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Constrained Multi-Objective Weapon Target Assignment for Area Targets by Efficient Evolutionary Algorithm

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ABSTRACT The weapon target assignment (WTA) problem is the crucial decision support in Command & Control (C2). In the classic WTA model, the point-to-point saturation salvo has a low efficiency-cost ratio when the swarming targets, which have the advantage of low casualty, low cost and recyclable, become the major operational units. The constraint is less studied for the operational intention of the decision-maker. In this paper, a constrained multi-objective weapon target assignment (CMWTA) model is formulated for area targets. The optimization objectives are minimizing collateral damage and resource consumption. The multi-constraint is derived from the actual operational requirements of security evasion, damage threshold, and preference assignment. To solve CMWTA efficiently, a novel multi-objective optimization evolutionary algorithm (MOEA) is proposed to obtain the non-dominated solutions as the alternative plans for the decision-maker. A self-adaptive sorting algorithm is proposed to guarantee the completeness of the Pareto-optimal set, and a cooperative evolutionary mechanism is adopted to strengthen the convergence. For handling multi-constraint, a repair mechanism is proposed to improve the quality of infeasible solutions, and the measurement of constraint violation is designed to evaluate the infeasible solutions. A variant of the convergence metric is introduced to evaluate the algorithms solving multi-objective weapon target assignment (MWTA) problem. The experimental results show the effectiveness and superiority of the proposed approaches.

when troops are in motion, efficiency reduces
so, we are studying about some constrained cases...

the algorithm is made self-adaptive for its completeness...
a repair mechanism is also proposed for solutions whose designs are infeasible...

INDEX TERMS Constrained weapon target assignment, collateral damage, multi-objective optimization algorithm, decision support system.

I. INTRODUCTION

The weapon target assignment (WTA), which is also known as Weapon Allocation or Weapon Assignment (WA), refers to the reactive assignment of defensive weapons to counter identified threats. With the development of intelligent over-the-horizon weapons, the WTA process is the critical decision support in Command & Control (C2). The optimality and real-time of WTA are consistent with the related own force mission objectives and compliant with the rules of engagement, weapon system characteristics and environment constraints [1].

The WTA problem can be divided into two categories by decision process: the static WTA (SWTA) and the dynamic WTA (DWTA). The difference between SWTA and DWTA is whether the time is considered as a dimension. In SWTA, weapons are launched in a salvo to maximize operational effectiveness. The DWTA assigns the sequence of weapons for the equilibrium plan during multi-stage. Manne introduced the WTA problem originally in 1959 [2]. In the following decades, the SWTA problem is investigated much more than the DWTA problem. This paper focuses on the SWTA problem, and the review of the DWTA problem can refer to [3], [4]. this paper focuses on the SWTA problem i.e. the static part

the WTA problem is divided into 2 parts - static & dynamic depending on the 'time' factor...
static - doesn't take time into account, just focuses on max destruction
dynamic - focuses on on how to destruct over a period of time using multiple attacks

In view of the popular objectives, the classic SWTA model is to maximize the expected damage effect of weapons or minimize the expected survival probability of targets, which is known as the target-based SWTA model [5]. The asset-based SWTA model is to maximize the expected value of surviving own-force assets by assigning the weapons

SWTA mainly has two objectives:
>> maximize the expected damage effect of the weapons
>> minimize the expected survival probability of the target

to the offensive targets [6]–[8]. Bogdanowicz [9] established the sensor-based SWTA model considering the benefit of assigning sensors to targets and weapons to targets. The subsequent research refined the utility function of sensors on weapons/targets [10], [11]. Besides, other operational effects are incorporated to formulate the multi-objective WTA. For example, Zeng *et al.* [12] presented a problem of cooperative salvo attack of multiple cruise missiles against targets in a group, which focus on the synchronization of the arrival time of missiles, minimization of time-consuming and maximization of expected damage. Li *et al.* [13] presented an MWTA problem considering the expected damage effect and ammunition consumption. Li *et al.* [14] designed the objectives maximizing the total effectiveness of attack and minimize the cost of missiles. In [15], Li, You, et al. added the residual weapons into previous work [14] to establish an MWTA formulation with three objectives. Volle and Rogers [16] presented the MWTA problem optimizing the operational effectiveness and the relative timing of agents' arrival.

math constraints & some work

The fundamental constraints of the SWTA model are mathematical constraints, namely integer constraint and quantity constraint. With the development of combat complexity, the more model constraints are constructed in WTA. For example, Lee [17] presented a constrained WTA (CWTA) model in which the number of interceptors assigned to each target has an upper bound. Li *et al.* [18] established a model of assigning interceptors to multiple waves of incoming ballistic missiles with shoot time constraints. Guo *et al.* [19] presented a multi-to-multi interception problem based on cooperative guidance in which the damage probability function is constructed by the miss-distance under heading error, the time-to-go, and the line-of-sight rate.

