particle swarm optimization

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>> nature inspired evolutionary algorithm to solve computationally hard optimization problems

>> robust technique based on intelligence of swarms

>> abstracts the working mechanism of swarms like - birds, fish, etc.

>> a swarm of 'n' particles communicate either directly or indirectly with one another using gradients

>> each particle updates its position according to its previous experience and neighbour's experience

>> each particle is composed of 3 vectors:

-> x-vector - current position of the particle in the search space

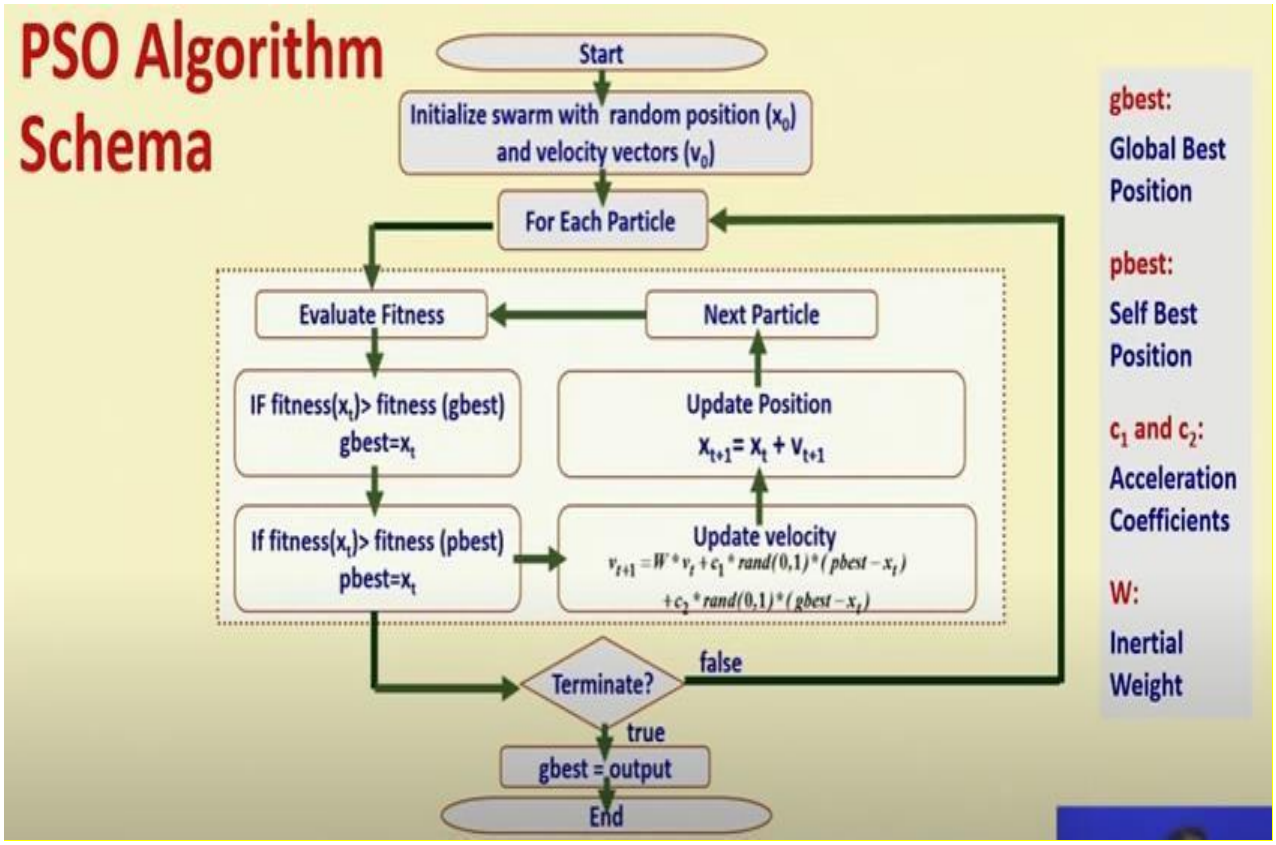
-> p-vector - records the best solution found so far

-> v-vector - contains a gradient in which particle will travel ideally

>> concept - every particle is moving towards their local best, and the local bests are moving towards the global best

>> no survival of the fittest algorithm is applied

>> all individuals are kept as members throughout the operation



›› basic concept is accelerating each particle toward the best position found by it so far (pbest) and the global best(gbest) obtained so far by any particle, with a random weighted acceleration each time step

