

Q1 Suppose we transform the original predictors  $X$  to  $\hat{Y}$  via linear regression. In detail let  $\hat{Y} = X(X^T X)^{-1} X^T Y = X \hat{\beta}$ ,

where  $Y$  is the indicator response matrix. Similarly for any point input  $x \in \mathbb{R}^p$ , we get a transformed vector  $\hat{y} = \hat{\beta}^T x \in \mathbb{R}^k$ .

Show that LDA using  $\hat{Y}$  is identical to LDA in the original space.

Proof:

We know that LDA equation.

$$\log \frac{\Pr(G=k | X=x)}{\Pr(G=l | X=x)} = \log \frac{\pi_k}{\pi_l} - \frac{1}{2} (\mu_k - \mu_l)^T \Sigma^{-1} (\mu_k - \mu_l) + x^T \Sigma^{-1} (\mu_k - \mu_l).$$

In this problem we are replacing  $X$  to  $\hat{Y}$

So our LDA equation will be look like.

$$\log \Pr \frac{(G=k | \hat{Y}=\hat{y})}{(G=l | \hat{Y}=\hat{y})} = \log \frac{\pi_k}{\pi_l} - \frac{1}{2} (\hat{\mu}_k^T + \hat{\mu}_l^T) (\hat{\Sigma}_{\hat{Y}})^{-1} + (\hat{y})^T (\hat{\Sigma}_{\hat{Y}})^{-1} (\hat{\mu}_k^T + \hat{\mu}_l^T).$$

$$\hat{Y} = \hat{B}^T \hat{x} \Rightarrow \hat{Y}^T = \hat{x}^T \hat{B} - \textcircled{1}$$

$$\hat{\mu}_k^y = \frac{\sum_{g_i=k} \hat{y}_i}{N_k} = \frac{\sum_{g_i=k} \hat{B}^T \hat{x}_i}{N_k} = \hat{B}^T \hat{\mu}_k^n$$

$$\hat{\mu}_k^y = \hat{B}^T \hat{\mu}_k^n - \textcircled{2}$$

Similarly

$$\hat{\mu}_e^y = \hat{B}^T \hat{\mu}_e^n - \textcircled{3}$$

$$\Sigma \hat{y} = \frac{\sum_{k=1}^N \sum_{g_i=k} (\hat{y}_i - \hat{\mu}_k^y)(\hat{y}_i - \hat{\mu}_k^y)^T}{N-k}$$

$$= \frac{\sum_{k=1}^N \sum_{g_i=k} \hat{B}^T (\hat{x}_i - \hat{\mu}_k^n)(\hat{x}_i - \hat{\mu}_k^n)^T \hat{B}}{N-k}$$

$$\hat{\Sigma} \hat{y} = \hat{B}^T \hat{\Sigma}_n \hat{B}$$

$$\hat{\Sigma}^{-1} = \left( \hat{B}^T \hat{\Sigma}_n \hat{B} \right)^{-1} - \textcircled{4}$$

from equation  $\textcircled{1}$   $\textcircled{2}$   $\textcircled{3}$  &  $\textcircled{4}$  we get

$$\begin{aligned} \log \frac{\Pr(G=k | \hat{Y}=\hat{y})}{\Pr(G=e | \hat{Y}=\hat{y})} &= \log \frac{\pi_k}{\pi_e} - \frac{1}{2} (\hat{\mu}_k^n + \hat{\mu}_e^n)^T \hat{B} (\hat{B}^T \hat{\Sigma}_n^{-1} \hat{B}) \\ &\quad \hat{B}^T (\hat{\mu}_k^n - \hat{\mu}_e^n) \\ &\quad + \hat{x}^T \hat{B} (\hat{B}^T \hat{\Sigma}_n^{-1})^{-1} \hat{B}^T (\hat{\mu}_k^n - \hat{\mu}_e^n) \end{aligned}$$

$$= \log \frac{\pi_k}{\pi_l} - \frac{1}{2} (\hat{u}_k^n + \hat{u}_l^n)^T \hat{B} (\hat{B}^T \hat{\Sigma}_n \hat{B})^{-1} \hat{B}^T (\hat{u}_k^n - \hat{u}_l^n) \\ + \pi_l^T \hat{B} (\hat{B}^T \hat{\Sigma}_n \hat{B})^{-1} \hat{B}^T (\hat{u}_k^n - \hat{u}_l^n)$$

Now we have to show that.

$$\hat{B} (\hat{B}^T \hat{\Sigma}_n \hat{B})^{-1} \hat{B}^T (\hat{u}_k^n - \hat{u}_l^n) = \\ \hat{\Sigma}_n (\hat{u}_k^n - \hat{u}_l^n)$$

So in this way we can say LDA using  $\hat{y}$  is identical to LDA in the original space.

