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Coati Optimization Algorithm: A new bio-inspired metaheuristic algorithm for solving optimization problems

Mohammad Dehghani [a], Zeinab Montazeri [b], Eva Trojovská [a], Pavel Trojovský [a,*]

[a] Department of Mathematics, Faculty of Science, University of Hradec Králové, Rokitanského 62, 500 03 Hradec Králové, Czech Republic
[b] Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz, Iran

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ABSTRACT

In this paper, a new metaheuristic algorithm called the Coati Optimization Algorithm (COA) is introduced, which mimics coati behavior in nature. The fundamental idea of COA is the simulation of the two natural behaviors of coatis: (i) their behavior when attacking and hunting iguanas and (ii) their escape from predators. The implementation steps of COA are described and mathematically modeled in two phases of exploration and exploitation. COA performance is evaluated on fifty-one objective functions, including twenty-nine functions from the IEEE CEC-2017 test suite and twenty-two real-world applications from the IEEE CEC-2011 test suite. COA's results are compared to those of eleven well-known metaheuristic algorithms. The simulation results indicate that COA has an evident superiority over the compared algorithms by balancing exploration in global search and exploitation in local search, and is far more competitive. To assess the COA's effectiveness in real-world applications, the proposed approach is implemented on the IEEE CEC-2011 test functions and four practical optimization problems, which the simulation results indicate the high capability of COA in dealing with these types of optimization problems.

1. Introduction

A problem with more than one feasible solution is known as an optimization problem, and the process of achieving the best solution among these existing solutions is called optimization [1]. An optimization problem is described using the three main parts of decision variables, constraints, and objective function [2]. With advances in science and technology, optimization issues have become more complex, and new optimization issues have emerged, which must be addressed using appropriate optimization tools. Optimization problem-solving methods fall into two groups of deterministic and stochastic methods [3]. Deterministic methods, which are divided into two categories of gradient-based and non-gradient-based methods, have good performance in dealing with linear, convex, and simple optimization problems. However, these approaches lose their effectiveness against complex problems, non-differentiable objective functions, nonlinear search spaces, non-convex problems, and NP-hard problems. However, these are the main features of optimization problems in real-world applications. These features and the inability of deterministic methods have led researchers to introduce stochastic approaches such as metaheuristic algorithms [4]. Metaheuristic algorithms using random operators, trial and error processes, as well as random scanning of problem-solving space are able to yield effective solutions to optimization problems. Simple concepts, easy implementation, no need for derivation process, efficiency in high-dimensional problems, efficiency in nonlinear and non-convex environments are some of the advantages that have led to the popularity and widespread use of metaheuristic algorithms [5].

The optimization process in metaheuristic algorithms is such that a certain number of randomly feasible solutions are initialized in the problem-solving space. Then, in a repetition-based process, the candidate solutions are updated and improved based on the algorithm instructions. After the full implementation of the algorithm, among the candidate solutions, the best one is introduced as the solution of the problem [6]. The important issue about the solution obtained from metaheuristic algorithms is that there is no guarantee that these solutions will be the optimal global solution. This is due to the nature of random search in this type of optimization approach. However, because these solutions are close to the original solution (sometimes even equal), they are acceptable as quasi-optimal solutions [7]. The main reason for the difference in the performance of metaheuristic algorithms in a remarkably similar problem is the difference in the process of searching and updating their candidate solutions. This issue and the desire to achieve better solutions to optimization

* Corresponding author.
E-mail addresses: mohammad.dehghani@uhk.cz (M. Dehghani), z.montazeri2002@gmail.com (Z. Montazeri), eva.trojovska@uhk.cz (E. Trojovská), pavel.trojovsky@uhk.cz (P. Trojovský).

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problems have led to the design and development of numerous optimization algorithms.

The main research question in the study of metaheuristic algorithms is that given that numerous optimization algorithms have been developed so far, there is still a need to introduce new methods? In response to this challenge, the No Free Lunch (NFL) theorem [8] states that the powerful performance of an algorithm in dealing with a group of optimization problems will not guarantee the same performance in other optimization issues. Therefore, it is a false claim that an algorithm works best in all optimization applications. The NFL theorem motivates authors to design newer algorithms to find better solutions to optimization problems.

The novelty and innovation of this paper are in introducing and designing a new optimization algorithm called Coati Optimization Algorithm (COA), which has application in dealing with optimization problems. The contributions of this study can be expressed as follows:

- A new optimization algorithm called Coati Optimization Algorithm (COA) is designed to model the natural behaviors of coatis.
- Twenty-nine standard benchmark functions from the CEC-2017 test suite have been employed to evaluate COA performance in solving optimization problems.
- The performance of the COA in presenting the optimization results is compared with eleven well-known metaheuristic algorithms.
- The performance of COA in real-world applications is tested in handling the CEC-2011 test suite including twenty-two test functions.
- The performance of COA in solving the practical optimization problems is tested on four engineering design problems.

The proposed COA approach has several advantages for global optimization problems. The first advantage of COA is that there is no control parameter in the design of this algorithm, and therefore there is no need to control the parameters in any way. The second advantage of COA is its high effective efficiency in dealing with a variety of optimization problems in various sciences as well as complex high-dimensional problems. The third advantage of the proposed method is that it shows its great ability to balance research and research in the search process, which allows it high-speed convergence to provide suitable values for decision variables in optimization tasks, especially in complex problems. The fourth advantage of the proposed COA is its powerful performance in handling real-world optimization applications.

In the following, the paper is organized so that in Section 2, the literature review is presented. The proposed COA algorithm is introduced and modeled in Section 3. Simulation studies and analyze of efficiency of COA in handling real-world applications is studied in Section 4. The efficiency of COA in solving practical optimization problems is studied in Section 5. Conclusions and several study directions for future research are provided in Section 5.

2. Literature review

The ideas employed in the design of metaheuristic algorithms are inspired by various natural phenomena, the behaviors of animals, insects, birds, living things, physical laws, biological sciences, human activities, rules of the games, and any other evolution-based process. For example, the swarm movement of birds or fish has been the main idea in the design of Particle Swarm Optimization (PSO) [9], the bee colony behavior in achieving food has been the source of inspiration in Artificial Bee Colony (ABC) design [10], the law of gravitational force in physics has been the idea behind the Gravitational Search Algorithm (GSA) design [11], the behavioral interactions between student and teacher have been the main idea in the design of Teaching–Learning Based Optimization (TLBO) [12], and the rules governing darts game have been the source of inspiration for the creation of Darts Game Optimization (DGO) [13]. Metaheuristic algorithms are classified into five groups based on the primary source of inspiration employed in their design: (i) swarm-based, (ii) evolutionary-based, (iii) physics-based, (iv) game-based, and (v) human-based algorithms.

2.1. Swarm-based metaheuristic algorithms

Swarm-based algorithms have been developed inspired by natural swarming phenomena, the behavior of animals, insects, birds, and other living things in nature. PSO inspired by the natural behavior of birds and fish and ABC inspired by the natural behavior of bee colonies are among the swarm-based algorithms introduced based on search behavior in finding food sources. Ant Colony Optimization (ACO) algorithm is a swarm-based approach inspired by the ants' natural behavior and ability to detect the shortest path between nest and food [14]. The process of searching for food and the strategy of attacking and hunting in the natural behaviors of animals, birds, reptiles, and aquatic animals have been the main source of inspiration in the design of numerous swarm-based algorithms. These methods include the Gray Wolf Optimization (GWO) algorithm inspired from the social life and strategy of gray wolves in hunting [15], Whale Optimization Algorithm (WOA) inspired from the humpback whales' natural behaviors and their bubble-net hunting strategy [16], Marine Predators Algorithm (MPA) inspired from the searching behavior of marine predators and their Brownian and Lévy movements in prey hunting [17], and Tunicate Search Algorithm (TSA) inspired from natural behaviors of tunicates during the navigation and foraging process [18]. Other swarm-based metaheuristic algorithms include: White Shark Optimizer (WSO) [19], Snake Optimizer (SO) [20], Horse herd Optimization Algorithm (HOA) [21], Monarch Butterfly Optimization (MBO) [22], Slime Mould Algorithm (SMA) [23], Hunger Games Search (HGS) [24], Colony Predation Algorithm (CPA) [25], Harris Hawks Optimizer (HHO) [26], Moth Search Algorithm (MSA) [27], RUNge Kutta Optimizer (RUN) [28], Emperor Penguin Optimizer (EPO) [29], Orca Predation Algorithm (OPA) [30], Artificial Hummingbird Algorithm (AHA) [31], Chameleon Swarm Algorithm (CSA) [32], and Reptile Search Algorithm (RSA) [33].

2.2. Evolutionary-based metaheuristic algorithms

Evolutionary-based algorithms have been developed based on modeling concepts in biology, genetics, natural selection law, and random operators. GA and Differential Evolution (DE) [34] are among the most widely used evolutionary algorithms designed based on modeling the reproductive process, natural selection, Darwin's theory of evolution, and the use of random operators of selection, crossover, and mutation. Mathematical modeling of the human body's defense system and its strategy against microbes and viruses has become the main idea in introducing the evolutionary algorithm of the Artificial Immune System (AIS) [35, 36]. Some other algorithms in this group are: Genetic Programming (GP) [37], Cultural Algorithm (CA) [38], Biogeography-Based Optimization (BBO) [39], and Evolution Strategy (ES) [40].

2.3. Physics-based metaheuristic algorithms

Physics-based algorithms are introduced based on mathematical modeling of various phenomena, concepts, laws, and forces in physics. The Simulated Annealing (SA) algorithm is one of the most famous physics-based algorithms, which is inspired by modeling the process of annealing metals [41]. Mathematical modeling of physical forces has led to the design of algorithms such as GSA inspired by gravitational force, Spring Search Algorithm (SSA) inspired by spring force and Hooke law [42], and Momentum Search Algorithm (MSA) inspired by impact force due to momentum [43]. The simulation of cosmological concepts, including wormhole, black hole, and white hole, has been the main source of inspiration for introducing of the Multi-Verse Optimizer (MVO) [44]. Some other physics-based algorithms are: Equilibrium Optimizer (EO) [45], Water Cycle Algorithm (WCA) [46], Archimedes Optimization Algorithm (AOA) [47], Henry Gas Solubility Optimization (HGSO) [48], weIghted meaN oF vectOrs (INFO) [49], and Thermal Exchange Optimization (TEO) [50].

2.4. Game-based metaheuristic algorithms

Game-based algorithms have been designed by imitating the rules and conditions governing various games as well as players' behavior. The behavior of players and coaches inspires Volleyball Premier League (VPL) method during volleyball matches [51]. Football Game-Based Optimizer (FGBO) is designed based on mathematical modeling of players' behavior and decisions of club managers in the football league [52]. Players' efforts to earn points and improve their performance have been led to the design of algorithms such as DGO inspired by darts game, Ring Toss Game Based Optimizer (RTGBO) inspired by ring throwing [53], and Puzzle Optimization Algorithm (POA) inspired by puzzle-solving [54].

2.5. Human-based metaheuristic algorithms

Human-based algorithms have been developed based on mathematical simulations of human behaviors, activities, and interactions in individual and social life. TLBO is one of the most widely used human-based algorithms designed based on mathematical modeling of a classroom educational space and interactions between teacher and students with the aim of improving the level of knowledge. Following Optimization Algorithm (FOA) is designed based on the mathematical simulation of the community leader's impact on people's progress [55]. Poor and Rich Optimization (PRO) [56], Brain Storm Optimization Algorithm (BSOA) [57], Dual-Population Social Group Optimization (DPSGO) [58], Human Eye Vision Algorithm (HEVA) [59], and Human Mental Search (HMS) [60] are some other physics-based algorithms.

Based on the best knowledge gained from the literature review, no metaheuristic optimization algorithm has been designed based on modeling the natural behaviors of coatis. However, the strategy of coatis when hunting iguana, as well as the behavior of coatis when confronting and fleeing from predators, are intelligent activities that can be the basis of an optimizer design. Therefore, in this study, to address this research gap, inspired by these two strategies of coatis, a new optimization algorithm has been developed, which is introduced in the next section.

3. Coati optimization algorithm

This section describes the proposed Coati Optimization Algorithm (COA), and we model its various steps mathematically.

3.1. Inspiration and behaviors of coatis

Coatis, also known as coatimundis, are members of the Nasua and Nasuella genera of the Procyonidae family. They are diurnal mammals native to the southwestern United States, Mexico, Central America, and South America [61]. A slim head with a flexible, extended, somewhat upward-turned nose, black paws, tiny ears, and a long non-prehensile tail utilized for signaling and balance are all features shared by all coatis. Adult coatis range from 33 to 69 cm in length from head to tail tip, which can be as long as their body. Coatis are roughly the size of a big house cat, weighing between 2 and 8 kg and standing around 30 cm tall at the shoulder. Males can grow to be nearly double the size of females, with sharp, large canine teeth. These measurements are for the South America coatis and white-nosed. The mountain coatis are the tinier of the two [62]. An image of coati is shown in Fig. 1. Coatis are omnivores; they eat invertebrates, such as tarantula, coatis also eat small vertebrate prey, such as small birds, lizards, rodents, crocodile eggs, and birds' eggs. One of the favorite foods of coatis is a green iguana. These large lizards (iguanas) are often found in trees, so coatis hunt them in groups. Some of them climb trees and scare the iguana into jumping to the ground, while other coatis attack it quickly. Nevertheless, coatis are at risk of being attacked by predators. Jaguars, ocelots, tayras, dogs, foxes, boa constrictors, maned wolves, anacondas, and jaguarundis are some of the coati's predators. They are also hunted by large raptors such as harpy eagles, black-and-chestnut eagles, and ornate hawk-eagles [63].

The strategy of the coatis when attacking the iguanas and their behavior when confronting and escaping from predators are intelligent processes. The simulation of these natural coatis' behaviors is the fundamental inspiration in designing the proposed COA approach.

3.2. Algorithm initialization process

The COA approach is a population-based metaheuristic in which the coatis are considered population members of this algorithm. The position of each coati in the search space determines the values for the decision variables. Hence, in the COA, coatis' position represents a candidate solution to the problem. At the beginning of the COA implementation, the position of the coatis in the search space is randomly initialized using Eq. (1).

$$X_i: \; x_{i,j} = lb_j + r \cdot \left(ub_j - lb_j\right), \quad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, m, \tag{1}$$

where X_i is the position of the ith coati in search space, $x_{i,j}$ is the value of the jth decision variable, N is the number of coatis, m is the number of decision variables, r is a random real number in the interval $[0, \; 1]$, and lb_j and ub_j are the lower bound and upper bound of the jth decision variable, respectively.

The population of coatis in the COA is mathematically represented using the following matrix X, called the population matrix,

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N\times m}. \tag{2}$$

The placement of candidate solutions in decision variables leads to the evaluation of different values for the objective function of the problem. These values are displayed using Eq. (3).

Fig. 1. Picture of a coati while hunting.

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N\times 1}, \tag{3}$$

where F is the vector of the obtained objective function and F_i is the objective function value obtained based on the ith coati.

In metaheuristic algorithms, such as the proposed COA, the measure of the quality of a candidate solution is the value of the objective function. Therefore, the member of the population that leads to the evaluation of the best value for the objective function is known as the best member of the population. Since the candidate solutions are updated during the algorithm iterations, the best member of the population is also updated in each iteration.

3.3. Mathematical model of COA

The process of updating the position of coatis (candidate solutions) in the COA is based on modeling two natural behaviors of coatis. These behaviors include:

(i) coatis' strategy when attacking iguanas,

(ii) coatis' escape strategy from predators.

Accordingly, the COA population is updated in two different phases.

3.3.1. Phase 1: Hunting and attacking strategy on iguana (exploration phase)

The first phase of updating the coatis' population in the search space is modeled based on simulating their strategy when attacking iguanas. In this strategy, a group of coatis climbs the tree to reach an iguana and scare it. Several other coatis wait under a tree until the iguana falls to the ground. After the iguana falls to the ground, the coatis attack it and hunt it. This strategy leads coatis to move to different positions in the search space, which demonstrates the COA's exploration ability in global search in the problem-solving space. The pattern diagram of this strategy is presented in Fig. 2.

In the COA design, the position of the best member of the population is assumed to be the position of the iguana. It is also assumed that half of the coatis climb the tree and the other half wait for the iguana to fall to the ground. Therefore, the position of the coatis rising from the tree is mathematically simulated using Eq. (4).

$$X_i^{P1}: x_{i,j}^{P1} = x_{i,j} + r \cdot \left(Iguana_j - I \cdot x_{i,j}\right), \text{ for } \; i = 1, 2, \ldots, \left\lfloor \frac{N}{2} \right\rfloor \text{ and } \; j = 1,\ 2,\ \ldots,\ m. \tag{4}$$

After the iguana falls to the ground, it is placed in a random position in the search space. Based on this random position, coatis on the ground move in the search space, which is simulated using Eqs. (5) and (6).

$$Iguana^G : Iguana_j^G = lb_j + r \cdot \left(ub_j - lb_j\right),\ j = 1,\ 2,\ \ldots,\ m, \tag{5}$$

$$X_i^{P1} : x_{i,j}^{P1} = \begin{cases} x_{i,j} + r \cdot \left(Iguana_j^G - I \cdot x_{i,j}\right), & F_{Iguana^G} < F_i, \\ x_{i,j} + r \cdot \left(x_{i,j} - Iguana_j^G\right), & else, \end{cases}$$
$$\text{for } i = \left\lfloor \frac{N}{2} \right\rfloor + 1, \left\lfloor \frac{N}{2} \right\rfloor + 2, \ldots, N \text{ and } j = 1,\ 2,\ \ldots,\ m. \tag{6}$$

The new position calculated for each coati is acceptable for the update process if it improves the value of the objective function, otherwise, the coati remains in the previous position. This update condition is for $i = 1,\ 2,\ \ldots,\ N$ simulated using Eq. (7).

$$X_i = \begin{cases} X_i^{P1}, & F_i^{P1} < F_i, \\ X_i, & else. \end{cases} \tag{7}$$

Here X_i^{P1} is the new position calculated for the ith coati, $x_{i,j}^{P1}$ is its jth dimension, F_i^{P1} is its objective function value, r is a random real number in the interval $[0,\ 1]$, $Iguana$ represents the iguana's position in the search space, which actually refers to the position of the best member, $Iguana_j$ is its jth dimension, I is an integer, which is randomly selected from the set $\{1,\ 2\}$, $Iguana^G$ is the position of the iguana on the ground, which is randomly generated, $Iguana_j^G$ is its jth dimension, F_{Iguana^G} is its value of the objective function, and $\lfloor \cdot \rfloor$ is the floor function (also known as the greatest integer function).

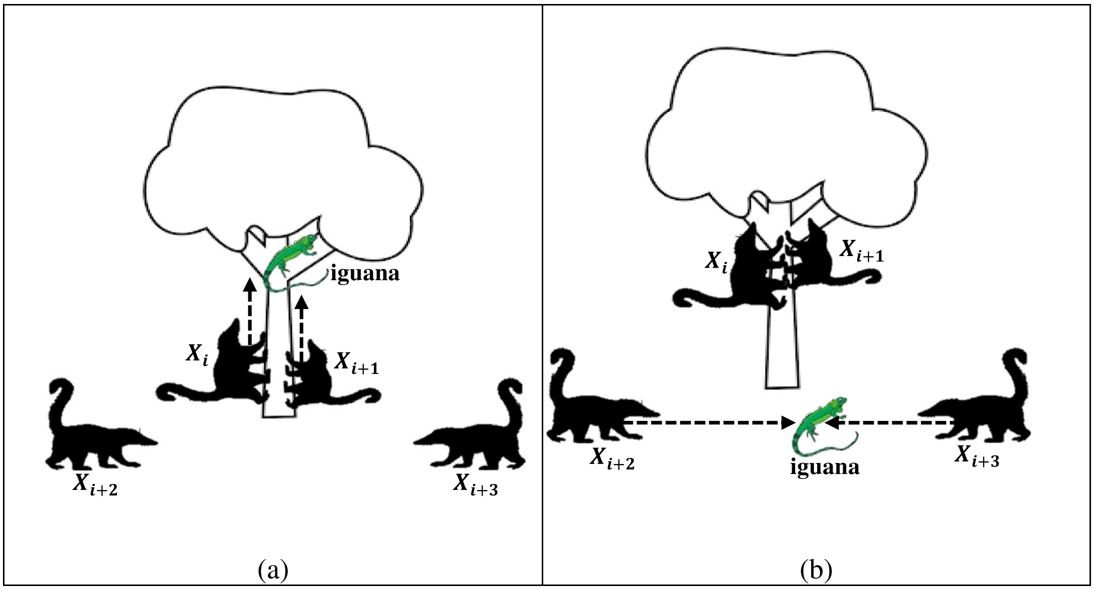

Fig. 2. Pattern diagram of the first phase of COA. (a) Attack of half of the coatis' population towards the iguana on the tree. (b) Hunting fallen iguana on the ground by the other half of the coatis' population.

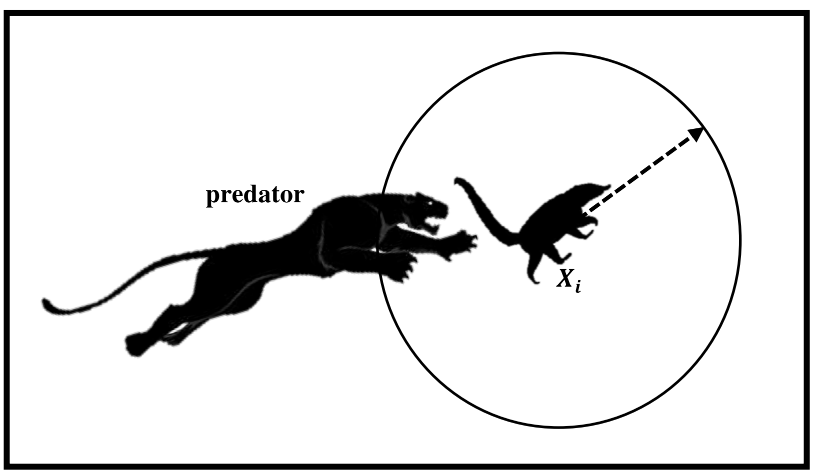

Fig. 3. Pattern diagram of coati escaping from a predator in the second phase of COA.

3.3.2. Phase 2: The process of escaping from predators (exploitation phase)

The second phase of the process of updating the position of coatis in the search space is mathematically modeled based on the natural behavior of coatis when encountering predators and escaping from predators. When a predator attacks a coati, the animal escapes from its position. Coati's moves in this strategy lead to it being in a safe position close to its current position, which indicates the COA's exploitation ability in local search. The pattern diagram of this strategy of coatis in escaping from predators is presented in Fig. 3.

To simulate this behavior, a random position is generated near the position in which each coati is located based on Eqs. (8) and (9).

$$lb_j^{local} = \frac{lb_j}{t}, ub_j^{local} = \frac{ub_j}{t}, \text{ where } t = 1, 2, \ldots, T. \tag{8}$$

$$X_i^{P2} : x_{i,j}^{P2} = x_{i,j} + (1 - 2r) \cdot \left(lb_j^{local} + r \cdot \left(ub_j^{local} - lb_j^{local}\right)\right), \quad i = 1, 2, \ldots, N, \ j = 1, 2, \ldots, m, \tag{9}$$

The newly calculated position is acceptable if it improves the value of the objective function, that this condition simulates using Eq. (10).

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} < F_i, \\ X_i, & else, \end{cases} \tag{10}$$

Here X_i^{P2} is the new position calculated for the ith coati, based on the second phase of COA, $x_{i,j}^{P2}$ is its jth dimension, F_i^{P2} is its objective function value, r is a random number in the interval [0, 1], t is the iteration counter, lb_j^{local} and ub_j^{local} are the local lower bound and local upper bound of the jth decision variable respectively, lb_j and ub_j are the lower bound and upper bound of the jth decision variable, respectively.

3.3.3. Repetition process, pseudocode, and flowchart of COA

After the position of all coatis in the search space is updated based on the first and second phases, a COA's iteration is completed. The process of updating the population, based on Eqs. (4) to (10), is repeated until the last iteration of the algorithm. Once

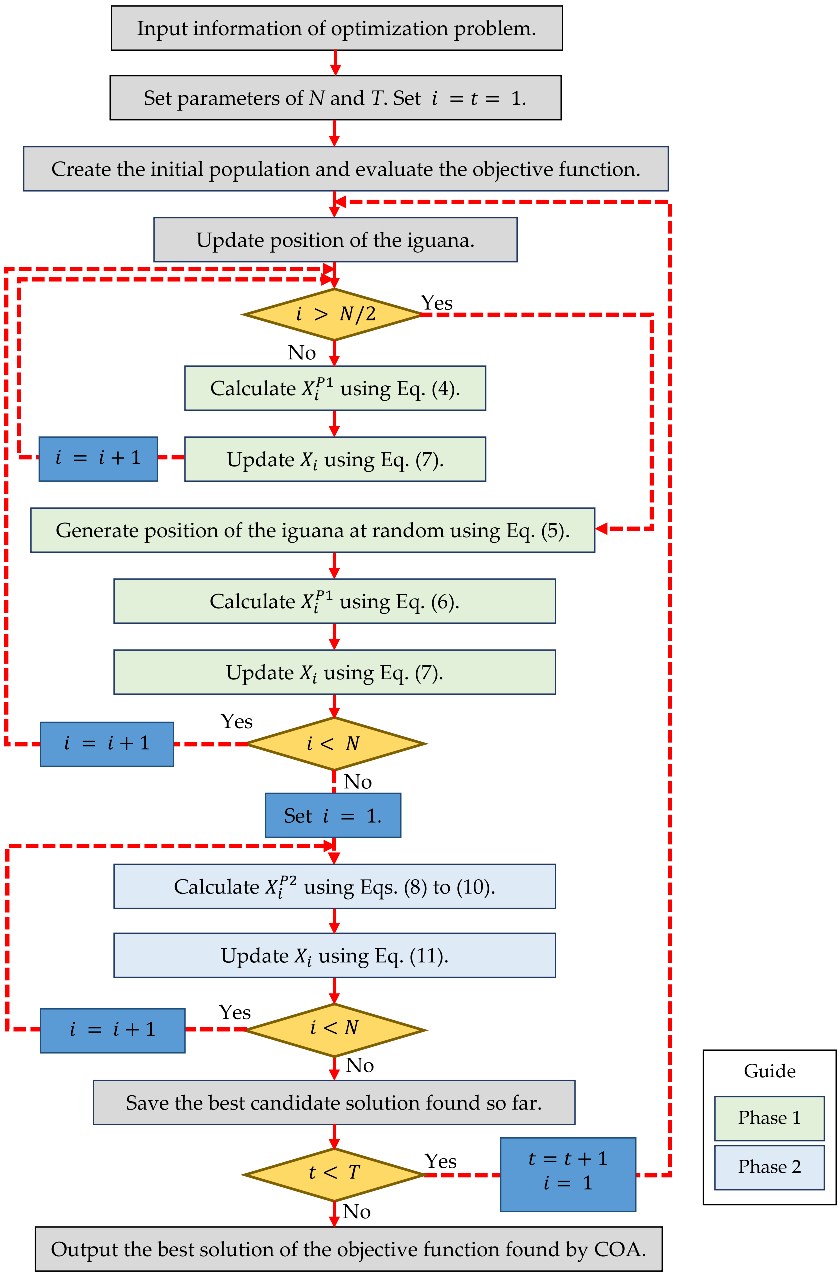

Fig. 4. The flowchart of the COA.

the run of COA completes, the best solution obtained during all algorithm iterations is returned as the output. The various stages of the COA implementation are presented as a flowchart in Fig. 4 and its pseudocode in Algorithm 1.

3.4. Computation complexity

This subsection investigates the computational complexity of the proposed algorithm. The COA preparation involves the production of the initial population of coatis and the initial calculation of the objective function having computational complexity equal to $O(Nm)$, where N is the number of coatis and m is the number of problem variables. The process of updating the COA population in the first phase has a computational complexity equal to $O(NmT)$, where T is the number of iterations of the algorithm. In the first phase of COA, the process of calculating the iguana's random position on the ground and calculating its objective function has a complexity equal to $O(Nm\ T/2)$. The second phase of the process of updating the position of the coatis has a computational complexity equal to $O(NmT)$. Accordingly, the total computational complexity of COA equals $O(Nm\ (1 + 5T/2))$.

Based on the analysis of total computational complexity of the proposed COA, it can be concluded that the main factors determining the computation complexity of the COA in addressing an optimization challenge are: the number of coatis population members (i.e., N), the number of decision variables of the problem (i.e., m), the number of algorithm iterations (i.e., T) and the cost of the objective function of the problem.

4. Simulation studies and results

In this section, simulation studies and evaluation of COA efficiency in optimization are performed. Experiments have been

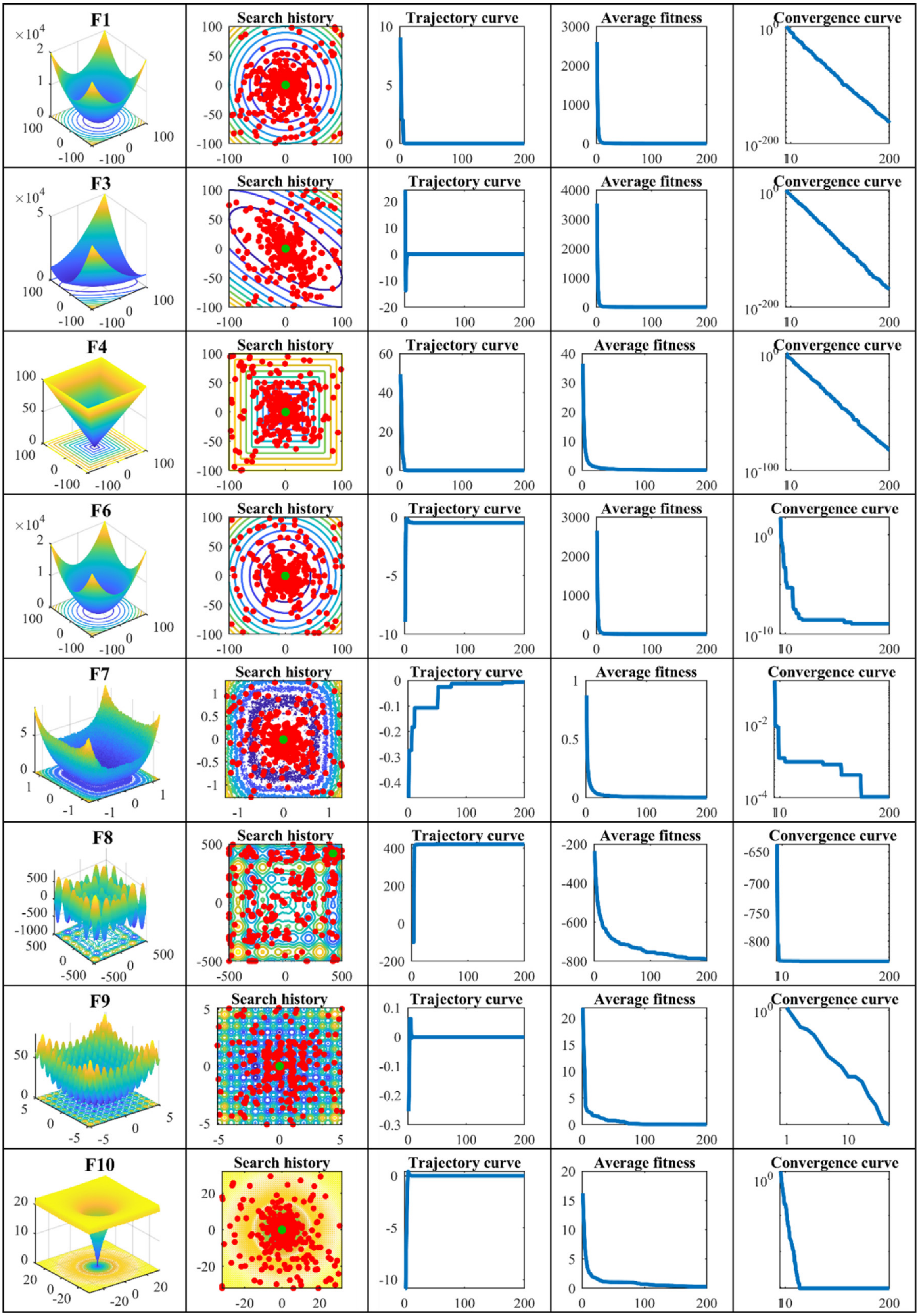

Fig. 5. Qualitative analysis of COA.

