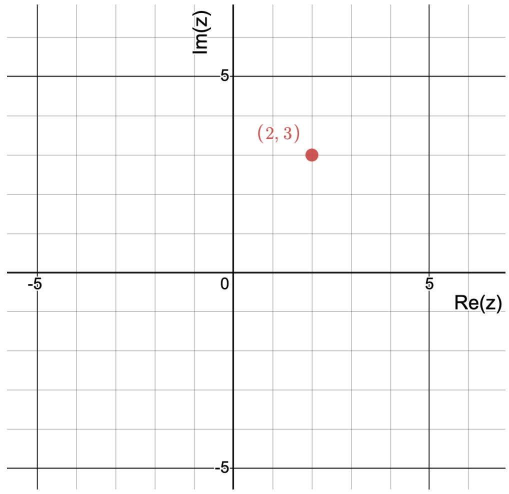
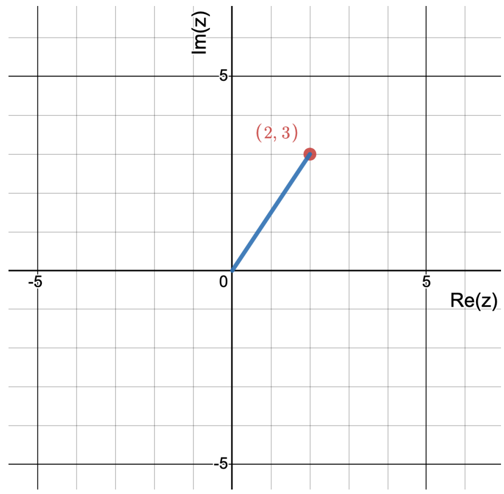
The Argand Diagram

A complex number has two components: the real part and the imaginary part.
We use these as "x" and "y" coordinates respectively to represent the complex number on the 2D plane.

e.g. $2 + 3i$ would correspond to $(2, 3)$ and be represented either as a point or a vector from (0, 0)





Note the axis labels:

- the $x -$ axis is $\mathrm{Re}(z)$ i.e. the real part of z
- the $y -$ axis is $\mathrm{Im}(z)$ i.e. the imaginary part of z

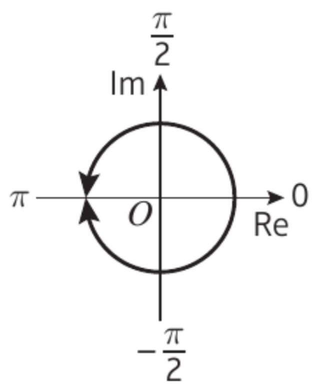
Modulus and Argument

The graphical representation of complex numbers suggests two geometric properties.

If $z = a + bi$ then:

- $|z|$ (the **modulus** of z) is calculated by $\sqrt{a^2 + b^2}$
  This is equivalent to the (Euclidean or Pythagorean) distance to a point from the origin on the Argand Diagram

- $\arg(z)$ (the **argument** of z) is the (anticlockwise) angle in radians between the positive real axis and the vector representing z. This is normally given as the principal argument i.e. $-\pi < \theta \leq \pi$.
  NB. $\arctan\left(\frac{b}{a}\right)$ will give the correct argument only if between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$
  so it is recommended to sketch on an Argand Diagram and use your brain!



e.g. Find $|z|$ and $\arg(z)$ if $z = 1 - \sqrt{3}i$

$$|z| = \sqrt{(1)^2 + \left(-\sqrt{3}\right)^2} = 2$$

$$\theta = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

So $\arg(z) = -\frac{\pi}{3}$