Tutorial 2 — Arinjay Aggarwal I, 35

Q1. What is the time complexity of below code and how?

```
void fun(int n){
  int j=1, i=0;
  while (i<n){
    i= i+j;
    j++;}}
```

$i = 0, 1, 3, 6, 10, 15$ let say k terms

so general form would be $k\frac{(k+1)}{2}$

k^{th} term $= n \Rightarrow \frac{k(k+1)}{2} = n$

$k^2 + k = 2n$

$k^2 = n \Rightarrow k = \sqrt{n}$

$\therefore$ Time complexity $= O(\sqrt{n})$ Ans

Q2. Write Recurrence relation for the recursive function that prints fibonacci series. Solve the recurrence relation to get the time complexity of this program and why?

Sol: Recurrence Relation →

Recursive Function

```
int fib(int n)
{  if(n<=1)        --> O(1) = c
     return n;
   return fib(n-1) + fib(n-2)   --> T(n-1) + T(n-2)
}
```

Recurrence Relation $T(n) = T(n-1) + T(n-2) + c$

Now $T(n-1) \simeq T(n-2)$

$T(n) = 2T(n-1) + c$

By backward substitution

$T(n-1) = 2T(n-1-1) + c \Rightarrow 2T(n-2) + c$

$T(n) = 2[2T(n-2) + c] + c$

$= 4T(n-2) + 3c$

Now $T(n-2) = 2T(n-2-1) + c$

$= 2T(n-3) + c$

$\therefore T(n) = 4T(n-2) + 3c$

$= 4(2T(n-3) + c) + 3c$

$T(n) = 8T(n-3) + 7c$

⋮

generalizing $2^k T(n-k) + (2^k - 1)c$

assume $n - k = 0 \Rightarrow n = k$

$2^n T(0) + (2^n - 1)c$

$= 2^n + (2^n - 1)c$

$= 2^n (1 + c) - c$

$= 2^n$

$\therefore$ Time complexity $= O(2^n)$

Space complexity

For fibonacci recursive implementation, the space required is directly proportional to the maximum depth of Recursion tree, since maximum depth is directly proportionate to number of elements so $O(n)$

Q3. Write programs which have complexity $\rightarrow n(\log n)$

[1]

```
for(i=1; i<=n; i++)
{
    for(j=1; j<=n; j=j*2)
    {
        sum = sum + i;
    }
}
```

(ii) n^3

```
for (i=0; i<n; i++)
{   for(j=0; j<n; j++)
    {   for(k=0; k<n; k++)
        {
            sum = sum+k;
        }
    }
}
```

(iii) $\log n(\log n)$

```
for (i=1; i<=n; i=i*2)
{   for (k=1; k<=n; k=k*2)
    {   sum = sum+j;
    }
}
```

Q 4. Solve the Recurrence relation $T(n) = T(n/4) + T(n/2) + cn^2$

Sol^n: $T(n/4) \simeq T(n/2)$

$T(n) = 2T(n/2) + cn^2$

as $a \geq 1$ and $b \geq k$

By using master's method.

$T(n) = aT(n/b) + f(n)$

$c = \log_b a \Rightarrow 1$

$f(n) > n^c \Rightarrow cn^2 > n^c$

$T(n) = O(f(n))$

$= O(n^2)$

Q 5. What is the time complexity of following fun()

```
int fun (int n)
{   for(int i=1; i<=n; i++)
    {   for(int j=1; j<n; j+=i) {
```

// Some O(1) task

}}}

Sol^n → for i=1 → 1+2+3+ ---- (n+1) ≈ n

for i=2 → 1+3+5+ --- n → n/2

for i=3 → 1+4+7+ --- n → n/3

$n + \frac{n}{2} + \frac{n}{3} + \cdots + 1$

$\Rightarrow n\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$

Now we know that $n\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right) \leq n\left(1+\frac{1}{2}+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\cdots\right)$

$n\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right) \leq n\left(\underbrace{1+0.5+0.5+\cdots}_{n}\right)$