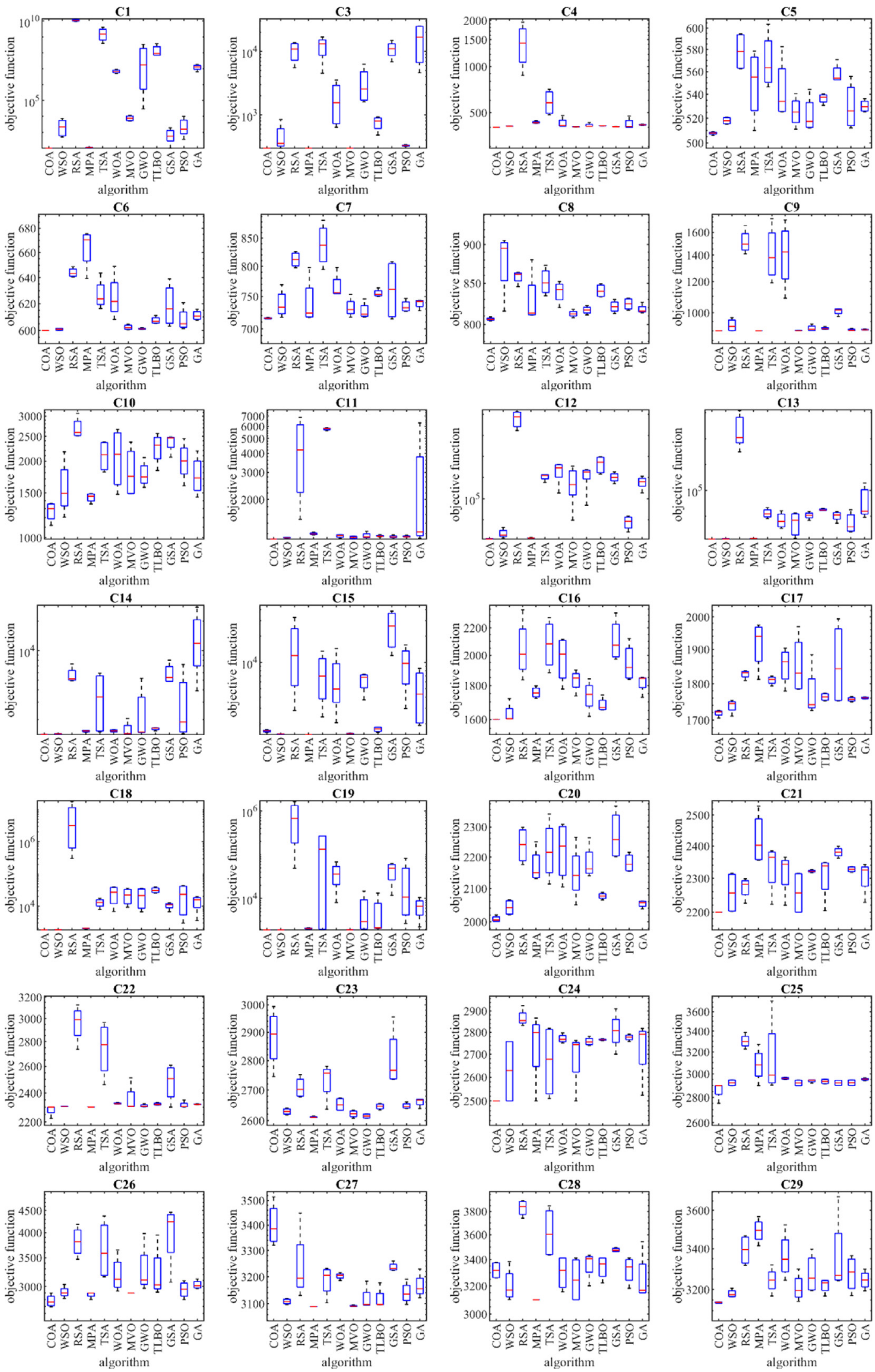

Fig. 6. Boxplot of COA and competitor algorithms in optimization of the CEC-2017 (the dimension $m = 10$).

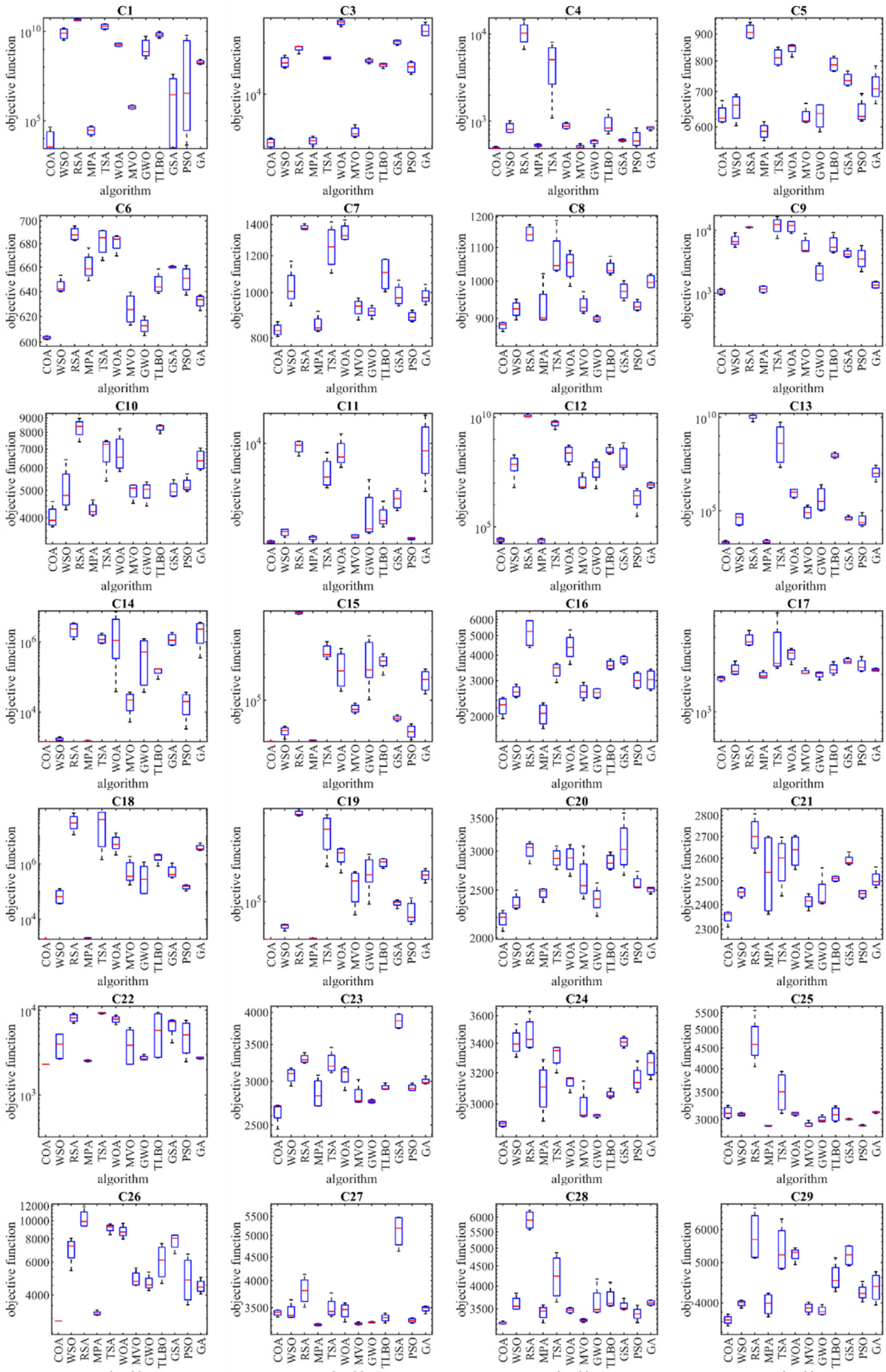

Fig. 7. Boxplot of COA and competitor algorithms in optimization of the CEC-2017 (the dimension $m = 30$).

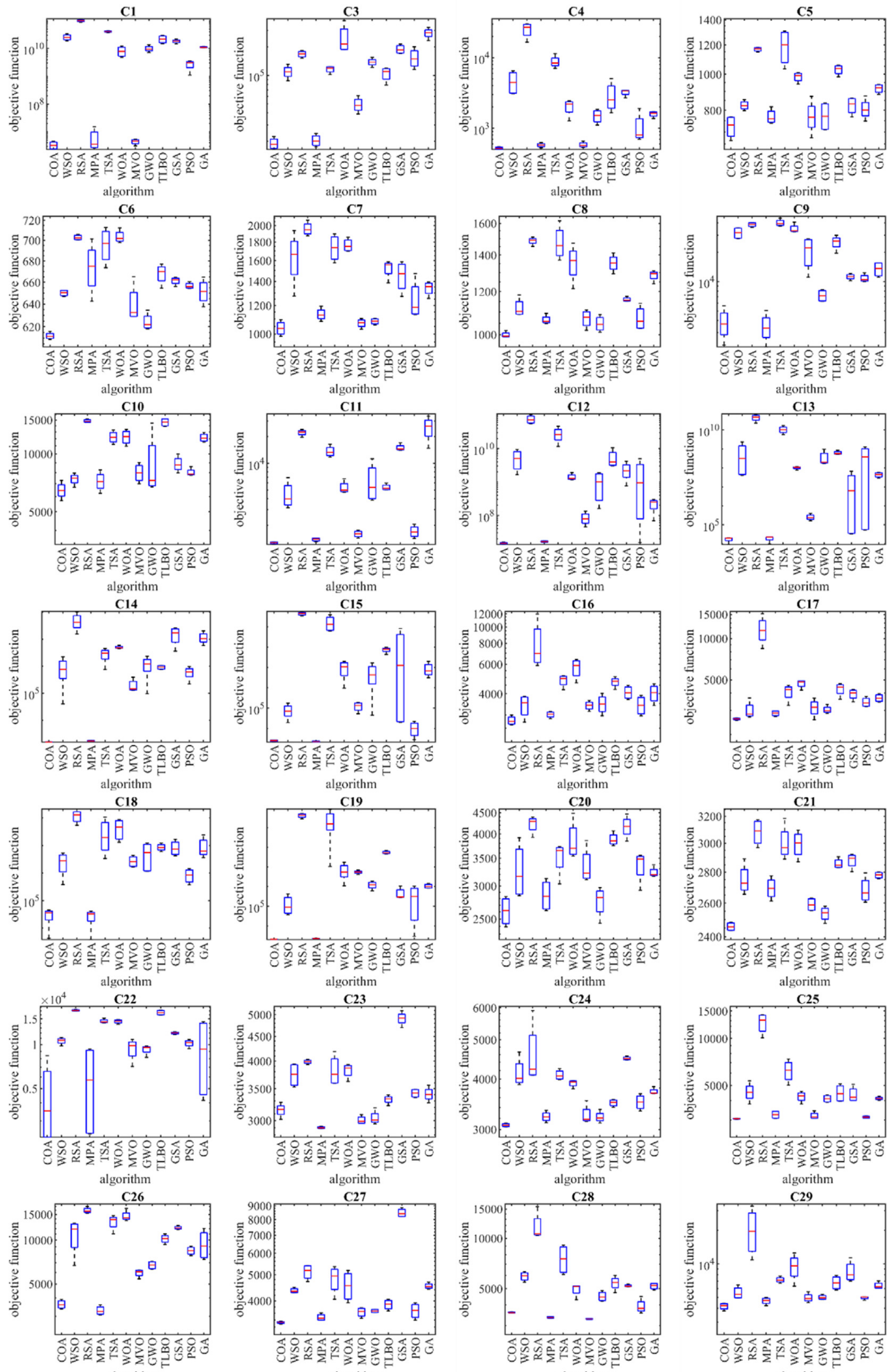

Fig. 8. Boxplot of COA and competitor algorithms in optimization of the CEC-2017 (the dimension $m = 50$).

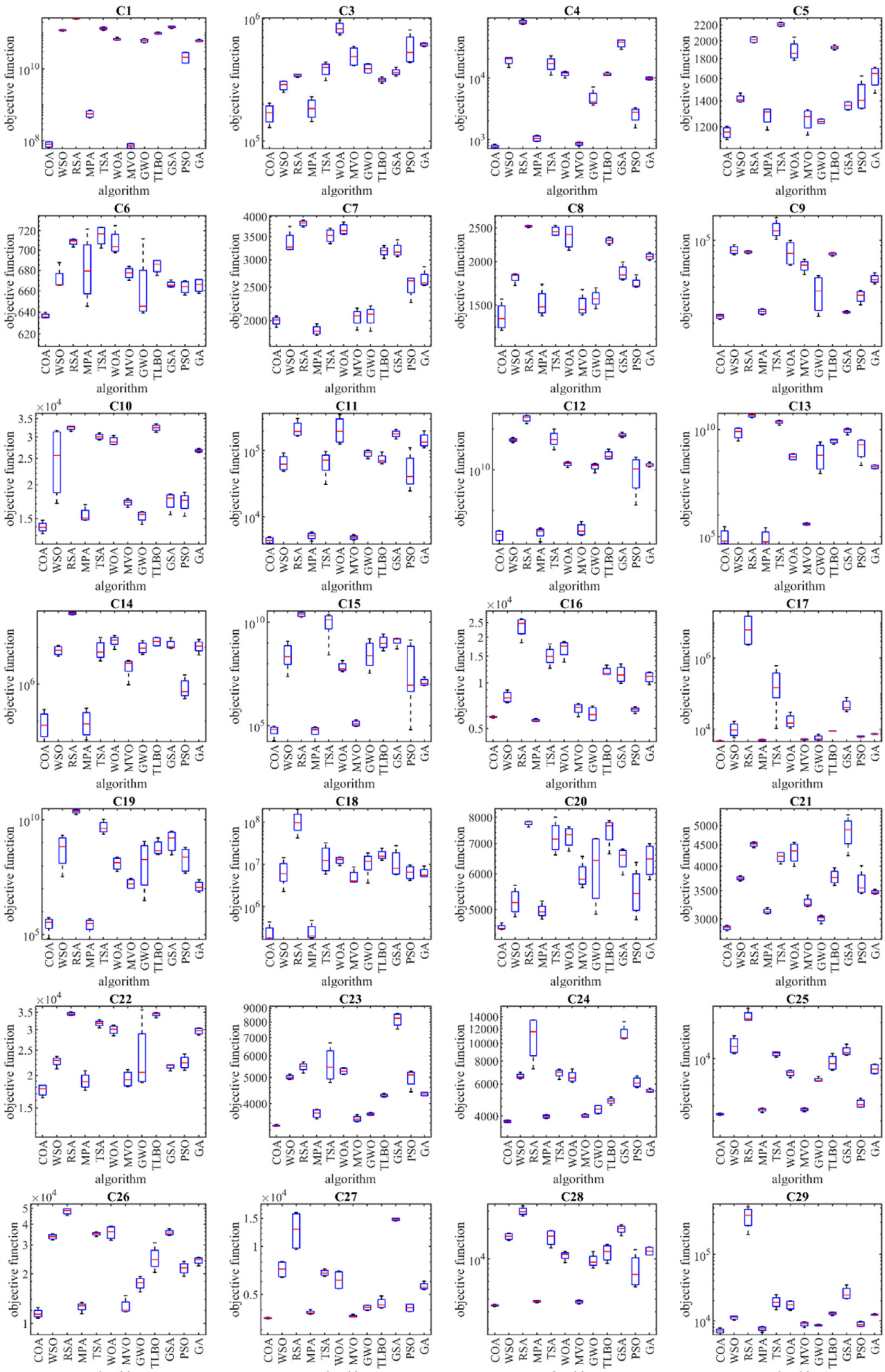

Fig. 9. Boxplot of COA and competitor algorithms in optimization of the CEC-2017 (the dimension $m = 100$).

Algorithm 1. Pseudo-code of COA.

Start COA.
1. Input the optimization problem information.
2. Set the number of iterations T and the number of coatis N.
3. Initialization of the positions of all coatis by Eq. (1) and evaluation of the objective function for this initial population.
4. For $t = 1{:}T$
5.   Update location of the iguana based on the location of the best member of the population.
6.   **Phase 1:** Hunting and attacking strategy on the iguana (**Exploration Phase**)
7.   For $i = 1 : \lfloor N/2 \rfloor$
8.   Calculate new position for the ith coati using Eq. (4).
9.   Update position of the ith coati using Eq. (7).
10.   End for
11.   For $i = 1 + \lfloor N/2 \rfloor : N$
12.   Calculate random position for the iguana using Eq. (5).
13.   Calculate new position for the ith coati using Eq. (6).
14.   Update position of the ith coati using Eq. (7).
15.   End for
16.   **Phase 2:** The process of escaping from predators (**Exploitation Phase**)
17.   Calculate the local bounds for variables using Eq. (8).
18.   For $i = 1{:}N$
19.   Calculate the new position for the ith coati using Eq. (9).
20.   Update the position of the ith coati using Eq. (10).
21.   End for
22.   Save the best candidate solution found so far.
23. End for
24. **Output** of the best obtained solution by COA for given problem.

End COA.

The algorithm of the proposed COA.

Table 1
Values set for control parameters of compared algorithms.

Algorithm	Parameter	Value
WSO	f_{min}	0.07
	f_{max}	0.75
	τ	4.125
	a_0	6.25
	a_1	100
	a_2	0.0005
RSA	Sensitive parameter	$\beta = 0.01$
	Sensitive parameter	$\alpha = 0.1$
	Evolutionary Sense (ES)	ES: randomly decreasing values between 2 and -2
MPA	Constant number	$P = 0.5$
	Random vector	$R \in [0,\ 1]$
	Fish Aggregating Devices (*FADs*)	$FADs = 0.2$
	Binary vector	$U = 0$ or 1
TSA	P_{min}	1
	P_{max}	4
	c_1, c_2, c_3	Random numbers stand in the interval $[0,\ 1]$.
WOA	a : Convergence parameter	Linear reduction from 2 to 0.
	r : random vector	$r \in [0,\ 1]$.
	l : random number	$l \in [-1,\ 1]$.
MVO	Wormhole existence probability (WEP)	$\min(WEP) = 0.2$ and $\max(WEP) = 1$.
	Exploitation accuracy over the iterations (p)	$p = 6$.
GWO	Convergence parameter (a)	a: Linear reduction from 2 to 0.
TLBO	T_F : teaching factor	T_F = round $[(1 + rand)]$
	Random number	$rand \in [0,\ 1]$.
GSA	Alpha	20
	R_{power}	1
	R_{norm}	2
	G_0	100
PSO	Topology	Fully connected.
	Cognitive constant	$C_1 = 2$
	Social constant	$C_2 = 2$
	Inertia weight	Linear reduction from 0.9 to 0.1.
	Velocity limit	10% of the dimensions range of the variables.
GA	Type	Real coded.
	Selection	Roulette wheel (Proportionate).
	Crossover	Whole arithmetic (Probability = 0.8. $\alpha \in [-0.5,\ 1.5]$).
	Mutation	Gaussian (Probability = 0.05).

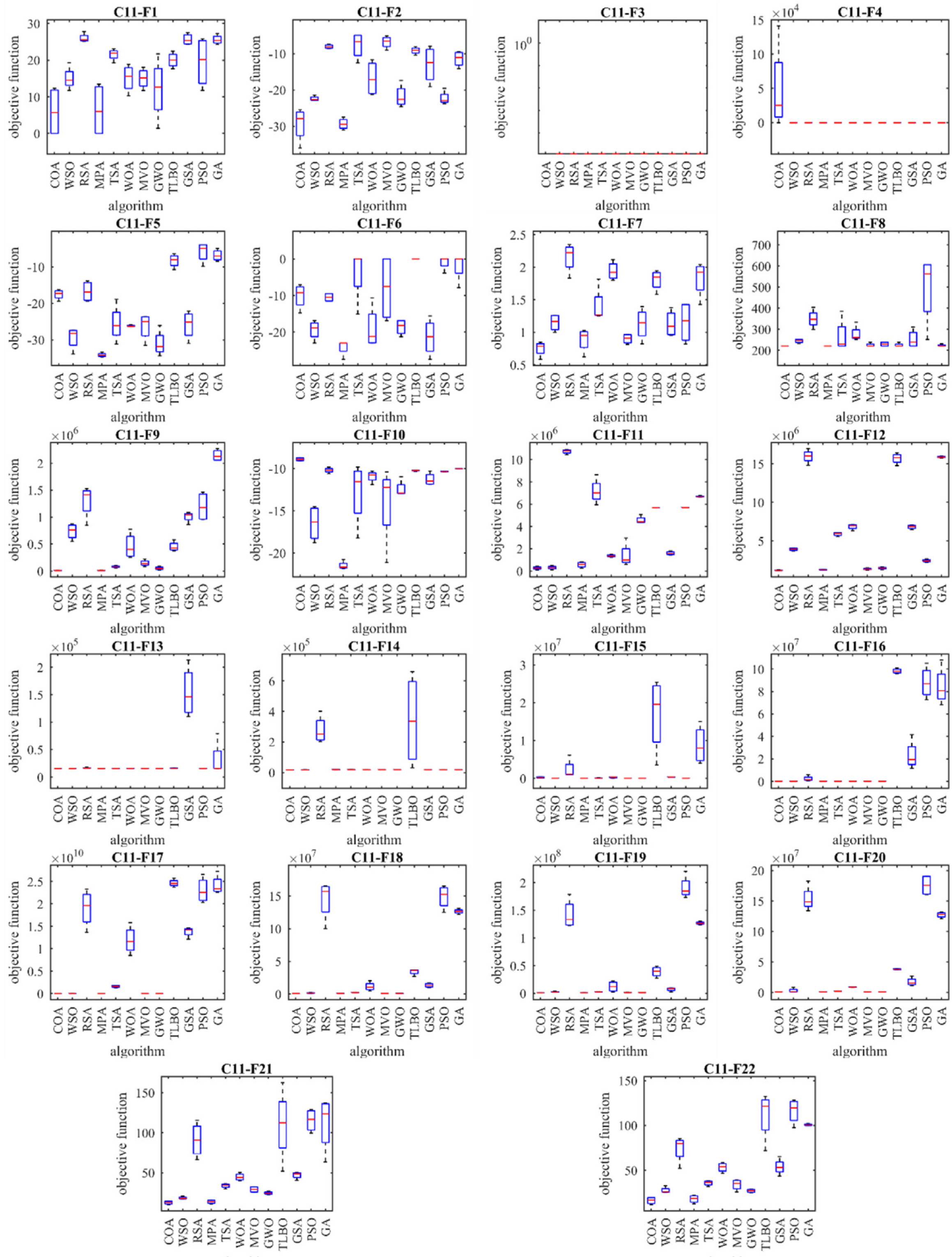

Fig. 10. Boxplot of COA and competitor algorithms in optimization of the CEC-2011 test suite.

implemented on MATLAB R2022a using a 64-bit Core i7 processor with 3.20 GHz and 16 GB of main memory.

4.1. Qualitative analysis of COA

The qualitative analysis results of the proposed COA approach in solving several common optimization problems of unimodal and multimodal types are presented in Fig. 5. These functions' details and complete information are stated in [64]. In this qualitative analysis, four well-known metrics are used to intuitively analyze COA performance: search history, the trajectory of the first coati in the 1st dimension, the average fitness of the coati population, and the convergence curve. The purpose of the search history metric is to display the position of coatis in the search

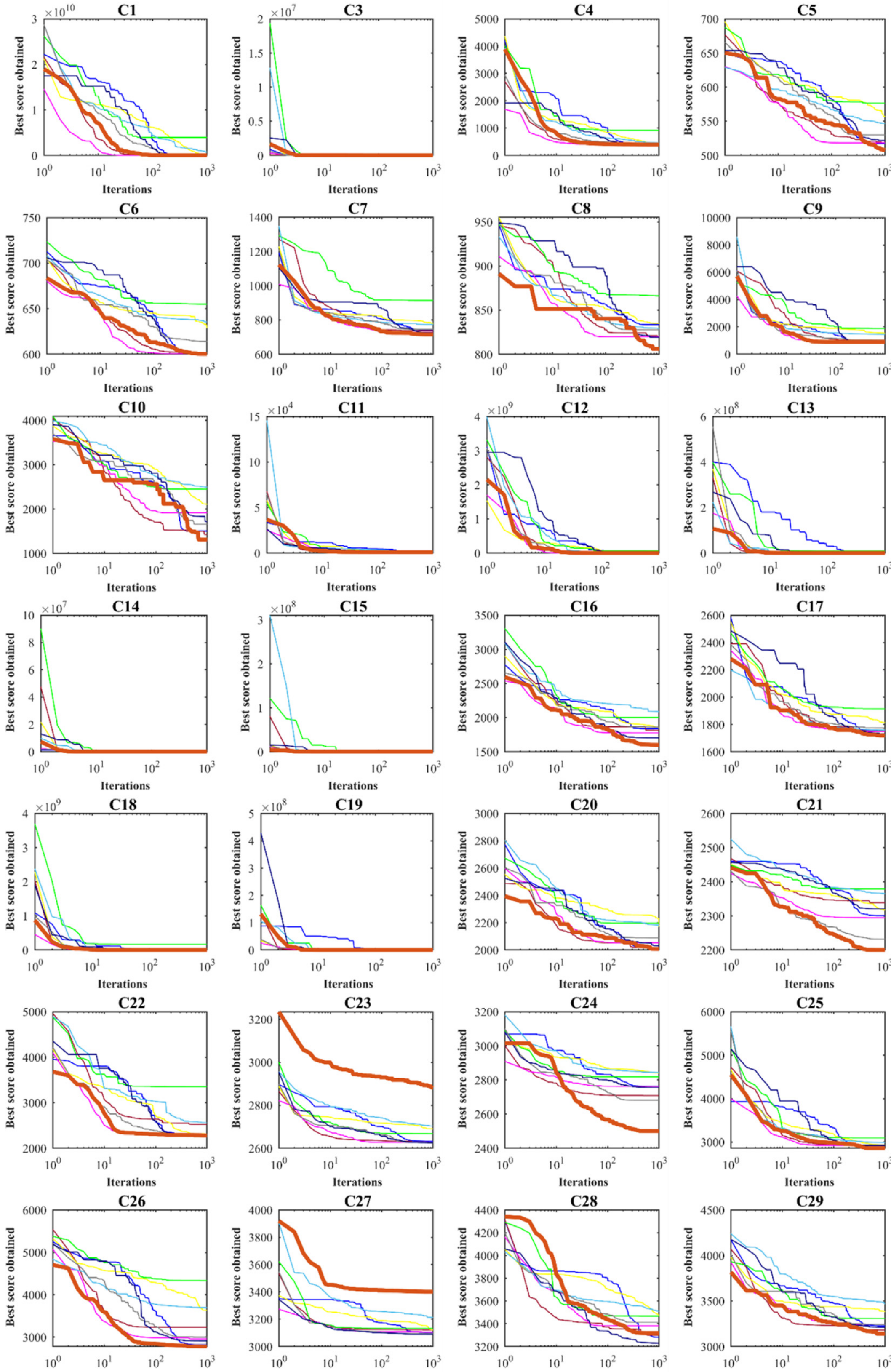

Fig. 11. Convergence curves of COA and some latest outstanding algorithms in the optimization of the CEC-2017 (the dimension $m = 10$).

space during the iterations of the algorithm. The goal of the trajectory of the coati metric is to observe position changes of the first coati in the first dimension during the iterations of the algorithm. The purpose of the average fitness metric is to show the average fitness of the coati population changes during the iterations of the algorithm. The purpose of the convergence curve

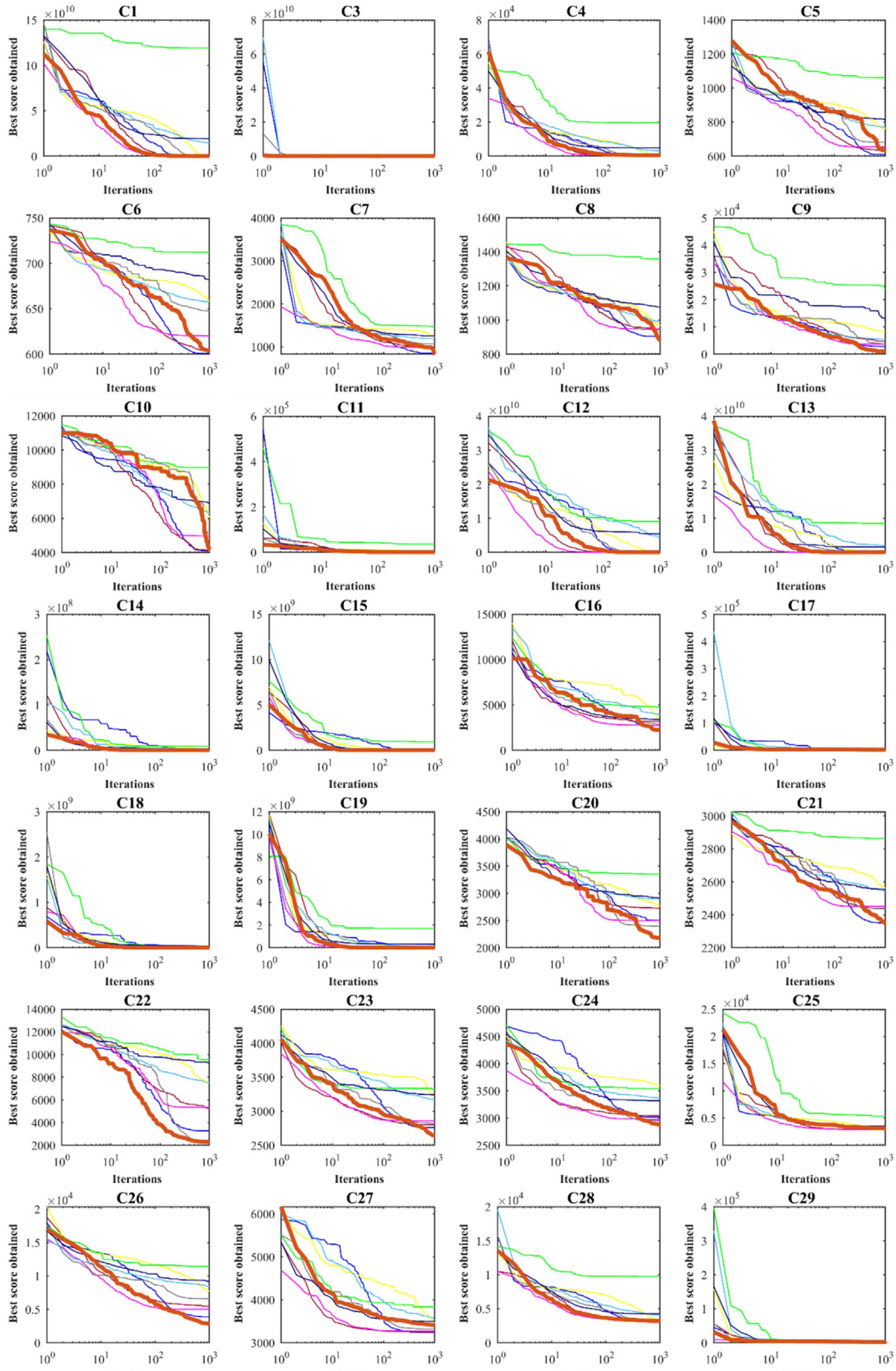

Fig. 12. Convergence curves of COA and some latest outstanding algorithms in the optimization of the CEC-2017 (the dimension $m = 30$).

metric is to show how the candidate solution improves and the reaching process of COA to the solution during the iterations of the algorithm.

