REDSUN
first class in class

Assigment 2 Arindaam PES1201800274

1) Equation be $A + bx + cx^2 = y$

at $(1,1)$ $1 = A + b + c$

$(2,-1)$ $-1 = A + 2b + 4c$

$(3,1)$ $1 = A + 3b + 9c$

$Ax = b$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$C = 2$, $B = -8$, $A = 7$

$y = 2x^2 - 8x + 7$

2.

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & 6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + 5R_1$

$R_4 \to R_4 - 5R_1$

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & +5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$
$R_4 \to R_4 + 2R_2$

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$U = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ -5 & -2 & 3 & 1 \end{bmatrix}$$

3

(i) $T(1,0,0) = (1,0,1)$, $T(0,1,0) = (2,1,1)$, $T(0,0,1) = (-1,1,-2)$

(ii) $$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_1} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Column Space basis $= \{(1,0,1), (2,1,1)\}$

Row Space basis $= C(T^T) = \{(1,2,-1), (0,1,1)\}$

Null Space: $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for $N(T) = (3,-1,1)$

Left Null Space: $N(T^T)$

$$T^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + R_1$
$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis $N(T^T) = (-1, 1, 1)$

iv)

$$\begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$(1-\lambda)([1-\lambda][-2-\lambda]-1) + 1(2-(1-\lambda)) = 0$$

$$\lambda^3 = 3\lambda$$

$$\lambda = \sqrt{3}, -\sqrt{3}, 0$$

Eigen vector for $\lambda = \sqrt{3}$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x}{3-\sqrt{3}} = \frac{y}{\sqrt{3}-1} = \frac{z}{4-2\sqrt{3}}$$

$$k_a(3-\sqrt{3}, \sqrt{3}-1, 4-2\sqrt{3}) \Rightarrow k_a\left(\frac{\sqrt{3}+3}{2}, \frac{\sqrt{3}+1}{2}, 1\right)$$

For $\lambda = 0$

$$\frac{x}{2+1} = \frac{y}{-1} = \frac{z}{1} = K_b$$

$$\underline{\underline{K_b\,(3, -1, 1)}}$$

For $\lambda = -\sqrt{3}$

$$\frac{x}{3+\sqrt{3}} = \frac{y}{-(1+\sqrt{3})} = \frac{z}{4+2\sqrt{3}} = K_c$$

$$\underline{\underline{K_c\left(-\frac{\sqrt{3}+3}{2}, -\frac{\sqrt{3}+1}{2}, 1\right)}}$$

iv) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$ $\quad q_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\quad q_2 = \frac{\ell_2}{|\ell_2|} \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \end{bmatrix}$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 1/2\sqrt{3/2} & -2/7 \\ 0 & 1/\sqrt{3/2} & 3/7 \\ 1/\sqrt{2} & -1/\sqrt{3/2} & -6/7 \end{bmatrix} \qquad R = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & \sqrt{3}/2 & \sqrt{3}/2 \\ 0 & 0 & 0 \end{bmatrix}$$

4.

x	-4	1	2	3
y	4	6	10	8

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C_x \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$A^T A x^n = A^T b$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 3 & 4 \end{bmatrix}$$

$\hat{c} = \frac{193}{27}$ $\qquad$ $\hat{d} = \frac{20}{29}$

$$y = \frac{193}{29} + \frac{20x}{29}$$

5. $V = [1 \; 1 \; 30 \; 4]$

$x_1 = -x_2 + (-3)x_3 - 4x_5$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = K_1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + K_2 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + K_3 \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\qquad P = A(A^TA)^{-1}A^T$

$P = \begin{bmatrix} 25/27 & -1/27 & -1/9 & 0 & -4/27 \\ -1/27 & 26/27 & -1/9 & 0 & -4/27 \\ -1/9 & -1/9 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -4/27 & -4/27 & -4/9 & 0 & 11/27 \end{bmatrix}$

$P + Q = I$

$Q = I - P = \begin{bmatrix} 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/9 & 1/9 & 1/3 & 0 & 4/9 \\ 0 & 0 & 0 & 1 & 0 \\ 4/27 & 4/27 & 4/9 & 0 & 16/27 \end{bmatrix}$

6.

$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$ (1) $|a| > 0$ $\qquad a > 0$

(2) $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} > 0 \Rightarrow a^2 - 4 > 0$

$a \in (-2, \infty) \cup (2, \infty)$

$(a-2)(a+2) > 0$ $\qquad a + 2 > 0$

$a - 2 > 0$ $\qquad a > -2$

$a > 2$

(3) $|A| > 0$

$a(a^2-4) - 2(2a-4) + 2(4-2a) > 0$

$(a+4)(a-2)^2 > 0$

$a \in (2, \infty)$ // (or) $a \in (-4, \infty)$

$F = x^TAx$

$F = 2x_1^2 \ldots + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2(x_3x_3)$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

7. $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$ $\quad B = A^T A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

$|B - \lambda I| = 0$

$(81 - \lambda)(9 - \lambda) - (27)^2 = 0$

$\lambda^2 = 90\lambda$

$\lambda = 0, 90$ [Eigen values]

(1) $\lambda = 0$ $\quad A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

$81x = 27y$ $\quad \frac{x}{1} = \frac{y}{3}$

Eigen vector $\rightarrow (1, 3)$

(2) $\lambda = 90$, $\quad A = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix}$

$9x = -27y$ $\quad \frac{x}{-3} = \frac{y}{1}$

Eigen vector $\rightarrow (-3, 1)$

$V = \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$ $\quad AA^T = \begin{bmatrix} -10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix}$

$|(AA^T) - \lambda I| = 0$

$(10-\lambda)(\lambda^2-80\lambda)+800\lambda=0$

$\lambda^2(\lambda-90)=0$

$\lambda=90,0$

$\sigma=3\sqrt{10},\ \sigma=0$

$$\Sigma=\begin{bmatrix}0&0\\0&0\\0&3\sqrt{10}\end{bmatrix}$$

$AV=U\Sigma$

$$\begin{bmatrix}0&\sqrt{10}\\0&-2\sqrt{10}\\0&-2\sqrt{10}\end{bmatrix}=U\begin{bmatrix}0&0\\0&0\\0&3\sqrt{10}\end{bmatrix}$$

$$U=\begin{bmatrix}0&0&1/3\\0&0&-2/3\\0&0&-2/3\end{bmatrix}$$

$A=U\Sigma V^T$

$$\begin{bmatrix}0&0&1/3\\0&0&-2/3\\0&0&-2/3\end{bmatrix}\begin{bmatrix}0&0\\0&0\\0&3\sqrt{10}\end{bmatrix}\begin{bmatrix}1/\sqrt{10}&3/\sqrt{10}\\-3/\sqrt{10}&1/\sqrt{10}\end{bmatrix}$$