$\gamma$  is a indicator response matrix, therefore.

$$N_k \hat{u}_k^n = \sum n_i = X^T Y_k$$

$$\hat{\Sigma}_n = \frac{1}{N-k} \left[ \sum_{i=1}^N x_i n_i^T - \sum N_k \hat{u}_k^n (\hat{u}_k^n)^T \right]$$

$$= \frac{1}{N-k} \left[ X^T X - X^T \sum_{K=1}^K (Y_K Y_K^T) X \right]$$

$$= \frac{1}{N-k} (X^T X - X^T Y Y^T X)$$

we know  $H = \hat{\Sigma}_x \hat{B} (\hat{B}^T \hat{\Sigma}_n \hat{B})^{-1} \hat{B}^T$   $\therefore HH = H$

$$H X^T Y = \hat{\Sigma}_x \hat{B} (\hat{B}^T \hat{\Sigma}_n \hat{B})^{-1} \hat{B}^T X^T Y$$

by definition of  $\hat{\beta}$

$$\hat{\beta}^T = ((x^T x)^{-1} x^T y)^T = y^T x (x^T x)^{-1}$$

$$\begin{aligned}\hat{\Sigma}_x \hat{\beta} &= \frac{1}{N-k} (x^T x - x^T y y^T x) (x^T x)^{-1} x^T y \\ &= \frac{1}{N-k} x^T y (I - y^T x (x^T x)^{-1} x^T y)\end{aligned}$$

$$\begin{aligned}(\hat{\beta}^T \hat{\Sigma}_x \hat{\beta})^{-1} &= (y^T x (x^T x)^{-1} \frac{1}{N-k} x^T y (I - y^T x (x^T x)^{-1} x^T y))^{-1} \\ &= (N-k) (y^T x (x^T x)^{-1} x^T y (I - \underbrace{y^T x (x^T x)^{-1} x^T y}_{\phi}))^{-1}\end{aligned}$$

$$\therefore \phi = \hat{\beta}^T x^T y = y^T x (x^T x)^{-1} x^T y$$

$$\hat{\Sigma}_x \hat{\beta} = \frac{1}{N-k} x^T y (I - \phi)$$

$$(\hat{\beta}^T \hat{\Sigma}_x \hat{\beta})^{-1} = (N-k) (\phi (I - \phi))^{-1}$$

$\phi(I - \phi)$  is invertible that means  $\phi$  and  $I - \phi$  are also invertible.

$$(\hat{\beta}^T \hat{\Sigma}_x \hat{\beta})^{-1} = (N-k) (I - \phi)^{-1} \phi^{-1}$$

$$H X^T Y = \frac{1}{N-k} X^T Y (I - Q) (I - Q)^{-1} Q^{-1} Y^T X$$

$$(X^T X)^{-1} X^T Y$$

$$= X^T Y Q^{-1} \underbrace{Y^T X (X^T X)^{-1} X^T Y}_Q$$

$$\boxed{Q = Y^T X (X^T X)^{-1} X^T Y}$$

$$= X^T Y Q^{-1} Q$$

$$= X^T Y$$

$$H X^T Y = X^T Y$$

$$\Rightarrow H X^T Y_K = X^T Y_K$$

$$H N_K \hat{U}_K^n = N_K \hat{U}_K^n$$

$$\sum_n \hat{B} (\hat{B}^T \sum_n \hat{B})^{-1} \hat{B}^T \hat{U}_K^n = \hat{U}_K^n$$

$$\hat{B} (\hat{B}^T \sum_n \hat{B})^{-1} \hat{B}^T \hat{U}_K^n = \sum_{n=1}^{N-1} \hat{U}_K^n$$

Subtracting  $\hat{U}_K^n$  both side.

$$\boxed{\hat{B} (\hat{B}^T \sum_n \hat{B})^{-1} \hat{B}^T (\hat{U}_K^n - \hat{U}_K^n) = \sum_{n=1}^{N-1} (\hat{U}_K^n - \hat{U}_K^n)}$$

fff

# Homework#4 Problem 2

Arinjay Jain

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```
library(class)
library(formatR)