›› this is done simply by adding the v-vector to the x-vector to get another x-vector, Xi = Xi + Vi

PSO - Stands for Particle Swarm Optimization

The movement of the particles is given by

$$v_{t+1} = w_t v_t + c_1 r_1 \left(p_t^b - x_t\right) + c_2 r_2 (p^g - x_t)$$

$$x_{t+1} = x_t + v_{t+1}$$

t: iteration number ;

w_t : inertia weight

r_1, r_2: random number between 0 & 1;

c_1, c_2: correction factor

p_t^b: pbest position at t iteration;

p^g: gbest position

v_t, x_t: velocity and position at t iteration

Maximize $f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

where $-5 \leq x_1, x_2 \leq 5$

PSO Parameter Setting (used only for illustration)

Randomly chosen

Population Size = 5 $c_1 = c_2 = 1.5$ Max. Iteration = 20

Dimension of the problem = 2 Inertia weight (w) = 0.9

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5

Goal is to update these values ALWAYS

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	.....	.....	.....
0.1908	0.2228	4.0579	−2.2150	.....	.....	.....
0.3828	0.3232	−3.7301	0.4688	.....	.....	.....
0.3976	0.3547	4.1338	4.5751	.....	.....	.....
0.0934	0.3773	1.3236	4.6489	.....	.....	.....

p^g (gbest) = [..... .....]

Maximize $f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):

Goal is to update these values ALWAYS

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	.....	.....
0.1908	0.2228	4.0579	−2.2150	32.6168	.....	.....
0.3828	0.3232	−3.7301	0.4688	13.2071	.....	.....
0.3976	0.3547	4.1338	4.5751	48.6753	.....	.....
0.0934	0.3773	1.3236	4.6489	41.4537	.....	.....

p^g (gbest) = [..... .....]

Maximize $f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 1:

- Randomly chosen Velocity between 0 & 1
- Randomly Initialize Position between -5 & 5
- Calculate fitness values f(x):
- Since there is NO previous iteration exists, so pbest is $p^b = x$
- Calculate gbest p^g

Maximum fitness value = 48.6753

Goal is to update these values ALWAYS

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2071	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [..... .....]

Maximize $f(X) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$

Iteration 2:

Velocity update:

For 1st particle, 1st component

$$v_{11} = wv_{11} + c_1r_1(p_{11}^b - x_{11}) + c_2r_2(p^g - x_{11})$$

$$= 0.9 \times 0.2194 + (1.5)(0.5949)(3.1472 - 3.1472) + (1.5)(0.0855)(4.1338 - 3.1472)$$

$$= 0.3240$$

Position update:

$$x_{11} = v_{11} + x_{11}$$

$$= 3.4712 \in (-5,5)$$

Iteration 1:

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2071	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	.....	3.4712	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....

p^g (gbest) = [..... .....]

Iteration 2:

Velocity update:

1st particle; 2nd component:

$$v_{12} = wv_{12} + c_1 r_1 (p_{12}^b - x_{12}) + c_2 r_2 (p^g - x_{12})$$

$$= (0.9)(0.2449) + (1.5)(0.5949)(-4.0246 + 4.0246) + (1.5)(0.0855)(4.5751 + 4.0246)$$

$$= 1.3233$$

Position update:

$$x_{12} = v_{12} + x_{12}$$

$$= -2.7013 \in (-5, 5)$$

Iteration 1:

v (velocity)	x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2071	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	1.3233	3.4712	−2.7013	------	------	------
------	------	------	------	------	------	------
------	------	------	------	------	------	------
------	------	------	------	------	------	------
------	------	------	------	------	------	------

p^g (gbest) = [------ ------]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

$$f(X) = x_1^2 - x_1 x_2 + x_2^2 + 2x_1 + 4x_2 + 3$$

Iteration 1:

v (velocity)		x (position) x_1	x_2	$f(x)$	p_i^b (pbest)	
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2071	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	1.3233	3.4712	−2.7013	27.8600	------	------
0.1815	1.0714	4.2394	−1.1436	31.0325	------	------
1.3531	0.8175	−2.3770	1.2863	13.7537	------	------
0.3578	0.3192	4.4916	4.8943	53.7063	------	------
0.4445	0.3301	1.7681	4.9790	45.5655	------	------

p^g (gbest) = [------ ------]

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

Calculate gbest (p^g):

Since 53.7063 > 48.6753 & its position is 4th.

