Homework 0 CS-577 (Spring 2020)

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QA $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

1). $2A - B \Rightarrow 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

2). $\|A\|$ and the angle of A relative to the positive X axis.

$$\|A\| = \sqrt{1^2+2^2+3^2} = \sqrt{14} = 3.74$$

Angle with (+)X axis.

$$\alpha = \cos^{-1}\left(\frac{A_x}{\|A\|}\right) = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right)$$

$$= 1.3005 \text{ Radians.}$$

3). A a unit vector in direction of A.

$$\text{unit vector of } A = \frac{\vec{A}}{\|A\|} = \left[\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right]$$

$$= [0.267, 0.534, 0.802]$$

4). Direction cosines of A.

$$\cos\alpha = \frac{1}{\sqrt{14}} = 1/3.74 = 0.267.$$

$\cos\beta = \frac{2}{\sqrt{14}} = 0.534$ $\cos\gamma = \frac{3}{\sqrt{14}} = 0.802$

5) A.B and B.A

$$A \cdot B = \begin{bmatrix}1\\2\\3\end{bmatrix} \cdot \begin{bmatrix}4\\5\\6\end{bmatrix} = 1\times4 + 2\times5 + 3\times6$$
$$= 4+10+18 = \underline{\underline{32}} \text{ Ans}$$

$$B \cdot A = \begin{bmatrix}4\\5\\6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\3\end{bmatrix} = 4\times1 + 10+18 = \cancel{32}\ \underline{\underline{32}}\ //$$

6) angle b/w A and B.

$$\cancel{\tan}\ \cos\theta = \frac{A \cdot B}{|A|\,|B|} \qquad |A| = 3.74$$
$$|B| = \sqrt{16+25+36}$$
$$A \cdot B = 32 \qquad = \sqrt{77}$$
$$\cos\theta = \frac{32}{3.74 \times 8.77} \qquad = 8.77 //$$
$$\theta = \cos^{-1}(0.975) \qquad \theta = 0.2213 \text{ Ans}$$

7) perpendicular to A.
for perpendicular vector then dot product should be zero $u \cdot A = 0$.
Let $u = [x, y, z]$ such that $[x, y, z] \cdot [1,2,3]$
$$= x + 2y + 3z = 0$$
Let $x = 5,\ y = 2$
$$5 + 2.2 + 3z = 0 \qquad 3z = -9 \quad \boxed{z = -3}$$

$$\vec{u} = [5, 2, -3] \rightarrow p.$$

8) A×B and B×A.

$$A \times B = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = i(12-15) - j(6-12) + k(5-8)$$
$$= -3i + 6j - 3k$$
$$= [-3, 6, -3] //$$

$$B \times A = \begin{vmatrix} i & j & k \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = i(15-12) - j(12-6) + k(8-5)$$
$$= i(3) - j(6) + k(3)$$
$$= 3i - 6j + 3k \quad = [3 -6\ 3] //$$

(9) a vector which is perpendicular to both A and B.

A cross product $A \times B$ is perpendicular to both A and B.

$A \times B = [-3, 6, -3]$.

(10) Linear dependency b/w A, B & C.

$$x\begin{bmatrix}1\\2\\3\end{bmatrix} + y\begin{bmatrix}4\\5\\6\end{bmatrix} + z\begin{bmatrix}-1\\1\\3\end{bmatrix} = 0$$

$$= \begin{bmatrix}1 & 4 & -1\\2 & 5 & 1\\3 & 6 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$$

OR $|D| = 1(15-6) - 4(6-3) - 1(12-15)$

$= 9 - 4.3 - 1(-3)$

$= -3 + 3 = 0$

Determinant is zero, there exists linear depency among the vectors.

