Communal QUBO: A Quantum Computing Approach to NP-Hard Optimization Problems

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Abstract

The cardinality-constrained portfolio optimization problem—selecting a fixed number of assets from a large universe to maximize risk-adjusted returns—is NP-hard. This intractability forces a trade-off: either use computationally cheap heuristics that ignore the market's correlation structure, or employ exact methods like Mixed-Integer Programming (MIP) that face exponential scaling. This research introduces ‘Communal QUBO,’ a novel hybrid quantum-classical framework designed to overcome this with structural diversification and scalability. We benchmarked this framework against both a heuristic and an exact MIP solver across three market scenarios. The results confirm that MIP solver's runtime increases exponentially. In contrast, Communal QUBO demonstrates superior scalability. Financially, the heuristic achieved higher returns by concentrating in top-performing sectors, while ‘Communal QUBO’ constructed more robustly diversified portfolios, leading to better risk management. This demonstrates a viable blueprint for tackling large-scale optimization problems on future quantum hardware.

1. Introduction

Selecting a fixed number of assets from a vast universe to construct an optimal portfolio is a cornerstone of modern finance. However, the cardinality constraint transforms the problem into an *NP-hard combinatorial* challenge, where the solution space expands exponentially, making exact classical solutions computationally intractable. Consequently, practitioners often resort to crude heuristics that can lead to poorly diversified portfolios vulnerable to concentration risk.

In contrast, quantum computing maps combinatorial problems to a Quadratic Unconstrained Binary Optimization (QUBO) model, using which quantum annealers can leverage quantum tunneling to find the minimum-energy state, or the optimal solution. However, the current Noisy Intermediate-Scale Quantum (NISQ) era is defined by hardware with limited qubits and high error rates, making it impossible to solve large, monolithic QUBO problems directly.

$$\text{QUBO: minimize } \mathcal{H}(\mathrm{x}) = \mathrm{x}^{\mathrm{T}}\mathrm{Qx} = \sum_{\mathrm{i}} \mathrm{R_i x_i} + \sum_{\mathrm{i<j}} \mathrm{C_{ij} x_i x_j}, \text{where } \mathrm{Q} = \begin{bmatrix} \mathrm{R_1} & \cdots & 0.5\mathrm{C_{ij}} \\ \vdots & \ddots & \vdots \\ 0.5\mathrm{C_{ij}} & \cdots & \mathrm{R_i} \end{bmatrix}$$

The QUBO Hamiltonian, where x are the nodes, linear terms Ri are the node coefficients (asset returns) and quadratic terms Cij are the nodes' covariance. The objective is to find a vector of binary variables x that minimizes the Hamiltonian.

This research proposes a framework, named 'Communal QUBO,' which uses a "divide and conquer" strategy. It leverages classical community detection to decompose the large-scale problem into smaller, independent sub-problems *tractable for near-term quantum hardware* and quantum QUBO sub-problems solved in parallel for each community. *The optimal asset within each community corresponds to the global minimum energy state of the Hamiltonian.*

Unlike classical methods that can get stuck in local minima (good portfolios), *quantum annealers use quantum tunneling to pass through energy barriers*, surpass the local minima and find the minimum energy state (the best portfolio).

This study tests the hypothesis that as the asset universe grows, Communal QUBO leads to diversified portfolios and demonstrate better scaling than exact classical MIP.

2. Methodology

2.1 Data and Scenarios To test robustness, we used Python to construct three 100-asset universes from historical S&P 500 lists to avoid survivorship bias:

- Scenario 1: 2008 Global Financial Crisis (Jan 2006 - Dec 2008).
- Scenario 2: 2015 Bull Market (Jan 2013 - Dec 2015).
- Scenario 3: 2025 Tech-Dominated Market (Jan 2023 - Dec 2025).

Historical data underwent cleaning: interpolation for missing values and Winsorization for outliers.

2.2 Experimental Design: For each scenario, we constructed portfolios of three cardinalities: 5, 10, and 15 assets, with the universe size growing in increments of 5.

2.3 The 'Communal QUBO' Framework consists of two stages:

1. **Community Detection & QUBO Selection.** The asset universe is modeled as a correlation-based graph. Leiden algorithm partitions this graph into a number of communities equal to the target portfolio size, enforcing *structural diversification*. Within each community, a QUBO problem is formulated to select the single asset with the best risk-adjusted return. These independent QUBOs are solved in parallel on a quantum simulator.
2. **Weight Allocation.** The selected assets form the final portfolio. Capital is allocated using Conditional Value-at-Risk (CVaR) optimization, which minimizes tail risk.

2.4 Benchmarks: We employ two classical benchmarks:

1. **Heuristic Benchmark:** A two-stage method that first selects assets with the highest Sharpe ratio, then allocates weights with CVaR. This mirrors industry practice for computational efficiency.
2. **MIP Benchmark:** A Mixed-Integer Program that formulates the cardinality-constrained CVaR problem to find the optimal classical solution and scales exponentially.

3. Results

Our framework demonstrates *superior 1) scalability and 2) risk management (diversification)*.

3.1 Computational Performance: The MIP benchmark hit the computational wall at 35 asset-universe when selecting 5 assets (for 10- and 15-asset portfolios, the exponential threshold is higher). The Communal QUBO's 'divide and conquer' approach has a vastly superior scaling profile up to 100 assets.

3.2 Financial Performance & Diversification: While the financial performance of the classical heuristic is better, it falls into a concentration trap. Communal QUBO enforces, by design, *structural diversification* by selecting assets from different market communities.

4. Discussion

Communal QUBO is a practical solution to the limitations of NISQ-era quantum hardware.

The scope of this study was designed to validate the framework's performance on universes up to 100 assets. While sufficient to demonstrate the exponential complexity of exact classical methods and the superior scaling architecture of Communal QUBO, future work should extend these experiments to industry-relevant scales to identify where the computational advantage becomes practical.

Furthermore, in the 2025 market, the heuristic produced a portfolio with greater sector diversification. This highlights that Communal QUBO's clustering is based on the market's true statistical correlation structure, which may not align with industry labels in the market. This suggests a valuable avenue for future research into the data-driven market structures that our framework uncovers.

References

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