Although the objectives and constraints are various, the point-to-point SWTA model is essentially a combinatorial optimization problem. The exact algorithms, such as implicit enumeration algorithm [20], the branch-and-bound algorithm [17] and dynamic programming [21], [22], are investigated in early research and suffering from exponential complexity because the SWTA problem has been proven to be NP-Complete [23]. Johansson and Falkman [24] presented an exhaustive search to solve the SWTA problem with seven weapons and six targets. Kline *et al.* [25] developed a branch and bound algorithm to find optimal solutions for up to ten weapons and ten targets. To improve the real-time of larger-scale problems, two techniques are presented: 1. Reformulate the SWTA model for reducing computational complexity; 2. Apply the swarm intelligent algorithm to obtain the approximate optimal solution. For example, Kwon *et al.* [26] formulated the SWTA as a nonlinear integer programming model which is solved by a branch and bound algorithm. Ahuja *et al.* [20] transformed SWTA into a linear integer programming model by logarithmic transformation. The swarm intelligent algorithms are extensively adopted to solve SWTA problem, such as genetic algorithm (GA) [27], [28], ant/bee colony algorithm [29]–[32], particle swarm optimization (PSO) [18], [33] and other swarming intelligence algorithms [34]. For balancing the optimality and real-time, some hybrid algorithms, which take advantage of exact and heuristic algorithms, are proposed [35], [36]. Because of the insensitivity of model scale and constraints, the evolutionary algorithm performs excellently on solving the WTA problem. Especially for MWTA, the multi-objective evolutionary algorithm (MOEA) has become the major solver due to the success in solving multi-objective optimization problems (MOPs). Zhou *et al.* [37] presented a PSO algorithm based on a two-stage evolutionary strategy to solve MWTA. Li *et al.* [7] proposed a novel MOEA based on decomposition (MOEA/D), which reformulates the mating restriction and selection operation, to solve the asset-based MWTA. In [15], an improved non-dominated sorting genetic algorithm (NSGA), which incorporates an online operator selection mechanism, is presented for MWTA.

In the research discussed thus far, the attack mode of SWTA is the point-to-point saturation salvo in which one weapon can only attack one target, and the number of weapons is more than the number of targets. As TSP and VPR, the SWTA model of this attack mode is also a combinatorial optimization problem (COP) and has been studied by the corresponding algorithms. Nevertheless, these SWTA models have two limitations: (1) The efficiency-cost ratio of this attack mode is unsatisfied when facing swarming agents which have the advantage of small size, low cost, and zero casualties. (2) As the decision support of C2, the constraints of the major models reflect the less intention of the decision-maker. For these requirements, an alternative strategy is employing weapons with lethal radius to attack the targets, which are viewed as area targets in Threat Evaluation (TE), by collateral damage. Bogdanowicz and Patel [38] designed a quick collateral damage estimate (QCDE) algorithm to support WTA capabilities with collateral damage consideration. Ma [39] established the principle of Constrained Target Clustering (CTC), but the number of clusters is set artificially and there is no overlap. Inspired by the above analysis, we formulate a constrained multi-objective weapon target assignment model.

The constraint handling method of the WTA problem is less studied since the mathematical constraint can be eliminated by encoding, and certain model constraints can be tackled by special representation schemes. However, only these two methods cannot satisfy CMWTA with different types of constraints. Deb *et al.* [40] and Peng [41] suggested a set of tunable test problems for constrained multi-objective optimization problems (CMOPs), and show the increasing difficulties in searching the Pareto-optimal sets. Coello [42], [43] gave a review of the constraint handling for COPs. The main constraint handling techniques of MOEAs can be divided into 1) penalty function; 2) repair algorithm; 3) dominance rule; 4) selection strategy. The penalty function is the most popular approach because of the simplicity and less restriction for constraint type. However, tuning penalty parameters is a difficult optimization problem itself. Hence the self-adaptive mechanism is proposed to improve the dynamic performance

techniques for large-scale problems:
-> reducing computational complexity
-> swarm intelligent algorithm to obtain approximate optimal soln.

TABLE 1. Notation declaration.