(11) $A^T B$ and $A B^T$.

$$A^T . B = [1\ 2\ 3]\begin{bmatrix}4\\5\\6\end{bmatrix}$$

$= 4 + 10 + 18$

$= 32.$

$$AB^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

$$A.B^T = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

Part ⇒ B

B: ⇒ $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$

B.1)

$$2A - B = 2 \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 3 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & +1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 7 \\ -3 & 12 & -3 \end{bmatrix}$$

B.2) AB and BA

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5+2 & 0-20-1 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$B.A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6-1 \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-8+0 & 6+4+5 & 9-6-1 \end{bmatrix}$$

$$B.A = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

B.3) $(AB)^T$ and $B^T.A^T$

Propority of transpose $(AB)^T = B^T.A^T$

$$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix} \Rightarrow (AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$\| \quad (B^T.A^T)$$

B.4) ⇒ $|A|$ and $|C|$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix} = 1(2-15) - 4(-2-15)$$

$$= -13 - 4(-17) = -13 + 68$$

$$|A| = 55$$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix} = 1.(15-6) - 4(6-3) + (-1)(12-15)$$

$$= 9 - 12 + 3$$

$$|C| = 0$$

B.5) The matrix (A, B, or C) in which the rows vectors form an orthogonal set.

Orthogonal sets ⇒ A set of vectors $\{u_1, u_2, \dots u_p\}$ in R^n is an orthogonal set if each pair of distinct vectors from the set is orthogonal that is $u_i . u_j = 0$ for all $i \neq j$

for $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$

$r_1 = \langle 1, 2, 3 \rangle \quad r_2 \langle 4, -2, 3 \rangle \quad r_3 \langle 0, 5, -1 \rangle$

$r_1 . r_2$, $r_2 r_3$ & $r_3 r_1$

$r_1 . r_2 = 4 + (-4) + 9 = 9 \neq 0$

for $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{matrix} \to r_1 \\ \to r_2 \\ \to r_3 \end{matrix}$ $\quad r_1 . r_2, r_2 r_3, r_3 r_1$

$r_1 r_2 = 2 + 2 - 4 = 0$ so B is orthogonal set.

$r_2 r_3 = 6 - 2 - 4 = 0$

$r_3 r_1 = 3 - 4 + 1 = 0$

for $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix} \begin{matrix} \to r_1 \\ \to r_2 \\ \to r_3 \end{matrix}$

$r_1 r_2 = 4 + 10 + 18 = 32 \neq 0$

Only B is orthogonal matrix.

B.6) A^{-1} and B^{-1}

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$

cofactor(1) $= \begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} = -13$

cofac(2) $= (-1) \begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} = +4$

cofac(3) $= \begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix} = 20$

cofac(4) $= (-1) \begin{vmatrix} 2 & 3 \\ 5 & -1 \end{vmatrix} = +17$

cofac(3) $= (-1) \begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} = -5$

cofac(-2) $= \begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} = -1$

cofac(0) $= \begin{vmatrix} 2 & 3 \\ -2 & 3 \end{vmatrix} = 12$

$\text{Cofac}(5) = (-1)\begin{vmatrix} 1 & 3 \\ 4 & 3 \end{vmatrix} = 9$

$\text{Cofac}(-1) = \begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} = -10$

$|A| = 55.$ $\quad$ Cofactor(C) matrix $= \begin{bmatrix} -13 & 4 & 20 \\ 17 & -1 & -5 \\ 12 & 9 & -10 \end{bmatrix}$

$\text{Adj}(A) = C^T$

$\text{Adj}(A) = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$

$$\left\{ A^{-1} = \frac{\text{Adj}(A)}{|A|} = \frac{1}{55}\begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix} \right\}$$

(6.2) $B^{-1} = ?$ $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \Rightarrow$ Cofactor matrix $= \begin{bmatrix} -7 & -14 & -7 \\ -4 & -2 & 8 \\ -9 & 6 & -3 \end{bmatrix}$

$|B| = -7 + 2(-14) - 7$
$= -42$

$\text{Adj}(B) = (\text{Cofac})^T = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$

$B^{-1} = \frac{-1}{42}\begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$

$$\left\{ B^{-1} = \frac{1}{42}\begin{bmatrix} 7 & 4 & 9 \\ 14 & 2 & -6 \\ 7 & -8 & 3 \end{bmatrix} \right\}$$

Part C)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

C.1) Eigenvalues of A.

1. An eigenvalue of

Eigenvalues.