The search history diagrams show that COA has high power in scanning the search space at global and local levels and can converge towards the optimal solution with a high convergence

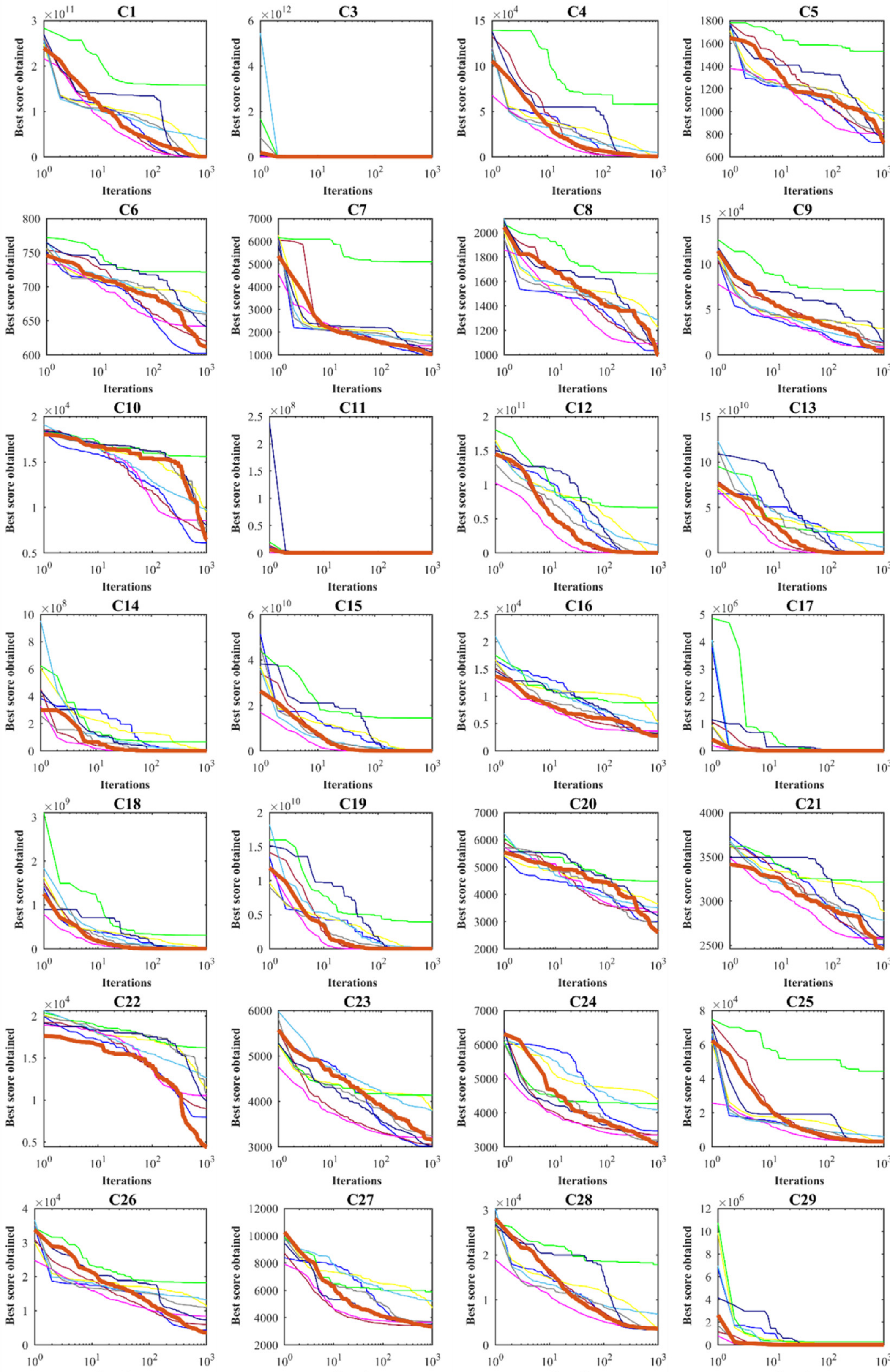

Fig. 13. Convergence curves of COA and some latest outstanding algorithms in the optimization of the CEC-2017 (the dimension $m = 50$).

speed after identifying the main optimal area. What is evident from the trajectory diagrams of the first coati is that in the initial iterations of the algorithm, based on the exploration capability, extensive changes have been made in the coati's position to identify the main optimal area. Then, by increasing the iterations of the algorithm, the changes in the position of coati are smaller to

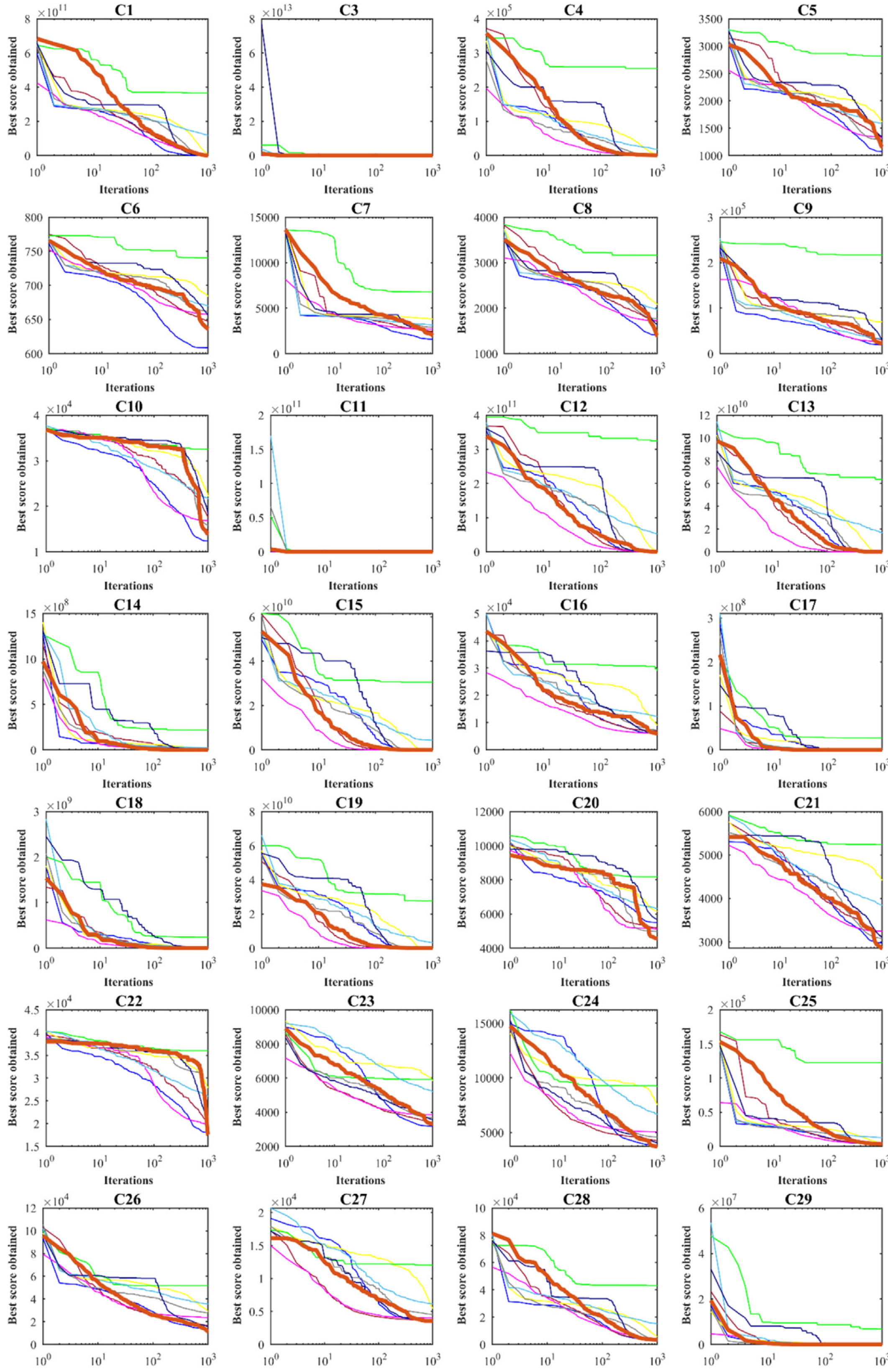

Fig. 14. Convergence curves of COA and some latest outstanding algorithms in the optimization of the CEC-2017 (the dimension $m = 100$).

converge to possible better solutions around promising solutions based on exploitation capability. The average fitness diagrams of the coatis' population indicate that according to these diagrams' decreasing trend, the coatis' population converges towards the optimal solution during the repetitions of the algorithm. What can be seen from the analysis of the convergence curves is that

Table 2
Assessment results of the CEC-2017 objective functions (the dimension $m = 10$).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_1	Mean	100	3072.047	1.09E+10	109.5751	1.86E+09	6 879 325	8014.981	94 083 630	1.57E+08	789.6176	3347.233	12 641 048
	std	1.33E−05	3233.023	1.61E+09	3.36854	1.62E+09	1 713 296	3146.963	1.66E+08	1.49E+08	779.9287	4427.81	4 849 091
	ET	2.448051	0.972822	6.405592	3.432174	1.303047	1.162363	2.05708	1.372409	4.741238	3.213548	1.517339	2.037542
	Rank	1	4	12	2	11	7	6	9	10	3	5	8
C_3	Mean	300	464.5135	10 267.95	300	11 925.54	1824.674	300.0582	3252.465	754.5378	10 918.38	329.3844	15 733.23
	std	4.64E−14	254.8057	3757.315	4.56E−11	5240.035	1362.066	0.052313	2144.538	197.091	3293.007	8.596052	10 584.32
	ET	2.390933	0.957824	6.16697	3.033797	1.256226	1.088733	1.522373	1.285341	4.677143	2.703576	1.259734	1.515914
	Rank	1	5	9	2	11	7	3	8	6	10	4	12
C_4	Mean	400.0017	407.2957	1414.847	431.148	588.2747	426.8392	403.5586	412.5268	409.7871	404.8591	421.6779	415.7083
	std	0.003425	0.610881	456.3131	7.439262	111.7747	34.54679	1.831915	11.82712	0.585883	1.230735	35.98526	3.157633
	ET	1.40523	0.944091	6.208042	3.154841	1.237309	1.08464	1.630819	1.291011	4.61799	2.657587	1.327702	1.472259
	Rank	1	4	12	10	11	9	2	6	5	3	8	7
C_5	Mean	507.7503	517.9215	578.3821	549.6645	569.274	544.0672	525.4579	522.7703	536.616	557.956	529.9867	530.1072
	std	1.305485	2.916079	17.75947	30.4356	25.38658	26.91797	12.46305	15.06522	4.272868	8.573295	20.21898	5.109001
	ET	2.524593	0.962253	6.273573	3.123378	1.323984	1.152157	1.683115	1.379372	4.862399	2.760354	1.345287	1.582547
	Rank	1	2	12	9	11	8	4	3	7	10	5	6
C_6	Mean	600.0005	600.8465	644.0479	663.9295	626.8685	625.0691	602.3263	601.2195	607.426	618.6179	608.0398	611.1025
	std	0.000358	0.916838	3.630547	16.71614	11.8317	17.16605	1.867883	0.50302	2.656369	16.6334	8.788191	3.64533
	ET	1.008633	1.161235	6.466014	3.494521	1.507921	1.328946	1.897401	1.59733	5.499674	2.886424	1.536365	1.738574
	Rank	1	2	11	12	10	9	4	3	5	8	6	7
C_7	Mean	715.9576	738.3029	812.0262	740.8463	838.1264	766.2735	732.5036	727.2388	755.4194	761.6679	734.5213	738.9913
	std	1.076312	22.44365	13.16748	38.53831	38.36098	21.26572	15.03417	12.98448	6.146001	50.10974	9.270855	7.625192
	ET	2.749575	1.043978	6.439517	3.251785	1.371907	1.213753	1.751752	1.444604	5.031541	2.8185	1.333314	1.606219
	Rank	1	5	11	7	12	10	3	2	8	9	4	6
C_8	Mean	806.2204	877.9534	858.0199	829.5	852.1631	839.2534	812.6909	817.042	840.7119	821.3916	824.5363	818.0629
	std	2.05445	41.79476	8.256772	33.8367	17.11863	13.90596	4.094321	4.670738	8.256642	7.197692	7.212869	5.753112
	ET	1.534467	1.023024	6.283318	3.154117	1.317752	1.171157	1.698311	1.397969	5.10731	2.722496	1.318552	1.641561
	Rank	1	12	11	7	10	8	2	3	9	5	6	4
C_9	Mean	900	929.3131	1514.186	900	1420.443	1414.578	900.8677	912.9217	912.8054	1008.252	904.5932	905.5344
	std	0	34.89239	107.5417	2.35E−08	234.679	265.3771	1.670271	16.53835	6.077825	20.74147	5.904611	3.075444
	ET	1.804356	1.063846	6.311677	3.261904	1.38957	1.236032	1.957807	1.41982	4.984022	2.833075	1.323421	1.669605
	Rank	1	8	12	2	11	10	3	7	6	9	4	5
C_{10}	Mean	1270.968	1590.578	2692.02	1437.894	2106.677	2098.525	1836.484	1777.094	2256.198	2369.988	2013.59	1766.646
	std	112.8325	412.6316	265.2632	60.84077	298.5021	570.468	429.5612	206.8288	309.9574	200.6485	348.9919	320.4534
	ET	1.702848	0.991027	6.264981	3.207363	1.375752	1.164882	2.280881	1.429785	5.111629	2.787736	1.357167	1.617746
	Rank	1	3	12	2	9	8	6	5	10	11	7	4
C_{11}	Mean	1102.144	1124.026	4189.355	1193.674	5769.669	1154.582	1129.463	1159.204	1154.533	1141.981	1146.624	2474.015
	std	0.58187	6.45982	2416.697	20.85742	109.2854	29.74835	23.20917	53.30428	15.94047	22.38728	15.80819	2568.811
	ET	1.505497	0.981483	6.37464	3.145267	1.320965	1.136778	1.697331	1.360972	4.933614	2.734688	1.363847	1.632728
	Rank	1	2	11	9	12	7	3	8	6	4	5	10
C_{12}	Mean	1227.936	2343.334	7.58E+08	1356.972	1 116 500	2 528 105	1 105 151	1 519 945	5 426 363	1 095 779	8588.212	649 676
	std	34.3984	1323.889	5.85E+08	42.20074	373 386.5	1 863 965	1 599 337	1 027 188	4 319 261	568 776.2	5574.514	393 704.8
	ET	1.522788	1.062879	6.209154	3.129779	1.315664	1.125769	1.800704	1.385309	4.994331	2.748337	1.39018	1.621577
	Rank	1	3	12	2	8	10	7	9	11	6	4	5
C_{13}	Mean	1306	1329.426	36 919 805	1382.314	13 582.32	8042.428	7129.179	10 963.57	17 865.5	10 719.01	7013.944	58 378.41
	std	0.894988	23.19983	57 222 140	6.114688	5836.786	5811.985	6115.004	3468.018	1645.737	4147.954	7306.492	89 972.56
	ET	1.739656	1.046108	6.281446	3.270871	1.383178	1.193222	1.937149	1.418381	5.075544	2.789648	1.404257	1.615838
	Rank	1	2	12	3	9	6	5	8	10	7	4	11
C_{14}	Mean	1403.981	1423.187	5643.2	1520.235	3535.701	1527.934	1584.776	2417.442	1605.133	5878.186	3115.468	13 829.69
	std	2.56885	12.24421	1119.612	32.63552	2342.208	42.30518	302.2913	1875.726	53.81531	1486.854	2780.418	10 065.91
	ET	1.609589	1.106832	6.304518	3.279898	1.391854	1.196807	1.848184	1.409921	5.105715	2.801147	1.424332	1.705614
	Rank	1	2	10	3	9	4	5	7	6	11	8	12
C_{15}	Mean	1650.381	1521.974	14 802.85	1500.3	7415.485	6572.399	1544.964	6138.038	1724.884	25 559.97	9559.728	4778.883
	std	53.30489	7.536983	12 967.07	0.186778	4724.37	5357.389	13.1269	1644.79	113.2876	12 641.58	5356.91	3272.896
	ET	1.532277	0.994289	6.31015	3.119683	1.313433	1.155879	2.205131	1.345113	4.878095	2.754639	1.35315	1.533785
	Rank	4	2	11	1	9	8	3	7	5	12	10	6
C_{16}	Mean	1601.117	1631.963	2047.53	1756.473	2080.719	1976.657	1832.371	1738.057	1682.598	2108.525	1947.807	1817.44
	std	0.408115	58.42219	213.7822	33.9662	180.1985	160.1949	68.84027	92.98471	40.19961	156.8351	129.9046	59.98088
	ET	1.679919	1.012746	6.354393	3.173488	1.345011	1.152327	2.155294	1.374953	4.979615	2.775434	1.379423	1.580402
	Rank	1	2	10	5	11	9	7	4	3	12	8	6
C_{17}	Mean	1719.14	1738.518	1827.274	1916.398	1809.876	1852.567	1853.514	1773.682	1762.775	1857.806	1756.302	1760.194
	std	9.27951	18.92729	12.48988	73.51088	12.05712	54.05302	87.55581	74.26512	10.69879	123.4642	6.142389	2.707453
	ET	2.138946	1.259462	6.445206	3.615316	1.566034	1.415488	2.318922	1.587765	5.741628	2.99679	1.547552	1.794696
	Rank	1	2	8	12	7	9	10	6	5	11	3	4

(*continued on next page*)

COA approaches the optimal solution with a high convergence speed and a decreasing trend during the iterations of the algorithm, which shows the high efficiency of the proposed COA approach in exploration, exploitation, and creating a balance between them during the searching process of the problem-solving space.

Table 2 (*continued*).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_{18}	Mean	1800.784	1821.362	6 109 172	2023.632	12 800.24	24 860.12	22 329.02	21 212.84	31 504.64	10 284.31	23 325.42	13 609.97
	std	0.646842	10.39659	8 077 112	58.58366	3933.784	15 582.56	12 624.87	14 825.22	6367.121	2500.631	20 955.05	7046.641
	ET	1.672837	1.021022	6.31286	3.201631	1.37345	1.18141	2.114435	1.432993	5.049462	2.774735	1.400416	1.592656
	Rank	1	2	12	3	5	10	8	7	11	4	9	6
C_{19}	Mean	1900.466	1906.001	754 594.9	2054.514	134 445.6	37 181.97	1915.789	5635.553	4898.742	43 205.63	26 610.54	6492.073
	std	0.365433	4.051826	709 246.5	60.77263	152 966	24 675.18	7.532073	6085.504	5570.514	22 823.33	37 542.78	3392.754
	ET	4.209786	2.015893	7.333061	5.241202	2.408426	2.185875	3.148839	2.444147	8.19639	3.898278	2.445585	2.6079
	Rank	1	2	12	4	11	9	3	6	5	10	8	7
C_{20}	Mean	2007.834	2042.536	2239.101	2170.251	2222.326	2221.485	2149.609	2182.156	2077.176	2271.99	2181.152	2053.815
	std	8.921894	23.49206	60.11148	54.94001	97.29023	97.15269	88.2677	55.62705	9.641058	82.95245	29.82688	10.95787
	ET	2.205817	1.251275	6.551243	3.652364	1.576621	1.443784	2.250428	1.671454	5.755097	3.013205	1.594801	1.821807
	Rank	1	2	11	6	10	9	5	8	4	12	7	3
C_{21}	Mean	2200	2257.378	2272.021	2422.388	2334.302	2317.873	2257.022	2321.56	2306.962	2380.593	2327.466	2305.331
	std	1.11E−05	63.34808	32.13155	82.59625	75.63138	66.25176	65.84355	4.049776	69.14021	15.60657	8.239775	51.87787
	ET	2.164595	1.284946	6.509065	3.641406	1.571497	1.383807	2.096905	1.619051	5.661371	3.025062	1.52408	1.808687
	Rank	1	3	4	12	10	7	2	8	6	11	9	5
C_{22}	Mean	2281.06	2304.053	2961.228	2300.351	2744.358	2325.549	2356.466	2309.229	2321.01	2480.373	2314.243	2319.245
	std	37.8806	1.78132	163.7741	0.315218	226.4641	5.908907	103.168	10.44165	8.848757	138.9628	23.1002	3.37026
	ET	2.656802	1.400821	6.626836	3.864473	1.772597	1.497466	2.13106	1.726702	5.981163	3.148835	1.693244	1.901028
	Rank	1	3	12	2	11	8	9	4	7	10	5	6
C_{23}	Mean	2881.61	2629.183	2708.158	2610.863	2732.714	2652.379	2621.726	2614.711	2645.742	2806.265	2647.616	2660.372
	std	104.6553	10.55426	35.0647	2.178856	64.86995	22.04341	11.58641	7.06418	9.556135	102.8952	9.372923	14.49226
	ET	2.907601	1.3512	6.619369	4.024382	1.771668	1.532794	2.16684	1.771407	6.15411	3.16642	1.730208	1.93059
	Rank	12	4	9	1	10	7	3	2	5	11	6	8
C_{24}	Mean	2500	2628.429	2866.473	2740.567	2671.146	2770.447	2687.309	2757.72	2765.308	2806.444	2775.772	2730.119
	std	6.49E−05	148.1166	40.74193	163.6047	164.5281	19.41927	125.0465	17.88321	3.526871	85.56431	14.04169	138.498
	ET	2.864576	1.404424	6.672468	4.04554	1.750173	1.593779	2.191942	1.941247	6.276043	3.1959	1.831297	2.006232
	Rank	1	2	12	6	3	9	4	7	8	11	10	5
C_{25}	Mean	2861.108	2922.277	3302.497	3082.712	3148.634	2960.642	2921.314	2939.149	2933.596	2921.498	2922.641	2953.708
	std	73.26884	27.1567	65.75238	153.584	381.3259	8.926043	27.14927	13.18703	20.74106	25.30253	27.11627	9.803424
	ET	2.373467	1.296469	6.583291	3.850655	1.751549	1.459254	2.094637	1.714968	5.942937	3.088336	1.63839	1.884882
	Rank	1	4	12	10	11	9	2	7	6	3	5	8
C_{26}	Mean	2779.982	2913.423	3820.788	2875.002	3675.06	3204.325	2900.159	3292.822	3229.716	4002.126	2953.29	3031.637
	std	91.636	90.24848	306.4302	49.9964	591.744	313.5334	0.038472	464.3667	482.8277	634.849	130.0464	69.84838
	ET	3.383959	1.486795	6.79877	4.180395	1.919745	1.651185	2.237936	1.87562	6.472393	3.240714	1.812288	2.125065
	Rank	1	4	11	2	10	7	3	9	8	12	5	6
C_{27}	Mean	3401.417	3108.049	3241.788	3089.081	3186.307	3202.862	3091.787	3118.113	3117.018	3236.309	3139.581	3165.314
	std	85.97019	9.908994	141.0072	0.151275	58.14708	12.41189	2.657404	43.53638	40.27925	16.13351	39.02481	45.27772
	ET	3.554295	1.435097	6.868196	4.266612	1.867463	1.694939	2.283099	1.930563	6.715525	3.329278	1.842092	2.078829
	Rank	12	3	11	1	8	9	2	5	4	10	6	7
C_{28}	Mean	3318.181	3206.371	3829.484	3100	3622.266	3300.63	3248.995	3363.069	3341.745	3476.706	3320.885	3257.183
	std	65.46377	125.1841	70.64911	7.42E−05	213.3328	131.4528	172.1895	108.4243	90.53327	15.77081	103.9299	191.9303
	ET	2.893927	1.359539	6.806764	4.061893	1.768222	1.598092	2.209172	1.823445	6.356397	3.247905	1.729504	2.013281
	Rank	6	2	12	1	11	5	3	9	8	10	7	4
C_{29}	Mean	3137.561	3179.875	3393.557	3492.725	3244.114	3365.261	3208.026	3275.244	3218.734	3362.024	3276.182	3245.186
	std	3.221487	18.65754	76.78127	63.00375	61.8325	117.2462	65.61926	96.88423	35.00934	208.2333	88.34778	44.37193
	ET	1.888947	1.538882	6.831717	4.165327	1.837098	1.635945	2.236849	1.847688	6.525595	3.281418	1.820307	2.044941
	Rank	1	2	11	12	5	10	3	7	4	9	8	6
C_{30}	Mean	3401.235	7103.166	3 935 476	3765.944	657 714.8	1 062 039	324 051.9	1 001 729	64 677.52	837 788.8	414 393.2	1 635 020
	std	6.935 657	5757.626	2 231 806	96.37005	540 064.4	1 967 742	608 283.3	664 431.3	37 893.81	176 988.7	469 896.9	1 490 657
	ET	2.459816	2.090883	7.659937	5.889492	2.689453	2.51453	3.216611	2.727053	9.068879	4.098566	2.715899	2.887235
	Rank	1	3	12	2	7	10	5	9	4	8	6	11
Sum Rank		59	96	317	150	273	238	125	183	192	252	182	195
Mean Rank		2.034483	3.310345	10.93103	5.172414	9.413793	8.206897	4.310345	6.310345	6.62069	8.689655	6.275862	6.724138
Total Rank		1	2	12	4	11	9	3	6	7	10	5	8

4.2. The benchmark set and compared algorithms

Fifty-one standard benchmark functions have been employed to test the COA's ability to handle various objective functions such as the IEEE CEC-2017 [65] and the IEEE CEC-2011 [66]. Complete information on these objective functions is provided in the Appendix. To analyze the quality of COA in providing the optimal solution, its performance is compared with eleven well-known algorithms, namely RSA, MVO, WSO, GSA, MPA, PSO, GA, TSA, GWO, WOA, and TLBO. Optimization results are reported using four indicators: mean, standard deviation (std), rank, and execution time (ET). The control parameters of all competitor algorithms are specified in Table 1.