Notation	Description
m	the number of hostile targets;
n	the number of available weapons;
l	the number of friendly or neutral targets;
$R = [r_i]_{1\times m}$	the lethal radius set of available weapons; r_i denotes the lethal radius of weapon i;
$V = [v_j]_{1\times n}$	the target survival value vector; v_j denotes the survival value of target j, $j = 1, 2, \ldots, n$;
$P = \{p_j\}_{1\times n}$	the position set of detected hostile targets; p_j denotes the position of target j;
$\rho = [\rho_j]_{1\times n}$	the survival probability threshold vector of hostile targets; ρ_j denotes the required maximum survival probability of target j, and $\rho_j = 1$ denotes target j has no requirement of survival probability threshold;
$S = s_k\}_{1\times l}$	the position set of friendly or neutral targets; s_k denotes the position of no-damage target k;
$E = [e_{ij}]_{m\times n}$	the preference matrix of weapon-target assignment; $e_{ij} = 1$ denotes the preference assignment of weapon-target pair (i, j), and $e_{ij} = 0$ denotes no preference assignment of (i, j).

of penalty item [44]. The repair algorithm improves the percentage of feasible solutions by enforcing the infeasible solutions within or near the feasible space [45]. The drawback is that the repair algorithm needs to be designed according to the application, and a rigid algorithm maybe threat the diversity of the population. For CMOPs, the modified dominance rule is proposed in the non-dominated sorting process of MOEAs. In the representation methods, Deb [46] proposed the constrained-dominated principle (CDP) in which the constraint violation takes priority over the fitness to tournament selection. Ray *et al.* [47] performed three non-dominated rankings of population, which is constructed by the values of constraint violation and objective fitness, on the evolutionary process. Cai and Wang [48] presented a dominated and replaced algorithm between the parent population and the offspring population, which is robust for types of constraints. A similar principle is also adopted in [49]–[52]. However, CDP is unsatisfied with solving the MOPs with the narrow feasible region in which the infeasible solutions play a crucial role in finding the global optimum. Hence technique 4) is increasingly applied in the tournament selection of MOEAs to shape search space by describing a relaxation on the constraint. The advantage is that this mechanism performs well on solving the COPs with narrow feasible regions. Takahama *et al.* [53] proposed the ϵ-constrained (EC) method to preserve diversity by an ϵ level. Runarsson and Yao [54] introduced a stochastic ranking (SR) approach to balance objective and penalty functions stochastically, which does not require a priori knowledge about the problem. The primary constraint handling methods are generic for solving constrained numerical optimization problems and not always efficient for MOPs with an application background. The proposed CMWTA problem is a CMOP deriving from the real-world, and the non-dominated solutions have a specific distribution. Hence it is necessary to investigate the constraint handling approach in solving the CMWTA according to the problem characteristics.

Based on the above research, we formulate a CMWTA problem and proposes an MOEA to obtain non-dominated solutions as alternative plans for the decision-maker. The contributions of this paper are described as follows:

- A constrained multi-objective WTA model, which is based on collateral damage, is established for area targets. The objectives are minimizing the expected survival probability of targets and weapon consumption. The constraints are derived from the operational requirements of security evasion, survival threshold of targets and preference assignment.
- For the challenges of solving CMWTA, a novel multi-objective evolutionary algorithm is proposed to obtain the Pareto-optimal set excellently.
- A variant of convergence metric is introduced to evaluate the performance of algorithms on solving MWTA which takes the operational effect and resource consumption as optimization objectives. Comparing several state-of-the-art MOEAs and constraint handling methods, the extensive experiments demonstrate that the proposed approaches are effective and promising.

The rest of this paper is organized as follows. Section II gives our motivations and formulates the CMWTA problem. Section III present the proposed MOEA for CMWTA. Section IV compare the proposed MOEA with several state-of-the-art MOEAs and constraint handling methods by experimental studies. The conclusion is finally summarized in Section V.

II. PROBLEM FORMULATION

With the development of military techniques, swarm agents, such as unmanned vehicles and intelligent robots, tend to be the central combat units having the advantage of low cost and recycling. The approved strategy is employing weapons with lethal radii, such as directed energy, electronic jamming, fragmentation warhead and continuous rod warhead. These weapons can damage the target within the lethal radius effectively and are useless to outside targets because of the sharp attenuation. Therefore this section formulates a constrained multi-objective weapon target assignment model to maximize the efficiency-cost ratio by optimizing the number and aiming point of weapons.

The notation employed in the context is listed in Table 1.

A. BASIC DEFINITIONS OF MOP

The multi-objective optimization problem, which has several objectives to be optimized simultaneously, can be stated as follows:

$$\begin{aligned} & \textit{minimize } F(x) = (f_1(x), f_2(x), \ldots, f_m(x))^T && (1)\\ & \textit{subject to } x \in \Omega && (2) \end{aligned}$$

where x is the decision vector, Ω is the decision space, $F : \Omega \rightarrow R^m$ consists of m objective functions.

