Roots of Polynomials: Quadratics

This topic examines relationships between the roots and coefficients of polynomials.

We have met this idea already when looking at quadratic equations and complex numbers.

If a quadratic equation has roots α and β then it can be written as $(x-\alpha)(x-\beta)=0$ which expands to give $x^2-(\alpha+\beta)x+\alpha\beta=0$.

In other words:

If α and β are roots of the equation $x^2+bx+c=0$ then:

- $\alpha+\beta=-b$
- $\alpha\beta=c$

You may see this written as:

If α and β are roots of the equation $ax^2+bx+c=0$ then:

- $\alpha+\beta=-\frac{b}{a}$
- $\alpha\beta=\frac{c}{a}$

I prefer my version – if the amount of x^2 is not 1, we can just divide through!

e.g. α and β are the roots of $x^2+4x-1=0$. Find the values of $\alpha+\beta$ and $\alpha\beta$

$\alpha+\beta=-(4)$
$\alpha\beta=(-1)$

We may also get asked to find the value of some expressions easily calculated from the above.

These two most frequently occur:

- $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-b}{c}$
- $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-b)^2-2c$

e.g. α and β are the roots of $x^2-9x-2=0$. Find the values of $\frac{1}{\alpha}+\frac{1}{\beta}$ and $\alpha^2+\beta^2$

$\alpha+\beta=9$ and $\alpha\beta=-2$
So $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-2}{9}$ and $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=81+4=85$