4.3. Evaluation of the CEC-2017 suite test

The performance of the COA in addressing optimization issues has been challenged on the up-to-date test functions of the CEC-2017 test suite. The CEC-2017 set has thirty standard benchmark functions of different types in four groups:

1. Three unimodal functions including C1 to C3.
2. Seven multimodal functions including C4 to C10.

Table 3
Assessment results of the CEC-2017 objective functions (dimension $m = 30$).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_1	Mean	13 192.16	8.63E+09	4.38E+10	29 972.79	1.91E+10	1.81E+09	572 915.6	1.78E+09	6.57E+09	11 193 569	1.5E+09	1.9E+08
	std	20 128.12	5.69E+09	6.83E+09	16 716.84	6.56E+09	4.2E+08	140 175.6	2.4E+09	2.36E+09	18 781 558	2.99E+09	52 024 553
	ET	3.218782	1.304884	16.48146	4.009595	1.943186	1.343995	2.564344	2.286843	5.168074	9.150027	1.627347	1.886391
	Rank	1	10	12	2	11	8	3	7	9	4	6	5
C_3	Mean	1155.064	42 375.1	78 405.91	1264.732	50 270.55	247 087.6	1872.07	44 375.79	36 933.57	102 114.9	33 960.65	178 162.3
	std	242.255	11 011.05	11 832.89	231.2389	2676.977	33 044.06	493.2663	4429.61	3855.066	11 070.13	8836.597	53 510.61
	ET	3.440744	1.263815	16.514	3.922917	1.97258	1.305722	2.236516	2.229595	5.213546	4.488007	1.593242	1.849118
	Rank	1	6	9	2	8	12	3	7	5	10	4	11
C_4	Mean	494.1826	841.7389	10 440.38	528.1157	4813.977	883.212	508.5892	579.3307	937.4558	603.7402	635.048	835.3451
	std	13.36359	127.7259	3289.65	16.89845	2929.477	71.14629	28.34223	40.74772	289.8164	20.33317	145.3182	37.92411
	ET	2.29552	1.221878	16.34037	3.842284	1.959314	1.291446	2.585777	2.137474	5.276378	4.353586	1.637337	1.748408
	Rank	1	8	12	3	11	9	2	4	10	5	6	7
C_5	Mean	632.7583	653.5151	909.9791	588.7215	813.769	845.022	627.3851	630.0453	788.5984	737.7933	641.4435	715.969
	std	27.7517	39.32124	30.80919	20.43314	31.33511	20.90352	25.16397	36.54903	25.31423	21.72216	35.74522	49.55192
	ET	3.79753	1.420104	16.30239	4.219738	2.135077	1.482399	2.743176	2.414575	5.709299	4.668204	1.765746	1.928986
	Rank	4	6	12	1	10	11	2	3	9	8	5	7
C_6	Mean	603.4788	644.2901	688.3251	660.6868	681.918	681.1026	625.952	612.6508	646.0847	660.113	649.9277	632.1073
	std	1.237184	6.086035	5.885883	11.54253	12.18317	7.929293	12.31866	6.264262	8.755362	0.814301	10.79808	5.394238
	ET	4.96856	1.971638	16.83683	5.491943	2.717874	2.070499	3.427208	2.927015	7.549411	5.089443	2.314395	2.519413
	Rank	1	5	12	9	11	10	3	2	6	8	7	4
C_7	Mean	832.3448	1028.891	1378.659	854.5801	1256.429	1344.785	929.3064	910.2218	1098.296	987.1371	888.3186	983.1542
	std	25.87207	97.43161	17.50672	38.98991	136.7428	61.58851	41.75679	25.27516	91.9537	55.15938	22.49433	42.12189
	ET	3.73344	1.573064	16.51654	4.44808	2.201025	1.568279	3.016487	2.40867	5.808502	4.540416	1.904321	1.982735
	Rank	1	8	12	2	10	11	5	4	9	7	3	6
C_8	Mean	881.3244	923.7029	1143.549	929.7729	1076.072	1045.627	934.4239	898.3978	1037.341	972.3755	932.0245	998.6785
	std	10.31683	22.36035	25.97783	61.08731	74.75508	44.93796	25.13817	7.129075	24.28483	24.10391	13.30378	19.84
	ET	11.17564	1.475028	16.44981	4.313289	2.171037	1.536237	2.78404	2.318408	5.805171	4.630729	1.834799	1.966238
	Rank	1	3	12	4	11	10	6	2	9	7	5	8
C_9	Mean	1055.207	6940.902	11 265.22	1160.117	12 185.44	11 692.96	5754.247	2155.322	6093.246	4288.668	3721.958	1353.996
	std	108.8082	1601.414	189.314	146.5822	3751.348	2531.489	2055.778	685.9403	2192.603	641.2085	1488.338	169.4089
	ET	3.20711	1.434278	16.50098	4.362066	2.203161	1.566925	2.798778	2.39086	5.963327	4.527136	1.796131	1.967704
	Rank	1	9	10	2	12	11	7	4	8	6	5	3
C_{10}	Mean	4032.979	5070.563	8287.882	4273.835	6852.655	6784.772	4974.673	4960.552	8308.86	5023.915	5228.975	6407.284
	std	376.7088	939.508	649.1055	244.9336	979.3841	1072.938	333.4433	400.2456	268.2536	323.6773	326.2744	547.0753
	ET	4.9485	1.43678	16.43479	4.532878	2.317967	1.623748	2.898713	2.461401	6.234178	4.604056	2.097938	2.083909
	Rank	1	6	11	2	10	9	4	3	12	5	7	8
C_{11}	Mean	1174.444	1448.116	9328.214	1281.2	5405.535	8274.198	1328.929	2269.803	2048.352	3021.119	1259.645	9720.26
	std	37.46779	121.068	1341.609	66.89519	1970.032	2772.506	51.28764	1527.23	536.6133	668.1668	29.94469	6346.306
	ET	2.7049	1.352691	16.33456	4.090852	2.100505	1.409705	2.371073	2.316734	5.643277	4.387027	1.736832	1.899494
	Rank	1	5	11	3	9	10	4	7	6	8	2	12
C_{12}	Mean	25 288.54	85 200 129	1.17E+10	23 008.16	5.42E+09	2.65E+08	12 003 293	56 178 065	3.23E+08	2.13E+08	2 740 246	8 216 781
	std	5951.198	77 647 077	2.04E+09	5537.264	1.86E+09	2.12E+08	11 376 457	48 983 938	1.61E+08	3.12E+08	2 222 416	2 297 663
	ET	3.59283	1.406575	16.48504	4.331025	2.173524	1.515228	2.748809	2.311146	5.874035	4.615434	1.906525	1.960998
	Rank	2	7	12	1	11	9	5	6	10	8	3	4
C_{13}	Mean	1929.051	42 943.25	1.13E+10	2162.399	1.56E+09	966 572.5	97 128.62	806 757.7	94 243 667	38 902.02	34 509.45	12 733 888
	std	392.7499	29 451.87	3.62E+09	490.2029	2.6E+09	507 379.9	73 433.77	1 145 937	31 811 620	12 177.81	29 394.28	10 257 654
	ET	2.30465	1.309867	16.42227	4.225391	2.156006	1.479912	2.704063	2.392655	5.71325	4.523365	1.798766	1.922749
	Rank	1	5	12	2	11	8	6	7	10	4	3	9
C_{14}	Mean	1441.59	1676.266	2 344 304	1561.608	1 253 660	2 373 386	21 610.64	568 944	149 211.9	1 220 811	19 931.08	2 142 684
	std	3.984674	205.8048	1 112 536	11.59089	401 150.7	3 312 115	13 625.13	600 535	41 707.85	494 689.9	14 495.67	1 502 574
	ET	4.56792	1.566447	16.5747	4.695981	2.306656	1.653 657	2.725056	2.794987	6.228647	4.771402	2.025846	2.116047
	Rank	1	3	11	2	9	12	5	7	6	8	4	10
C_{15}	Mean	1626.687	4778.296	6.38E+08	1778.637	15 334 685	5 381 347	45 540.28	16 883 687	5 476 665	17 042.7	4998.083	1 019 371
	std	26.87863	2216.528	75 373 693	83.41449	13 677 702	8 171 053	21277	30 903 101	3 717 340	4629.296	3294.051	959 602.1
	ET	3.59765	1.377813	16.25109	4.082402	2.056317	1.513005	2.611331	2.442912	5.436479	4.393152	1.74878	1.871535
	Rank	1	3	12	2	10	8	6	11	9	5	4	7
C_{16}	Mean	2247.447	2652.321	5197.758	2052.194	3380.028	4417.333	2650.651	2603.495	3585.431	3800.762	3016.217	3036.419
	std	239.1718	180.599	848.5357	264.8006	316.2617	710.3575	237.4922	149.9208	203.7173	165.3951	285.388	362.9403
	ET	3.52186	1.474035	16.5913	4.26574	2.150607	1.613688	2.761878	2.422901	5.790602	4.47891	1.835975	1.976617
	Rank	2	5	12	1	8	11	4	3	9	10	6	7
C_{17}	Mean	1863.83	2208.309	3850.265	1986.07	3381.412	2928.1	2108.117	2006.705	2229.567	2581.32	2380.227	2188.707
	std	80.49175	262.7278	511.0619	129.575	1966.658	368.9942	107.5831	133.7297	238.7003	131.3936	303.0418	57.50817
	ET	5.73065	2.078272	17.03146	5.538777	2.733683	2.095749	3.3172	2.935717	7.503896	5.352274	2.447915	2.534663
	Rank	1	6	12	2	11	10	4	3	7	9	8	5
C_{18}	Mean	1901.733	73 105.14	34 857 872	2087.727	38 762 645	6 294 812	682 499.4	447 364.1	1 776 900	549 137.3	146 230.4	3 888 623
	std	17.295	43 259.8	24 260 902	47.4737	40 002 539	4 671 695	781 823	501 738.7	647 867.6	351 297.8	30 394.44	1 221 489
	ET	3.6461	1.386549	16.50794	4.439156	2.169175	1.520467	2.629998	2.305029	5.727457	4.481437	1.855193	1.974733
	Rank	1	3	11	2	12	10	7	5	8	6	4	9

(continued on next page)

Table 3 (*continued*).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_{19}	Mean	1924.713	7634.682	1.04E+09	2135.514	3.14E+08	15 282 801	1 001 928	4 302 077	6 134 774	87 089.66	47 334.27	1 729 099
	std	3.692 442	2126.603	3.68E+08	65.0383	4E+08	11 127 256	1 084 106	6 424 126	2 723 212	29 169.63	63 346.56	1 007 456
	ET	10.1975	4.370609	19.42097	10.65424	5.221357	4.589672	5.773174	5.404539	14.96195	7.703112	4.994248	5.057154
	Rank	1	3	12	2	11	10	6	8	9	5	4	7
C_{20}	Mean	2184.966	2360.68	3018.944	2464.392	2908.489	2895.216	2645.569	2398.388	2852.875	3079.815	2583.379	2506.176
	std	88.64055	94.38145	133.7757	70.50871	132.8081	181.5602	295.8071	151.1479	109.4276	379.9235	97.0379	35.02764
	ET	7.60978	2.004038	17.23006	5.746742	2.850805	2.23519	3.468045	3.209074	7.935644	5.209856	2.463929	2.661997
	Rank	1	2	11	4	10	9	7	3	8	12	6	5
C_{21}	Mean	2350.368	2451.525	2708.062	2534.461	2584.239	2633.329	2412.664	2446.374	2509.744	2590.171	2444.263	2506.648
	std	28.27442	20.94728	79.98545	185.2684	112.0961	75.95313	29.47147	74.80206	11.56261	25.92502	17.52546	38.25462
	ET	6.8376	2.275384	17.31182	6.164841	3.042045	2.413057	3.652132	3.205279	8.44079	5.405171	2.760043	2.857065
	Rank	1	5	12	8	9	11	2	4	7	10	3	6
C_{22}	Mean	2303.148	3959.433	8152.075	2533.335	9262.883	7791.534	4079.421	2734.521	5940.652	6631.627	5079.773	2732.552
	std	1.320014	1461.304	954.9404	60.29863	171.9551	809.1836	2075.353	194.3766	3656.965	1678.839	2362.05	70.80185
	ET	6.19179	2.542078	17.58437	6.58743	3.251819	2.626755	3.880485	3.404116	9.152478	5.615678	2.885135	3.054332
	Rank	1	5	11	2	12	10	6	4	8	9	7	3
C_{23}	Mean	2641.551	3077.843	3294.999	2857.005	3240.91	3079.343	2825.783	2759.202	2924.384	3858.628	2920.73	3001.7
	std	123.5185	100.1635	62.73727	181.4107	150.9637	133.0817	131.4231	18.74713	36.94432	123.4437	42.27833	46.18844
	ET	7.20132	2.621138	17.69085	6.897547	3.458376	2.866457	4.104512	3.657416	9.707524	5.821798	3.089022	3.298424
	Rank	1	8	11	4	10	9	3	2	6	12	5	7
C_{24}	Mean	2879.028	3408.484	3463.698	3099.812	3318.291	3140.134	2983.509	2928.416	3059.868	3408.418	3156.023	3259.174
	std	16.47476	96.59341	122.5211	164.2185	81.32298	46.62727	108.4055	9.558574	27.79208	35.88437	88.21771	87.43201
	ET	7.11195	2.809718	17.86842	7.378802	3.654539	3.042872	4.275243	3.846216	10.42713	6.222771	3.2371	3.457234
	Rank	1	11	12	5	9	6	3	2	4	10	7	8
C_{25}	Mean	3119.933	3083.84	4702.903	2891.658	3517.16	3097.791	2911.381	3002.021	3090.058	3004.331	2895.632	3125.703
	std	108.486	26.82682	635.4398	6.330068	407.6933	28.34088	46.42401	54.66619	133.6655	10.77907	12.50025	13.78007
	ET	6.04937	2.658217	17.86822	6.815261	3.390773	2.765187	4.046097	3.565367	9.740856	5.808113	2.964355	3.242644
	Rank	9	6	12	1	11	8	3	4	7	5	2	10
C_{26}	Mean	2900.361	7030.026	10 262.03	3184.06	9145.999	8760.018	4883.754	4643.924	6111.459	7785.852	4950.062	4459.208
	std	0.181233	1138.534	1181.029	93.97554	501.4022	704.9311	491.6056	464.182	1336.487	805.9973	1437.638	388.5703
	ET	8.30017	3.181723	18.20593	7.909905	3.931616	3.485475	4.563493	4.192108	11.37455	6.425178	3.491959	4.0234
	Rank	1	8	12	2	11	10	5	4	7	9	6	3
C_{27}	Mean	3412.903	3427.03	3811.95	3215.237	3495.63	3445.775	3233.418	3253.37	3327.109	5117.272	3284.576	3480.593
	std	53.55879	143.5802	264.1424	17.30317	185.2389	137.5661	20.67544	10.96328	66.74832	413.4343	37.8571	58.09403
	ET	8.6109	3.129562	18.85547	8.60177	4.290813	3.762545	4.927789	4.498036	12.8161	6.693306	3.975765	4.528151
	Rank	6	7	11	1	10	8	2	3	5	12	4	9
C_{28}	Mean	3218.819	3608.074	5904.207	3425.277	4245.736	3466.595	3269.546	3638.419	3716.866	3556.346	3399.175	3622.987
	std	25.17824	160.5476	328.4469	150.0103	566.0832	54.84804	30.83298	354.1798	251.5693	109.5052	140.7534	56.07654
	ET	9.95368	3.066995	18.28915	7.850114	3.871084	3.309712	4.519823	4.079317	11.3554	6.226844	3.509667	3.948564
	Rank	1	7	12	4	11	5	2	9	10	6	3	8
C_{29}	Mean	3637.205	3986.726	5807.492	3973.563	5397.736	5230.816	3883.662	3826.885	4606.718	5204.786	4236.031	4364.774
	std	97.55547	74.30865	799.9387	247.1793	724.6104	205.3706	113.1259	100.0287	375.219	309.7752	199.105	357.878
	ET	8.3956	2.819085	17.98335	7.133749	3.572183	3.105965	4.303773	3.728723	10.63919	6.304657	3.10057	3.456765
	Rank	1	5	12	4	11	10	3	2	8	9	6	7
C_{30}	Mean	7524.216	137 769.6	3.03E+09	8384.435	41 221 542	42 064 196	3 317 671	6 841 959	40 608 917	2 426 811	292 115.4	752 650.9
	std	1292.389	56 915.13	5.7E+08	1573.644	37 323 575	24 600 435	1 853 120	7 828 761	29 882 407	345 090	543 106.1	599 911.5
	ET	10.58424	4.850729	20.51795	12.15729	6.131017	5.462438	6.672822	6.23661	18.04703	8.628704	5.622371	6.123689
	Rank	1	3	12	2	10	11	7	8	9	6	4	5
Sum Rank		47	168	335	81	300	276	125	138	230	223	139	200
Mean Rank		1.62069	5.793103	11.55172	2.793103	10.34483	9.517241	4.310345	4.758621	7.931034	7.689655	4.793103	6.896552
Total Rank		1	6	12	2	11	10	3	4	9	8	5	7

3. Ten hybrid functions including C11 to C20.
4. Ten composition functions including C21 to C30.

We do not use the test function C2 from the CEC-2017 set due to its unstable behavior (similarly as other authors in their papers). The details and complete information of these test functions are stated in [65]. The proposed COA approach and competitor algorithms are implemented on CEC-2017 test functions in fifty-one independent implementations where each execution contains $10,000 \cdot m$ of function evaluations (FEs). Experiments are performed for different dimensions of test functions equal to 10, 30, 50, and 100. The optimization results of benchmark functions of the CEC-2017 set using COA and competitor algorithms are reported in Tables 2, 3, 4, and 5. The boxplots of the performance of COA and competitor algorithms in handling the CEC-2017 benchmark set functions for different values of dimension are shown in Figs. 6, 7, 8, and 9. Analysis of the simulation results for dimensions equal to ten ($m = 10$) shows that the proposed COA approach, in dealing with C1, C3 to C14, C16 to C22, C24 to C26, C29, and C30 functions, has been ranked as the first best optimizer compared to competitor algorithms. The implementation results of COA and competitor algorithms on CEC-2017 for dimensions equal to thirty ($m = 30$), show that the proposed COA approach is the first best optimizer to solve C1, C3, C4, C6 to C11, C13 to C15, C17 to C24, C26, and C28 to C30 functions. What is evident from the results of the CEC-2017 set optimization for dimensions equal to fifty ($m = 50$) is that the COA is the first best optimizer to solve C1, C3 to C8, C10 to C14, C16, C17, C19 to C22, C24, C25, C27, and C29. Based on the results obtained from the recruitment of COA and competitor algorithms on CEC-2017 for dimensions equal to one hundred ($m = 100$), it is concluded that the proposed COA approach is the first best optimizer to handle

Table 4
Assessment results of the CEC-2017 objective functions (the dimension $m = 50$).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_1	Mean	3 315 269	2.57E+10	9.96E+10	6 231 529	4.05E+10	8.19E+09	4 427 552	9.95E+09	2.21E+10	1.82E+10	2.69E+09	1.11E+10
	std	1 075 756	6.64E+09	9.48E+09	6 021 780	2.6E+09	3.51E+09	987 966.2	2.69E+09	7.16E+09	2.97E+09	1.09E+09	6.48E+08
	ET	4.13013	2.059145	28.36328	5.273381	2.952156	2.034547	3.944988	3.213452	6.750656	6.569285	2.657902	2.457211
	Rank	1	10	12	3	11	5	2	6	9	8	4	7
C_3	Mean	19 036.24	109 166.7	167 728.7	20 498.07	115 914.1	248 566.9	49 123.19	138 065	104 420.9	189 072.7	153 776	279 869.1
	std	2706.178	17 915.62	13 624.19	2986.359	10 051.85	90 299.03	9239.722	13 766.59	18 348.63	20 728.75	36 788.11	36 170.69
	ET	5.60337	1.917142	30.99464	5.280301	2.970793	1.870235	3.278145	3.204619	6.799276	6.592748	2.194544	2.317156
	Rank	1	5	9	2	6	11	3	7	4	10	8	12
C_4	Mean	520.8665	4628.003	25 211.16	576.142	8801.481	2031.626	581.638	1496.227	2939.819	3225.817	1050.067	1591.141
	std	21.64596	1751.262	6158.629	40.42898	1828.486	525.392	52.90703	329.7703	1494.052	357.6048	570.9406	154.3357
	ET	4.55062	1.928033	27.98853	5.349814	3.049928	1.863984	3.835268	3.194857	6.76349	6.626663	2.163599	2.298814
	Rank	1	10	12	2	11	7	3	5	8	9	4	6
C_5	Mean	723.5615	825.0179	1166.95	769.1347	1185.912	983.3821	770.1379	772.1105	1028.885	824.3543	806.2324	915.3559
	std	51.09782	23.67232	15.67259	35.31616	132.3679	31.00758	80.82595	70.13627	32.92175	44.50277	51.93749	25.75455
	ET	5.81933	2.37249	27.62056	5.81124	3.37138	2.246185	4.144463	3.478511	7.676686	6.844373	2.4271	2.607201
	Rank	1	7	11	2	12	9	3	4	10	6	5	8
C_6	Mean	612.0016	650.0934	702.8348	673.5121	695.0745	703.5169	639.8201	624.1169	667.9101	661.2626	656.6625	651.3333
	std	2.927746	2.809484	2.578301	24.30849	17.41024	6.2449	17.25255	7.379143	9.676579	3.64112	2.792805	11.30702
	ET	5.97599	3.39029	28.89353	7.869561	4.273647	3.156906	5.138662	4.504524	10.65626	7.780686	3.491476	3.640336
	Rank	1	4	11	9	10	12	3	2	8	7	6	5
C_7	Mean	1037.816	1634.936	1959.322	1133.771	1737.276	1764.136	1071.383	1083.897	1521.168	1449.5	1244.488	1339.57
	std	48.52962	272.8645	83.88886	45.53244	149.2211	73.90752	30.75051	20.87074	91.14827	142.0339	158.5392	60.65699
	ET	6.56278	2.407271	27.61007	6.153847	3.377765	2.333907	4.45428	3.612121	7.827493	6.877402	2.499854	2.656767
	Rank	1	9	12	4	10	11	2	3	8	7	5	6
C_8	Mean	999.1181	1119.935	1485.684	1064.58	1474.849	1355.599	1070.418	1047.52	1352.89	1160.173	1071.442	1284.63
	std	12.78859	43.54379	23.96556	20.53437	106.7519	106.3228	40.10384	34.47188	49.09583	10.62307	54.7899	30.845
	ET	6.37259	2.289046	27.35975	6.141061	3.30834	2.329043	4.238619	3.583135	7.823168	6.965961	2.488757	2.776774
	Rank	1	6	12	3	11	10	4	2	9	7	5	8
C_9	Mean	3807.053	31 574.54	38 384.66	3463.694	40 053.85	34 861.7	20 744.86	7215.716	25 373.65	11 230.46	10 828.21	13 478.69
	std	1409.831	4276.902	1998.844	1200.261	3346.549	3963.398	7704.472	1018.247	4293.626	792.2221	1028.623	2360.658
	ET	6.62697	2.214192	27.50515	6.138456	3.380792	2.297848	4.362373	3.646226	7.813568	6.872742	2.473953	2.66579
	Rank	2	9	11	1	12	10	7	3	8	5	4	6
C_{10}	Mean	6474.525	7403.704	14 818.96	7217.105	12 231.33	12 239.55	7990.934	8939.629	14 594.19	8868.741	8011.769	12 156.26
	std	642.9051	515.452	286.0404	834.3217	897.3872	1074.015	892.1587	3701.027	705.0441	837.357	388.2711	621.4052
	ET	8.1653	2.257842	27.66271	6.422074	3.509917	2.410516	4.683054	3.761256	8.420045	7.178027	2.707651	2.9727
	Rank	1	3	12	2	9	10	4	7	11	6	5	8
C_{11}	Mean	1263.128	4491.299	22 214.14	1380.542	13 659.7	5341.405	1596.298	6438.624	5358.279	15 005.37	1705.645	25 425.49
	std	37.26383	1664.8	1809.233	64.20378	2019.416	921.7387	141.0849	3416.178	438.455	1355.471	272.4961	7978.965
	ET	4.84691	2.005484	27.85837	5.617448	3.130142	2.038194	3.732305	3.37602	7.292931	6.67387	2.368867	2.68521
	Rank	1	5	11	2	9	6	3	8	7	10	4	12
C_{12}	Mean	14 734 349	5.27E+09	7.57E+10	16 412 091	2.75E+10	1.41E+09	84 254 444	1.02E+09	5.37E+09	2.31E+09	1.71E+09	2.17E+08
	std	1 056 181	3.48E+09	2.24E+10	953 588.9	1.43E+10	3.46E+08	37 283 653	8.64E+08	3.53E+09	1.4E+09	2.29E+09	1.02E+08
	ET	6.50826	2.262422	27.51985	6.062683	3.312017	2.223965	4.210764	3.558431	7.861787	6.9194	2.654364	2.802153
	Rank	1	9	12	2	11	6	3	5	10	8	7	4
C_{13}	Mean	17 284.75	7.45E+08	4.58E+10	19 097.95	1.07E+10	1.01E+08	256 531.1	3.8E+08	6.23E+08	19 712 046	5.08E+08	44 165 345
	std	5458.85	1.08E+09	1.79E+10	6259.464	4.65E+09	17 133 428	102 257.1	3.84E+08	1.55E+08	31 690 714	6.25E+08	13 504 471
	ET	5.82105	2.44872	27.28862	5.763	3.217029	2.114405	4.242541	3.474091	7.502669	6.748075	2.515707	2.58898
	Rank	1	10	12	2	11	6	3	7	9	4	8	5
C_{14}	Mean	1571.45	954 429.1	51 521 240	1711.819	2 860 340	5 075 501	203 088	1 225 810	921 246	16 125 876	610 875.9	11 934 319
	std	18.22786	929 949.3	37 650 671	68.63317	1 567 865	664 096.5	127 906.7	926 827.3	158 215.7	10 346 083	310 664.5	6 192 749
	ET	7.25617	2.28374	27.36955	6.409105	3.549084	2.426521	4.2535	3.838332	8.41478	7.047098	2.838305	2.888709
	Rank	1	6	12	2	8	9	3	7	5	11	4	10
C_{15}	Mean	2538.389	84 644.13	4.46E+09	2307.959	1.82E+09	10 563 106	129 124.5	6 333 825	75 128 116	2.1E+08	11 457.38	9 130 463
	std	242.9841	66 115.44	7.97E+08	158.2228	1.55E+09	8 241 610	61 896.63	7 259 774	22 473 942	4.04E+08	8781.041	7 391 546
	ET	14.79132	2.006763	26.89541	5.60423	3.125149	2.026338	4.000995	3.399974	7.227762	6.804952	2.329487	2.568064
	Rank	2	4	12	1	11	8	5	6	9	10	3	7
C_{16}	Mean	2776.232	3413.809	7943.425	3014.868	4809.635	5715.393	3410.118	3485.642	4705.426	4079.109	3424.383	4034.449
	std	172.4686	530.6386	2755.255	131.7112	401.0982	783.4614	213.6592	450.749	333.7934	375.0628	453.8043	512.5131
	ET	6.13545	2.168005	27.28086	5.838524	3.253143	2.164184	4.341446	3.481773	8.050005	7.06325	2.472514	2.643974
	Rank	1	4	12	2	10	11	3	6	9	8	5	7
C_{17}	Mean	2593.645	3002.378	11 639.65	2855.9	4083.25	4696.997	3138.658	3029.826	4287.214	3937.182	3437.399	3686.236
	std	48.0614	464.2714	2763.849	128.0975	571.6828	337.675	459.8173	197.6878	471.9461	348.598	279.2906	241.5093
	ET	6.82358	3.279516	28.97495	7.712769	4.210812	3.104564	5.18683	4.411553	10.48736	8.238313	3.405478	3.57624
	Rank	1	3	12	2	9	11	5	4	10	8	6	7
C_{18}	Mean	30 548.72	2 805 300	1.2E+08	27 838.33	37 345 298	48 133 103	2 813 167	6 098 300	8 737 390	8 959 584	877 999.5	10 091 026
	std	18 042.09	2 146 247	55 161 299	16 523.08	47 492 083	36 644 851	1 302 315	5 740 922	2 595 996	5 701 618	488 766.7	9 534 743
	ET	5.91065	2.047649	27.16928	5.922221	3.258925	2.147991	4.263039	3.466858	7.574409	6.982974	2.565446	2.65009
	Rank	2	4	12	1	10	11	5	6	7	8	3	9

(*continued on next page*)

Table 4 (*continued*).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_{19}	Mean	2092.182	159 032.8	4.09E+09	2308.363	2.85E+09	7 289 229	5 459 947	1 238 917	54 012 474	481 438.7	419 243.9	1 056 490
	std	44.64316	178 299	1.02E+09	70.46027	3.72E+09	6 911 694	1 067 739	542 949.2	10 120 202	382 629.3	497 981.2	285 419.2
	ET	18.45071	7.113569	32.21721	16.04018	8.341345	7.271696	9.803923	8.6811	24.11799	12.13466	7.579072	7.912256
	Rank	1	3	12	2	11	9	8	7	10	5	4	6
C_{20}	Mean	2615.441	3273.289	4223.731	2852.916	3518.297	3854.667	3352.622	2760.307	3881.775	4165.34	3362.401	3233.289
	std	210.8097	507.0025	204.2398	248.8347	324.1626	432.7806	345.1429	229.051	136.691	255.2723	290.8029	95.60353
	ET	6.41316	3.219183	28.25861	8.188441	4.448716	3.327005	5.605411	4.676373	11.69909	8.00799	3.614669	3.802783
	Rank	1	5	12	3	8	9	6	2	10	11	7	4
C_{21}	Mean	2458.266	2750.03	3080.695	2694.11	3002.147	2992.409	2591.155	2535.186	2856.992	2878.604	2682.334	2779.856
	std	25.03703	100.6273	97.29591	69.4487	127.3505	96.22577	41.02789	44.48903	34.80494	53.10801	83.82384	22.0109
	ET	10.40283	4.170407	29.33839	10.19363	5.345634	4.241401	6.332394	5.69715	14.44177	8.949361	4.481806	4.821397
	Rank	1	6	12	5	11	10	3	2	8	9	4	7
C_{22}	Mean	4441.218	10 601.63	17 156.72	5807.981	14 474.94	14 396.62	9400.445	9267.124	16 567.01	12 006.95	10 212.34	9283.242
	std	2878.434	587.4619	202.7537	3858.758	494.792	441.4524	1624.951	719.5638	658.3144	208.2034	598.0722	5491.014
	ET	12.5758	4.408645	29.66646	10.83081	5.685521	4.609728	6.547461	5.968059	15.27601	9.208929	4.874647	5.142972
	Rank	1	7	12	2	10	9	5	3	11	8	6	4
C_{23}	Mean	3157.258	3742.803	3984.37	2900.93	3819.804	3822.535	3007.637	3040.969	3322.401	4909.23	3425.242	3409.601
	std	105.4818	215.66	41.04961	15.42863	281.7046	136.2482	59.59619	103.4839	70.91921	161.8938	71.34462	118.3325
	ET	11.61551	4.687739	30.57443	12.09766	6.401262	5.302998	7.469741	6.647262	17.54353	10.04646	5.525315	5.679863
	Rank	4	8	11	1	9	10	2	3	5	12	7	6
C_{24}	Mean	3075.367	4142.676	4611.799	3227.629	4092.17	3902.947	3258.678	3221.341	3490.673	4499.5	3506.655	3724.005
	std	30.27965	355.0229	869.707	95.14867	116.7817	82.71185	185.4599	103.791	68.70402	44.92477	152.9553	71.36583
	ET	14.68311	5.233271	30.62493	12.74456	6.690299	5.620654	7.720118	6.944134	19.02458	10.27779	5.857397	6.289918
	Rank	1	10	12	3	9	8	4	2	5	11	6	7
C_{25}	Mean	3053.663	4544.506	12 627.24	3245.97	6239.607	4243.786	3196.936	4114.156	4479.892	4376.108	3132.178	4129.964
	std	22.9098	632.2521	1922.174	187.9645	1015.467	328.6974	167.5165	225.3022	586.7835	511.0707	52.01119	102.4012
	ET	12.37096	5.392232	30.53703	12.60585	6.584492	5.493316	7.485713	6.778405	18.16804	10.14696	5.779912	5.918255
	Rank	1	10	12	4	11	7	3	5	9	8	2	6
C_{26}	Mean	3625.379	10 887.89	15 965.14	3292.299	13 326.48	14 612.08	5950.186	6732.172	10 194.77	12 162.55	8468.28	9401.688
	std	249.5009	2918.176	735.8058	240.7666	1586.645	1271.489	370.4272	429.3325	707.5114	338.5611	551.4051	2204.682
	ET	14.84107	6.404236	31.61121	14.37638	7.502224	6.384111	8.41198	7.73557	20.81697	11.0707	6.623204	6.873124
	Rank	2	8	12	1	10	11	3	4	7	9	5	6
C_{27}	Mean	3328.512	4366.514	5125.112	3476.931	4824.111	4555.356	3634.317	3670.887	3876.584	8470.607	3676.8	4538.63
	std	37.02017	94.88507	330.3539	102.877	565.6199	586.9991	147.9302	56.91339	171.439	322.7935	246.3906	117.7455
	ET	20.40176	6.418829	751.4049	16.45292	8.501632	7.367071	9.311285	8.67136	23.61473	12.10706	7.552317	8.20048
	Rank	1	7	11	2	10	9	3	4	6	12	5	8
C_{28}	Mean	3595.247	5927.069	11 797.14	3359.728	7567.641	4944.583	3288.148	4493.132	5405.52	5201.752	3920.642	5180.476
	std	28.94284	374.6972	2452.27	43.38815	1531.288	439.7597	21.63108	307.9493	518.4822	88.70492	389.9513	230.8519
	ET	16.12046	6.557746	31.50602	46.7972	7.808279	6.632258	8.691254	7.967651	21.51879	11.34388	6.840787	7.841928
	Rank	3	10	12	2	11	6	1	5	9	8	4	7
C_{29}	Mean	4160.974	5508.011	20 771.09	4655.216	7188.91	9499.927	4963.308	5003.798	6795.121	8566.63	4966.871	6378.735
	std	293.423	720.6455	9862.853	332.5938	436.399	2557.072	475.1588	249.0193	970.0283	1938.806	165.3959	464.5458
	ET	9.00653	4.733514	30.34651	14.81977	6.187883	5.028518	7.096739	6.355616	16.67771	9.757751	5.312877	5.610104
	Rank	1	6	12	2	9	11	3	5	8	10	4	7
C_{30}	Mean	1 902 810	35 167 304	5.86E+09	1 727 553	1.77E+09	1.69E+08	75 133 132	1.49E+08	3.2E+08	1.97E+08	5 091 401	62 286 725
	std	850 536	8 325 550	2.41E+09	747 146.9	1.74E+09	59 743 782	8 044 904	75 103 242	76 443 862	44 980 788	1 754 250	17 227 322
	ET	21.54887	7.975459	34.26105	19.98942	10.3383	9.198505	11.26854	10.54938	29.16824	14.0752	9.443989	9.82425
	Rank	2	4	12	1	11	8	6	7	10	9	3	5
Sum Rank		39	192	339	70	291	260	108	137	239	244	143	200
Mean Rank		1.344828	6.62069	11.68966	2.413793	10.03448	8.965517	3.724138	4.724138	8.241379	8.413793	4.931034	6.896552
Total Rank		1	6	12	2	11	10	3	4	8	9	5	7

the C3 to C6, C8 to C12, C14, C17, C18, and C20 to C30 functions. A comparison of the simulation results shows that the proposed approach is superior to competitor algorithms in most of the test functions, and overall, the COA ranks first in considering the CEC-2017 benchmark functions for different dimensions equal to 10, 30, 50, and 100. Analysis of simulation results indicates the high ability of COA in exploration and exploitation, which has led to the apparent superiority of the proposed approach over competitor algorithms.