## Warning: package 'formatR' was built under R version 3.6.3

admit_df <- read.table(file = "C:/Arinjay_Personal/Statistical Learning/Homework#4/admit.txt",
                      header = T)

admit_df <- data.frame(admit_df)

# Using the logit model: The code below estimates a logistic regression model using the glm (generalized linear model) function. First, we convert rank to a factor to indicate that rank should be treated as a categorical variable.

admit_df$rank <- factor(admit_df$rank)

log_model <- glm(admit~., data= admit_df, family = "binomial")
```

## Part A

```
model_summary <- summary(log_model)
coff_table <- model_summary$coefficients[,-4]

print("Results from a logistic regression fit to the admit data.")
```

```
## [1] "Results from a logistic regression fit to the admit data."
coff_table
```

```
##             Estimate Std. Error   z value
## (Intercept) -3.989979073 1.139950936 -3.500132
## gre          0.002264426 0.001093998  2.069864
## gpa          0.804037549 0.331819298  2.423119
## rank2        -0.675442928 0.316489661 -2.134171
## rank3        -1.340203916 0.345306418 -3.881202
## rank4        -1.551463677 0.417831633 -3.713131
```

## Part B Write log-ratio equation:

```
print("log-ratio equation:")  
  
## [1] "log-ratio equation:"  
  
print("log(p/1-p) = -3.9899 + (0.0022*gre) + (0.804*gpa) + (-0.6754*rank2) + (-1.3402*rank3) + (-1.5514*rank4)")  
  
## [1] "log(p/1-p)  
=-3.9899+(0.0022*gre)+(0.804*gpa)+(-0.6754*rank2)+(-1.3402*rank3)+(-1.5514*rank4)"  
  
## now how we will use this equation, explanation with example  
# estimated log-odds of graduate school admission for a student  
#with a GPA of 3.2 and a GRE score of 670 who attended a rank 1  
#school?  
  
newdata <- data.frame(gre = 670, gpa = 3.2, rank = as.factor(1))  
  