So updated gbest is [4.4916 4.8943]

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2971 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) $= [4.1338 \quad 4.5751]$

v (velocity)

$$\begin{bmatrix} 0.3240 & 1.3233 \\ 0.1815 & 1.0714 \\ 1.3531 & 0.8175 \\ 0.3578 & 0.3192 \\ 0.4445 & 0.3301 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.4712 & -2.7013 \\ 4.2394 & -1.1436 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 27.8600 \\ 31.0325 \\ 13.7537 \\ 53.7063 \\ 45.5655 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

p^g (gbest) $= [4.4916 \quad 4.8943]$

Iteration 2:

Similarly, we can found for other pairs the

Velocity & Position updates.

Calculate the fitness values:

Calculate gbest (p^g):

Since 53.7063 > 48.6753 & its position is 4th.

So updated gbest is [4.4916 4.8943]

Calculate pbest (p^b):

For 1st particle:

Since 27.8600 < 31.9645, so pbest position remains same

For 3rd particle:

Since 13.7537 < 13.2971, so updated pbest position

Iteration 1:

v (velocity)

$$\begin{bmatrix} 0.2194 & 0.2449 \\ 0.1908 & 0.2228 \\ 0.3828 & 0.3232 \\ 0.3976 & 0.3547 \\ 0.0934 & 0.3773 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 31.9645 \\ 32.6168 \\ 13.2971 \\ 48.6753 \\ 41.4537 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -3.7301 & 0.4688 \\ 4.1338 & 4.5751 \\ 1.3236 & 4.6489 \end{bmatrix}$$

p^g (gbest) $= [4.1338 \quad 4.5751]$

v (velocity)

$$\begin{bmatrix} 0.3240 & 1.3233 \\ 0.1815 & 1.0714 \\ 1.3531 & 0.8175 \\ 0.3578 & 0.3192 \\ 0.4445 & 0.3301 \end{bmatrix}$$

x (position)

$$\begin{bmatrix} 3.4712 & -2.7013 \\ 4.2394 & -1.1436 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$$

$f(x)$

$$\begin{bmatrix} 27.8600 \\ 31.0325 \\ 13.7537 \\ 53.7063 \\ 45.5655 \end{bmatrix}$$

p_i^b (pbest)

$$\begin{bmatrix} 3.1472 & -4.0246 \\ 4.0579 & -2.2150 \\ -2.3770 & 1.2863 \\ 4.4916 & 4.8943 \\ 1.7681 & 4.9790 \end{bmatrix}$$

p^g (gbest) $= [4.4916 \quad 4.8943]$

Iteration 2:

Velocity update:

For 1st particle, 1st component

$$v_{11} = wv_{11} + c_1 r_1 (p_{11}^b - x_{11}) + c_2 r_2 (p^g - x_{11})$$

$$= 0.9 \times 0.2194 + (1.5)(0.5949)(3.1472 - 3.1472) + (1.5)(0.0855)(4.1338 - 3.1472)$$

$$= 0.3240$$

Position update:

$$x_{11} = v_{11} + x_{11}$$

$$= 3.4712 \in (-5, 5)$$

Iteration 1:

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
		x_1	x_2			
0.2194	0.2449	3.1472	−4.0246	31.9645	3.1472	−4.0246
0.1908	0.2228	4.0579	−2.2150	32.6168	4.0579	−2.2150
0.3828	0.3232	−3.7301	0.4688	13.2071	−3.7301	0.4688
0.3976	0.3547	4.1338	4.5751	48.6753	4.1338	4.5751
0.0934	0.3773	1.3236	4.6489	41.4537	1.3236	4.6489

p^g (gbest) = [4.1338 4.5751]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	.....	3.4712	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....

p^g (gbest) = [..... .....]

Iteration 3:

Update the velocity & position:

Calculate the fitness value:

Update gbest (p^g):

Since 56.7306 > 53.7063, so updated gbest is

Gbest = [4.8137 5.0000]

Max

Iteration 2: results

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.3240	1.3233	3.4712	−2.7013	27.8600	3.1472	−4.0246
0.1815	1.0714	4.2394	−1.1436	31.0325	4.0579	−2.2150
1.3531	0.8175	−2.3770	1.2863	13.7537	−2.3770	1.2863
0.3578	0.3192	4.4916	4.8943	53.7063	4.4916	4.8943
0.4445	0.3301	1.7681	4.9790	45.5655	1.7681	4.9790

p^g (gbest) = [4.4916 4.8943]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1334	0.9843	3.6045	−1.7170	25.4710	.....	.....
0.0337	0.7826	4.2731	−0.3611	30.0346	.....	.....
2.0987	1.1985	−0.2784	2.4848	19.3259	.....	.....
0.3221	0.2873	4.8137	5.0000	56.7306	.....	.....
0.7493	0.2862	2.5174	5.0000	46.7851	.....	.....