4.4. Evaluation of the CEC-2011 objective functions

This subsection challenges the COA's ability to address real-world optimization applications. For this purpose, the CEC-2011 test suite is employed, which includes twenty-two up-to-date test functions of real-world problems. The different types of objective functions and the high number of local optimizations are features that make these problems suitable for evaluating the exploration and exploitation capabilities of metaheuristic algorithms. Details and complete information of the CEC-2011 test suite are stated in [66]. The proposed COA approach and each of the competitor algorithms is implemented on the CEC-2011 functions in twenty-five independent implementations where each implementation contains 150,000 FEs.

The implementation results of COA and competitor algorithms on the CEC-2011 set functions are reported in Table 6. In addition, the boxplots of performance of COA and competitor algorithms are shown in Fig. 10. What can be deduced from the analysis of the simulation results is that the proposed COA approach is the first best optimizer to handle the C11-F1, C11-F2, C11-F3, C11-F4, C11-F7, C11-F8, C11-F9, C11-F11, C11-F12, C11-F14, C11-F17, C11-F18, C11-F19, C11-F20, C11-F21, and C11-F22 functions. The

Table 5
Assessment results of the CEC-2017 objective functions (the dimension $m = 100$).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_1	Mean	77 405 257	1.19E+11	2.48E+11	5.54E+08	1.34E+11	6.68E+10	70 079 931	6.08E+10	9.71E+10	1.45E+11	2.13E+10	5.97E+10
	std	15 022 042	5.84E+09	2.86E+09	1.35E+08	1.32E+10	5.38E+09	11 560 881	7.66E+09	6.73E+09	9.25E+09	8.06E+09	5.22E+09
	ET	12.72053	4.656825	56.39666	11.53558	6.761015	4.556632	8.365783	7.24913	14.96532	13.36212	4.896176	5.327995
	Rank	2	9	12	3	10	7	1	6	8	11	4	5
C_3	Mean	168 018.9	284 175.9	343 395.2	185 387.6	386 995.3	840 008.9	496 104	391 924.8	315 226.7	365 510	575 710.7	614 498.5
	std	32 449.11	28 313.85	9036.029	37 530.05	55 836.8	102 421	92 573.97	37 752.87	15 476.44	25 622.02	172 280.6	19 960.28
	ET	12.83754	4.550569	54.33139	11.43952	6.747196	4.550615	7.193599	7.37274	14.72922	13.19979	4.901213	5.146623
	Rank	1	3	5	2	7	12	9	8	4	6	10	11
C_4	Mean	770.5338	19 140.58	79 883.06	1045.411	16 993.82	11 602.74	856.0337	4714.567	11 390.42	36 254.68	2592.373	9756.097
	std	51.359	3087.898	6886.744	117.0599	4752.389	1220.077	55.23923	1608.593	698.1713	5899.575	746.4594	535.1329
	ET	10.08412	4.541598	55.26566	11.48673	6.761333	4.611475	8.24082	7.379116	14.90168	13.33273	5.161936	5.048013
	Rank	1	10	12	3	9	8	2	5	7	11	4	6
C_5	Mean	1159.534	1416.959	2014.712	1282.832	2207.365	1885.857	1255.328	1239.614	1922.424	1360.571	1441.885	1618.198
	std	41.56693	34.79746	37.35075	73.64593	26.35613	113.3929	84.57141	17.74502	24.27371	35.6516	134.3458	110.5399
	ET	12.36161	5.185661	56.21442	12.58825	7.288918	5.108913	8.77272	7.909036	16.22265	14.20052	5.458182	5.584455
	Rank	1	6	11	4	12	9	3	2	10	5	7	8
C_6	Mean	636.3933	671.2647	708.4028	681.3705	714.9346	707.6007	677.2441	660.4498	684.1197	666.219	663.5625	665.261
	std	2.368568	10.94671	3.608402	32.40881	10.44185	12.63285	5.815079	34.43224	7.014111	3.123273	6.464293	6.760768
	ET	20.56597	6.996824	58.8152	16.60051	9.297162	7.13046	10.77563	9.977606	22.16249	15.7976	7.380006	7.599076
	Rank	1	6	11	8	12	10	7	2	9	5	3	4
C_7	Mean	1995.873	3374.351	3828.487	1873.719	3527.022	3677.963	2044.624	2060.686	3174.858	3201.209	2530.903	2632.596
	std	62.29099	257.1896	75.52237	63.09679	163.8364	142.108	123.831	143.5882	113.6426	158.3012	190.6207	154.7131
	ET	12.17208	5.355389	57.56033	12.73284	7.407609	5.195627	8.873078	7.999521	16.81509	13.96065	5.501317	5.710446
	Rank	2	9	12	1	10	11	3	4	7	8	5	6
C_8	Mean	1393.982	1802.604	2528.82	1522.421	2448.625	2365.793	1496.537	1570.277	2298.315	1857.552	1733.675	2070.2
	std	128.9828	63.86616	18.33389	141.9043	78.12869	189.3343	115.1692	89.3735	52.68324	99.1555	66.67368	45.97973
	ET	12.4164	5.299902	57.47095	12.79873	7.492479	5.23961	8.872122	7.956285	16.68273	13.92364	5.600977	5.804097
	Rank	1	6	12	3	11	10	2	4	9	7	5	8
C_9	Mean	22 768.17	82 669.58	79 373.17	24 990.11	123 790	78 872.92	60 829.71	36 901.78	76 467.06	24 708.36	33 731.02	47 213.7
	std	1051.544	7174.092	2100.287	1354.899	22 237.37	19 030.59	7377.634	13 379.93	2261.417	548.6017	4032.062	4385.168
	ET	10.0161	5.150724	58.32623	12.75466	7.426297	5.195485	8.842833	7.982558	16.55273	14.04435	5.491215	5.636551
	Rank	1	11	10	3	12	9	7	5	8	2	4	6
C_{10}	Mean	13 963.28	25 000.53	32 371.07	15 458.72	30 050.41	28 969.7	17 189.73	15 322.29	32 381.09	17 432.37	17 274.3	26 654.68
	std	645.9373	7406.577	647.8442	974.2234	762.4971	1055.231	528.8473	742.7296	956.0097	1344.659	1430.116	384.2499
	ET	12.43456	4.781109	61.24723	13.7027	7.979256	5.674736	9.529146	8.458111	18.24622	14.36243	5.994234	6.183812
	Rank	1	7	11	3	10	9	4	2	12	6	5	8
C_{11}	Mean	4420.523	66 539.98	215 441.7	5158.694	68 164.47	217 448.1	4889.742	90 937.47	74 844.98	180 254.9	54 266.91	145 318.4
	std	513.5259	19 508.04	64 071.68	670.1345	27 574.46	104 274.2	426.1793	11 534.4	14 176.23	24 827.6	38 625.67	39 211.88
	ET	14.07226	4.620642	55.82035	12.13102	7.270974	4.877049	7.426876	7.748179	15.63841	13.65734	5.176525	5.608402
	Rank	1	5	11	3	6	12	2	8	7	10	4	9
C_{12}	Mean	2.5E+08	5.33E+10	1.79E+11	2.94E+08	5.93E+10	1.38E+10	3.54E+08	1.19E+10	2.29E+10	6.98E+10	1.05E+10	1.29E+10
	std	79 804 035	6.22E+09	3.4E+10	84 947 675	2.84E+10	1.92E+09	1.35E+08	2.56E+09	6.2E+09	8.97E+09	8.5E+09	1.58E+09
	ET	14.71234	5.121601	58.48422	12.85632	7.538511	5.260312	9.031602	8.065777	16.70683	13.88842	5.618708	5.96717
	Rank	1	9	12	2	10	7	3	5	8	11	4	6
C_{13}	Mean	111 930.2	7.73E+09	4.46E+10	101 142.6	2.23E+10	5.46E+08	369 852.4	9.89E+08	2.94E+09	9.12E+09	1.84E+09	1.83E+08
	std	114 248.1	4.32E+09	7.42E+09	100 721.1	4.6E+09	1.8E+08	46 099.12	1.17E+09	6.96E+08	2.56E+09	1.54E+09	39 675 342
	ET	12.0608	4.6506	56.90503	12.27883	7.24645	5.015775	8.769536	7.846623	15.83409	13.85038	5.246276	5.386116
	Rank	2	9	12	1	11	5	3	6	8	10	7	4
C_{14}	Mean	95 013.85	8 398 313	83 370 942	103 967.6	9 309 367	15 226 441	3 174 913	10 065 302	14 555 757	12 032 635	852 890.2	10 991 690
	std	78 323.57	2 526 169	7 314 111	85 066.23	6 139 922	4 953 974	1 521 147	3 823 609	4 055 464	4 054 374	621 661.3	4 180 142
	ET	16.11798	5.14819	57.31802	13.72988	7.933819	5.692025	9.126907	8.546136	18.14142	14.38219	6.094356	6.209456
	Rank	1	5	12	2	6	11	4	7	10	9	3	8
C_{15}	Mean	61 998.36	4.19E+08	2.46E+10	58 512.51	1.26E+10	73 362 817	132 300.9	5.25E+08	1.25E+09	1.3E+09	3.49E+08	13 279 712
	std	32 307.71	5.44E+08	6.5E+09	30 434.63	1.01E+10	45 706 550	45 899.08	7.12E+08	9.85E+08	5.28E+08	6.86E+08	6 389 017
	ET	14.28005	4.503007	56.88235	11.92176	7.097925	4.797472	8.441369	7.671604	15.40191	13.51557	5.141028	5.269202
	Rank	2	7	12	1	11	5	3	8	9	10	6	4
C_{16}	Mean	5974.812	8075.619	23 585.12	5661.854	15 125.42	16 875.49	6784.499	6266.523	11 912.22	11 453.36	6667.282	10 915.39
	std	74.25068	704.2064	3635.585	115.5176	2303.077	2224.298	574.2097	695.6854	826.8669	1523.488	271.435	927.0944
	ET	15.01932	4.969765	55.32919	12.51733	7.465189	5.14283	8.809536	7.929018	16.17491	13.82993	5.445556	5.693101
	Rank	2	6	12	1	10	11	5	3	9	8	4	7
C_{17}	Mean	4626.489	10 409.14	8 696 852	4974.237	229 005.1	17 717.28	5207.819	5735.767	9028.661	48 494.12	6255.591	7364.395
	std	146.1274	4852.693	8 306 042	306.1371	261 379.9	8623.931	234.6543	1109.149	137.643	20 914.62	264.1113	207.5571
	ET	21.13927	6.56474	58.48177	16.31319	10.68463	6.987761	10.45312	9.780839	21.86651	15.70102	7.306115	7.456471
	Rank	1	8	12	2	11	9	3	4	7	10	5	6
C_{18}	Mean	243 060.2	7 212 606	1.08E+08	264 725.8	15 614 257	12 576 039	5 142 944	11 483 632	16 978 914	12 318 085	6 744 491	6 326 356
	std	130 447.2	5 140 198	65 562 281	140 633.7	11 741 593	2 526 619	2 339 909	6 147 568	4 934 944	10 232 189	2 579 591	1 923 115
	ET	16.53821	4.766383	56.72284	12.53637	7.645209	5.255334	8.743703	7.952184	16.20433	13.72077	5.467938	5.664581
	Rank	1	6	12	2	10	9	3	7	11	8	5	4

(continued on next page)

Table 5 (*continued*).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C_{19}	Mean	325 050.6	8.76E+08	2.34E+10	293 442.2	5.29E+09	1.4E+08	17 445 101	3.78E+08	7.01E+08	1.66E+09	2.83E+08	13 418 617
	std	199 512	9.6E+08	4.98E+09	180 757.6	3.61E+09	83 874 380	8 665 700	5.3E+08	6.15E+08	1.41E+09	2.74E+08	7 729 218
	ET	24.4647	13.75878	68.29124	33.09738	17.48625	15.39419	18.96095	18.26364	46.70063	24.32194	15.61414	15.88095
	Rank	2	9	12	1	11	5	4	7	8	10	6	3
C_{20}	Mean	4566.422	5209.792	7771.453	4964.841	7233.151	7246.388	5952.747	6229.274	7461.71	6488.834	5490.106	6437.625
	std	70.39281	355.8995	106.793	189.0951	599.9526	395.995	422.8979	1133.323	557.3596	377.6008	686.3791	553.7011
	ET	18.03894	6.697903	60.20271	17.2582	9.685805	7.542347	11.13922	10.36337	23.18623	16.36276	7.950868	7.962873
	Rank	1	3	12	2	9	10	5	6	11	8	4	7
C_{21}	Mean	2856.475	3745.407	4508.594	3127.506	4205.852	4313.733	3279.91	3006.17	3772.867	4826.617	3641.135	3469.032
	std	35.856	43.77476	61.58535	46.56761	125.5021	251.7109	94.92984	60.92987	155.1119	440.5266	259.8198	38.85816
	ET	21.248	11.78442	65.44038	28.04148	15.01686	12.88325	16.61197	15.65352	39.34907	21.54734	13.16289	13.4443
	Rank	1	7	11	3	9	10	4	2	8	12	6	5
C_{22}	Mean	17 560.96	22 635.78	34 479.41	19 042.03	31 677.7	29 958.83	19 403.54	23 907.13	34 350.47	21 626.57	22 428.56	29 603.04
	std	935.1663	1037.438	491.9479	1398.375	935.7031	1221.34	1410.489	8026.013	678.5577	533.6613	1371.156	885.9392
	ET	22.2343	12.94516	65.52973	29.34387	15.71686	13.49964	17.35543	16.31821	41.40921	22.23516	13.77283	14.15693
	Rank	1	6	12	2	10	9	3	7	11	4	5	8
C_{23}	Mean	3320.684	5009.335	5475.558	3678.247	5600.97	5275.989	3518.824	3660.413	4287.317	8176.084	4980.687	4342.684
	std	22.55816	101.5099	210.9503	142.5672	853.0103	146.6653	94.77814	38.1779	59.73447	490.4698	382.2972	76.26294
	ET	25.2888	10.73635	66.17142	35.51952	18.66019	16.57562	20.25858	19.37997	1326.56	25.21985	16.77714	16.84864
	Rank	1	8	10	4	11	9	2	3	5	12	7	6
C_{24}	Mean	3740.195	6619.833	10 952.73	3982.671	6881.667	6571.716	4008.708	4355.301	4842.054	11 286.94	6126.276	5504.299
	std	57.75833	240.8529	2980.522	84.80273	370.2203	493.9121	91.0319	251.1859	206.1132	1225.03	409.4878	129.3864
	ET	26.4639	13.39375	67.70093	36.73724	19.35246	17.22645	20.81343	19.80846	52.53937	26.01052	17.42242	17.61044
	Rank	1	9	11	2	10	8	3	4	5	12	7	6
C_{25}	Mean	3422.69	12 914.68	22 463.79	3717.65	10 932.7	7562.908	3750.675	6636.297	9267.746	11 525.62	4201.16	8184.254
	std	59.70878	2106.496	2514.302	130.8935	518.6866	484.1265	125.4368	292.7752	1398.912	1128.823	357.02	800.929
	ET	26.7336	18.40703	68.49214	39.34139	20.6831	18.54472	22.25741	21.42674	56.27984	27.35005	18.87771	18.95531
	Rank	1	11	12	2	9	6	3	5	8	10	4	7
C_{26}	Mean	11 512.07	33 589.15	47 915.4	12 627.72	35 024.25	35 690.63	12 683.5	17 561.05	24 996.58	35 586.58	21 641.4	24 078.25
	std	737.8098	1291.344	2110.57	835.2709	945.3891	3410.818	1405.09	1562.864	4358.679	1444.208	1863.093	1255.696
	ET	27.5755	19.71464	74.59587	42.9371	22.54353	20.36853	24.20819	23.06341	62.13359	31.59323	1266.628	20.82038
	Rank	1	8	12	2	9	11	3	4	7	10	5	6
C_{27}	Mean	3549.793	7203.719	12 874.15	3856.168	6822.127	6185.898	3645.456	4133.961	4397.238	14 757.55	4125.828	5631.426
	std	34.36312	874.2169	3625.498	110.9491	315.9009	855.2059	71.90072	158.6322	352.2487	296.3277	240.8069	302.7769
	ET	30.1962	20.03896	75.07449	50.08541	25.96096	23.95681	27.61783	26.66497	73.20217	35.49329	25.63387	24.56765
	Rank	1	10	11	3	9	8	2	5	6	12	4	7
C_{28}	Mean	3468.207	16 810.98	29 879.23	3801.356	16 518.83	10 841.18	3765.615	9664.276	11 692.29	19 823.16	7931.201	12 026.92
	std	73.72614	1313.643	2965.76	95.56287	3043.15	1139.129	146.2277	1554.274	2291.278	2022.435	3232.697	1239.906
	ET	30.9242	21.96862	74.30412	47.22546	24.64309	22.39027	26.21548	25.22822	68.47812	33.33656	22.66888	22.7536
	Rank	1	10	12	3	9	6	2	5	7	11	4	8
C_{29}	Mean	7031.672	11 200.52	371 294.3	7611.455	19 336.1	17 313.05	8945.818	8543.82	12 928.31	26 243.8	8907.336	12 297.62
	std	685.616	754.6761	134 866.2	672.1105	4116.746	2696.084	757.849	241.7365	655.2789	6030.707	804.7772	334.1728
	ET	26.0094	14.44892	65.29449	30.96446	16.63042	14.33984	17.74328	17.11014	43.88943	24.59769	14.58347	14.78207
	Rank	1	6	12	2	10	9	5	3	8	11	4	7
C_{30}	Mean	4 983 073	2.47E+09	4E+10	5 349 477	1.42E+10	1.59E+09	1.09E+08	1.95E+09	4.02E+09	7.8E+09	6.42E+08	7.07E+08
	std	2 734 432	2.6E+09	2.53E+09	2 851 121	3.92E+09	3.84E+08	29 923 832	7.86E+08	2.98E+09	1.64E+09	9E+08	78 710 028
	ET	28.4775	19.8555	73.79778	47.71889	24.95313	22.64243	26.40534	25.54835	68.85316	33.08901	22.9091	23.04624
	Rank	1	8	12	2	11	6	3	7	9	10	4	5
Sum Rank		35	217	330	72	285	251	103	144	236	259	145	185
Mean Rank		1.206897	7.482759	11.37931	2.482759	9.827586	8.655172	3.551724	4.965517	8.137931	8.931034	5	6.37931
Total Rank		1	7	12	2	11	9	3	4	8	10	5	6

simulation results show that the proposed COA approach is highly efficient in dealing with real-world optimization applications and has an advantage over competitor algorithms.

4.5. Statistical analysis

In the previous subsection, the results of the implementation of competitor optimization algorithms and COA are presented by four statistical criteria, i.e., mean, std, rank, and ET, which show which method is superior in each benchmark function. This superiority may be evident, or in a tough competition, one method is slightly different and superior to the other methods. This fact indicates the need for a more detailed statistical study of the performance of COA and the employed algorithms. In this subsection, a statistical analysis is presented on the simulation results obtained from the implementation of COA and competitor algorithms to show how significant the superiority of COA over competitor algorithms is from a statistical point of view. For this purpose, the Wilcoxon sum rank test [67], which is a non-parametric test, is used. In this test, an index called p-value indicates whether the mean of the two data sets have a significant difference. The results of Wilcoxon statistical analysis on COA performance against each of the alternative algorithms are presented in Table 7. What can be deduced from the implementation results is that in cases where the p-value is less than 0.05, the COA has a statistically significant superiority over the corresponding algorithm.

Table 6
Assessment results of the CEC-2011 objective functions.

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C11-F1	Mean	2.843588	15.02557	26.00412	6.499366	21.56497	15.09655	15.02735	12.10703	20.03647	25.65497	19.48093	25.589
	std	5.687176	3.129599	1.209479	7.537959	1.628633	3.734856	2.718165	8.369631	2.04283	1.515837	6.93741	1.273165
	ET	2.04562	0.621113	1.30252	1.685247	0.737185	0.766996	0.991598	0.752646	2.475677	1.699408	0.786887	0.790768
	Rank	1	4	12	2	9	6	5	3	8	11	7	10
C11-F2	Mean	−28.8108	−22.3944	−8.06028	−28.7894	−7.71267	−16.7919	−6.80337	−21.762	−9.13516	−12.9774	−22.2721	−11.4007
	std	5.507744	0.674967	0.550565	1.430853	3.620175	4.87226	1.679744	3.127631	0.967906	5.100808	1.858832	2.1547
	ET	1.630126	1.219837	4.733408	2.995567	1.435136	1.401353	1.511874	1.526617	4.026616	3.163467	1.308459	1.347199
	Rank	1	3	10	2	11	6	12	5	9	7	4	8
C11-F3	Mean	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05	1.15E−05
	std	0.00035	2.03E−19	5.81E−11	1.9E−19	2.77E−14	7.54E−19	1.04E−12	4.34E−15	8.21E−14	2.38E−19	6.03E−20	2.9E−18
	ET	868.00905	743.8869	585.0686	1217.047	609.0948	607.9165	607.454	608.6231	1748.186	818.5322	575.8977	617.9594
	Rank	1	2	12	3	9	6	11	8	10	5	4	7
C11-F4	Mean	0	0	0	0	0	0	0	0	0	0	0	0
	std	0	0	0	0	0	0	0	0	0	0	0	0
	ET	14.15682	15.07985	18.81958	30.37307	16.72845	16.36406	18.36957	14.71633	39.0127	12.24346	14.55213	11.4772
	Rank	1	1	1	1	1	1	1	1	1	1	1	1
C11-F5	Mean	−17.5227	−29.4024	−16.7272	−34.1274	−25.5476	−26.1607	−26.2509	−30.9979	−8.28938	−25.8141	−5.892	−6.84436
	std	1.37731	3.068482	2.848936	0.561306	5.055006	0.23178	3.69706	3.523914	1.829833	3.885845	2.768865	1.535021
	ET	28.38912	17.66267	23.08973	40.65085	20.24999	19.94252	20.18764	20.07313	56.89843	20.81698	23.56863	19.47957
	Rank	8	3	9	1	7	5	4	2	10	6	12	11
C11-F6	Mean	−25.2178	−19.3848	−10.5136	−25.6718	−3.76673	−19.0147	−7.9834	−18.6181	0	−22.4159	−0.95891	−1.96296
	std	2.554107	2.647173	1.153367	4.027751	7.53347	5.79744	9.246332	2.168807	0	6.056873	1.917828	3.925929
	ET	16.0921	13.60279	23.08044	41.60155	20.42178	20.71275	20.61406	20.66935	58.46636	21.3468	24.64162	19.49718
	Rank	2	4	7	1	9	5	8	6	12	3	11	10
C11-F7	Mean	0.749017	1.14754	2.155071	0.895811	1.399862	1.93967	0.901048	1.123129	1.804349	1.128163	1.149726	1.82829
	std	0.115638	0.129835	0.227085	0.187289	0.278022	0.139802	0.073573	0.249063	0.160025	0.20338	0.319642	0.277389
	ET	2.024755	1.204608	3.572109	3.413572	1.623746	1.513652	1.66462	1.532956	4.603534	1.91725	1.702331	1.42789
	Rank	1	6	12	2	8	11	3	4	9	5	7	10
C11-F8	Mean	220	246.4328	349.25	220	265.75	276.5	224.5	229	224.5	252.3177	495.4306	222.75
	std	0	4.799943	43.06874	0	79.95155	37.96051	9	10.3923	9	42.67888	168.2035	5.5
	ET	10.51563	3.190836	4.784833	8.056244	3.951239	3.889735	4.004715	3.940096	11.53115	4.247251	4.428498	3.938002
	Rank	1	5	9	1	7	8	3	4	3	6	10	2
C11-F9	Mean	8789.286	735 450.4	1 301 689	9555.373	79 300.39	457 982.2	146 438.6	50 812.21	450 581.2	1 008 711	1 195 008	2 145 417
	std	3700.105	147 088.4	308 299.6	3930.041	18 679.42	239 495.7	57 363.3	29 636.9	90 228.33	100 073.3	269 705.5	105 998.2
	ET	10.296050	8.138154	26.02624	23.28378	11.61257	11.01663	11.85829	11.5064	32.24512	13.08107	13.25627	10.74148
	Rank	1	8	11	2	4	7	5	3	6	9	10	12
C11-F10	Mean	−21.4889	−16.4886	−10.1972	−23.3822	−12.7862	−10.9262	−14.0006	−12.4356	−10.232	−11.2701	−10.3421	−10.016
	std	0.474376	2.099664	0.317858	0.763803	3.774754	0.690993	4.82948	0.971772	0.059014	0.740622	0.035282	0.015903
	ET	38.89083	21.08799	22.99103	43.64193	21.9544	21.68244	22.13131	21.43308	61.8121	21.51456	20.48358	20.6292
	Rank	2	3	11	1	5	8	4	6	10	7	9	12
C11-F11	Mean	275 953.6	305 973.6	10 733 179	571 712.3	7 159 507	1 360 696	1 384 293	4 570 346	5 689 705	1 600 704	5 701 929	6 697 096
	std	133 300.2	150 265.3	203 662.6	248 237.1	1 115 398	108 481.8	1 068 784	343 310	0	151 894.6	14 114.72	58 982.11
	ET	6.728003	3.396937	17.17506	8.026048	3.913679	13.50512	4.331815	4.005783	10.27428	5.514548	3.521359	3.619429
	Rank	1	2	12	3	11	4	5	7	8	6	9	10
C11-F12	Mean	1 186 100	3 917 084	15 954 845	1 312 927	5 894 635	6 857 336	1 357 796	1 474 699	15 685 253	6 826 485	2 428 911	15 862 796
	std	42 666.96	176 574.8	882 680.7	74 021.43	244 355.5	360 392.8	109 218.5	146 090.4	695 442.8	269 442.2	176 214.5	113 798.6
	ET	12.720862	3.793065	36.57603	14.09973	7.279686	26.13292	7.768194	7.321105	18.25782	10.3244	6.274536	6.328642
	Rank	1	6	12	2	7	9	3	4	10	8	5	11
C11-F13	Mean	15 454.67	15 453.44	16 506.66	15 444.2	15 500.62	15 556.16	15 514.52	15 514.01	15 984.05	153 930.6	15 495.68	31 517.17
	std	2.451103	15.66896	852.4752	0.008649	13.6631	58.55158	30.06305	10.27854	434.1253	46 276.94	27.16982	31 851.35
	ET	6.315078	0.606276	1.473014	1.430873	0.722044	1.266859	0.730559	0.680644	2.216298	0.972618	0.681876	0.713066
	Rank	3	2	10	1	5	8	7	6	9	12	4	11
C11-F14	Mean	18 251.34	18 709.96	276 941.8	19 776.67	19 811.79	19 436.86	19 536.54	19 445.38	341 308.2	19 273.65	19 211.78	19 197.69
	std	38.80248	491.6072	88 734.65	1063.496	460.4729	149.5547	83.40401	179.0924	302 008.6	266.3652	137.4755	265.683
	ET	1.014952	0.427157	2.07299	1.127994	0.553375	1.603479	0.579655	0.515422	1.60509	0.890782	0.497711	0.507926
	Rank	1	2	11	9	10	6	8	7	12	5	4	3
C11-F15	Mean	239 954.4	33 006.71	2 343 498	32 883.58	59 488.5	260 914.7	33 122.71	33 122.1	17 036 307	360 564.8	33 330.48	8 766 184
	std	203 421.2	134.8848	2 527 893	73.20611	52 571.93	155 126.1	65.05175	46.84925	9 930 539	33 146.7	8.572658	5 060 984
	ET	1.862347	0.565858	2.30257	1.666847	0.827405	2.011513	0.85081	0.784439	2.276938	1.166119	0.731841	0.761925
	Rank	7	2	10	1	6	8	4	3	12	9	5	11
C11-F16	Mean	138 044.1	140 692.8	2 362 861	133 550	147 761.5	144 124.1	142 673.9	148 677.3	98 220 886	22 956 478	87 910 864	84 408 504
	std	3367.715	7711.282	2 413 071	2275.9	2797.151	5593.217	8238.548	4216.938	2 236 256	12 934 800	13 938 126	16 885 692
	ET	2.163795	0.433626	4.868135	1.597066	0.731681	3.918149	0.892749	0.743205	1.834396	1.395037	0.553988	0.56492
	Rank	2	3	8	1	6	5	4	7	12	9	11	10
C11-F17	Mean	1 926 615	4 080 675	1.91E+10	2 074 061	1.57E+09	1.19E+10	3 236 561	3 268 550	2.47E+10	1.38E+10	2.3E+10	2.42E+10
	std	11 419.96	3 793 542	4.12E+09	24 672.76	2.58E+08	3.09E+09	738 307.1	1 572 618	8.32E+08	1.12E+09	2.84E+09	2.14E+09
	ET	6.395614	1.727172	18.59715	5.198236	2.575721	14.0566	3.044497	2.670038	7.715739	4.519795	2.566988	2.630842
	Rank	1	5	9	2	6	7	3	4	12	8	10	11

(*continued on next page*)

Table 6 (*continued*).

		COA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C11-F18	Mean	942 057.5	1 616 414	1.45E+08	1 040 748	2 302 603	11 547 520	992 950.7	1 051 413	34 159 342	13 448 732	1.49E+08	1.26E+08
	std	2639.272	494 288.8	30 701 419	30 276.27	355 558.6	6 581 970	18 274.1	138 562.1	4 756 083	3 144 962	18 065 452	3 804 565
	ET	4.649493	1.678574	12.92761	5.425479	2.665727	11.02835	2.981397	2.831208	6.959598	4.026526	2.3726	2.426632
	Rank	1	5	11	3	6	7	2	4	9	8	12	10
C11-F19	Mean	1 025 341	2 590 116	1.42E+08	1 142 135	2 791 055	12 312 863	1 524 075	1 439 674	39 223 360	7 461 696	1.91E+08	1.27E+08
	std	94 829.24	879 404.3	26 103 571	108 617.1	381 864.2	9 499 136	385 678.1	141 575.6	9 322 602	3 277 745	20 606 580	2 840 728
	ET	5.82334	2.354316	14.45363	7.592415	3.778641	11.9253	4.349197	3.795235	9.93465	5.329499	3.338571	3.43572
	Rank	1	5	11	2	6	8	4	3	9	7	12	10
C11-F20	Mean	941 250.4	3 058 450	1.54E+08	1 038 964	2 028 256	8 725 656	976 129.4	1 011 592	38 105 472	17 308 187	1.76E+08	1.27E+08
	std	4769.814	3 580 771	20 568 980	13 978.4	284 994.2	515 841.7	10 681.17	20 853.76	725 639.4	6 766 980	16 864 071	4 570 779
	ET	4.619890	2.231574	15.13121	7.498915	3.893597	11.4622	4.305707	3.855959	9.956753	5.153071	3.370197	3.421131
	Rank	1	6	11	4	5	7	2	3	9	8	12	10
C11-F21	Mean	12.71443	18.43484	91.01355	14.12941	33.73053	44.74353	29.10064	24.583	109.9653	47.09098	115.531	112.0717
	std	2.295373	1.636928	21.7381	2.39395	2.783762	4.538794	3.743287	1.829596	45.37279	4.49623	14.34348	34.36794
	ET	47.38995	58.27956	28.47101	51.89277	25.96351	25.78215	27.82912	25.74904	433.5071	26.45769	24.85448	324.6405
	Rank	1	3	9	2	6	7	5	4	10	8	12	11
C11-F22	Mean	16.12513	27.48089	74.30445	17.47382	35.79601	53.23544	34.00153	27.0194	111.7436	53.67425	116.2026	100.6545
	std	3.993717	3.475444	15.10545	4.297623	2.874281	5.253638	6.149356	1.900382	27.19142	8.858149	13.87778	1.032337
	ET	47.41507	23.87322	25.70583	47.58784	23.85285	23.52593	24.27957	23.53612	348.8059	24.34236	22.87887	272.1996
	Rank	1	4	9	2	6	7	5	3	11	8	12	10
Sum Rank		41	84	217	48	150	146	108	97	201	156	183	201
Mean Rank		1.863636	3.818182	9.863636	2.181818	6.818182	6.636364	4.909091	4.409091	9.136364	7.090909	8.318182	9.136364
Total Rank		1	3	11	2	7	6	5	4	10	8	9	10

Table 7
Obtained results for *p*-value from Wilcoxon sum Rank test.