$= (\lambda$

$= \lambda$

$=$

$= -$

2. Eigenvectors of
nonzero solutions
for $\lambda = -1$,

$\left(\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right.$

$\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$=$

Let $x_1 = t$

$\left\{ \vec{x} = t\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right.$

Part C)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$.

C.1) Eigenvalues and corresponding eigenvectors of A.

1. An eigenvalue of A is a scaler λ $\det(\lambda I - A) = 0$

Eigenvalues. $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{vmatrix} \lambda-1 & -2 \\ -3 & \lambda-2 \end{vmatrix}$

$$= (\lambda-1)(\lambda-2) - 6 = 0$$
$$= \lambda^2 - 3\lambda + 2 - 6 =$$
$$= \lambda^2 - 3\lambda - 4 = 0$$
$$= \cancel{\lambda^2 (} \lambda^2 - 4\lambda + \lambda - 4$$
$$(\lambda - 4)(\lambda + 1) = 0 \quad \boxed{\begin{matrix} \lambda = 4 \\ \lambda = -1 \end{matrix}}$$

2. Eigenvectors of A corresponding to λ are the nonzero solutions of $(\lambda I - A)\vec{x} = \vec{0}$

for $\lambda = -1$

$$\left(\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -2 & 0 \\ -3 & -3 & 0 \end{bmatrix} \begin{matrix} -\frac{1}{2} R_1 \\ -\frac{2}{3} R_2 + R_1 \end{matrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= x_1 + x_2 = 0 \quad \boxed{x_1 = -x_2}$$

Let $x_1 = t$

$$\left\{ \vec{x} = t \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

for $\lambda_2 = 4$

$$\left(\begin{bmatrix}4&0\\0&4\end{bmatrix} - \begin{bmatrix}1&2\\3&2\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}3&-2\\-3&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}3&-2&0\\-3&2&0\end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix}3&-2&0\\0&0&0\end{bmatrix}$$

$$3x_1 - 2x_2 = 0$$

Let $x_1 = t$ $\qquad \frac{3}{2}x_1 = x_2$

$$\vec{x} = t\begin{bmatrix}1\\3/2\end{bmatrix}$$

2. Matrix $V^{-1}AV$, V is composed of the eigenvectors of A.

$$V = \begin{bmatrix}1&1\\-1&3/2\end{bmatrix} \qquad V^{-1} = \frac{1}{3/2+1}\begin{bmatrix}3/2&-1\\1&1\end{bmatrix}$$

$$V^{-1} = \frac{2}{5}\begin{bmatrix}3/2&-1\\1&1\end{bmatrix} = \begin{bmatrix}3/5&-2/5\\2/5&2/5\end{bmatrix}$$

$$\begin{bmatrix}3/5&-2/5\\2/5&2/5\end{bmatrix}\begin{bmatrix}1&2\\3&2\end{bmatrix}\begin{bmatrix}1&1\\-1&3/2\end{bmatrix} =$$

$$= \frac{1}{5}\begin{bmatrix}3&-2\\2&2\end{bmatrix}\begin{bmatrix}-1&4\\1&6\end{bmatrix} = \frac{1}{5}\begin{bmatrix}-5&0\\0&20\end{bmatrix} = \begin{bmatrix}-1&0\\0&4\end{bmatrix}$$

C. Dot product b/w the eigenvectors of A.

$$\langle 1, -1\rangle \cdot \langle 1, 3/2\rangle = 1\times1 + (-1)(3/2)$$
$$= 1 - 3/2 = -1/2$$

4) Dot ~~product~~ product

eigenvalue of B

$$\begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix} - \ldots$$

$(\lambda-2)(\ldots$

$\lambda^2 - \ldots$

$= \ldots$

① for $\lambda = 1$

$$\left(\begin{bmatrix}1&0\\0&1\end{bmatrix} - \ldots\right.$$

$$\begin{bmatrix}-1&2&\ldots\\2&-\ldots\end{bmatrix}$$

$$\begin{bmatrix}-1&2&\ldots\\2&-4&\ldots\end{bmatrix}$$

$-x_1 \ldots$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}\ldots$$

② for $\lambda = 6$

$$\left(\begin{bmatrix}6&0\\0&6\end{bmatrix} - \begin{bmatrix}2&\ldots\\-\ldots\end{bmatrix}\right.$$

$$\begin{bmatrix}4&2\\2&1\end{bmatrix}\ldots$$

$$\begin{bmatrix}4&2&0\\2&1&0\end{bmatrix}\ldots$$

4) Dot ~~product~~ product of eigenvectors of B.