In a minimization MOP, it is said that a decision vector $x_A \in \Omega$ dominates another vector $x_B \in \Omega$, which is written as $x_A \prec x_B$, if and only if $f_i(x_A) \le f_i(x_B), \forall i = 1, 2, \ldots, m$ and $f_i(x_A) < f_i(x_B), \exists i = 1, 2, \ldots, m$. So $x* \in \Omega$ is defined as a Pareto-optimal solution or non-dominated solution if there is no solution $x \in \Omega$ satisfying $x \prec x*$, and $F(x*)$ is called Pareto-optimal vector. The set of all Pareto-optimal solutions is called the Pareto-optimal set (PS), and the set of all Pareto-optimal vectors is called Pareto-optimal front (PF). The algorithm of MOP is to obtain a set of Pareto-optimal solutions approximating the true Pareto-optimal front.

B. OBJECTIVES OF CMWTA

As a minimization MOP, the survival probability of targets and weapon consumption, which conflict with each other, are employed to be objectives of CMWTA.

$$\begin{aligned} \min F(X, A) &= (f_1, f_2) \\ &= \left(\sum_{j=1}^{n} v_j \prod_{i=1}^{m} \left(1 - D\left(\|p_j - a_i\|\right)\right) \cdot x_i, \sum_{i=1}^{m} x_i \right) && (3) \\ s.t\ x_i &\in \{0, 1\}, \quad for\ i = 1, 2, \ldots, m && (4) \end{aligned}$$

where f_1 is the objective that denotes the expected kill probability of the targets; f_2 is the objective that denotes the weapon consumption; m is the number of weapons and n is the number of targets; v_j is the survival value of target j which is evaluated by threat evaluation; $\{p_1, p_2, \cdots, p_n\}$ is the position set of detected hostile targets; $A = \{a_1, a_2, \cdots, a_m\}$ is the position set of aiming points of weapons; $X = (x_1, x_2, \cdots, x_m)$ is the decision vector; $D(\cdot)$ is the function of damage effect by the distance between blast points and targets. Based on research involving collateral damage estimate [38], [39], [55]–[58], we assume no firing error and approximate the conditional damage law of target coordinate as

$$D(r) = 1 - \exp\left(-\frac{\delta^2}{r^2}\right) \qquad (5)$$

where δ is a parameter of damage law in target condition, which is related to environmental resistance. For example, in low altitude dense atmosphere, $G_0(r)$ has a sharp attenuation owing to the rapid decrease of the explosive fragment speed. r is the projection of miss distance on dispersion plane.

Constraint (4) is the binary constraint and can be eliminated by the encoding method. Considering the actual operational requirement, we present the constraint set as follows.

C. CONSTRAINTS OF CMWTA

In CMWTA, the following constraints, which deriving from actual operational requirements, are considered:

1) SECURITY EVASION

The no-damage objects, such as friendly units and civilians, should be outside of the lethal radius of weapons. The safe distance should be apart from aiming points to friendly or neutral targets. The constraint of security evasion is denoted as

$$C_1 : \|s_k - a_i\| > R_i, \quad for\ k = 1, 2, \ldots, l;\ i = 1, 2, \ldots, m$$

constraint for not damaging certain targets (6)

where $S = \{s_1, s_2 \cdots, s_l\}$ is the set of no-damage targets; R_i is the lethal radius of weapon i.

2) SURVIVAL THRESHOLD

In the intention of the decision-maker, the key nodes of hostile architecture are expected to be a lower survival probability. The constraint of survival threshold is constructed as

$$C_2 : \prod_{i=1}^{m} \left(1 - D\left(\|p_j - a_i\|\right)\right) \cdot x_i \le \rho_j, \quad for\ j = 1, 2, \ldots, n \qquad (7)$$

where $\rho = (\rho_1, \rho_2, \cdots, \rho_n)$ is the predetermined threshold vector, namely ρ_j is the expected maximum survival probability of target j. The components are set to 1 for the targets which have no requirement of damage threshold.