# solution:  
predict(log_model, newdata)  
  
## 1  
## 0.1001064
```

Q3 part(A)

a) According to LDA Discriminant function

$$S_k(x) = x^T \Sigma^{-1} \mu_k - \frac{1}{2} \mu_k^T \Sigma^{-1} \mu_k + \log \pi_k$$

According to LDA if  $S_2(x) > S_1(x)$  then LDA rule classifies to class 2 and class 1 otherwise

$$S_2(x) > S_1(x)$$

$$\Rightarrow x^T \Sigma^{-1} \mu_2 - \frac{1}{2} \mu_2^T \Sigma^{-1} \mu_2 + \log \pi_2$$

$$> x^T \Sigma^{-1} \hat{\mu}_1 - \frac{1}{2} \hat{\mu}_1^T \Sigma^{-1} \hat{\mu}_1 + \log \pi_1$$

$$\Rightarrow x^T \Sigma^{-1} \mu_2 - x^T \Sigma^{-1} \hat{\mu}_1 > \frac{1}{2} \hat{\mu}_2^T \Sigma^{-1} \hat{\mu}_2 - \frac{1}{2} \hat{\mu}_1^T \Sigma^{-1} \hat{\mu}_1 \\ + \log \pi_1 - \log \pi_2$$

$$= \therefore \pi_1 = N_1/N \quad \pi_2 = N_2/N$$

$$\pi_1^T \Sigma^{-1} (\hat{\mu}_2 - \hat{\mu}_1) > \frac{1}{2} \hat{\mu}_2^T \Sigma^{-1} \hat{\mu}_2 - \frac{1}{2} \hat{\mu}_1^T \Sigma^{-1} \hat{\mu}_1$$

$$+ \log \left( \frac{N_1}{N} \right) - \log \left( \frac{N_2}{N} \right)$$

(b) Consider min of the least squares criterion

$$\sum_{i=1}^N (y_i - \beta_0 - \beta^T x_i)^2$$

Show that the solution  $\hat{\beta}$  satisfies

$$[(N-2) \hat{\Sigma} + \frac{N_1 N_2}{N} \hat{\Sigma}_B] \beta = N(\hat{u}_2 - \hat{u}_1)$$

Solution :

$$RSS = \sum_{i=1}^N (y_i - \beta_0 + \beta^T x_i)^2$$

$$\beta' = (\beta_0 \ \beta_1)^T$$

Let find  $\beta_0$  first

$$\frac{\partial RSS(\beta')}{\partial \beta_0} = \frac{\partial}{\partial \beta_0} ((y_i - \beta_0 - x_i \beta)^T (y - \beta_0 - x \beta))$$

$$= -2 \sum (y_i - \beta_0 - \beta^T x_i) = 0$$

$$\left\{ \beta_0 = \frac{1}{N} \sum (y_i - \beta^T x_i) \right\} - \textcircled{1}$$

Similarly for  $\beta$

$$\frac{\partial}{\partial \beta} (y_i - \beta_0 - x_i \beta)^T (y - \beta_0 - x \beta)$$

$$= 2 x^T x \beta - 2 x^T y + 2 \beta_0 x^T = 0$$

$$x^T x \beta = x^T y - \beta_0 x^T - \textcircled{2}$$

Substituting value from ① to ②

$$x^T x \beta = x^T y - \frac{1}{N} (y - x\beta) x^T$$

$$x^T x \beta = x^T y - \frac{1}{N} y x^T - \frac{1}{N} x^T x \beta$$

$$(x^T x - \frac{1}{N} x^T x) \beta = x^T y - \frac{1}{N} x^T y. \quad \text{--- } ③$$

we know that  $\bar{x} = \frac{1}{N} \sum x$

so we can write,

$$x^T = \bar{x}^T (N_1 + N_2) \quad \text{--- } ④$$

or

$$x^T = N_1 \bar{u}_1^T + N_2 \bar{u}_2^T$$

And we have  $y$  labels  $-N_1 \bar{u}_1 + N_2 \bar{u}_2$

$$y = -\frac{N_1}{N_1 + N_2} + \frac{N_2}{N_1 + N_2} \quad \text{--- } ⑤$$

putting equation (4) & (5) back to equation ③

$$\begin{aligned} x^T x - \frac{1}{N} (N_1^2 \bar{u}_1 \bar{u}_1^T + N_2^2 \bar{u}_2 \bar{u}_2^T + N_1 N_2 \bar{u}_1 \bar{u}_2^T \\ + N_1 N_2 \bar{u}_2 \bar{u}_1^T) \beta \end{aligned}$$

$$= x^T \left( -\frac{N_1}{N_1 + N_2} + \frac{N_2}{N_1 + N_2} \right) - \frac{1}{N} \left( N_1 \bar{u}_1 + N_2 \bar{u}_2 \right) \left( -\frac{N_1}{N_1 + N_2} + \frac{N_2}{N_1 + N_2} \right)$$

$$\Rightarrow x^T x - \frac{N_1^2}{N} \hat{u}_1 \hat{u}_1^T - \frac{N_2^2}{N} \hat{u}_2 \hat{u}_2^T - \frac{N_1 N_2}{N} \hat{u}_1 \hat{u}_2^T - \frac{N_1 N_2}{N} \hat{u}_2 \hat{u}_1^T$$

$$- \frac{N_1 N_2}{N} \hat{u}_1 \hat{u}_2^T \Big) \beta$$

$$= -\frac{N}{N_1} N_1 \hat{u}_1 + \frac{N}{N_2} N_2 \hat{u}_2 = 0$$

$$\begin{aligned} \Rightarrow x^T x - N_1 \hat{u}_1 \hat{u}_1^T - N_2 \hat{u}_2 \hat{u}_2^T + & \left[ \frac{N_1 N_2}{N} \hat{u}_1 \hat{u}_1^T \right. \\ & \left. + \frac{N_1 N_2}{N} \hat{u}_2 \hat{u}_2^T - \frac{N_1 N_2}{N} \hat{u}_1 \hat{u}_2^T - \frac{N_1 N_2}{N} \hat{u}_2 \hat{u}_1^T \right] \beta \\ = & N_1 (\hat{u}_1 - \hat{u}_2) \end{aligned}$$

$$\Rightarrow ((N-2) \hat{\Sigma} + \frac{N_1 N_2}{N} \hat{\Sigma}_B) \hat{\beta} = N (\hat{u}_1 - \hat{u}_2)$$

$$(N-2) \hat{\Sigma} = \sum (x_i - \hat{u}_1) (x_i - \hat{u}_1)^T$$

$$+ \sum (x_i - \hat{u}_2) (x_i - \hat{u}_2)^T$$

$$= x^T x - N_1 \hat{u}_1 \hat{u}_1^T - N_2 \hat{u}_2 \hat{u}_2^T$$

$$\hat{\Sigma}_B = (\hat{u}_1 - \hat{u}_2) (\hat{u}_1 - \hat{u}_2)^T$$

(c)

$$\Sigma_B \hat{\beta} = (\vec{u}_2 - \vec{u}_1) (\vec{u}_2 - \vec{u}_1)^T \beta$$

$(\vec{u}_2 - \vec{u}_1)^T \beta$  is a scalar  $\Sigma_B \beta$ , in the

direction  $(\vec{u}_2 - \vec{u}_1)$  and

$$\hat{\beta} = \frac{1}{N-2} \sum_{i=1}^{N-1} \left[ N(\vec{u}_2^T - \vec{u}_1) - \frac{N_1 N_2}{N} \Sigma_B \beta \right]$$

$$= \frac{1}{N-2} \left[ N - \frac{N_1 N_2}{N} (\vec{u}_2 - \vec{u}_1)^T \beta \right] \sum_{i=1}^{N-1} (\vec{u}_2^T - \vec{u}_1)$$

$$\propto \sum_{i=1}^{N-1} (\vec{u}_2^T - \vec{u}_1)$$

# Homework#4 Problem#3-B

Arinjay Jain

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```
classification.fun=function(train,test,i=4){

  temp      = train[,-1,with=F]
  temp$y[temp$y!=i]=0
  temp$y[temp$y==i]=1

  model = lm(y~.,data=temp)
  #summary(model)
  pred.train = predict(model,temp[,-1,with=F])
  pred.test   = predict(model,test[,c(-1,-2),with=F])

  return(list(model))
}

# Finally predicted final prediction
pred.fun=function(total.model,train){

  pred.train =
    sapply(c(1:length(total.model)) ,
    function(x)
      predict(total.model[[x]],train[,c(-1,-2),with=F])
    ) %>% data.table

  colnames(pred.train) =
    paste("y=",c(1:length(total.model)),sep="")

  #apply( pred.train , 1 , sum )
  return(pred.train)
}

library(MASS)
library(data.table)

## Warning: package 'data.table' was built under R version 3.6.3

library(dplyr)

## Warning: package 'dplyr' was built under R version 3.6.3
```

```

## 
## Attaching package: 'dplyr'

## The following objects are masked from 'package:data.table':
##   between, first, last

## The following object is masked from 'package:MASS':
##   select

## The following objects are masked from 'package:stats':
##   filter, lag

## The following objects are masked from 'package:base':
##   intersect, setdiff, setequal, union

library(readr)

## Warning: package 'readr' was built under R version 3.6.3

train_df <- read_csv("C:/Arinjay_Personal/Statistical Learning/Homework#4/vowel.train.txt",
                     trim_ws = FALSE)

## 
## -- Column specification -----
## cols(
##   row.names = col_double(),
##   y = col_double(),
##   x.1 = col_double(),
##   x.2 = col_double(),
##   x.3 = col_double(),
##   x.4 = col_double(),
##   x.5 = col_double(),
##   x.6 = col_double(),
##   x.7 = col_double(),
##   x.8 = col_double(),
##   x.9 = col_double(),
##   x.10 = col_double()
## )

#train_df <- data.frame(train_df)[-1]

test_df <- read_csv("C:/Arinjay_Personal/Statistical Learning/Homework#4/vowel.test.txt",
                     trim_ws = FALSE)

## 
## -- Column specification -----
## cols(