Compared algorithms	Test function type				
	CEC- 2017				CEC-2011
	Dimension				
	10	30	50	100	
COA vs. WSO	2.86E−09	3.32E−20	1.97E−21	1.97E−21	1.98E−07
COA vs. RSA	3.51E−19	1.97E−21	1.97E−21	1.97E−21	8.01E−14
COA vs. TSA	1.37E−08	7.03E−09	2.54E−05	6.50E−10	0.004054
COA vs. MPA	5.14E−18	2.35E−21	1.97E−21	1.97E−21	1.08E−09
COA vs. WOA	5.26E−16	3.88E−21	1.97E−21	1.97E−21	5.06E−10
COA vs. MVO	2.02E−11	2.41E−17	1.86E−18	5.77E−17	2.82E−08
COA vs. GWO	3.87E−15	2.40E−18	1.02E−20	6.88E−21	2.12E−08
COA vs. TLBO	2.79E−14	1.63E−20	1.97E−21	1.97E−21	1.74E−13
COA vs. GSA	4.00E−18	2.30E−20	1.97E−21	1.97E−21	2.61E−11
COA vs. PSO	7.52E−15	1.55E−18	2.54E−21	1.10E−20	5.37E−11
COA vs. GA	9.44E−15	3.88E−21	1.97E−21	1.97E−21	2.12E−13

4.6. Comparison proposed COA with the latest outstanding algorithms

In this subsection, the performance of the proposed COA approach has been compared with the performance of the eight latest outstanding algorithms, including Monarch Butterfly Optimization (MBO) [22], Slime Mould Algorithm (SMA) [23], Hunger Games Search (HGS) [24], Colony Predation Algorithm (CPA) [25], Harris Hawks Optimizer (HHO) [26], Moth Search Algorithm (MSA) [27], RUNge Kutta Optimizer (RUN) [28], and weIghted meaN oF vectOrs (INFO) [49]. The proposed COA approach and these eight latest outstanding algorithms are implemented on the CEC-2017 test suite for dimensions 10, 30, 50, and 100 and also on the CEC-2011 test suite.

The simulation results of the CEC-2017 test suite are reported in Tables 8, 9, 10, and 11. What is concluded from the analysis of the simulation results is that for dimensions equal to ten ($m = 10$), the proposed COA approach is the first best optimizer compared to the eight latest outstanding algorithms in solving C1, C3 to C14, C16 to C22, C24 to C26, C29, and C30 functions. Comparison of simulation results in the CEC-2017 for dimensions equal to thirty (m = 30) shows that COA is the first best optimizer in dealing with C2, C7 to C10, C12 to C21, C23, C24, C26, C28, and C30 functions. The implementation results on the CEC-2017 for dimensions equal to fifty ($m = 50$) show that COA is the first best optimizer to solve C3 to C5, C8, C9, C11, C14 to C22, C24 to C27, and C29 functions. The results of applying COA and the eight latest outstanding algorithms to solve the CEC-2017 test suite for dimensions equal to one hundred ($m = 100$) show that the proposed approach is the first best optimizer to solve C3, C4, C8, C14, C17, C18, C20 to C22, C24, and C25 to C28 functions. Based on the comparison of the simulation results, it is evident that the proposed COA approach has performed better in solving most of the CEC-2017 functions compared to the eight latest outstanding algorithms and has been able to win first place in the optimization of the CEC-2017 test suite for dimensions 10, 30 50, and 100. The convergence curves of COA and the eight latest outstanding algorithms while achieving the solution during algorithm iterations are presented in Figs. 11, 12,13 , and 14. The simulation results concluded that the proposed approach with high exploration, exploitation, and balancing ability had superior performance compared with the latest outstanding algorithms. Also, the results of the Wilcoxon sum rank statistical test show that the superiority of COA against each of the corresponding algorithms is significant from a statistical point of view.

The optimization results of the CEC-2011 test suite using COA and the eight latest outstanding algorithms are reported in Table 12. What is concluded from the simulation results is

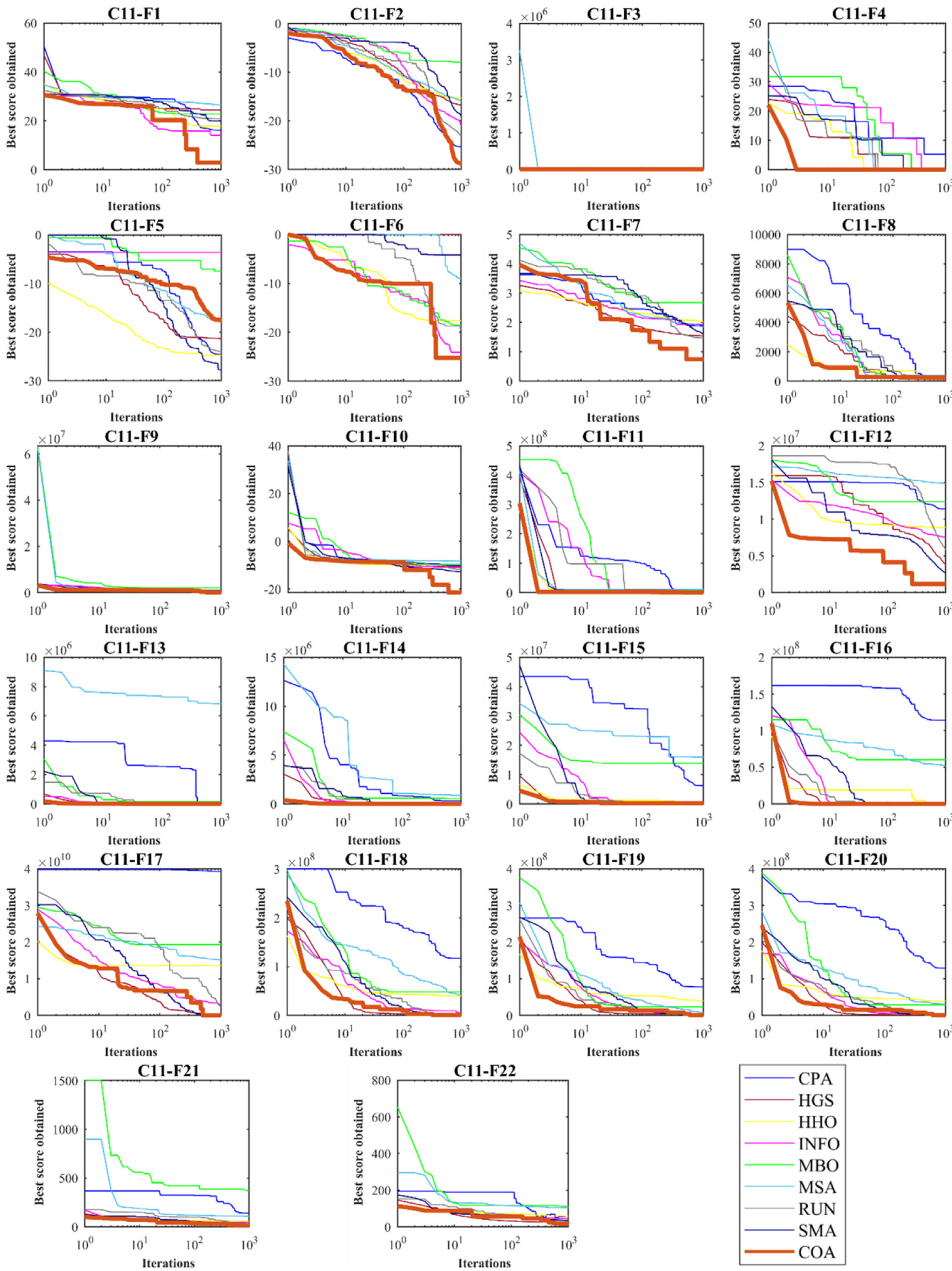

Fig. 15. Convergence curves of COA and some latest outstanding algorithms in the optimization of the CEC-2011 test suite.

that the proposed COA approach is the first best optimizer for C11-F1, C11-F2, C11-F4, C11-F6 to C11-F14, and C11-F16 to C11-F22 functions. The convergence curves of COA and the eight latest outstanding algorithms while achieving the solution for the CEC-2011 functions during algorithm iterations are presented in Fig. 15. The analysis of the simulation results shows that the proposed COA approach has provided better performance in solving most of the CEC-2011 functions. It has ranked as the best optimizer for handling the CEC-2011 test suite compared to the eight latest outstanding algorithms. What is concluded from the analysis of the results is that the proposed COA approach has acceptable efficiency in handling optimization tasks in real-world applications. In addition, the results of the Wilcoxon sum rank statistical test indicate the significant statistical superiority

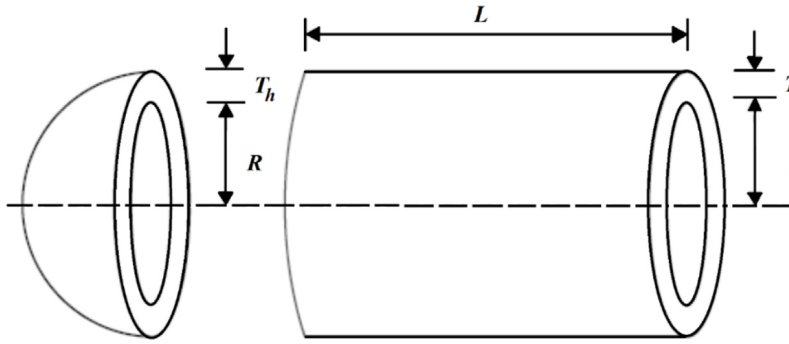

Fig. 16. Schematic of the pressure vessel design.

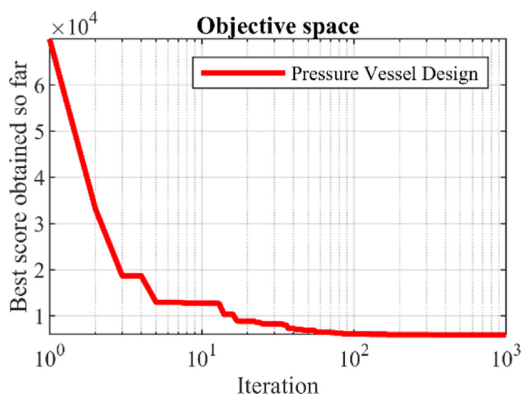

Fig. 17. COA's performance convergence curve on the pressure vessel design.

of COA against the eight latest outstanding algorithms in solving the CEC-2011 functions.

5. COA for engineering optimization problems

This section tests the efficiency of COA in handling real-world optimization applications in four engineering design challenges.

5.1. Pressure vessel design problem

Pressure vessel design is an engineering optimization problem to minimize the total cost of the design, a schematic of which is presented in Fig. 16, see [68]. The mathematical formulation of this optimization problem is as follows:

$$\textit{Consider}: X = [x_1,\ x_2,\ x_3,\ x_4] = [T_s,\ T_h,\ R,\ L].$$

$$\textit{Minimize}: f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3.$$

Subject to :

$$g_1(x) = -x_1 + 0.0193x_3 \le 0, \quad g_2(x) = -x_2 + 0.00954x_3 \le 0,$$

$$g_3(x) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0, \quad g_4(x) = x_4 - 240 \le 0.$$

With

$$0 \le x_1, x_2 \le 100 \quad \text{and} \quad 10 \le x_3, x_4 \le 200.$$

The proposed COA and competitor algorithms are employed to achieve the optimal values of pressure vessel design variables; the results are presented in Table 13. The simulation results show that the COA has provided the optimal solution to the pressure vessel design problem with design variables equal to (0.783194, 0.3874305, 40.5794, 196.3323) values, and the corresponding value of the objective function is equal to 5893.1336. The values obtained for statistical indicators in optimizing this problem are reported in Table 14, which indicates the superiority of COA performance in handling pressure vessel design compared to competitor algorithms. The COA convergence curve while achieving the optimal solution for this engineering challenge is presented in Fig. 17.

5.2. Speed reducer design problem

Speed reducer design is an optimization challenge in engineering sciences to minimize the weight of the speed reducer, the schematic of which is shown in Fig. 18 [69,70]. The mathematical formulation of the speed reducer design optimization problem is as follows:

$$\textit{Consider}: X = [x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7] = [b,\ m,\ p,\ l_1,\ l_2,\ d_1,\ d_2].$$

$$\textit{Minimize}: f(x) = 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) - 1.508x_1\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right) + 0.7854(x_4x_6^2 + x_5x_7^2).$$

Subject to :

$$g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \le 0, \quad g_2(x) = \frac{397.5}{x_1x_2^2x_3} - 1 \le 0,$$

$$g_3(x) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0, \quad g_4(x) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0,$$

$$g_5(x) = \frac{1}{110x_6^3}\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9 \cdot 10^6} - 1 \le 0,$$

$$g_6(x) = \frac{1}{85x_7^3}\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5 \cdot 10^6} - 1 \le 0,$$

$$g_7(x) = \frac{x_2x_3}{40} - 1 \le 0, \quad g_8(x) = \frac{5x_2}{x_1} - 1 \le 0,$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \le 0, \quad g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0,$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0.$$

With

$$2.6 \le x_1 \le 3.6,\ 0.7 \le x_2 \le 0.8,\ 17 \le x_3 \le 28,\ 7.3 \le x_4 \le 8.3,\ 7.8 \le x_5 \le 8.3,\ 2.9 \le x_6 \le 3.9,\ \text{and}\ 5 \le x_7 \le 5.5.$$

Table 8
Assessment results of the CEC-2017 objective functions (the dimension $m = 10$).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_1	Mean	100	2912.141	9641.143	7.32E+08	3.92E+09	100.0012	889 658.4	8967.873	1720.982
	std	1.33E−05	303.0946	4080.171	8.13E+08	1.09E+09	0.002393	663 485.1	4371.004	1295.115
	ET	2.448051	15.2716	10.78089	7.826498	34.52774	5.487068	3.00459	1.232735	2.420179
	Rank	1	4	6	8	9	2	7	5	3
C_3	Mean	300	300.0002	300.0094	9716.036	38 331.37	300	313.2439	300.3495	300
	std	4.64E−14	3.18E−05	0.007169	1829.89	46 476.45	8.04E−14	20.67584	0.693476	3.03E−08
	ET	2.390933	16.24	10.005	7.289024	31.31529	4.837077	2.761794	1.115136	2.064268
	Rank	1	4	5	8	9	2	7	6	3
C_4	Mean	400.0017	400.9132	405.6514	455.0945	918.4111	400.1271	404.9965	405.8339	402.4177
	std	0.003425	1.064478	2.351443	45.58161	487.0423	0.073926	3.579808	0.503775	2.302799
	ET	1.40523	8.2425	4.9725	7.27248	31.58291	4.821349	2.599707	1.083697	1.863614
	Rank	1	3	6	8	9	2	5	7	4
C_5	Mean	507.7503	529.8487	521.8991	546.7115	576.5469	518.3706	556.6996	518.8474	517.163
	std	1.305485	12.02213	11.84087	11.65546	36.34125	5.1098	21.68988	11.09447	4.477277
	ET	2.524593	7.545	5.1575	7.268235	37.08972	4.810261	2.846811	1.237938	2.039916
	Rank	1	6	5	7	9	3	8	4	2
C_6	Mean	600.0005	613.982	600.1553	635.5052	654.963	600.4409	628.7179	601.0439	600.1038
	std	0.000358	6.369081	0.073793	10.08763	22.25632	0.834656	11.75362	0.983498	0.128843
	ET	1.008633	7.8525	5.195	7.589936	31.01186	5.12717	3.342429	1.432484	2.342885
	Rank	1	6	3	8	9	4	7	5	2
C_7	Mean	715.9576	744.674	726.3513	773.3439	913.8595	728.9878	780.4687	736.6973	741.7148
	std	1.076312	16.00222	4.199218	32.7712	87.62654	9.005402	34.09176	12.34712	10.0317
	ET	2.749575	8.8975	5.18	7.353696	31.68518	4.861625	3.077045	1.206789	2.392808
	Rank	1	6	2	7	9	3	8	4	5
C_8	Mean	806.2204	827.6101	819.1544	829.2079	866.244	819.8991	834.7593	819.1531	833.5798
	std	2.05445	5.879228	2.739866	6.247167	32.87217	9.882992	13.64961	3.756033	2.857782
	ET	1.534467	8.845	5.0925	7.39334	33.50534	4.832062	2.916022	1.179748	2.188155
	Rank	1	5	3	6	9	4	8	2	7
C_9	Mean	900	1020.648	900.0018	1455.494	1867.939	910.7464	1529.668	978.8444	905.0519
	std	0	62.83255	0.000919	232.1487	302.9304	20.22944	260.975	157.0687	5.611629
	ET	1.804356	8.0925	5.1575	7.277705	41.63315	4.863786	3.06965	1.216617	2.357807
	Rank	1	6	2	7	9	4	8	5	3
C_{10}	Mean	1270.968	1659.409	1715.431	2495.512	2448.109	1913.516	2108.504	1426.359	1504.543
	std	112.8325	189.115	234.8695	393.28	205.7444	199.3574	132.381	234.9249	299.7481
	ET	1.702848	8.2775	5.1625	7.394618	35.09581	4.8814	3.003159	1.281756	2.343831
	Rank	1	4	5	9	8	6	7	2	3
C_{11}	Mean	1102.144	1128.772	1159.703	1324.295	2361.857	1112.616	1236.168	1121.421	1114.004
	std	0.58187	6.384293	91.37488	140.1763	1270.901	8.979398	153.1781	21.61761	6.889904
	ET	1.505497	8.2175	5.3225	7.394299	32.51192	4.830267	2.852156	1.179799	2.162012
	Rank	1	5	6	8	9	2	7	4	3
C_{12}	Mean	1227.936	6090.758	81 748.33	1 073 560	65 334 640	2265.339	2 022 847	40 613.87	2 408 058
	std	34.3984	2834.899	58 067.32	1 010 967	86 900 724	1051.452	2 050 148	30 202.58	3 395 530
	ET	1.522788	8.195	4.94	7.325999	31.18516	5.02418	2.912054	1.181971	2.073327
	Rank	1	3	5	6	9	2	7	4	8
C_{13}	Mean	1306	8386.29	8247.278	5854.744	9 020 384	1520.633	24 299.81	6784.299	10 112.54
	std	0.894988	1297.892	12 610.91	3257.318	14 141 890	265.5243	19 716.13	6654.848	823.4693
	ET	1.739656	8.1975	4.9425	7.322805	31.68107	4.923738	2.981222	1.226778	2.182497
	Rank	1	6	5	3	9	2	8	4	7
C_{14}	Mean	1403.981	1515.613	1446.651	3127.083	309 594.5	1436.713	1621.368	1923.554	2389.092
	std	2.56885	98.33118	18.75711	2796.724	286 625.7	21.10596	206.6674	305.1665	983.7388
	ET	1.609589	9.68	5.295	7.312443	30.87592	4.915877	3.030448	1.228818	2.221781
	Rank	1	4	3	8	9	2	5	6	7
C_{15}	Mean	1650.381	1534.227	2886.885	8757.552	1 178 934	1517.654	6376.375	4137.305	3634.008
	std	53.30489	10.78798	1250.772	7252.626	1 671 869	10.53487	2920.333	2882.202	475.1664
	ET	1.532277	9.1175	5.315	7.256547	30.96852	4.907426	2.984912	1.176307	2.039147
	Rank	3	2	4	8	9	1	7	6	5
C_{16}	Mean	1601.117	1858.422	1704.823	2088.462	2000.539	1777.09	1848.299	1814.726	1839.562
	std	0.408115	176.0716	57.99604	120.236	85.26253	65.50944	238.0386	54.98389	168.3268
	ET	1.679919	9.6575	5.125	7.322302	31.53896	5.042127	2.998669	1.217337	2.134137
	Rank	1	7	2	9	8	3	6	4	5
C_{17}	Mean	1719.14	1765.924	1731.17	1758.202	1914.078	1748.107	1813.198	1751.158	1752.514
	std	9.27951	23.19057	22.2943	8.281349	195.5116	66.54414	44.3905	33.2337	34.0012
	ET	2.138946	9.0325	5.47	7.700601	32.78816	5.201819	3.453889	1.419872	2.492107
	Rank	1	7	2	6	9	3	8	4	5
C_{18}	Mean	1800.784	23 834.08	32 790.54	17 976.09	1.68E+08	1848.941	14 588.22	32 129.76	8904.663
	std	0.646842	9284.296	14 674.45	7519.339	3.33E+08	23.68528	5452.815	7984.251	8689.415
	ET	1.672837	8.3225	5.45	7.380296	31.32978	4.941225	2.954281	1.248893	2.127021
	Rank	1	6	8	5	9	2	4	7	3

(continued on next page)

Table 8 (*continued*).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_{19}	Mean	1900.466	1910.029	7302.813	75 510.23	1 347 280	1926.495	16 749.26	10 011.66	3526.403
	std	0.365433	4.317602	9335.172	95 912.69	2 655 336	8.597216	15 390.88	6264.492	1624.438
	ET	4.209786	8.8725	5.905	8.200627	32.08453	5.944816	5.499856	2.437326	3.965151
	Rank	1	2	5	8	9	3	7	6	4
C_{20}	Mean	2007.834	2089.381	2031.076	2183.378	2196.824	2053.657	2171.901	2013.442	2016.464
	std	8.921894	58.00548	8.006934	112.4959	84.48242	67.63574	66.96642	12.31494	9.719319
	ET	2.205817	8.66	6.01	7.632884	32.18217	5.114406	3.569298	1.422033	2.498372
	Rank	1	6	4	8	9	5	7	2	3
C_{21}	Mean	2200	2232.184	2320.642	2365.181	2378.71	2294.259	2317.462	2338.939	2300.731
	std	1.11E−05	64.36385	5.019333	30.00768	15.62305	62.96438	80.25571	11.80078	67.8594
	ET	2.164595	7.995	5.705	7.67592	30.91818	5.093766	3.446597	1.550895	2.456032
	Rank	1	2	6	8	9	3	5	7	4
C_{22}	Mean	2281.06	2309.059	2301.533	2557.964	3353.759	2301.647	2310.614	2522.3	2302.843
	std	37.8806	4.943361	0.66621	318.5123	290.3335	1.966896	2.218272	440.1517	2.23971
	ET	2.656802	8	5.6	7.691468	32.47053	5.219412	3.650461	1.485458	2.647322
	Rank	1	5	2	8	9	3	6	7	4
C_{23}	Mean	2881.61	2625.078	2625.741	2703.122	2667.124	2626.906	2689.764	2626.989	2632.371
	std	104.6553	11.92366	6.161892	29.22992	23.92093	4.394539	21.62766	3.838097	9.177014
	ET	2.907601	8.44	5.9925	7.540476	31.96432	5.243154	3.80728	1.556376	2.712267
	Rank	9	1	2	8	6	3	7	4	5
C_{24}	Mean	2500	2681.511	2756.395	2844.502	2818.813	2762.964	2842.203	2707.59	2763.583
	std	6.49E−05	121.0459	11.20021	57.90647	19.13374	30.62347	45.61578	138.4949	25.72395
	ET	2.864576	8.79	6.1025	7.583012	31.50968	5.287799	3.982074	1.599807	2.793504
	Rank	1	2	4	9	7	5	8	3	6
C_{25}	Mean	2861.108	2923.571	2922.878	2989.874	3096.027	2922.878	2934.449	2924.698	2923.133
	std	73.26884	27.66195	27.80449	46.80657	27.81794	28.04559	23.17837	28.82089	27.44369
	ET	2.373467	8.87	6.2525	7.55964	30.81478	5.235113	3.623797	1.555178	2.730574
	Rank	1	5	3	8	9	2	7	6	4
C_{26}	Mean	2779.982	2996.725	2911.805	3693.708	4334.765	2954.486	3637.886	3233.734	2825
	std	91.636	132.0475	23.56343	553.8564	127.3249	300.7383	701.149	403.4561	50
	ET	3.383959	8.9	5.6175	7.752859	33.15123	5.37428	4.027064	1.643292	3.277667
	Rank	1	5	3	8	9	4	7	6	2
C_{27}	Mean	3401.417	3094.491	3089.918	3213.005	3133.005	3104.248	3120.537	3121.697	3102.957
	std	85.97019	3.107031	0.637406	89.17928	20.51004	23.95163	13.84502	33.02832	11.62274
	ET	3.554295	8.5725	5.9725	7.862544	33.08424	5.534568	4.1001	1.704705	3.056976
	Rank	9	2	1	8	7	4	5	6	3
C_{28}	Mean	3318.181	3411.842	3228.875	3509.079	3466.503	3381.584	3469.922	3296.815	3280.077
	std	65.46377	0.024089	121.9704	210.4539	113.3702	54.55919	81.28454	133.3557	157.1532
	ET	2.893927	8.61	6.06	7.660696	31.47488	5.461973	3.432892	1.60551	3.219265
	Rank	4	6	1	9	7	5	8	3	2
C_{29}	Mean	3137.561	3240.941	3210.616	3469.044	3307.968	3242.911	3382.392	3177.151	3224.907
	std	3.221487	44.41818	85.33189	95.99438	82.71836	46.00763	203.6293	16.56955	41.64273
	ET	1.888947	9.045	7.1275	7.726333	30.68658	5.993476	4.189407	1.624118	2.860956
	Rank	1	5	3	9	7	6	8	2	4
C_{30}	Mean	3401.235	4842.217	392 251.8	4 029 402	15 643 943	208 214.7	1 711 523	294572	39 359.4
	std	6.935657	714.3725	754 598.5	4 595 904	13 002 120	408 242.4	2 042 804	468 180.2	21 217.88
	ET	2.459816	9.75	6.85	8.489675	33.35524	6.420917	6.072103	2.492744	4.417325
	Rank	1	2	6	8	9	4	7	5	3
Sum Rank		50	127	112	220	248	94	199	136	119
Mean Rank		1.724138	4.37931	3.862069	7.586207	8.551724	3.241379	6.862069	4.689655	4.103448
Total Rank		1	5	3	8	9	2	7	6	4
P-value			1.35E−12	7.75E−12	7.77E−18	2.64E−18	1.09E−08	1.7E−16	3.78E−12	5.19E−13

The optimization results obtained from the employment of COA and competitor algorithms in determining the optimal values for the design variables are presented in Table 15. What is clear from the simulation results is that it has provided the optimal solution to the speed reducer design problem with the values of the design variables equal to (3.5, 0.7, 17, 7.3, 7.8, 3.35021, 5.28668), and the objective function value equal to 2996.3482. The statistical results obtained from COA and competitor algorithms implementation are released in Table 16. These results show that COA has superior performance over competitor algorithms due to better values of statistical indicators. The COA convergence curve in achieving the optimal values of the design variables for the speed reducer design problem is presented in Fig. 19.

