Intersections with Parametric Curves

In general, when finding intersections with parametric curves it is a bad idea to convert to Cartesian first. The best approach is almost always using the parametric equations to find t and then substitute to find the appropriate $x-$ and $y-$ coordinates.

Common scenario 1: Intersection with a coordinate axis

- Set the y or x parametric equation equal to zero
- Solve to find t
- Substitute into the other parametric equation and state the coordinates

e.g. Find the coordinates at which $x = t^2 - 1, y = 2t + 3$ meets the $y-$ axis

- The $y-$ axis means $x = 0$ so $t^2 - 1 = 0$
- $t = -1$ or $t = 1$
- When $t = -1, y = 1$ giving the coordinates $(0, 1)$
  When $t = 1, y = 5$ giving the coordinates $(0, 5)$

Common scenario 2: Intersection with a Cartesian equation

- Substitute the parametric equations for x and y into the Cartesian equation
- Solve to find t
- Substitute into both parametric equations and state the coordinates

e.g. Find the coordinates at which $x = 2t^2 - 1, y = 3 - t^2$ meets $x^2 + y^2 = 10$

- $(2t^2 - 1)^2 + (3 - t^2)^2 = 10$
- $4t^4 - 4t^2 + 1 + 9 - 6t^2 + t^4 = 10$
  $5t^4 - 10t^2 = 0$
  $5t^2(t^2 - 2) = 0$
  $t = 0, \sqrt{2}, -\sqrt{2}$
- When $t = 0, x = -1$ and $y = 3$ giving coordinates $(-1, 3)$
  When $t = \sqrt{2}, x = 3$ and $y = 1$ giving coordinates $(3, 1)$
  When $t = -\sqrt{2}, x = 3$ and $y = 1$ giving coordinates $(3, 1)$ again!