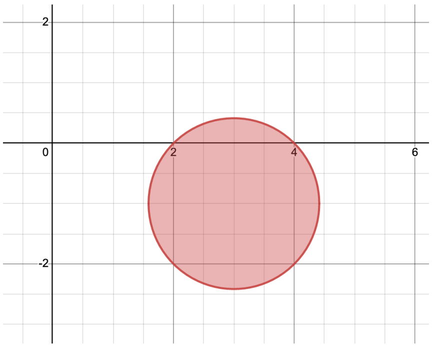
Regions on the Complex Plane

A natural extension of the three loci we have studied is to change the equations to inequalities and define regions on the complex plane.

$\|z - z_1\| \leq r$	The inside of a circle centre z_1 radius r
$\|z - z_1\| \geq r$	The outside of a circle centre z_1 radius r
$\|z - z_1\| < \|z - z_2\|$	"Nearer to z_1 than z_2" i.e. shade the z_1 side of the perpendicular bisector
$\theta_1 \leq \arg(z - z_1) \leq \theta_2$	The region between half-lines starting at z_1 at angles θ_1 and θ_2

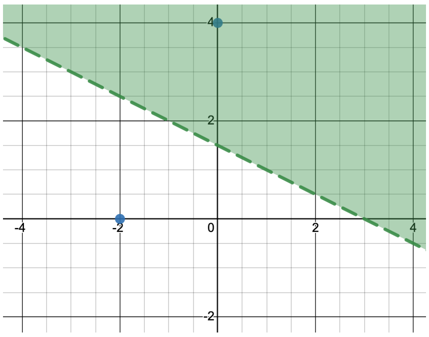
e.g. $|z - 3 + i| \leq 2$

Rewrite as $|z - (3 - i)| \leq 2$
Inside a circle centre $(3, -1)$ radius 2
Note that the circle will cross through the $x -$ axis but not the $y -$ axis



e.g. $|z - 4i| < |z + 2|$

Rewrite as $|z - 4i| < |z - (-2)|$
The chord connecting $(-2, 0)$ and $(0, 4)$ has gradient 2 and midpoint $(-1, 2)$
The perpendicular bisector is therefore $y - 2 = -\frac{1}{2}(x - -1)$ i.e. $y = -\frac{1}{2}x + \frac{3}{2}$
We want the region closer to $4i$ than -2 i.e the region above $y = -\frac{1}{2}x + \frac{3}{2}$



e.g. $-\frac{\pi}{4} \leq \arg\big(z - (1 + i)\big) \leq \frac{\pi}{2}$

Between half-lines from $(1, 1)$ going at angle $-\frac{\pi}{4}$ (i.e. parallel to $y = -x$) and $\frac{\pi}{2}$ (straight up)