3) PREFERENCE ASSIGNMENT

There is the requirement of weapon-target pair in the following situations: a) situation assessment based on domain knowledge; b) type matching of weapon to target; c) the preference of decision-maker. The preference matrix $E = [e_{ij}]_{m \times n}$ is introduced, and the element e_{ij} is represented by

$$e_{ij} = \begin{cases} 1, & \textit{prior weapon } i \textit{ to target } j \\ 0, & \textit{otherwise} \end{cases} \qquad (8)$$

The constraint of preference assignment is denoted as

$$C_3 : e_{ij} / \|p_j - a_i\| > 1/R_i, \quad for\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \qquad (9)$$

The CMWTA model is written as

$$\min \begin{cases} f_1 = \sum_{j=1}^{n} v_j \prod_{i=1}^{m} \left(1 - D\left(\|p_j - a_i\|\right)\right) \cdot x_i \\ f_2 = \sum_{i=1}^{m} x_i \end{cases} \qquad (10)$$

$$s.t. \begin{cases} \|s_k - a_i\| > R_i \\ \prod_{i=1}^{m} \left(1 - D\left(\|p_j - a_i\|\right)\right) \cdot x_i \le \rho_j \\ e_{ij} / \|p_j - a_i\| > 1/R_i \\ x_i \in \{0, 1\} \\ \sum_{i=1}^{m} x_i \le m \end{cases} \qquad (11)$$

The CMWTA model has the following difference with the point-to-point WTA model: (1) In the CMWTA model, the decision variable X is a binary vector and A is a continuous matrix.f_1 and f_2 not only are piecewise continuous and integer variables respectively but also have the gradients in different orders of magnitudes. In the point-to-point WTA, the input is only a decision vector X. Besides the identical objective of weapon consumption, the objective of the fitness is a discrete variable. (2) The true Pareto-optimal set of CMWTA is almost impossible to get, while the optimal solution of classic WTA is only restricted to the enumeration scale.

III. MOEA-CMWTA

The multi-objective evolutionary algorithms, which are based on the natural evolution mechanism, have shown an increasing application in MOPs owing to the less restriction for problem scale and constraints. For the characteristics analyzed so far, the framework of MOEA is proposed to solve CMWTA.

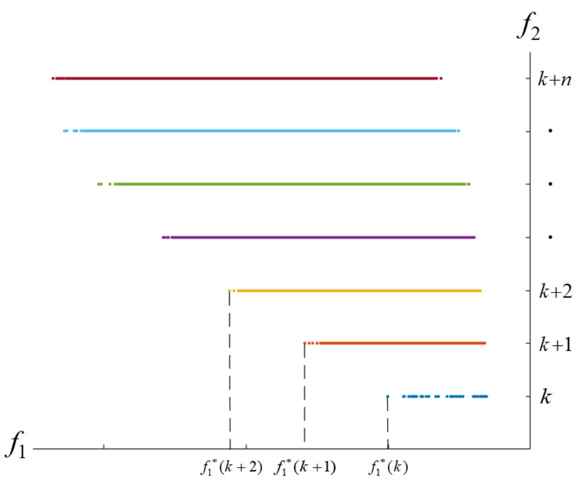


FIGURE 1. Distribution diagram of solutions of MOEA solving CMWTA.

The distribution of solutions, which are generated in the evolutionary iterations of MOEA, is shown in Figure 1. In Figure 1, the X-axis shows the excepted survival probability of targets; the Y-axis shows the weapon consumption. One solution represents one decision plan which can be identified by weapon consumption. Therefore the solutions can be divided into several layers by weapon consumption. The Pareto-optimal set of MWTA is consist of the leftmost solution of each layer and has the following characteristics:

- The size of the Pareto-optimal set is no more than the number of weapons.
- The non-dominated solutions of neighboring layers show the high similarity empirically.
- As shown in Figure 1, let $\left(f_1^*(i), i\right)$ denotes the fitness of non-dominated solutions where $i = k, k+1, \ldots, k+n$. The difference $f_1^*(i) - f_1^*(i+1)$ decreases with i increases.

There are two major challenges in solving CMWTA: (1) The completeness of Pareto-optimal set is unable to be guaranteed by a rigorous selection method, which is verified

solving the CMWTA poses lots of problems, so we take a novel MWTA to address those problems

in the experimental section. (2) Owing to multi-constraint, the high percentage of infeasible solutions reduces search efficiency. For the above obstacles, a novel multi-objective evolutionary algorithm for CMWTA (MOEA-CMWTA) is proposed with the following contributions:

- For the completeness of the Pareto-optimal set, a self-adaptive sorting algorithm, which has low computational complexity and does not require any parameter, is proposed to preserving the diversity of the population.
- A cooperative evolutionary strategy, which utilizes the similarity of the neighboring non-dominated solutions and contributes to the good information transferring in neighboring subpopulations, is presented to strengthen the convergence performance. similar to try & catch block
- For handling multi-constraint, a detection and repair mechanism is introduced to improve the infeasible solutions. A measurement of constraint violation is designed to evaluate the infeasible solutions.

The main loop of MOEA-CMWTA is outlined in Algorithm 1.