5.3. Welded beam design

Welded beam design is a minimization challenge aimed at reducing the fabrication cost of the welded beam, the schematic of which is shown in Fig. 20, see [16]. The mathematical formulation of this optimization problem is as follows:

$$Consider: \; X = [x_1, \; x_2, \; x_3, \; x_4] = [h, \; l, \; t, \; b].$$

$$Minimize: \; f(x) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 \left(14.0 + x_2\right).$$

Subject to :

$$g_1(x) = \tau(x) - 13600 \le 0, \quad g_2(x) = \sigma(x) - 30000 \le 0,$$
$$g_3(x) = x_1 - x_4 \le 0,$$

$$g_4(x) = 0.10471 x_1^2 + 0.04811 x_3 x_4 \left(14 + x_2\right) - 5.0 \le 0,$$

Table 9
Assessment results of the CEC-2017 objective functions (the dimension $m = 30$).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_1	Mean	13 192.16	6834.324	1.91E+10	1.45E+10	1.19E+11	12 369.14	29 927 690	3 292 570	8808.362
	std	20 128.12	3258.115	6.56E+09	3.3E+09	2.07E+10	8882.596	10 397 847	6 540 532	7550.58
	ET	3.218782	18.3497	11.47835	16.43225	86.13094	5.428928	3.31186	1.880483	3.379116
	Rank	4	1	8	7	9	3	6	5	2
C_3	Mean	1155.064	1410.108	50 270.55	71 266.42	311 978.4	4554.144	36 523.02	24 254.23	28 465.39
	std	242.255	1089.162	2676.977	14 084.36	225 479.3	3830.5	6864.904	6134.094	4013.591
	ET	3.440744	17.33	12.4875	16.40225	91.12376	5.539506	3.396387	2.059457	3.551207
	Rank	1	2	7	8	9	3	6	4	5
C_4	Mean	494.1826	498.6949	4813.977	2893.754	19 701.77	493.5752	574.2664	502.0038	521.5817
	std	13.36359	20.54551	2929.477	1328.191	5756.467	25.11836	39.60236	12.82021	38.76644
	ET	2.29552	8.9375	6.235	16.57796	94.73134	5.390731	3.298446	1.625923	3.330338
	Rank	2	3	8	7	9	1	6	4	5
C_5	Mean	632.7583	680.3417	813.769	765.9957	1062.28	654.9641	757.5132	626.8799	610.6887
	std	27.7517	26.06507	31.33511	19.43874	132.0501	25.57847	45.57821	31.17762	26.98461
	ET	3.79753	9.5675	6.4	16.46336	86.20747	5.5862	3.956965	2.009782	4.045952
	Rank	3	5	8	7	9	4	6	2	1
C_6	Mean	603.4788	647.7238	681.918	657.4201	712.4569	620.3918	658.8722	602.515	600.8239
	std	1.237184	9.159851	12.18317	3.529809	15.75672	6.42238	6.891111	1.793651	0.424316
	ET	4.96856	9.9625	6.8275	16.84262	95.47662	6.307297	5.598972	2.4179	5.528779
	Rank	3	5	8	6	9	4	7	2	1
C_7	Mean	832.3448	1072.087	1256.429	1190.422	1477.364	993.0221	1240.659	995.8591	852.1594
	std	25.87207	75.25951	136.7428	56.16735	186.8651	152.5607	48.33755	65.03424	14.62477
	ET	3.73344	9.875	6.5275	17.68813	98.01842	5.662265	3.957543	1.866193	3.84117
	Rank	1	5	8	6	9	3	7	4	2
C_8	Mean	881.3244	942.2807	1076.072	988.4608	1359.15	947.7558	968.0795	947.7817	904.9677
	std	10.31683	16.26651	74.75508	29.59179	83.03775	32.55049	14.15261	39.93544	16.93361
	ET	11.17564	9.73	6.455	16.78675	98.53333	5.623088	3.988445	1.833051	3.722885
	Rank	1	3	8	7	9	4	6	5	2
C_9	Mean	1055.207	3754.914	12 185.44	5447.032	24 068.44	3697.012	8312.999	4672.748	2785.546
	std	108.8082	894.4392	3751.348	775.4573	14 882.59	920.739	1132.435	980.4118	633.5971
	ET	3.20711	9.1275	6.3975	17.2146	98.78208	5.840488	3.998224	1.909363	3.85741
	Rank	1	4	8	6	9	3	7	5	2
C_{10}	Mean	4032.979	5070.512	6852.655	6334.542	8989.464	4977.696	6049.238	4111.449	4097.49
	std	376.7088	856.0735	979.3841	1108.06	587.8434	714.1025	708.4935	429.2742	876.3847
	ET	4.9485	8.925	6.365	16.91384	98.75047	5.793806	4.41101	2.097676	4.180843
	Rank	1	5	8	7	9	4	6	3	2
C_{11}	Mean	1174.444	1248.114	5405.535	5156.018	36 521.86	1294.21	1299.659	1172.203	1206.552
	std	37.46779	10.79267	1970.032	1614.353	43 836.54	59.29824	51.34188	16.66207	33.40919
	ET	2.7049	9.19	6.44	16.87838	99.23629	5.743342	3.75735	1.789888	4.20798
	Rank	2	4	8	7	9	5	6	1	3
C_{12}	Mean	25 288.54	3 149 972	5.42E+09	4.41E+09	9.07E+09	274 477.1	19 543 825	4 066 819	2 249 596
	std	5951.198	1 922 496	1.86E+09	2.3E+09	3.89E+09	211 991.3	9 719 632	2 252 913	2 406 859
	ET	3.59283	8.7	6.2725	17.14905	98.67863	5.704228	4.109145	2.027968	4.414334
	Rank	1	4	8	7	9	2	6	5	3
C_{13}	Mean	1929.051	27 503.47	1.56E+09	1.9E+09	8.5E+09	23 134.37	625 538.4	40 918.94	5601.93
	std	392.7499	13 539.44	2.60E+09	2.78E+09	3.61E+09	28 062.93	202 440.6	31 985.55	4289.611
	ET	2.30465	8.72	6.4275	17.47737	97.85723	5.705427	3.803851	2.120537	4.348225
	Rank	1	4	7	8	9	3	6	5	2
C_{14}	Mean	1441.59	7676.16	1 253 660	2 245 171	8 576 887	3719.271	1 065 499	330810	51 989.62
	std	3.984674	4583.662	401 150.7	1 395 717	5 231 155	1229.062	656 596.5	471 163.9	38 914.46
	ET	4.56792	8.735	7.1225	17.21265	98.57253	5.831434	4.136545	2.056431	4.100411
	Rank	1	3	7	8	9	2	6	5	4
C_{15}	Mean	1626.687	16 898.75	15 334 685	460153	9.22E+08	11 484.57	93 428.18	13 860.77	8985.011
	std	26.87863	3225.086	13 677 702	591 085.5	7.71E+08	16 407.39	74 420.35	19 381.82	14 549.95
	ET	3.59765	8.735	7.1625	16.68643	98.02372	5.543987	3.665856	1.750559	3.49478
	Rank	1	5	8	7	9	3	6	4	2
C_{16}	Mean	2247.447	2898.495	3380.028	3938.411	4778.03	2750.455	3376.06	3201.13	2735.798
	std	239.1718	277.9592	316.2617	1075.297	380.0955	231.445	230.8674	263.7755	285.6483
	ET	3.52186	8.86	7.0475	16.9993	97.52749	5.742731	4.027208	1.828306	3.729537
	Rank	1	4	7	8	9	3	6	5	2
C_{17}	Mean	1863.83	2360.081	3381.412	2669.197	3457.172	2281.661	2500.963	2344.249	2419.006
	std	80.49175	355.0427	1966.658	449.9754	141.8733	348.2781	154.6543	251.5582	305.9124
	ET	5.73065	9.16	7.495	17.27437	99.46528	6.400678	5.378896	2.431481	5.143885
	Rank	1	4	8	7	9	2	6	3	5
C_{18}	Mean	1901.733	57 684.47	38 762 645	4 475 548	38 475 863	38 505.95	1 613 975	1 759 275	1 097 197
	std	17.295	15 975.22	40 002 539	2 759 094	26 621 174	12 725.52	1 824 680	1 008 128	1 745 000
	ET	3.6461	9.14	8.8	16.97522	113.9871	5.671528	3.986927	1.851991	3.714166
	Rank	1	3	9	7	8	2	5	6	4

(continued on next page)

Table 9 (*continued*).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_{19}	Mean	1924.713	7031.78	3.14E+08	84 096 333	1.71E+09	19 009.77	694 628.7	5778.088	3865.876
	std	3.692442	2140.138	4.00E+08	1.64E+08	1.6E+09	18 890.59	474 410.2	4410.03	514.7364
	ET	10.1975	12.005	12.435	19.20614	102.0908	8.868973	11.65898	4.956911	11.11792
	Rank	1	4	8	7	9	5	6	3	2
C_{20}	Mean	2184.966	2394.506	2908.489	2895.383	3353.181	2497.84	2772.515	2725.398	2503.788
	std	88.64055	105.7484	132.8081	261.6491	214.0679	234.1132	171.3568	215.4237	68.51425
	ET	7.60978	11.69	10.93	17.3619	100.3842	6.509156	5.617558	3.013091	5.457871
	Rank	1	2	8	7	9	3	6	5	4
C_{21}	Mean	2350.368	2436.43	2552.29	2541.87	2863.985	2450.214	2562.559	2439.239	2351.506
	std	28.27442	24.59018	171.0179	65.92515	109.5826	46.03706	26.49848	22.37526	90.97219
	ET	6.8376	10.315	8.2425	17.58072	100.2111	6.606322	6.048824	3.471319	5.815199
	Rank	1	3	7	6	9	5	8	4	2
C_{22}	Mean	2303.148	2300.499	9262.883	7485.314	9552.094	5354.949	7527.103	5295.391	3278.934
	std	1.320014	0.245434	171.9551	611.015	468.7319	2146.634	435.9927	2012.143	1955.08
	ET	6.19179	10.9425	7.8425	17.80606	99.81453	6.860423	6.687391	3.148215	6.209978
	Rank	2	1	8	6	9	5	7	4	3
C_{23}	Mean	2641.551	2821.125	3240.91	3172.875	3325.841	2859.882	3239.452	2802.059	2760.729
	std	123.5185	39.65883	150.9637	103.4973	88.48584	86.62251	98.7694	22.10149	10.66424
	ET	7.20132	12.135	8.075	17.87434	101.3341	7.117329	7.111117	3.339705	6.924079
	Rank	1	4	8	6	9	5	7	3	2
C_{24}	Mean	2879.028	2947.651	3318.291	3364.852	3541.283	2972.409	3594.201	3043.991	3017.349
	std	16.47476	61.7323	81.32298	64.50831	102.1246	68.74434	6.910262	63.65891	22.67671
	ET	7.11195	12.1475	8.945	17.9629	100.5939	7.198784	7.384945	3.431541	7.274516
	Rank	1	2	6	7	8	3	9	5	4
C_{25}	Mean	3119.933	2895.971	3517.16	3240.544	5366.143	2893.681	2954.639	2905.63	2907.268
	std	108.486	11.06199	407.6933	133.713	721.6637	12.81998	38.5276	21.21039	23.90155
	ET	6.04937	13.03	9.0175	17.65292	100.019	6.954525	6.717033	3.133043	6.801693
	Rank	6	2	8	7	9	1	5	3	4
C_{26}	Mean	2900.361	6504.46	9145.999	8461.809	11 459.01	5020.642	7676.89	5470.911	3874.659
	std	0.181233	905.5686	501.4022	66.81249	2691.837	845.1924	976.7057	327.8804	1929.825
	ET	8.30017	13.7625	8.8875	18.29147	100.3662	7.519331	8.226178	3.744161	8.299498
	Rank	1	5	8	7	9	3	6	4	2
C_{27}	Mean	3412.903	3316.9	3495.63	3539.545	3836.009	3272.215	3462.921	3250.379	3236.998
	std	53.55879	8.005529	185.2389	69.03988	134.0435	49.62406	119.2647	14.29706	1.149267
	ET	8.6109	12.57	9.575	18.69611	101.9383	8.030756	8.530925	3.971858	9.047374
	Rank	5	4	7	8	9	3	6	2	1
C_{28}	Mean	3218.819	3224.432	4245.736	4066.904	9767.343	3219.147	3351.283	3269.706	3273.843
	std	25.17824	26.52424	566.0832	127.7742	3422.132	23.31549	16.37631	53.61776	68.30171
	ET	9.95368	12.335	8.825	18.10061	100.077	7.484028	7.805466	3.597163	8.07657
	Rank	1	3	8	7	9	2	6	4	5
C_{29}	Mean	3637.205	4266.781	5397.736	5171.172	6733.607	4217.868	4598.84	3932.081	3594.673
	std	97.55547	205.8735	724.6104	363.7938	1582.384	213.5489	506.9326	237.8881	174.5654
	ET	8.3956	11.6425	8.5325	18.03109	101.6079	7.257766	7.253381	3.26448	8.087574
	Rank	2	5	8	7	9	4	6	3	1
C_{30}	Mean	7524.216	115 354.3	41 221 542	25 382 608	7.74E+08	11 671.43	5 733 549	129 614.8	61 556.14
	std	1292.389	37 343.13	37 323 575	12 183 538	6.87E+08	5623.693	2 182 426	168 100.3	42 705.64
	ET	10.58424	14.1775	10.39	20.21186	101.8412	9.676731	13.53821	5.729067	16.46926
	Rank	1	4	8	7	9	2	6	5	3
Sum Rank		49	103	225	202	259	92	182	113	80
Mean Rank		1.689655	3.551724	7.758621	6.965517	8.931034	3.172414	6.275862	3.896552	2.758621
Total Rank		1	4	8	7	9	3	6	5	2
P-value			5.5E−13	3.44E−21	2.02E−21	1.97E−21	1.03E−12	3.02E−20	7.61E−14	3.93E−09

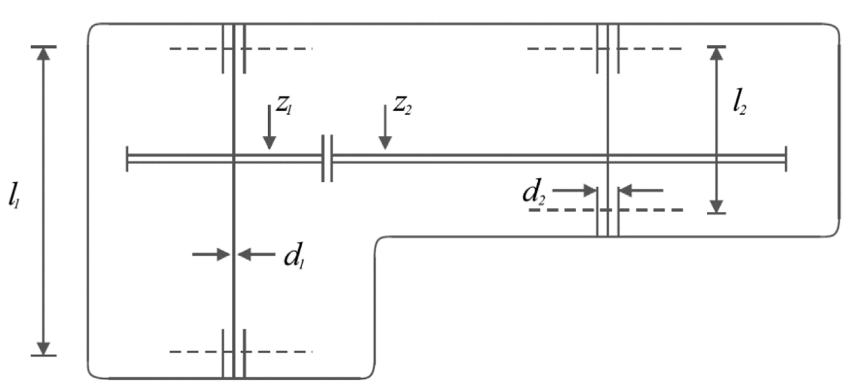

Fig. 18. Schematic of speed reducer design.

Table 10
Assessment results of the CEC-2017 objective functions (the dimension $m = 50$).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_1	Mean	3 315 269	42 842.82	405 210.6	3.86E+10	1.58E+11	252 252	3.12E+08	1.82E+08	1748.654
	std	1 075 756	12 838.42	67 030.79	8.56E+09	8.59E+10	274 007.4	14 943 768	3.14E+08	1585.226
	ET	4.13013	27.61751	20.43712	26.20912	161.9226	6.569379	4.544283	2.569475	7.911928
	Rank	5	2	4	8	9	3	7	6	1
C_3	Mean	19 036.24	25 316.46	80 998.24	178 126.4	433 994.5	48 905.85	137 172.5	116 505	95 130.43
	std	2706.178	7280.511	18 530.18	20 509.16	221 153.7	27 618.48	8507.933	49 454.33	20 877.52
	ET	5.60337	20.7425	18.295	25.6966	160.7663	6.380577	4.827886	2.477386	6.610373
	Rank	1	2	4	8	9	3	7	6	5
C_4	Mean	520.8665	536.804	658.4521	4880.397	57 862.04	642.3212	820.9545	601.9624	611.6765
	std	21.64596	39.1547	102.803	758.7308	41 856.6	108.6563	147.9188	16.69119	86.56252
	ET	4.55062	11.1225	9.08	25.54619	162.9919	6.613335	4.529132	2.479982	6.577678
	Rank	1	2	6	8	9	5	7	3	4
C_5	Mean	723.5615	846.7622	777.2264	958.8824	1529.609	805.7	914.3806	801.3027	728.0931
	std	51.09782	22.32041	57.9801	25.43934	178.4865	37.33739	46.5467	44.19212	15.2061
	ET	5.81933	11.4525	9.0575	26.18504	163.4574	6.6495	5.726731	2.754316	7.483692
	Rank	1	6	3	8	9	5	7	4	2
C_6	Mean	612.0016	659.3903	641.5586	661.2122	721.3811	642.0786	676.1133	619.4905	601.9702
	std	2.927746	3.495261	9.586024	5.083751	30.65998	4.821438	1.764442	3.919541	0.769677
	ET	5.97599	12.5225	9.685	27.56717	152.5744	7.684058	8.216784	3.717213	11.15986
	Rank	2	6	4	7	9	5	8	3	1
C_7	Mean	1037.816	1476.312	1129.128	1610.911	5093.482	1407.681	1809.914	1234.734	1023.755
	std	48.52962	74.95805	96.8263	42.29354	1298.529	120.2135	94.7745	80.68742	78.30643
	ET	6.56278	11.3675	10.3125	26.01934	139.976	6.667532	5.612421	2.860109	7.690668
	Rank	2	6	3	7	9	5	8	4	1
C_8	Mean	999.1181	1115.417	1066.612	1284.93	1665.783	1094.257	1225.864	1072.979	1033.068
	std	12.78859	33.42453	48.44566	27.55233	206.0551	58.64504	9.063994	53.79039	15.14104
	ET	6.37259	10.9275	9.745	25.86033	142.3547	6.752656	5.665634	2.907547	7.554506
	Rank	1	6	3	8	9	5	7	4	2
C_9	Mean	3807.053	10 415.74	14 257.04	14 930.69	69 861.7	8679.325	28 481.41	13 257.87	6779.496
	std	1409.831	834.971	4157.913	1951.765	27 668.83	2166.377	750.8706	5549.205	2791.339
	ET	6.62697	10.9175	9.1225	26.04296	141.3945	6.688472	5.675774	2.867966	7.943198
	Rank	1	4	6	7	9	3	8	5	2
C_{10}	Mean	6474.525	7750.663	8079.263	9742.126	15 572.56	8561.54	9535.194	7192.878	6102.468
	std	642.9051	899.8296	1241.837	999.4984	1381.539	923.4614	521.4451	1701.341	426.8156
	ET	8.1653	11.21	9.6225	25.87596	141.1085	6.927257	6.216079	3.04137	7.652248
	Rank	2	4	5	8	9	6	7	3	1
C_{11}	Mean	1263.128	1301.234	1415.114	12 647.05	50 789.17	1431.419	1806.023	1367.406	1369.452
	std	37.26383	34.69649	67.5818	4268.664	37 992.09	53.89609	144.0292	43.88708	62.68602
	ET	4.84691	10.49	9.3925	25.89098	141.3677	6.574947	5.151527	2.706422	6.643485
	Rank	1	2	5	8	9	6	7	3	4
C_{12}	Mean	14 734 349	14 523 775	28 970 678	1.15E+10	6.64E+10	6 514 769	1.94E+08	21 804 971	15 233 433
	std	1 056 181	5 590 039	14 911 280	4.74E+09	1.23E+10	1 493 309	1.08E+08	12 560 789	13 685 632
	ET	6.50826	10.71	9.295	26.26017	141.1668	6.809979	5.571421	2.793689	7.143096
	Rank	3	2	6	8	9	1	7	5	4
C_{13}	Mean	17 284.75	33 254.95	107 376.8	6.54E+09	2.25E+10	18 645.75	4 211 674	55 917.83	13 988.33
	std	5458.85	5577.723	33 628.58	5.6E+09	4.36E+09	15 137.23	2 072 599	16 894.32	11 148.99
	ET	5.82105	10.7625	8.5675	26.59835	141.7889	6.67793	5.305679	2.672377	7.160787
	Rank	2	4	6	8	9	3	7	5	1
C_{14}	Mean	1571.45	72 882.26	484 252.2	4 459 589	66 008 390	54 941.63	4 480 979	995 551.4	683 427.1
	std	18.22786	51 249.07	290 621.6	3 468 428	48 997 430	70 590.58	3 767 931	459 667	339 592.3
	ET	7.25617	11.2125	8.8725	33.73479	141.1293	6.938014	6.14169	3.055636	8.605737
	Rank	1	3	4	7	9	2	8	6	5
C_{15}	Mean	2538.389	22 117.17	30 167.94	4.22E+08	1.45E+10	15 278.19	870 400.8	15 267.5	11 552.02
	std	242.9841	4421.737	19 155.36	3.53E+08	4.23E+09	8933.205	553 181.8	12 632.59	5868.08
	ET	14.79132	10.8625	8.91	36.88921	141.5932	6.842865	5.024685	2.579032	7.822418
	Rank	1	5	6	8	9	4	7	3	2
C_{16}	Mean	2776.232	3290.828	3133.017	5078.6	8645.303	3624.084	5261.493	3662.142	3246.869
	std	172.4686	423.4437	335.9539	466.8744	1716.103	772.3387	361.8172	411.5865	313.7888
	ET	6.13545	11.24	9.1	27.28298	141.0615	6.747998	5.689725	2.796426	8.458575
	Rank	1	4	2	7	9	5	8	6	3
C_{17}	Mean	2593.645	3622.423	3460.929	4207.636	22 516.2	3479.48	3666.198	3500.166	2946.517
	std	48.0614	261.2782	471.8638	230.2988	25 331.61	480.5379	460.7234	369.8376	586.8719
	ET	6.82358	12.2725	9.5	27.35111	141.1197	7.630416	7.730899	3.938573	10.26146
	Rank	1	6	3	8	9	4	7	5	2
C_{18}	Mean	30 548.72	285 408.8	4 728 235	33 265 307	3.09E+08	144 490.1	5 194 829	6 150 916	2 736 152
	std	18 042.09	107 201.9	2 842 407	23 766 615	3.41E+08	84 874.22	4 536 852	3 718 236	1 013 375
	ET	5.91065	11.43	9.265	26.60523	140.4356	6.656553	5.358059	3.035908	7.119003
	Rank	1	3	5	8	9	2	6	7	4

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Table 10 (*continued*).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_{19}	Mean	2092.182	187 713	14 682.08	71 199 665	3.96E+09	13 924.36	1 346 619	21 370.92	27 125.77
	std	44.64316	77 646.77	20 052.98	22 973 522	3.06E+09	10 154.77	795 814.7	20 174.08	8321.168
	ET	18.45071	16.42	12.5925	30.43693	149.1319	11.86309	17.80513	8.005873	23.67321
	Rank	1	6	3	8	9	2	7	4	5
C_{20}	Mean	2615.441	2975.217	3201.152	3511.344	4481.263	3351.881	3653.435	3352.375	2999.659
	std	210.8097	221.5988	235.4402	229.5363	724.7672	547.7326	337.8715	277.0634	216.1621
	ET	6.41316	15.91	11.595	27.45428	140.6535	8.01566	8.464354	4.032267	10.79614
	Rank	1	2	4	7	9	5	8	6	3
C_{21}	Mean	2458.266	2563.268	2590.342	2780.428	3212.666	2578.918	2875.403	2583.652	2505.552
	std	25.03703	32.7608	54.60647	46.76724	89.26708	42.67194	80.42838	39.87448	49.40136
	ET	10.40283	14.39	11.525	28.08641	142.663	9.101136	10.65395	5.016092	13.85557
	Rank	1	3	6	7	9	4	8	5	2
C_{22}	Mean	4441.218	10 769.28	9861.138	12 669.9	16 234.87	10 514.74	11 372.25	8995.997	7978.785
	std	2878.434	808.8176	526.2107	1284.472	1057.112	1099.41	1060.88	810.8149	884.9973
	ET	12.5758	15.4125	11.845	28.88192	142.7082	9.195117	11.48886	5.236989	16.76211
	Rank	1	6	4	8	9	5	7	3	2
C_{23}	Mean	3157.258	3250.112	3019.89	3795.253	4134.235	3208.919	3802.545	3056.003	3014.028
	std	105.4818	46.01476	27.95624	168.2833	108.2145	74.36035	226.1346	77.2273	59.61279
	ET	11.61551	15.955	12.98	29.00058	142.9817	9.982539	13.10981	5.92802	19.74762
	Rank	4	6	2	7	9	5	8	3	1
C_{24}	Mean	3075.367	3202.16	3141.655	4077.713	4274.258	3344.262	4391.214	3352.968	3471.09
	std	30.27965	36.52006	78.20428	50.21555	103.2999	91.83818	464.6094	135.8725	55.09225
	ET	14.68311	16.8325	12.5825	29.71366	144.4749	10.54873	13.46267	6.39756	20.47672
	Rank	1	3	2	7	8	4	9	5	6
C_{25}	Mean	3053.663	3119.685	3072.296	5927.904	44 345.66	3087.517	3260.813	3098.619	3126.829
	std	22.9098	17.21472	9.069501	372.4434	32 415.98	36.36447	53.24515	26.02493	34.75452
	ET	12.37096	17.5375	12.92	30.08949	143.4648	10.10314	13.32678	6.142383	19.45077
	Rank	1	5	2	8	9	3	7	4	6
C_{26}	Mean	3625.379	11 003.77	7346.474	13 207.6	18 251.97	8406.027	11 272.24	5983.638	4310.282
	std	249.5009	1714.184	736.4798	353.0066	2094.699	2695.247	1183.283	2700.448	2644.019
	ET	14.84107	18.7125	13.18	30.49171	143.9118	11.02709	15.24959	6.927317	21.71514
	Rank	1	6	4	8	9	5	7	3	2
C_{27}	Mean	3328.512	3708.432	3513.203	5202.322	5894.718	3654.683	4703.264	3398.644	3522.27
	std	37.02017	87.72919	90.28107	253.0724	645.2934	22.77417	458.5873	55.06845	123.9673
	ET	20.40176	20.575	14.035	31.43948	144.3136	12.04256	17.6981	7.88417	25.52838
	Rank	1	6	3	8	9	5	7	2	4
C_{28}	Mean	3595.247	3350.738	3349.943	6781.478	17 862.2	3348.262	3840.206	3539.514	3378.84
	std	28.94284	25.51792	20.45934	776.95	5032.823	15.62518	98.47132	198.2283	35.77909
	ET	16.12046	20.3025	14.0025	30.66428	144.4589	11.2621	15.53376	7.218618	24.26196
	Rank	6	3	2	8	9	1	7	5	4
C_{29}	Mean	4160.974	5891.789	4802.263	7508.055	270 291.9	5191.122	6358.622	4497.028	4484.107
	std	293.423	249.0131	147.7708	599.8184	325 333.2	116.2426	546.3355	531.112	294.6761
	ET	9.00653	18.5925	12.43	29.45915	142.417	9.84777	12.33984	5.578563	18.82964
	Rank	1	6	4	8	9	5	7	3	2
C_{30}	Mean	1 902 810	29 734 435	7 438 553	5.44E+08	6E+09	998 867.9	84 102 948	2 131 384	1 060 620
	std	850 536	4 031 364	678 397.1	1.53E+08	3E+09	229 849.7	28 752 488	752 313.4	138 286.5
	ET	21.54887	22.7	15.1775	32.96077	146.4922	13.96169	22.37302	9.778476	32.1624
	Rank	3	6	5	8	9	1	7	4	2
Sum Rank		49	125	116	223	260	112	212	125	83
Mean Rank		1.689655	4.310345	4	7.689655	8.965517	3.862069	7.310345	4.310345	2.862069
Total Rank		1	5	4	7	8	3	6	5	2
P-value			7.03E−09	7.54E−10	1.97E−21	1.97E−21	0.000196	1.97E−21	1.16E−14	0.043919

$$g_5(x) = 0.125 - x_1 \le 0, \quad g_6(x) = \delta(x) - 0.25 \le 0,$$

$$g_7(x) = 6000 - p_c(x) \le 0.$$

where

$$\tau(x) = \sqrt{(\tau')^2 + (2\tau\tau')\frac{x_2}{2R} + (\tau")^2}, \quad \tau' = \frac{6000}{\sqrt{2}x_1x_2}, \quad \tau" = \frac{MR}{J},$$

$$M = 6000\left(14 + \frac{x_2}{2}\right), \quad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2},$$

$$J = 2\left\{x_1x_2\sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2}\right)^2\right]\right\}, \quad \sigma(x) = \frac{504000}{x_4x_3^2},$$

$$\delta(x) = \frac{65856000}{(30\cdot 10^6)\,x_4x_3^3},$$

$$p_c(x) = \frac{4.013\,(30\cdot 10^6)\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1 - \frac{x_3}{28}\sqrt{\frac{30\cdot 10^6}{4(12\cdot 10^6)}}\right).$$

With

$$0.1 \le x_1,\ x_4 \le 2 \text{ and } 0.1 \le x_2,\ x_3 \le 10.$$

The optimization results of welded beam design variables using COA and competitor algorithms are presented in Table 17. Based on the results, COA has provided the optimal solution to the welded beam design problem with the values of the design variables equal to (0.20573, 3.4705, 9.0366, 0.20573), and

Table 11
Assessment results of the CEC-2017 objective functions (the dimension $m = 100$).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_1	Mean	77 405 257	6 652 259	53 550 561	1.17E+11	3.66E+11	5.38E+09	8.06E+09	4.21E+09	7 749 289
	std	15 022 042	2 844 142	11 881 381	2.46E+09	1.27E+11	4.68E+09	1.33E+09	2.01E+09	3 570 613
	ET	12.72053	32.32173	29.76239	52.31149	280.6746	10.04899	10.54983	5.968778	24.58238
	Rank	4	1	3	8	9	6	7	5	2
C_3	Mean	168 018.9	221 703.6	540 528.5	343 547.6	2 591 096	244 019.9	326 598.9	404 341.3	304 639.7
	std	32 449.11	19 404.1	312 134	14 684.2	4 218 512	40 559.84	4594.905	154 610.7	15 455.73
	ET	12.83754	35.9075	30.755	51.71869	279.4147	10.05519	11.64342	6.002042	25.92682
	Rank	1	2	8	6	9	3	5	7	4
C_4	Mean	770.5338	1047.664	874.8845	17 532.97	255 147	1054.363	2806.475	1210.298	967.8153
	std	51.359	94.22896	29.26596	892.4654	82 550.1	222.5295	185.9697	26.70695	137.7038
	ET	10.08412	16.07	15.2725	52.37745	279.1109	9.97022	10.57799	5.754225	25.6676
	Rank	1	4	2	8	9	5	7	6	3
C_5	Mean	1159.534	1297.865	1340.461	1581.03	2821.148	1331.672	1615.54	1330.894	1076.116
	std	41.56693	41.22007	35.79038	61.09321	485.9184	104.7801	106.9889	89.45892	108.9174
	ET	12.36161	16.4175	15.9725	52.08178	280.6587	10.79397	12.6818	6.314778	28.71218
	Rank	2	3	6	7	9	5	8	4	1
C_6	Mean	636.3933	661.8228	655.7855	669.8956	740.4929	657.7108	683.9256	647.2944	608.6615
	std	2.368568	1.666763	5.160623	2.737058	24.77902	5.58988	2.326721	2.692698	2.608521
	ET	20.56597	18.71	17.3875	53.82728	283.7645	12.65433	17.48451	8.332883	40.27261
	Rank	2	6	4	7	9	5	8	3	1
C_7	Mean	1995.873	2892.27	2118.126	3146.098	6787.941	2687.366	3809.293	2425.588	1580.557
	std	62.29099	191.0618	106.1042	93.87899	1844.783	431.7171	111.2355	137.3137	186.2498
	ET	12.17208	18.0175	16.965	52.1415	282.3977	10.72964	12.81611	6.434585	28.42653
	Rank	2	6	3	7	9	5	8	4	1
C_8	Mean	1393.982	1770.807	1634.366	1971.045	3165.053	1726.328	2073.262	1708.053	1412.464
	std	128.9828	33.83947	123.811	69.62971	374.2206	102.9452	46.2196	126.9794	59.72827
	ET	12.4164	17.5175	16.8125	52.4591	284.1283	10.93991	12.83868	6.541607	28.99947
	Rank	1	6	3	7	9	5	8	4	2
C_9	Mean	22 768.17	24 488.14	30 457.14	32 350.69	216 860.8	24 859.23	70 177.61	29 891.58	19 116.37
	std	1051.544	1397.584	3126.702	3420.436	55 984.65	2270.317	5322.684	2585.196	2537.958
	ET	10.0161	18.4475	16.53	53.03951	280.8296	10.60503	13.10753	6.326379	27.2745
	Rank	2	3	6	7	9	4	8	5	1
C_{10}	Mean	13 963.28	15 774.67	18 245.96	21 612.58	32 552.71	16 866.85	22 182.38	17 660.02	12 399.59
	std	645.9373	1575.541	1192.44	1659.223	1306.313	286.3837	348.7854	608.5102	1054.92
	ET	12.43456	22.515	16.4375	52.40375	289.7703	11.14534	14.24492	6.964459	30.21424
	Rank	2	3	6	7	9	4	8	5	1
C_{11}	Mean	4420.523	3764.265	7395.784	97 286.59	436 685.2	6459.598	85 067.91	38 201.58	14 820.81
	std	513.5259	880.9741	3576.74	8990.158	343 056.3	2407.4	25 593.69	12 399.92	7781.148
	ET	14.07226	20.165	16.4225	52.08797	286.7005	10.29599	12.16451	6.011349	25.48237
	Rank	2	1	4	8	9	3	7	6	5
C_{12}	Mean	2.5E+08	2.96E+08	3.14E+08	5.26E+10	3.25E+11	89 477 277	1.75E+09	1.4E+09	1.37E+08
	std	79 804 035	1.28E+08	1.23E+08	9.95E+09	3.94E+10	43 621 271	3.91E+08	1.05E+09	55 263 037
	ET	14.71234	17.62	16.415	52.48044	303.4965	10.93244	12.88288	6.399252	28.80564
	Rank	3	4	5	8	9	1	7	6	2
C_{13}	Mean	111 930.2	34 112.28	925 581.8	1.65E+10	6.38E+10	24 107.97	22 893 716	5 477 161	8290.541
	std	114 248.1	6564.774	1 673 613	5.52E+09	2.49E+10	5778.677	5 940 592	8 253 840	2845.271
	ET	12.0608	18.7025	16.74	52.81704	305.1645	10.71318	12.59859	6.227414	25.6453
	Rank	4	3	5	8	9	2	7	6	1
C_{14}	Mean	95 013.85	378 691.9	2 311 950	23 404 263	2.2E+08	904 886.1	5 509 122	5 448 575	2 649 777
	std	78 323.57	137 548.5	1 033 002	9 961 126	1.42E+08	457 299.7	3 572 724	2 375 593	1 566 053
	ET	16.11798	18.51	15.74	52.90491	358.1617	11.15509	14.3087	7.211028	29.61405
	Rank	1	2	4	8	9	3	7	6	5
C_{15}	Mean	61 998.36	23 448.85	283 946.9	4.3E+09	3.05E+10	4660.313	3 678 275	623 479.8	5871.829
	std	32 307.71	9281.963	455 844.5	2.15E+09	6.39E+09	2097.459	1 070 312	710 389.3	3064.439
	ET	14.28005	17.4925	15.36	51.77933	309.7246	10.25323	12.01953	6.407818	24.41751
	Rank	4	3	5	8	9	1	7	6	2
C_{16}	Mean	5974.812	7287.714	5736.294	12 300.79	29 175.36	5957.653	9046.773	6897.642	6370.269
	std	74.25068	841.4534	616.223	1256.537	11 214.87	814.0841	1115.37	348.9503	632.8713
	ET	15.01932	17.905	15.5225	52.163	330.1629	10.73461	12.65645	6.289483	26.09917
	Rank	3	6	1	8	9	2	7	5	4
C_{17}	Mean	4626.489	5613.772	5917.958	595 325.2	27 389 897	5609.819	7277.368	5478.58	4725.716
	std	146.1274	278.5767	573.4387	961 268.3	48 017 659	728.8229	216.3915	834.972	995.7761
	ET	21.13927	20.7625	17.105	54.36595	360.1523	12.61897	17.42717	8.132109	40.3022
	Rank	1	5	6	8	9	4	7	3	2
C_{18}	Mean	243 060.2	981 709.8	12 460 849	24 453 733	2.35E+08	1 424 603	6 435 022	10 343 040	3 829 066
	std	130 447.2	359 338.8	8 382 157	4 962 378	1.5E+08	985 603.2	1 489 242	10 191 496	2 166 081
	ET	16.53821	19.3675	15.915	53.19953	374.4038	10.75781	12.8708	6.255659	25.96581
	Rank	1	2	7	8	9	3	5	6	4

