Course Code: BCA57203(T)

Course Name: Artificial Intelligence

Class: BCA 2023

Academic Session: 2025-26



Study Material (Fuzzy Logic)

Table of Contents

Module No.	Module Name	Content
Module 2	Fuzzy Logic	Introduction to Fuzzy Logic
		Fuzzy Sets and Membership Functions
		Fuzzy Logic Operations
		Fuzzy Arithmetic

1. Introduction to Fuzzy Logic

1.1 What is Fuzzy Logic?

Fuzzy Logic is a form of many-valued logic in which the truth value of variables may be any real number between 0 and 1, inclusive. It is a departure from the classic, crisp logic (Boolean logic) that has dominated for centuries.

• Crisp Logic (Traditional Logic):

- Based on bivalence: a statement is either completely true or completely false.
- Uses values of 1 (True) and 0 (False).
- Example: A person is either "tall" (1) or "not tall" (0). There is a sharp, defined boundary. If the boundary is 6 feet, a person who is 5'11.9" is "not tall."

• Fuzzy Logic:

- Deals with degrees of truth. A statement can be partially true and partially false.
- Allows for modeling the kind of imprecise or vague reasoning that humans use.
- Example: A person can be "somewhat tall," "very tall," or "not very tall." A person who is 5'11.9" might be considered "tall" to a degree of 0.95.

1.2 A Brief History

- **Aristotle:** Laid the foundation for classical two-valued logic.
- **George Boole (19th Century):** Formalized binary logic, which is the basis for modern computers.
- Lotfi A. Zadeh (1965): Professor at UC Berkeley, introduced the concept of "fuzzy sets" and "fuzzy logic" in his seminal paper. He is considered the "father of fuzzy logic." He proposed it as a way to model the uncertainty of natural language.

Course Code: BCA57203(T)

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1.3 Why Do We Need Fuzzy Logic?

The real world is often ambiguous and imprecise. Human decision-making is not typically based on crisp, binary choices.

- **Handling Imprecision:** Many concepts we use daily are fuzzy: "hot" weather, "fast" car, "young" person. Fuzzy logic provides a mathematical framework to handle this vagueness.
- **Control Systems:** It is highly effective in control systems where the model is complex or unknown. Examples include:
 - Anti-lock Braking Systems (ABS)
 - Washing machines (adjusting water level and wash cycle based on "dirtiness" of clothes)
 - Air conditioners (maintaining a "comfortable" temperature)
- **Artificial Intelligence:** Used in expert systems, pattern recognition, and machine learning to create more "human-like" reasoning.

2. Fuzzy Sets and Membership Functions

2.1 Fuzzy Sets

A fuzzy set is a set containing elements that have varying degrees of membership. In classical set theory, an element either belongs to a set or it does not. In fuzzy set theory, an element belongs to a set to a certain degree.

- **Crisp Set Example:** Let U be the set of people. Let A be the crisp set of "tall" people, defined as anyone with a height of 6 feet or more.
 - A person who is 6'1" is in set A.
 - A person who is 5'11" is not in set A.
 - **Fuzzy Set Example:** Let B be the fuzzy set of "tall" people.
 - A person who is 6'1" might have a membership of 1.0 in set B.
 - A person who is 5'11" might have a membership of 0.9 in set B.
 - A person who is 5'5" might have a membership of 0.2 in set B.

2.2 Membership Functions (μ)

A membership function maps each element of a universe of discourse (the set of all possible values) to a membership value (or degree of membership) between 0 and 1.

Let X be the universe of discourse. A fuzzy set A in X is defined by a membership function μA (x) for all $x \in X$.

- $\mu A(x)=1$ if x is completely in A.
- $\mu A(x)=0$ if x is not in A.
- $0 \le \mu A(x) \le 1$ if x is partially in A.

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Course Code: BCA57203(T)

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Academic Session: 2025-26



2.3 Types of Membership Functions

There are several common shapes for membership functions. The choice depends on the application and the nature of the concept being modeled.

- Triangular Membership Function: Defined by three points (a, b, c).
 - Formula:

$$\mu(x) = egin{cases} 0 & x \leq a \ rac{x-a}{b-a} & a < x \leq b \ rac{c-z}{c-b} & b < x < c \ 0 & x \geq c \end{cases}$$

- Example: Fuzzy set for "average temperature."
- Trapezoidal Membership Function: Defined by four points (a, b, c, d).
 - · Formula:

$$\mu(x) = egin{cases} 0 & x \leq a \ rac{x-a}{b-a} & a < x \leq b \ 1 & b < x < c \ rac{d-x}{d-c} & c \leq x < d \ 0 & x \geq d \end{cases}$$

- Gaussian Membership Function: A smooth, bell-shaped curve defined by a center (c) and a width (σ).
 - · Formula:

$$\mu(x)=e^{-\frac{(x-c)^2}{2\sigma^2}}$$

· Example: Often used in neural networks and adaptive systems.

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Course Code: BCA57203(T)

Course Name: Artificial Intelligence

Class: BCA 2023

Academic Session: 2025-26



3. Fuzzy Logic Operations

Fuzzy logic uses operations analogous to Boolean logic's NOT, AND, and OR. These are defined using the membership functions of the fuzzy sets.