Algorithm 1 Main Loop of MOEA-CMWTA

1: Initialization: Generate an initial population P_0 which consists of m subpopulations.
2: Termination: If $t \leq T$ or another stopping criterion is satified then obtain the non-dominated solutions of P_t as output. Stop.
3: Environmental selection: Perform the self-adaptive sorting algorithm on P_t to create the effective population $\bar{P}_t$.
4: Variation: Implement the cooperative evolutionary procedure on $\bar{P}_t$ to create a new population Q_t.
5: Fitness assignment: Calculate the fitness and constraint violation of individuals in Q_t. Set $P_{t+1} = \bar{P}_t \cup Q_t$ and $t = t + 1$. Go to Step 2.

A. SOLUTION BUILDING

1) SOLUTION ENCODING

To indicate the weapon usage and aiming points, we encode the solution of CMWTA by the real-value set $A = \{a_1, a_2, \cdots, a_m\}$ where the component a_i shows the aiming point of weapon i in the scenario. $a_i = \emptyset$ when the weapon i is not used and a_i represents the aiming point of weapon i otherwise. In initialization, solution A is generated by the corresponding decision vector $X = (x_1, x_2, \ldots, x_m)$ where $x_i = 1$ represents the weapon i is employed and $x_i = 0$ conversely. Hence the solutions can be divided into m subpopulations by $|X|$ which is corresponding to the layers in Fig. 1. The algorithm of population initialization is shown in Algorithm 2.

In line 5 of Algorithm 2, the random integer u is the number of the employed weapons. In line 6, L is the index list of employed weapons. Line 11 gives the detection and repair algorithm which is described as follows.

Algorithm 2 Population Initialization

Input: pop: the size of the population; m: the number of weapons; n: the number of targets; B: the battlefield domain.

Output: $P = \{SP_1, SP_2, \ldots, SP_m\}$: the initial population where SP_i is the ith subpopulation.

$P = \{SP_1, SP_2, \ldots, SP_m\}$, $SP_i = \emptyset$ initialy, no weapon is aiming anywhere
for $k = 1$ to pop **do**
  $X = [x_i]_{1\times m}$, $x_i = 0$
  $A = \{a_1, a_2, \ldots, a_m\}$, $qa_i = \emptyset$
  $u = \lceil rand\,(0, 1) \times m \rceil$
  Generate a vector $L = (l_1, l_2, \ldots l_u)$ containing u unique integers selected randomly from $[0, m]$;
  for $i = 1$ to u **do**
    $x_{l_i} = 1$
    $a_{l_i} = B \cdot rand$
  end for
  Detect and repair (X, A);
  Add solution (X, A) to subpopulation $SP_{|X|}$;
end for

Algorithm 3 Detection and Repair Algorithm of Consistency

Input: (X, A): the raw solution; m: the number of weapons; n: the number of targets; $\{p_1, p_2, \ldots, p_n\}$: the location of targets in the scenario.

Output: (X, A): the repaired solution.

1: $D = \left[d_{ij}\right]_{m\times n}$, $d_{ij} = 0$
2: **for** $i = 1$ to m **do**
3:   **if** $x_i = 1$ **then**
4:     **for** $j = 1$ to n **do**
5:       **if** $\left\|p_j - a_i\right\| < R_i$ **then**
6:         $d_{ij} = 1$
7:       **end if**
8:     **end for**
9:   **end if**
10: **end for**
11: **for** $i = 1$ to m **do**
12:   **if** $x_i + \sum_{j=1}^{n} d_{ij} = 1$ **then**
13:     $x_i = 0$
14:     $a_i = \emptyset$
15:   **end if**
16: **end for**

Algorithm 4 Detection and Repair Algorithm of Survival Threshold

Input: (X, A): the raw solution; m: the number of weapons; n: the number of targets; ρ: the damage threshold vector; E: the preference matrix; D: the decision matrix.

Output: A: the repaired solution.

1: Generate the set of weapon index $U = \left\{ i \,\middle|\, x_i = 1 \ and \ \sum_{j=1}^{n} e_{ij} = 0 \right\}$;
2: **for** $j = 1$ to n **do**
3:   **if** $\rho_j + \sum_{i=1}^{m} d_{ij} < 1$ **then**
4:     **if** $U \neq \emptyset$ **then**
5:       Randomly select an index k from U and delete k from U;
6:       Randomly generate a_k by $\left\|p_j - a_k\right\| < R_k$;
7:     **else**
8:       **return** A
9:     **end if**
10:   **end if**
11: **end for**

2) EVOLUTIONARY OPERATOR

To avoid the accuracy decline and the Hamming Cliff problem [59], [60], the Simulated Binary Crossover named SBX, and Polynomial Mutation is adopted on the real-valued solutions without coding and decoding. Apply the evolutionary operators on the equal genes of solutions by the strategy:

1 Execute the SBX operation when two parent genes are the nonempty set.
2 Exchange two parent genes when only one parent gene is the nonempty set.
3 Randomly select a nonempty gene in the parent as the mutation gene.