O(n log n) Ans

Q6. What should be the time complexity of

```
for (int i=2; i<=n; i == pow(i, k))
{   // some O(1) expressions or statements
}   where k is constant
```

for first iteration i=2

for second iteration $i = 2^k$

for third iteration $i = (2^k)^k = 2^{k^2}$

n^{th} iteration, loop ends when $2^{k^i} = n$

Take log on both sides

$\log n = \log_2 2^{k^i}$

$\log n = k^i \rightarrow$ **$i = \log(\log n)$**

$O(\log(\log n))$

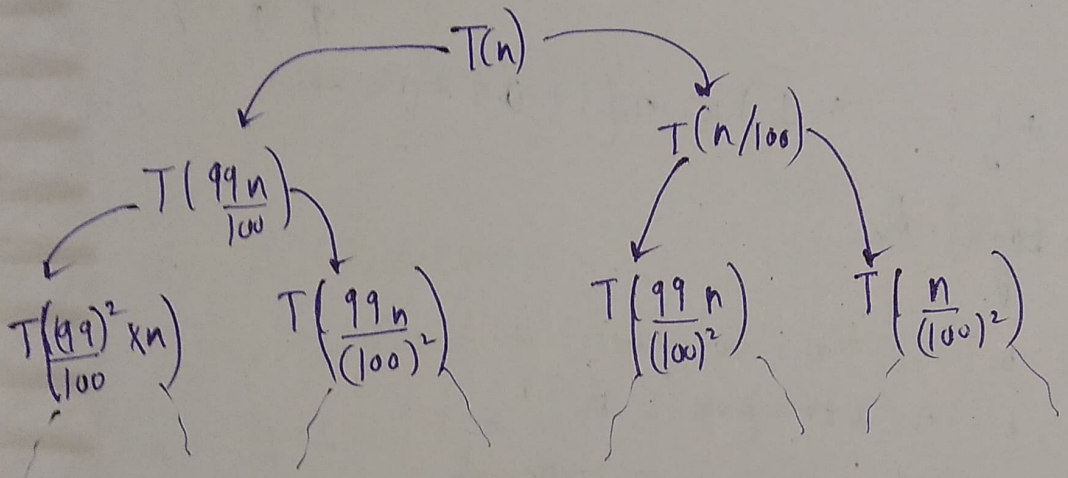
Q7. Write a recurrence relation when quick sort repeatedly divides the array in two parts of 99% and 1%. Derive the time complexity in this case. Show the recurrence tree while deriving time complexity and find the difference in heights of both the extreme parts! What do you understand by the analysis? *

99 to 1 in quick sort

when pivot is where from front or end always.

so,

$$T(n) = T(99n/100) + T(n/100) + O(n)$$



$$n = \left(\frac{99}{100}\right)^k$$

$$\log n = k \log \frac{99}{100}$$

$$k = \log^n \frac{100}{99}$$

$\therefore$ Time Complexity $= n * \log\left(\frac{100}{99} n\right)$

Ques 8 Arrange the following in increasing order of rate of growth.

(a) n, $n!$, $\log n$, $\log\log n$, $\text{root}(n)$, $\log(n!)$, $n\log n$, $\log^2(2n)$, 2^n, 2^{2^n}, 4^n, n^2, 100

$100 < \log(\log n) < \log^2 n < \log^2 n < \sqrt{n} < n < \log n! < n\log n < \log^{2^n} < n^2 < 2^n < 4^n < 2^{2^n} < n!$

(b) $2(2^n)$, $4n$, $2n$, 1, $\log(n)$, $\log(\log(n))$, $\sqrt{\log(n)}$, $\log 2n$, $2\log(n)$, n, $\log(n!)$, $n!$, n^2, $n\log(n)$

$1 < \log(\log n) < \sqrt{\log(n)} < \log n < 2\log(n) < \log(2n) < n < 2n < 4n < \log(n!) < n\log(n) < 2(2^n) < n^2 < 2n! < n^2 < 2\cdot(2^n) < n!$