(continued on next page)

Table 11 (*continued*).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C_{19}	Mean	325 050.6	933 086.7	195 464.4	3.09E+09	2.77E+10	5120.695	11 973 306	38 066.79	7723.994
	std	199 512	269 154.8	149 321.1	2.03E+09	5.58E+09	1867.731	2 797 158	26 762.99	4283.078
	ET	24.4647	30.355	22.415	61.8386	362.8368	21.01764	38.19421	16.5128	84.09295
	Rank	5	6	4	8	9	1	7	3	2
C_{20}	Mean	4566.422	4948.474	5675.428	6317.018	8177.305	5117.673	6151.932	5191.428	5505.808
	std	70.39281	301.568	288.1634	662.0971	358.4526	723.8248	1014.118	801.6889	178.5296
	ET	18.03894	28.5825	20.8825	55.34823	354.6389	13.01604	18.46468	8.627666	38.54546
	Rank	1	2	6	8	9	3	7	4	5
C_{21}	Mean	2856.475	3082.292	3101.232	3847.085	5240.004	3247.269	4418.118	3110.895	2976.934
	std	35.856	94.0634	65.48749	166.4613	524.3842	170.5796	152.5915	99.17849	81.69652
	ET	21.248	27.325	21.655	59.77126	332.3327	19.33273	31.97972	13.99663	64.95225
	Rank	1	3	4	7	9	6	8	5	2
C_{22}	Mean	17 560.96	21 813.98	20 409.61	26 033.84	36 032.02	19 810.8	27 887.99	19 801.47	18 090.89
	std	935.1663	1525.059	298.4085	481.522	482.1705	941.3289	948.7086	2370.764	1998.956
	ET	22.2343	31.985	23.88	59.45096	308.259	19.74433	34.52864	14.7396	69.58938
	Rank	1	6	5	7	9	4	8	3	2
C_{23}	Mean	3320.684	3674.101	3553.42	5204.827	5925.285	3853.559	5847.054	3375.795	3193.435
	std	22.55816	40.53096	93.07469	204.0372	213.3863	126.917	413.3709	71.26698	33.78684
	ET	25.2888	35.935	25.8975	61.47077	293.1445	24.13588	40.30186	17.99268	83.54918
	Rank	2	5	4	7	9	6	8	3	1
C_{24}	Mean	3740.195	4593.698	4076.577	6664.68	9223.837	5082.126	7535.477	4248.49	3833.585
	std	57.75833	256.903	98.40505	220.7467	808.3508	153.5245	484.2995	93.61931	61.30861
	ET	26.4639	39.3175	26.9175	61.39859	294.3377	26.05021	41.18798	18.26514	89.77189
	Rank	1	5	3	7	9	6	8	4	2
C_{25}	Mean	3422.69	3619.605	3513.548	12 797.88	122 764	3711.239	4516.334	4182.961	3555.433
	std	59.70878	17.37803	78.22261	1183.302	53 581.49	91.66989	257.3957	366.8607	60.25021
	ET	26.7336	40.0225	27.825	62.64393	297.6942	27.05369	42.87095	19.58886	91.46303
	Rank	1	4	2	8	9	5	7	6	3
C_{26}	Mean	11 512.07	27 802.38	15 919.85	35 508.61	51 916.17	23 662.87	28 945.45	16 624.01	13 940.89
	std	737.8098	668.8902	3644.047	1374.056	4588.424	3080.363	936.7875	2350.801	2197.828
	ET	27.5755	42.615	29.555	64.0579	295.8426	29.09365	48.30442	21.53098	98.37307
	Rank	1	6	3	8	9	5	7	4	2
C_{27}	Mean	3549.793	4568.285	3721.42	6150.628	12 037.88	4027.586	5461.593	3694.274	3653.321
	std	34.36312	224.1683	121.4235	407.9372	1438.469	320.7989	621.1477	131.434	137.4355
	ET	30.1962	49.2325	32.715	66.78838	300.3753	33.25373	55.65682	25.46694	115.3875
	Rank	1	6	4	8	9	5	7	3	2
C_{28}	Mean	3468.207	3802.592	3580.114	15 095.93	43 261.04	4026.29	5732.341	3840.452	3646.016
	std	73.72614	33.58999	21.39515	1781.467	19 331.26	104.925	140.8126	79.46468	51.46395
	ET	30.9242	50.065	32.87	66.9469	297.7879	31.41532	51.83289	23.64146	107.5348
	Rank	1	4	2	8	9	6	7	5	3
C_{29}	Mean	7031.672	10 424.39	8587.656	25 094.17	6 855 137	8291.693	10 723.36	6828.818	7569.797
	std	685.616	1119.807	621.3203	11 147.68	6 106 536	731.561	1083.554	520.7025	1353.774
	ET	26.0094	40.495	27.635	59.61373	290.4891	23.14888	34.4463	15.3793	70.68116
	Rank	2	6	5	8	9	4	7	1	3
C_{30}	Mean	4 983 073	14 547 929	4 749 821	7.55E+09	5.01E+10	326 665.4	2.23E+08	4 068 352	4 906 400
	std	2 734 432	4 948 453	3 626 812	1.11E+09	2.23E+10	182 057.3	74 700 064	3 494 690	2 882 164
	ET	28.4775	44.615	30.305	66.25074	298.4729	33.10657	53.95788	23.81396	111.8294
	Rank	5	6	3	8	9	1	7	2	4
Sum Rank		58	119	123	220	261	113	209	130	72
Mean Rank		2	4.103448	4.241379	7.586207	9	3.896552	7.206897	4.482759	2.482759
Total Rank		1	4	5	8	9	3	7	6	2
P-value			3.75E−06	6.67E−07	1.97E−21	1.97E−21	7.5E−05	1.97E−21	7.89E−12	0.042953

the value of the objective function equals 1.7249. The statistical results obtained from the performance of COA and competitor algorithms are released in Table 18, indicating COA's favorable conditions in providing statistical indicators. The COA convergence curve while optimizing the welded beam design problem is shown in Fig. 21.

5.4. Tension/compression spring design

Tension/compression spring design is an optimization problem in engineering sciences to reduce the weight of tension/compression spring, the schematic of which is shown in Fig. 22 [16]. The mathematical formulation of this engineering design is as follows:

$$Consider:\ X = [x_1,\ x_2,\ x_3\ ] = [d,\ D,\ P].$$

$$Minimize:\ f(x) = (x_3 + 2)\, x_2 x_1^2.$$

Subject to :

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0,$$

$$g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3)} + \frac{1}{5108 x_1^2} - 1 \le 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0, \qquad g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \le 0.$$

Table 12
Assessment results of the CEC-2011 objective functions.

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C11-F1	Mean	2.843588	20.81105	19.93745	26.54347	22.80713	14.09116	17.68907	24.44839	16.15227
	std	5.687176	1.282434	6.609033	2.710176	1.198068	6.186749	3.230161	1.666718	7.231641
	ET	2.04562	2.31252	2.97783	5.292984	29.20915	5.15573	2.95091	1.888171	4.722715
	Rank	1	6	5	9	7	2	4	8	3
C11-F2	Mean	−28.8108	−23.1363	−19.0467	−16.0028	−7.94423	−20.8255	−15.2941	−16.7812	−25.3168
	std	5.507744	1.753088	7.79832	4.31307	2.076318	5.12237	3.970742	6.278191	1.208623
	ET	1.630126	2.31297	2.97810	11.89551	95.75669	7.145472	2.367971	4.032026	7.634852
	Rank	1	3	5	7	9	4	8	6	2
C11-F3	Mean	1.15E−05	1.15E−05	1.15E−05	1.08E−05	1.15E−05	1.15E−05	1.17E−05	1.15E−05	1.15E−05
	std	0.00035	6.98E−18	5.1E−09	1.51E−06	9.7E−14	2.29E−19	1.22E−11	5.82E−09	4.63E−19
	ET	868.0091	1374.9	1417.2	1552.026	1549.041	1553.726	670.6252	3838.272	2152.557
	Rank	2	5	7	1	6	3	9	8	4
C11-F4	Mean	0	0	0	0	0	0	0	0	5.239349
	std	0	0	0	0	0	0	0	0	10.4787
	ET	14.15682	24.2458	26.4140	16.87746	13.34398	10.2566	17.19681	249.9868	97.17605
	Rank	1	1	1	1	1	1	1	1	2
C11-F5	Mean	−17.5227	−23.9891	−27.7773	−17.8577	−7.44621	−3.58581	−24.7029	−21.3207	−24.5453
	std	1.37731	1.189697	4.367602	5.981184	5.731588	4.26441	0.682568	14.87202	2.942181
	ET	28.38912	42.6064	46.4203	16.85133	32.56863	9.517845	22.91466	56.44468	43.83191
	Rank	7	4	1	6	8	9	2	5	3
C11-F6	Mean	−25.2178	−19.394	−4.21135	−4.47773	0	0	−17.6098	0	0
	std	2.554107	3.398835	8.422701	5.77869	0	0	4.887593	0	0
	ET	16.0921	34.2753	38.4266	16.90792	32.83394	9.949495	23.44931	43.6938	33.00644
	Rank	1	2	5	4	6	6	3	6	6
C11-F7	Mean	0.749017	1.470027	1.598288	1.855356	2.674961	1.927268	2.023698	1.542722	1.891998
	std	0.115638	0.287654	0.234438	0.299029	0.799883	0.250582	0.148675	0.260706	0.402927
	ET	2.024755	4.28136	4.60428	3.337664	13.50524	1.782937	2.129804	4.710615	3.693627
	Rank	1	2	4	5	9	7	8	3	6
C11-F8	Mean	220	380.4759	318.1466	244.5	229	305.1134	297.3075	220	224.5
	std	0	99.08035	112.2878	7.505553	10.3923	56.94067	31.29607	0	9
	ET	10.51563	24.28429	26.04308	10.96358	28.40884	3.197	4.550901	40.1736	6.562568
	Rank	1	8	7	4	3	6	5	1	2
C11-F9	Mean	8789.286	899 441.7	78 247.66	109 702.8	1 998 032	213 402	644 320.2	252 278.4	1 057 279
	std	3700.105	317 668	26 200.22	56 346.4	1 095 674	55 401.3	225 156.3	162 784.6	282 458.9
	ET	10.29605	23.29151	25.43448	18.09356	79.85985	9.503477	15.82127	39.79276	63.17633
	Rank	1	7	2	3	9	4	6	5	8
C11-F10	Mean	−21.4889	−11.81	−12.7755	−8.30137	−9.78566	−10.9439	−10.9916	−10.4679	−10.2392
	std	0.474376	0.641541	1.740318	0.691048	0.50113	1.47866	0.576699	1.223408	0.119208
	ET	38.89083	74.30429	79.04406	16.73373	24.04803	15.86703	24.11789	67.33133	28.03311
	Rank	1	3	2	9	8	5	4	6	7
C11-F11	Mean	275 953.6	2 266 818	725 464.4	10 469 475	9 216 363	3 644 049	3 356 131	394 760.5	5 689 705
	std	133 300.2	412 735.4	79 800.66	241 278.6	2 348 826	3 313 946	102 946.1	170 396.3	0
	ET	6.728003	12.43182	12.90241	11.59426	69.07697	3.52682	7.211967	15.20003	20.43834
	Rank	1	4	3	9	8	6	5	2	7
C11-F12	Mean	1 186 100	5 844 903	2 645 075	14 890 580	12 384 883	7 558 349	8 904 960	3 801 184	11 417 782
	std	42 666.96	326 022.8	591 700.6	1 110 208	3 475 067	1 360 402	289 952.1	452 386.5	243 078
	ET	12.72086	24.31761	25.44781	22.59032	138.6727	7.12864	14.26996	28.81052	61.91513
	Rank	1	4	2	9	8	5	6	3	7
C11-F13	Mean	15 454.67	15 540.6	15 546.19	6 843 366	155 272.4	15 521.6	16 066.89	15 565.87	15 519.41
	std	2.451103	63.37666	41.30702	6 186 928	278 018.7	36.11072	154.9063	28.92438	11.95263
	ET	6.315078	12.43207	12.84965	1.53521	5.355945	1.14119	0.921913	18.317081	1.710161
	Rank	1	4	5	9	8	3	7	6	2
C11-F14	Mean	18 251.34	19 019.61	19 234.39	866 559.5	537 176.1	19 228.33	73 901.64	19 383.09	323 976.5
	std	38.80248	2.1E−12	235.4537	1 398 374	309 620.6	341.3748	18 557.54	191.7411	91 542.49
	ET	1.014952	2.43212	2.59122	1.863927	8.666232	0.979323	0.899208	2.023471	1.047816
	Rank	1	2	4	9	8	3	6	5	7
C11-F15	Mean	239 954.4	144 926.3	33 073.62	16 004 244	13 929 078	103 209.9	703 475.4	32 983.86	6 286 721
	std	203 421.2	119 857.2	118.7887	8 007 748	9 945 369	105 600.8	586 905.3	50.6518	5 348 133
	ET	1.862347	2.81624	3.06254	2.210702	10.26877	1.225892	1.189535	3.529188	2.385632
	Rank	5	4	2	9	8	3	6	1	7
C11-F16	Mean	138 044.1	143 651.7	149 712.5	50 897 289	60 282 729	152 164	612 941.2	147 920.9	1.14E+08
	std	3367.715	8010.091	10 095.71	20 418 144	30 122 129	10 366.65	510 494.4	10 782.88	59 719 791
	ET	2.163795	4.62018	4.95201	3.606738	23.40972	1.107552	1.691193	2.583211	1.694327
	Rank	1	2	4	7	8	5	6	3	9
C11-F17	Mean	1 926 615	2.34E+09	2 178 073	1.51E+10	1.93E+10	3.15E+09	1.36E+10	1.25E+08	3.93E+10
	std	11 419.96	9.54E+08	447 778.9	3.63E+09	5.77E+09	2.74E+09	1.64E+09	1.58E+08	8.13E+09
	ET	6.395614	14.23263	15.06294	13.77417	81.14803	3.808496	6.308436	14.21291	18.96868
	Rank	1	4	2	7	8	5	6	3	9

(*continued on next page*)

Table 12 (*continued*).

		COA	RUN	SMA	MSA	MBO	INFO	HHO	HGS	CPA
C11-F18	Mean	942 057.5	1 382 177	966 485	41 767 268	48 154 519	5 219 134	39 842 227	1 538 924	1.17E+08
	std	2639.272	399 065.6	25 605.52	49 418 096	56 732 188	4 906 068	10 275 333	759 919.6	65 785 148
	ET	4.649493	8.62371	9.18524	8.970916	56.05316	2.805942	5.262885	10.44125	11.57237
	Rank	1	3	2	7	8	5	6	4	9
C11-F19	Mean	1 025 341	2 640 186	1 384 299	7 745 229	23 518 586	4 335 919	39 823 035	3 010 646	77 686 064
	std	94 829.24	1 832 458	362 402.8	4 349 432	7 203 699	3 059 321	11 477 029	1 291 226	39 469 003
	ET	5.82334	11.06472	12.61547	9.874736	57.40189	3.735707	6.450973	14.99991	13.99214
	Rank	1	3	2	6	7	5	8	4	9
C11-F20	Mean	941 250.4	1 132 312	1 122 215	30 723 932	29 293 318	3 307 606	39 315 316	1 217 907	1.29E+08
	std	4769.814	267 460.9	212 982.4	33 695 226	28 170 768	1 619 828	4 680 564	519 904.2	39 738 364
	ET	4.61989	8.96247	10.27965	9.960833	57.7106	3.705372	6.647916	15.69455	13.50264
	Rank	1	3	2	7	6	5	8	4	9
C11-F21	Mean	12.71443	34.05445	22.67698	107.4986	372.2746	47.28543	55.3551	24.07516	140.8684
	std	2.295373	12.10513	4.174117	13.80526	385.8301	4.676531	8.221969	9.676234	77.56987
	ET	47.38995	91.63472	95.61429	39.53072	56.82145	38.3812	29.15304	159.6545	103.3241
	Rank	1	4	2	7	9	5	6	3	8
C11-F22	Mean	16.12513	31.63733	34.9258	104.3459	112.0603	45.02295	58.72464	24.65206	59.35846
	std	3.993717	3.942067	4.030006	23.83522	37.69032	13.26943	4.728237	2.395053	55.8951
	ET	47.41507	102.62846	105.92462	37.96693	50.09898	36.35808	26.58073	146.8143	81.42794
	Rank	1	3	4	8	9	5	6	2	7
Sum Rank		33	81	73	143	161	102	126	89	133
Mean Rank		1.5	3.681818	3.318182	6.5	7.318182	4.636364	5.727273	4.045455	6.045455
Total Rank		1	3	2	8	9	5	6	4	7
p-value			1.35E−09	3.65E−08	5.37E−15	8.84E−15	1.21E−11	5.36E−13	5.65E−08	7.62E−13

Table 13
Performance of optimization algorithms on the pressure vessel design problem.

Algorithm	Optimum variables				Optimum cost
	T_s	T_h	R	L	
COA	0. 783194	0. 3874305	40.5794	196.3323	5893.1336
RUN	0.788364	0.389911	40.84104	200.0000	5922.697
SMA	0.789199	0.389678	40.85395	200.0000	5926.513
MSA	0.819006	0.441004	42.43535	178.0534	5928.544
MBO	0.856754	0.424026	44.38794	158.4219	6049.427
INFO	0.828244	0.423385	42.2941	185.9678	6176.079
HHO	1.099967	0.962004	49.98904	171.6986	11 623.14
HGS	0.762178	0.404753	40.9803	200.0000	5927.478
CPA	1.113869	0.918407	45.03642	182.0029	6591.333

Table 14
Statistical results of optimization algorithms on the pressure vessel design problem.

Algorithm	Best	Mean	Worst	Std. Dev.	Median
COA	5893.134	5897.061	5899.220	25.2068	5896.761
RUN	5922.697	5898.047	5902.933	28.9821	5896.829
SMA	5926.513	5902.134	5905.239	13.9351	5901.258
MSA	5928.544	6075.940	7407.905	66.7386	6427.669
MBO	6049.427	6488.970	7263.975	327.596	6409.002
INFO	6176.079	6338.155	6524.083	126.837	6329.696
HHO	11 623.14	6852.862	7172.184	5801.053	6849.947
HGS	5927.478	6275.286	7018.367	497.021	6123.699
CPA	6591.333	6655.952	8019.857	658.707	7599.671

Table 15
Performance of optimization algorithms on speed reducer design problem.

Algorithm	Optimum variables							Optimum cost
	b	m	p	l_1	l_2	d_1	d_2	
COA	3.500000	0.7	17	7.3	7.8	3.35021	5.28668	2996.348
RUN	3.507125	0.7	17	7.307812	7.8078	3.356534	5.29705	3004.852
SMA	3.512233	0.7	17	7.388958	7.8236	3.363121	5.29507	3007.596
MSA	3.505551	0.7	17	8.308882	7.8079	3.357677	5.29502	3012.079
MBO	3.514048	0.7	17	7.418166	7.8239	3.363347	5.29508	3009.238
INFO	3.514301	0.7	17	7.307301	7.8078	3.466456	5.29752	3036.931
HHO	3.605690	0.7	17	8.308882	7.8078	3.374951	5.29753	3057.532
HGS	3.515801	0.7	17	8.358936	7.8078	3.367481	5.29603	3074.007
CPA	3.525688	0.7	17	8.378957	7.8078	3.372258	5.29702	3035.367

Table 16
Statistical results of optimization algorithms on speed reducer design problem.

Algorithm	Best	Mean	Worst	Std. Dev.	Median
COA	2996.348	3000.100	3001.261	1.160348	2998.816
RUN	3004.852	3006.845	3011.104	1.936571	3006.390
SMA	3007.596	3013.065	3015.979	5.851962	3011.736
MSA	3012.079	3112.711	3218.887	79.82938	3112.711
MBO	3009.238	3036.116	3068.310	13.04987	3034.302
INFO	3036.931	3073.281	3112.236	18.11761	3072.972
HHO	3057.532	3177.949	3371.953	92.79495	3164.334
HGS	3074.007	3194.176	3321.158	17.15971	3205.869
CPA	3035.367	3303.244	3628.159	57.16046	3296.556

Table 17
Performance of optimization algorithms on the welded beam design problem.

Algorithm	Optimum variables				Optimum cost
	h	l	t	b	
COA	0.20573	3.4705	9.0366	0.20573	1.7249
RUN	0.205769	3.478321	9.044835	0.206017	1.729384
SMA	0.205884	3.478878	9.046	0.206435	1.730721
MSA	0.197608	3.318376	10.008	0.201596	1.824323
MBO	0.205817	3.475574	9.049972	0.205915	1.729194
INFO	0.2049	3.539827	9.013294	0.210235	1.762968
HHO	0.147245	5.496235	10.0	0.217943	2.177546
HGS	0.164335	4.036574	10.0	0.223871	1.878014
CPA	0.206693	3.639508	10.0	0.203452	1.840211

Table 18
Statistical results of optimization algorithms on welded beam design problem.

Algorithm	Best	Mean	Worst	Std. Dev.	Median
COA	1.7249	1.726405	1.72861	0.004124	1.725596
RUN	1.729384	1.73059	1.730826	0.000287	1.730549
SMA	1.730721	1.731893	1.73233	0.00116	1.731852
MSA	1.824323	2.236462	3.056641	0.325421	2.250856
MBO	1.729194	1.734452	1.746456	0.00488	1.732185
INFO	1.762968	1.822671	1.878577	0.027619	1.825149
HHO	2.177546	2.551258	3.011943	0.256565	2.501997
HGS	1.878014	2.125086	2.326525	0.034916	2.102834
CPA	1.840211	1.367289	2.040862	0.13987	1.941088

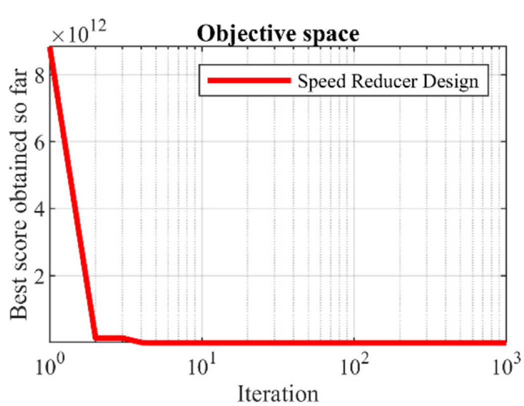

Fig. 19. COA's performance convergence curve on speed reducer design.

With

$$0.05 \leq x_1 \leq 2,\ 0.25 \leq x_2 \leq 1.3 \text{ and } 2 \leq x_3 \leq 15.$$

The implementation results of COA and competitor algorithms in achieving the optimal solution for the tension/compression spring design variables are reported in Table 19. Based on these results, COA has provided the optimal solution to the tension/compression spring problem with the values of the design variables equal to (0.0519129, 0.362122, 10.9792) and the value of the objective function equal to 0.012666. The statistical results obtained from using COAs and competitor algorithms in optimizing this design challenge are presented in Table 20, which indicates the superior performance of COA due to having better values of statistical indicators. The COA convergence curve in achieving the optimal values of the design variables for the tension/compression spring problem is presented in Fig. 23.

6. Conclusions and future works

In this paper, a new bioinspired optimization algorithm, called the Coati Optimization Algorithm (COA), was designed to model coatis' activities in nature. The strategy of coatis when attacking iguanas and the escape mechanism of coatis from predators are the two natural behaviors of coatis that are the fundamental inspirations used in the design of the COA. The phases of COA implementation were mathematically modeled in the two-phase exploration based on a hunting strategy simulation and in an exploitation phase based on an escape strategy simulation. The performance of the COA in optimization was evaluated on a set of fifty-one standard benchmark functions of the types the CEC-2017 and the CEC-2011. The quality of the optimization results obtained from the COA is compared with the performance of eleven well-known algorithms. The simulation results showed

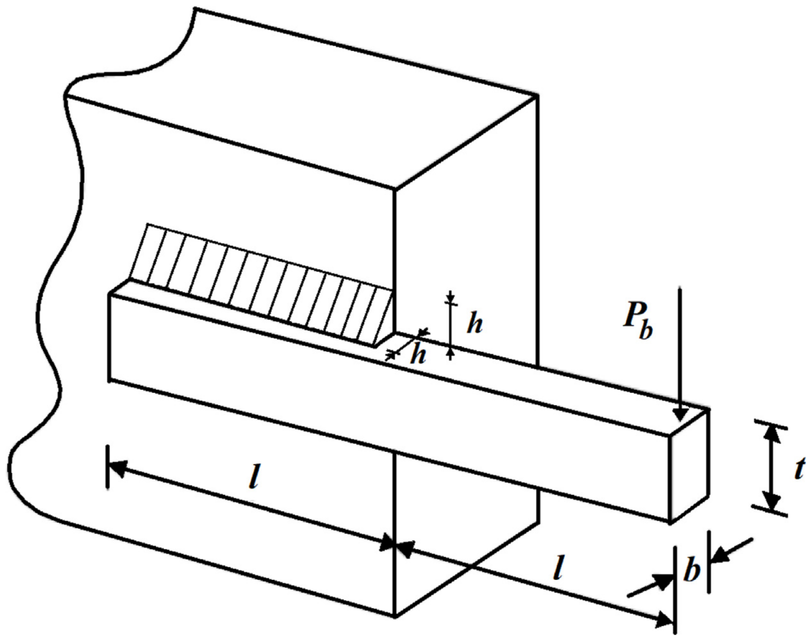

Fig. 20. Schematic of welded beam design.

Table 19
Performance of optimization algorithms on the tension/compression spring design problem.

Algorithm	Optimum variables			Optimum cost
	d	D	P	
COA	0.0519129	0.362122	10.9792	0.012666
RUN	0.050693	0.340722	11.98892	0.012791
SMA	0.049736	0.338531	11.9671	0.012795
MSA	0.04956	0.307678	14.86782	0.013314
MBO	0.04956	0.313172	14.10088	0.012935
INFO	0.050332	0.331829	12.61058	0.012827
HHO	0.04956	0.314515	14.10329	0.012992
HGS	0.049659	0.307378	13.87663	0.013156
CPA	0.049807	0.313563	15.10531	0.012894

Table 20
Statistical results of optimization algorithms on the tension/compression spring design problem.

Algorithm	Best	Mean	Worst	Std. Dev.	Median
COA	0.012666	0.012688	0.012697	0.001023	0.012685
RUN	0.012791	0.012808	0.012839	0.005674	0.012811
SMA	0.012795	0.012821	0.012845	0.004194	0.012824
MSA	0.013314	0.014962	0.018036	0.002294	0.013321
MBO	0.012935	0.014605	0.018013	0.001638	0.014157
INFO	0.012827	0.012965	0.013125	0.007834	0.01297
HHO	0.012992	0.01357	0.01435	0.000289	0.013497
HGS	0.013156	0.014172	0.016409	0.002093	0.013128
CPA	0.012894	0.013197	0.015362	0.000378	0.013078

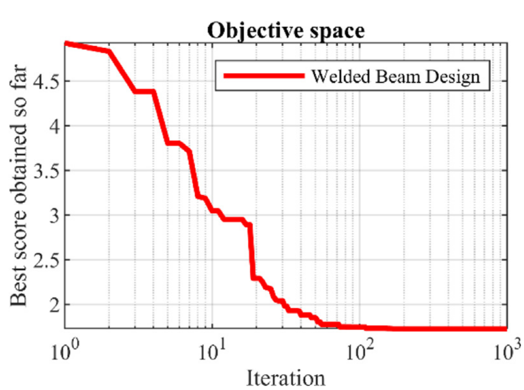

Fig. 21. COA's performance convergence curve on welded beam design.

that the COA provides appropriate solutions to optimization problems by striking the suitable balance between exploration in global search and exploitation in local search. COA analysis versus compared algorithms showed COA competitive superiority in optimization applications. In addition, the implementation of COA in addressing the CEC-2011 test suite demonstrated the high capability of COA in real-world applications. In addition, the implementation of COA in addressing four engineering design optimization issues demonstrated the high capability of COA in practical optimization problems.

The authors open several research directions for future studies of this paper, including the design of the binary and the multi-modal version of the COA. The application of COA in optimization problems in various sciences and other real-world applications is another research proposal of this paper.

Fig. 22. Schematic of tension/compression spring design.

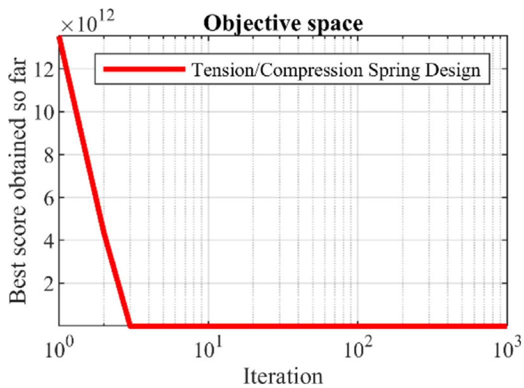

Fig. 23. COA's performance convergence curve on the tension/compression spring.

CRediT authorship contribution statement

Mohammad Dehghani: Conceptualization, Methodology, Software. **Zeinab Montazeri:** Conceptualization, Investigation, Writing – original draft, Formal analysis. **Eva Trojovská:** Validation, Funding acquisition, Formal analysis. **Pavel Trojovský:** Writing – original draft, Supervision, Validation, Funding acquisition, Project administration, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Code availability

The Matlab source code of the COA is available at: https://uk.mathworks.com/matlabcentral/fileexchange/116965-coa-coati-optimization-algorithm

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.knosys.2022.110011.

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