The advantage of this strategy is that the solutions are still in the neighboring subpopulations after the evolutionary operators.

B. CONSTRAINT HANDLING

1) REPAIR OPERATOR

For the diversity of the population, the repair operator has a level of relaxation on constraints. The repair operator consists of three detection and repair procedures of consistency, survival threshold, and preference assignment.

(1) Consistency. Calculating (f_1, f_2) by raw (X, A) may lead to the inconsistency of f_1 and f_2 which causes the redundancy of mapping from search space to the objective function. Hence the decision matrix is proposed to update (X, A). The detection and repair algorithm of consistency is presented in Algorithm 3.

In Algorithm 3, lines 1-10 show the initialization of decision matrix D. Lines 11-16 show the redundant weapon is pruned by the decision matrix.

(2) Survival threshold. To prevent local optimum, elastic detection is adopted on the constraint of damage threshold, namely have the target, which has the requirement of damage threshold, within the lethal radius of any weapon. The detection and repair algorithm of the survival threshold is presented in Algorithm 4.

Line 1 of Algorithm 4 obtains the employed weapons which are out of preference assignment and available to be reassigned. Line 3 obtains the target which has the requirement of damage threshold but is not within the lethal radius of any weapon. Line 4-9 shows an available weapon is selected randomly to the target which is unsatisfied with the survival threshold. As shown in line 8, the algorithm stops when all unsatisfied targets are repaired, or no available weapon can be reassigned.

(3) Preference Assignment. For the diversity of the population, the following strategy is adopted in the case of preference assignment of weapon-target pair $i-j$.

1 If weapon i is not available, no operator is performed to the solution.

2 If weapon i is available and target j is not within the lethal radius of weapon i, randomly generate the aiming point of weapon i within a certain range centered on target j.

The detection and repair algorithm of preference assignment is presented in Algorithm 5. Line 3-5 show the weapon i, which is available but not for the prior target j, is reassigned to threaten target j.

Algorithm 5 Detection and Repair Algorithm of Preference Assignment

Input: (X, A): the raw solution; D: the decision matrix; E: the preference matrix; m: the number of weapons; n: the number of targets.

Output: A: the repaired solution.

1: **for** $j = 1$ to n **do**
2:     **for** $i = 1$ to m **do**
3:         **if** $e_{ij} = 1$ and $x_i + d_{ij} = 1$ **then**
4:             Randomly generate a_i by $\left\|p_j - a_i\right\| < R_i$;
5:             break
6:         **end if**
7:     **end for**
8: **end for**

Considering the mutual interaction, the sequence of three algorithms is preference assignment, damage threshold, and consistency. The procedure of repair operator is described as follows.

Step 1 Generate the raw solution (X, A) then calculate the decision matrix D;

Step 2 Implement the preference assignment algorithm on (X, A) then update the decision matrix D;

Step 3 Implement the damage threshold algorithm on (X, A) then update the decision matrix D;

Step 4 Implement the consistency algorithm on (X, A).

2) MEASUREMENT OF CONSTRAINT VIOLATION

Because the relaxation of the repair operator, the most repaired solutions also violate the constraints. Preparing for the selection algorithm, a measurement of constraint violation is designed to evaluate the infeasible solutions.

The constraint violation of a solution is defined as

$$\phi(X, A) = \sum_{i=1}^{|C|} s_i \cdot g_i(X, A) \tag{12}$$

where C is the set of constraints; s_i is the constant coefficient of the ith constraint; $g_i(X, A)$ represents the constraint violation of solution (X, A) on the ith constraint and is designed by the following principles.

- $g_i(X, C)$ reflects the deviation of the infeasible solution X to the feasible region on C_i.
- $g_i(X, C)$ is proportional to the distance from the infeasible solution X to the nearest feasible solution on C_i and is normalized.

In this paper, $C = \{C_1, C_2, C_3\}$ represents the constraint of security evasion, survival threshold, and preference assignment respectively; s_i is set to 1 since $g_i(X, C)$ is normalized. The constraint violation of security evasion is designed as

$$g_1 = \frac{1}{|S|} \sum_{k=1}^{l} \sum_{i=1}^{m} \max\left(0, 1 - \frac{\|s_k - a_i\|}{R_i}\right) \tag{13}$$

$g_1 = 0$ when the friendly/neutral targets are not within the lethal radius of any weapon. Conversely, g_1 is inverse proportional to the distance from friendly/neutral targets to the aiming points of weapons violating C_1.