```

```

##   row.names = col_double(),
##   y = col_double(),
##   x.1 = col_double(),
##   x.2 = col_double(),
##   x.3 = col_double(),
##   x.4 = col_double(),
##   x.5 = col_double(),
##   x.6 = col_double(),
##   x.7 = col_double(),
##   x.8 = col_double(),
##   x.9 = col_double(),
##   x.10 = col_double()
## )

#test_df <- data.frame(test_df)[-1]

# Probability -> classification
class.pred.fun=function(tem){
  class.y = which( ( tem %in% max(tem) ) ==1 )
  return(class.y)
}

#misclassification , balance data, 11 classification
table(train_df$y)

## 
##   1   2   3   4   5   6   7   8   9   10  11
## 48  48  48  48  48  48  48  48  48  48  48

# Do more models, 1 vs non-1
set.seed(100)
temp = sapply(c(1:11), function(x) classification.fun(train_df,test_df,i=x))

total.model = temp

pred.train = pred.fun(total.model,train_df)

# Probability classification change
pred.train.class = apply( pred.train,1,class.pred.fun)
# confusion matrix
t.train.matrix = table(train_df$y,pred.train.class)
t.train.matrix

##      pred.train.class
##      1   2   3   4   5   6   7   8   9   10  11
## 1   39  3  0  0  0  0  0  4  0  2  0
## 2   18  21  9  0  0  0  0  0  0  0  0
## 3   1   6  30  7  0  0  0  0  0  0  4
## 4   1   0  5  40  0  2  0  0  0  0  0
## 5   0   0  0  1  32  1  10  3  0  1  0
## 6   2   0  2  10  14  5  10  3  0  0  2

