Arinjay Aggarwal 'I' 35 Tutorial-4

Ques1. $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a \geq 1, b > 1$

On comparing

$a = 3, b = 2, f(n) = n^2$

Now $c = \log_b a = \log_2 3 \Rightarrow 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n^2)$

Ques3. $T(n) = T(n/2) + \cancel{n}\, 2^n$

$a = 1, b = 2, f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

Q5. $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 4^2 \Rightarrow 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

Ques2. $T(n) = 4T(n/2) + n^2$

$a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$\therefore$ ~~n~~ $c = \log_2 4 \Rightarrow 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log n)$

Q4. $T(n) = 2^n T(n/2) + n^n$

here Master's Theorem can't be applied as 'a' must be constant.

Q6. $T(n) = 2T(n/2) + n \log n$

$a = 2, b = 2$

$f(n) = n \log n$

$c = \log_2 2 \Rightarrow 1$

$\therefore n^c = n^1 = n$

Since $n \log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n \log n)$

Q7. $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 \rightarrow 1$

$\therefore n^c = n^1 = n$

Since $n/\log n < n \therefore T(n) = \theta(n)$

Q8 → $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = \theta n^{0.5}$

since $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

Q10 → $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a \Rightarrow \log_4 16 = 2$

Now, $n^c = n^2$

as $n! > n^2$

$\therefore T(n) = \theta(n!)$

Q12 → $T(n) = sqrt(n)\, T(n/2) + \log n$

here MT cannot be applied as a must be constant.

Q14 → $T(n) = 3T(n/3) + sqrt(n)$

$a = 3, b = 3, c = \log_3 3 \Rightarrow 1$

$\therefore n^c = n^1 = n$

as $sqrt(n) < n$

$\therefore f(n) < n^c$

$\theta(n)$

Q9 → $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5, b = 2$

$\because$ A/c to Master's Theorem

$a \geq 1$, but here a is 0.5

so M.T cannot be applied.

Q11 → $4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 4 \Rightarrow 2$

$\therefore n^c = n^2$ and $f(n) = \log n$

$\because \log n < n^2$

$T(n) = \theta(n^2)$

Q13 → $T(n) = 3T(n/2) + n$

$a = 3, b = 2, f(n) = n$

$c = \log_b a = \log_2 3 \Rightarrow 1.5849$

$\therefore n < n^{1.5489} \Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.5489})$

Q15 → $T(n) = 4T(n/2) + Cn$

$a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$Cn < n^2$ (for any constant)

$\therefore T(n) = \theta(n^2)$

Q16 $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a \Rightarrow \log_4 3 \Rightarrow 0.792$

$n^c = n^{0.792}$

$\because n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n\log n)$

Q18 $T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

as $n^{1.6309} < n^2\log n$

$\therefore T(n) = \Theta(n^2\log n)$

Q19 $T(n) = 4T(n/2) + n/\log n$

$a = 4\ b = 2;\ f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^c = n^2 > n/\log n$

$T(n) = \Theta(n^2)$

Q20 $T(n) = 64T(n/8) - n^2\log n$

$a = 64,\ b = 8$

MT can't be applied here

as $f(n)$ is -ve.

Q17 $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

as $\frac{n}{2} < n$

$T(n) = \Theta(n)$

Q21 $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712} < n^2$

$\therefore T(n) = \Theta(n^2)$

Q22 $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1,\ b = 2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$