e

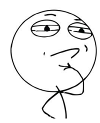
Sometimes known as Euler's number, e is an irrational, transcendental constant (like π) which turns up surprisingly often in higher maths. For now, the crucial points are:

- $e \approx 2.7$
- The function e^x differentiates to itself i.e. $\frac{d}{dx}(e^x) = e^x$
- Following the principles of the chain rule (which you will study soon), $\frac{d}{dx}(e^{kx}) = ke^{kx}$ for any constant k. i.e. multiply by the constant multiple of x and make sure you leave the k in the power.

Dr Frost has some interesting background information on e, for interest only (although Further Mathematicians will prove this result in FP1 using l'Hôpital's rule):

Just for your interest…

Where does e come from, and why is it so important?



$e = 2.71828\ldots$
is known as **Euler's Number**, and is considered one of the five fundamental constants in maths:
$0,\ 1,\ \pi,\ \ e,\ \ i$

Its value was originally encountered by Bernoulli who was solving the following problem:
You have £1. If you put it in a bank account with 100% interest, how much do you have a year later? If the interest is split into 2 instalments of 50% interest, how much will I have? What about 3 instalments of 33.3%? And so on…

No. Instalments	Money after a year
1	$1\times 2^1 = £2$
2	$1\times 1.5^2 = £2.25$
3	$1\times 1.\dot{3}^3 = £2.37$
4	$1\times 1.25^4 = £2.44$
n	$\left(1+\frac{1}{n}\right)^n$

As n becomes larger, the amount after a year approaches £2.71…, i.e. e!

Thus:
$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

But we have seen that differentiation by first principles uses 'limits'. It is therefore possible to prove from the definition above that $\frac{d}{dx}(e^x) = e^x$, and **these two definitions of *e* are considered to be equivalent***.

e therefore tends to arise in problems involving limits, and also therefore crops up all the time in anything involving differentiation and integration. Let's see some applications…

*You can find a full proof here in my Graph Sketching/Limits slides:
http://www.drfrostmaths.com/resources/resource.php?rid=163

Modelling with exponentials

Situations in which growth or decay is proportional to the current value (bacterial growth, compound interest etc) can be modelled using exponential functions. This is precisely because the differential/integral of an exponential function is a multiple of the original function.

e.g. The mass in grams of Carbon 14 isotope in a sample after t years is modelled by $m = 1000e^{-0.000121t}$

By using simple transformations of exponentials, we can also deal with situations in which growth or decay is proportional to the difference between the current value and some fixed "background value".

e.g. The temperature in °C of a cup of tea left to cool is modelled by $T = 24 + 59.2e^{-0.0274t}$

What might you be asked to do?

- Sketch the graph

- State the initial value i.e. substitution $t = 0$. **Just remember that $\boldsymbol{a^0 = 1}$ for any $\boldsymbol{a \neq 0}$** so e.g. the initial temperature of the cup of tea is $T = 24 + 59.2 \times 1 = 83.2$°C. Don't forget UNITS!!!!!

- Interpret the constants in an equation e.g. in the equation describing the Carbon 14, the constant 1000 represents the initial mass of Carbon 14 being 1000g. **Include UNITS and NUMBERS in your answer!!!!**

  With an exponential function (that doesn't have a constant added), it is important to note that the base of the exponential function is a **multiplicative** constant e.g. suppose my IQ follows the function $I = (1.2)^{t+25}$ at t years of age. The 1.2 represents that my IQ is multiplied by 1.2 each year. It would not be acceptable to write e.g. that 1.2 is the amount my IQ increases by each year.

- Use known values and a given equation form to find the constants (we practised the same skill in a previous lesson when we were given coordinate points for equations of the form $y = ka^x$)

- Describe the long-term behaviour by considering what happens when $t \to \infty$. Crucially, the exponential term e^{kx} tends to infinity if $k > 1$ and tends to zero if $0 < k < 1$.
  e.g. describe and interpret the long-term behaviour of the tea's temperature: as $t \to \infty$, $59.2e^{-0.0274t} \to 0$ so $T \to 24$°C. This can be considered the room temperature. (side note for you Physicists: this is related to Newton's Law of Cooling). **UNITS!!!**

- Evaluate the validity of a model (Is infinite unbounded growth realistic? Is it possible for the value to be less than 1 in the context of the question? etc). Try to give answers which refer to the specifics of the context in the question. **UNITS!!!**

- Find implied properties of the function's behaviour such as how long it takes for the value to halve e.g. calculating the half life of Carbon 14 from our molecule. However, this typically requires logs so we will come back to it later.