Let A and B be two fuzzy sets on the universe X.

3.1 Complement (NOT)

The complement of a fuzzy set A, denoted as A', is defined as:

$$\mu A'(x)=1-\mu A(x)$$

• **Example:** If the membership value for "hot" is 0.8, the membership value for "not hot" is 1-0.8=0.2.

3.2 Intersection (AND)

The intersection of two fuzzy sets A and B, denoted as $A \cap B$, combines the sets by taking the minimum of their membership values. This is the most common definition (Zadeh's definition).

$$\mu A \cap B(x) = \min(\mu A(x), \mu B(x))$$

- **Example:** Consider a decision for a smart fan: "If the temperature is **high** AND the humidity is **high**, then set the fan to **high**."
 - Let μ high_temp(x)=0.9
 - \circ Let μ high humidity(y)=0.7
 - The degree to which both conditions are true is min(0.9,0.7)=0.7.

3.3 Union (OR)

The union of two fuzzy sets A and B, denoted as AUB, combines the sets by taking the maximum of their membership values.

$$\mu A \cup B(x) = max(\mu A(x), \mu B(x))$$

- **Example:** Consider a rule for an irrigation system: "If the soil is **dry** OR the weather forecast is **sunny**, then water the plants."
 - \circ Let μ dry soil(x)=0.4
 - \circ Let μ sunny forecast(y)=0.8
 - The degree to which at least one condition is true is max(0.4,0.8)=0.8.

Course Code: BCA57203(T)

Course Name: Artificial Intelligence

Class: BCA 2023

Academic Session: 2025-26



4. Fuzzy Arithmetic

Fuzzy arithmetic is an extension of standard arithmetic to fuzzy numbers. A fuzzy number is a fuzzy set that is both convex and normalized. They are often represented by triangular or trapezoidal membership functions.

Let's use triangular fuzzy numbers (TFNs) for our examples. A TFN is represented as A = (a1, a2, a3), where a2 is the peak and a1 and a3 are the endpoints.

4.1 Addition of Fuzzy Numbers

If A = (a1, a2, a3) and B = (b1, b2, b3) are two TFNs, their sum is:

$$A+B=(a1+b1,a2+b2,a3+b3)$$

- **Example:** Let A be the fuzzy number "about 5," represented as (4, 5, 6). Let B be "about 10," represented as (9, 10, 11).
 - \circ A+B=(4+9,5+10,6+11)=(13,15,17)
 - The result is a new fuzzy number, "about 15."

4.2 Subtraction of Fuzzy Numbers

If A = (a1, a2, a3) and B = (b1, b2, b3) are two TFNs, their difference is:

$$A-B=(a1-b3,a2-b2,a3-b1)$$

Note the crossover of terms.

- **Example:** Using A = (4, 5, 6) and B = (9, 10, 11).
 - \circ A-B=(4-11,5-10,6-9)=(-7,-5,-3)
 - The result is "about -5."

4.3 Multiplication of Fuzzy Numbers

If A and B are two positive TFNs, their product is approximately:

$$A\times B\approx (a1\times b1, a2\times b2, a3\times b3)$$

This is an approximation but is commonly used for simplicity.

- **Example:** Let A = "about 2" (1, 2, 3) and B = "about 4" (3, 4, 5).
 - \circ A×B≈(1×3,2×4,3×5)=(3,8,15)
 - The result is "about 8," but with a wider spread, reflecting increased uncertainty.

Course Code: BCA57203(T)

Course Name: Artificial Intelligence

Class: BCA 2023

Academic Session: 2025-26



4.4 Division of Fuzzy Numbers

If A and B are two positive TFNs, their quotient is approximately:

 $A \div B \approx (a1 \div b3, a2 \div b2, a3 \div b1)$

- **Example:** Let A = "about 10" (8, 10, 12) and B = "about 2" (1, 2, 3).
 - \circ A÷B \approx (8÷3,10÷2,12÷1)=(2.67,5,12)
 - The result is "about 5," again with a wide spread.

Fuzzy Logic

What is Fuzzy Logic?

- Traditional Logic:
 A statement is either true
 (1) or false (0)
- Fuzzy Logic: can be partially true or partially false – e g. 0.2, 0.5, 0.75

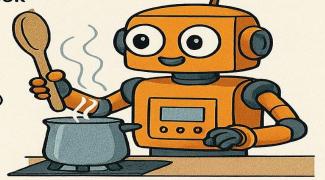
Story-Based Example: RoboChef – The Smart Cook

RoboChef initially followed rigid rules to prepare tea.

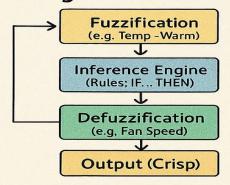
- *Now evaluating RobCh: when evaluating sugar:
- IF temperature is 0.3 Add little sugar 0.6)
- IF temperature is 0.9 Hot "heater is 0

Real-Life Example: Room Temperature Control

Temperature	Furzy System Say
15°C	Cold Cold
22°C	Normal (0.4,01)
30°C	Warm (0.6, Hot (0.33)



Diagrammatic Note



Fuzzy Rule Example

- IF temperature is "cold" THEN heater is "high"
- IF temperature is "warm" THEN heater is "medium"
- IF temperature is "hot" THEN heater is "off"