```

```

##   7   0   0   3   1  12   0  11  15   1   5   0
##   8   0   0   0   0   0   0   0  36   3   9   0
##   9   1   0   0   0   0   0   0  13  12  22   0
##  10   1   0   0   0   0   0   0   2   8  37   0
##  11  10   2   5   5   5   2   1   1   2   2  13

# Correct percent
train_acc <- sum( diag( t.train.matrix ) )/sum(t.train.matrix)
cat("Train accuracy: ", train_acc*100, "% \n")

## Train accuracy: 52.27273 %

# test before
pred.test = pred.fun(total.model, test_df)
# Probability classification change
pred.test.class = apply( pred.test, 1, class.pred.fun)
# confusion matrix
t.test.matrix = table(test_df$y, pred.test.class)

t.test.matrix

##      pred.test.class
##      1   2   3   4   5   6   7   8   9  10  11
##  1   41   0   1   0   0   0   0   0   0   0   0
##  2   25   5   9   0   0   0   0   0   0   0   3
##  3   4   5  21   8   0   4   0   0   0   0   0
##  4   0   0   4  26   6   6   0   0   0   0   0
##  5   0   0   0  15   9  12   3   3   0   0   0
##  6   1   0   6  13   9   8   2   0   0   3   0
##  7   0   5   0   6  18   3   0   1   2   7   0
##  8   0   0   0   0   4   0   0  18   1  19   0
##  9   0   0   2   0   0   0   0   6   3  31   0
## 10  12   0   4   0   0   0   0   0   4  22   0
## 11  11   1   8   9   0   1   0   0   2   9   1

# Correct percent
test_acc <- sum( diag( t.test.matrix ) )/sum(t.test.matrix)

cat("\n Misclassification error for the test data:", (1 - test_acc)*100, "%")

## 
## Misclassification error for the test data: 66.66667 %

```

## Using QDA

```

qda_fit <- qda(y~. -row.names, data = train_df)

pred <- predict(qda_fit, newdata = test_df)

classes <- pred$class

```

```

conf_mat <- table(classes, test_df$y)
conf_mat

## 
##    classes 1 2 3 4 5 6 7 8 9 10 11
##    1       37 18 9 0 0 0 0 0 0 2 0
##    2        4 22 13 2 0 0 0 0 0 4 1
##    3        0 1 12 3 0 0 0 0 0 0 0
##    4        0 0 5 12 0 1 0 0 0 0 2
##    5        0 0 0 5 16 0 11 0 0 0 0
##    6        0 0 2 17 7 22 1 0 0 0 1
##    7        0 0 0 2 19 14 22 15 3 4 2
##    8        0 0 0 0 0 0 0 6 1 0 0
##    9        1 1 1 0 0 0 3 21 38 21 15
##   10       0 0 0 0 0 0 0 0 0 11 1
##   11       0 0 0 1 0 5 5 0 0 0 20

# Correct percent
qda_acc <- sum(diag(conf_mat))/sum(conf_mat)

cat("\n Misclassification error for the test data using qda:", (1 -qda_acc)*100, "%")

##
## Misclassification error for the test data using qda: 52.81385 %

print("we are getting better result using QDA MASS R function with 52.81% Misclassification but is compu
## [1] "we are getting better result using QDA MASS R function with 52.81% Misclassification but is computer program we are getting higer misclassification which is 66.667%"
```