eigenvalue of B

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} = \begin{vmatrix} \lambda-2 & 2 \\ 2 & \lambda-5 \end{vmatrix} = 0$$

$$(\lambda-2)(\lambda-5) - 4 = 0$$

$$\lambda^2 - 7\lambda + 10 - 4 = 0 \Rightarrow \lambda^2 - 7\lambda + 6 = 0$$

$$= \lambda^2 - 6\lambda - \lambda + 6 = 0$$

$$(\lambda-6)(\lambda-1) = 0$$

$$\boxed{\lambda_1 = 1 \text{ or } \lambda_2 = 6}$$

① for $\lambda = 1$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 0 \\ 2 & -4 & 0 \end{bmatrix} \; \tfrac{1}{2}R_2 + R_1 \Rightarrow \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$-x_1 + 2x_2 = 0 \qquad \boxed{2x_2 = x_1} \qquad x_2 = 1/2\, x_1$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ \tfrac{1}{2}x_1 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$$

Eigen vector

② for $\lambda = 6$

$$\left(\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix} \; \begin{matrix} 1/2\,R_1 \\ 2R_2 - R_1 \end{matrix} \Rightarrow \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad 2x_1 + x_2 = 0$$

$x_2 = -2x_1$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ -2x_1 \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ Eigen vector

Dot product. $\Rightarrow \langle 1, 1/2 \rangle \cdot \langle 1, -2 \rangle$

$= 1 \times 1 + \frac{1}{2} \times (-2) = \cancel{1+1} = \cancel{2}$

$= 1 - 1 = 0$

C.5) $\Rightarrow$ we found the ~~dop ro~~ dot product of eigenvector of B ($ev_1 \cdot ev_2 = 0$) is zero, that ~~meens~~ mean's they are orthogonal to each other.

The eigenvectors of a symmetric metrix ~~AB~~ B corresponding to different eigenvalues are orthogonal to each other.

$B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \rightarrow$ Symmetric matrix.

part D Let $f(x) = x^2 + 3$, $g(x,y) = x^2 + y^2$

1) $f'(x)$ & $f''(x)$

$f'(x) = \frac{\partial f(x)}{\partial x} = \frac{\partial (x^2+3)}{\partial x} = 2x$

$f''(x) = \frac{\partial f(x)}{\partial x} = \frac{\partial (2x)}{\partial x} = 2$

2) partial derivatives : $\frac{\partial g}{\partial x}$ and ~~dg~~ $\frac{\partial g}{\partial y}$.

$\frac{\partial g(x,y)}{\partial x} = \frac{\partial (x^2+y^2)}{\partial x} = 2x$

$\frac{\partial g(x,y)}{\partial y} = \frac{\partial (x^2+y^2)}{\partial y} = 2y$

3) Gradient vector $\nabla g(x, y)$.

$$\nabla g(x,y) = \langle g_x(x,y), g_y(x,y) \rangle$$

$$\nabla g(x,y) = \langle 2x, 2y \rangle$$

4) Univariate Gaussian Distributions.

The Gaussion (Normal) distribution is a continuous probability distribution with the follow probability function (pdf):

$$p(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2} \right\}$$

we then say that a random variable X is normally (Gaussion) distribution and write $X \sim N(\mu, \sigma^2)$

$X \sim N(\mu, \sigma^2)$ μ - mean

σ^2 - variance.

The parameters of the pdf represent the first two moments of the distribution

$$\mu = E_x[X] = \int_{-\infty}^{\infty} x\, p(x; \mu, \sigma^2)\, dx$$

$$\sigma^2 = E_x[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 p(x; \mu, \sigma^2)\, dx.$$