p^g (gbest) = [4.8137 5.0000]

Iteration 4:

Update the velocity & position:

Calculate the fitness value:

Update gbest (p^g):

Since 58.000 > 56.7306

gbest = [5.0000 5.0000]

Update pbest (p^b):

Iteration 3: results

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1334	0.9843	3.6045	−1.7170	25.4710	3.4712	−2.7013
0.0337	0.7826	4.2731	−0.3611	30.0346	4.2394	−1.1436
2.0987	1.1985	−0.2784	2.4848	19.3259	−0.2784	2.4848
0.3221	0.2873	4.8137	5.0000	56.7306	4.8137	5.0000
0.7493	0.2862	2.5174	5.0000	46.7851	2.5174	5.0000

p^g (gbest) = [4.8137 5.0000]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1561	0.8690	3.7606	−0.8480	25.1798	.....	.....
0.0696	0.6935	4.3427	0.3325	30.5409	.....	.....
2.5419	1.4012	2.2635	3.8860	34.4996	.....	.....
0.2899	0.2586	5.0000	5.0000	58.0000	.....	.....
0.9689	0.2576	3.4863	5.0000	49.6953	.....	.....

p^g (gbest) = [5.0000 5.0000]

Iteration 5:

Update the velocity & position:

Calculate the fitness value:

Update gbest (p^g):

Update pbest (p^b):

Iteration 4: results

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1561	0.8690	3.7606	−0.8480	25.1798	3.6045	−1.7170
0.0696	0.6935	4.3427	0.3325	30.5409	4.3427	0.3325
2.5419	1.4012	2.2635	3.8860	34.4996	2.2635	3.8860
0.2899	0.2586	5.0000	5.0000	58.0000	5.0000	5.0000
0.9689	0.2576	3.4863	5.0000	49.6953	3.4863	5.0000

p^g (gbest) = [5.0000 5.0000]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1601	0.7567	3.9208	−0.0914	26.2152	3.9208	−0.0914
0.1469	1.2228	4.4896	1.5553	33.7933	4.4896	1.5553
2.6386	1.4040	4.9021	5.0000	57.3244	4.9021	5.0000
0.2609	0.2327	5.0000	5.0000	58.0000	5.0000	5.0000
1.0661	0.2319	4.5524	5.0000	55.0672	4.5524	5.0000

p^g (gbest) = [5.0000 5.0000]

optim

we stop

$f(x) - f(x) \simeq 0$

Iteration 5:

Update the velocity & position:

Calculate the fitness value:

Update gbest (p^g):

Update pbest (p^b):

Since there is no change in the fitness value (58.000) and hence optimum will reach.

$x_1 = 5, \quad x_2 = 5$
$f(x_1, x_2) = 58$

Iteration 4: results

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1561	0.8690	3.7606	−0.8480	25.1798	3.6045	−1.7170
0.0696	0.6935	4.3427	0.3325	30.5409	4.3427	0.3325
2.5419	1.4012	2.2635	3.8860	34.4996	2.2635	3.8860
0.2899	0.2586	5.0000	5.0000	58.0000	5.0000	5.0000
0.9689	0.2576	3.4863	5.0000	49.6953	3.4863	5.0000

p^g (gbest) = [5.0000 5.0000]

v (velocity)		x (position)		$f(x)$	p_i^b (pbest)	
0.1601	0.7567	3.9208	−0.0914	26.2152	3.9208	−0.0914
0.1469	1.2228	4.4896	1.5553	33.7933	4.4896	1.5553
2.6386	1.4040	4.9021	5.0000	57.3244	4.9021	5.0000
0.2609	0.2327	5.0000	5.0000	58.0000	5.0000	5.0000
1.0661	0.2319	4.5524	5.0000	55.0672	4.5524	5.0000

p^g (gbest) = [5.0000 5.0000]

x_1 x_2

Summary:

Update the velocity & position:

$$v_{t+1} = w_t v_t + c_1 r_1 (p_t^b - x_t) + c_2 r_2 (p^g - x_t)$$
$$x_{t+1} = x_t + v_{t+1}$$

Calculate the fitness value:

Calculate the objective function values f(X)

Update gbest (p^g):

Compare the fitness value

Update pbest (p^b):

Compare the individual particle fitness value & select the best one.