The constraint violation of the survival threshold is designed as

$$g_2 = \sum_{j=1}^{n} \frac{\max\left(0, \prod_{i=1}^{n} \left(1 - D\left(\left\|p_j - a_i\right\|\right)\right) \cdot x_{ij} - \rho_j\right)}{\left(1 - \rho_j\right) \sum_{j=1}^{n} \left\lceil 1 - \rho_j \right\rceil} \tag{14}$$

$g_2 = 0$ when survival probability vector P and survival threshold vector satisfy $P_i \leq \rho_i, i = 1, 2, \cdots, n$; otherwise g_2 is proportional to the deviation $|P - \rho|$.

The constraint violation of preference assignment is designed

$$g_3 = \frac{1}{\|E\|_1} \sum_{i=1}^{m} \sum_{j=1}^{n} \max(0, \frac{\left\|p_j - a_i\right\| \cdot e_{ij} - R_i}{B - R_i}) \tag{15}$$

where $\|E\|_1$ is the l1-norm of the preference matrix E, which denotes the number of preferred weapon-target pairs. $g_3 = 0$ when the preference pair $i - j$ is satisfied; otherwise g_3 is proportional to the distance from target j to the aiming point of weapon i.

C. ENVIRONMENT SELECTION

One challenge of solving CMWTA is obtaining the non-dominated solutions of fewer weapons. The reason is that fewer weapons are difficult to satisfy the constraints, and more weapons are easy to violate constraints. The situation of more weapons is better than fewer weapons in the evolutionary process because adding one weapon to the feasible plan of the middle subpopulation is more likely to approach a feasible plan than removing one weapon from the feasible plan of the middle subpopulation. Hence the diversity of the subpopulation of small f_2 is more important than the other subpopulations. Hence this paper presents a self-adaptive selection algorithm that gives more relaxation on the subpopulations of smaller f_2.

First, we defines the solution of CMWTA, which has a low constraint violation and acceptable fitness, as an effective solution. The self-adaptive selection algorithm selects the effective solutions to an elitist environment called effective population. According to the characteristic of the WTA problem, the survival probability of target declines obviously when one more weapon is used, especially the one more weapon violates more constraints. If a solution uses more weapons and violates more constraints, but leads to less damage effect, we consider this solution contributes less effective information. Inspired by this principle, the procedure of the self-adaptive sorting algorithm is as follows: Pick the first front consisting of the non-dominated solution of each subpopulation. In this front, if a solution is dominated by any solution with smaller f_2 on (f_1, ϕ), then this solution is removed from the current front and is left to the next front. When this detection of each solution of the current front completes, copy the existing solutions to the effective population. Pick the next front and repeat the above steps until the effective population is filled. Besides the low computational complexity and no parameter tuning, this sorting method has a self-adaptive mechanism making the solutions with smaller f_2 be more likely to be selected to the effective population. The self-adaptive selection algorithm is detailed in Algorithm 6.

Algorithm 6 Selection Algorithm of the Effective Population

Input: $P = \{SP_1, SP_2, \ldots, SP_m\}$: the population; sep: the size of the effective population; m: the number of weapons.

Output: EP: the effective population.

1: $EP = \emptyset$, $k = 1$;
2: **if** $k < sep$ **then**
3: $S = (sp_1*, sp_2*, \ldots, sp_m*)$ where sp_i* is the non-dominated solution of SP_i on (f_1, ϕ).
4: **for** $i = m$ to 2 **do**
5: **if** $sp_i* = \emptyset$ **then**
6: continue;
7: **end if**
8: **for** $j = i - 1$ to 1 **do**
9: **if** $sp_j* \neq \emptyset$ and $sp_j * .f_1 < sp_i * .f_1$ and $sp_j * .\phi < sp_i * .\phi$ **then**
10: Delete sp_i* from S;
11: break;
12: **end if**
13: **end for**
14: Copy S to EP and delete S from P;
15: $k = k + |S|$;
16: **end for**
17: **end if**

As shown in line 3 of Algorithm 6, S is the front prepared for the effective population in which $s(p_i)*$ is the non-dominated solution on (f_1, ϕ) of SP_i rather than the entire population. Line 4-9 gives the filter procedure of effective solutions. The solutions of S is checked from SP_m to SP_1, and one solution is only compared with the solutions with smaller f_2. For example, solution sp_i and sp_j of the front S come from the subpopulation SP_i and SP_j $(i < j)$ respectively. If solution i dominates solution j on (f_1, ϕ), namely $sp_i.f_1 < sp_j.f_1$ and $sp_i.\phi < sp_j.\phi$, we believe the solution j provides the less effective information than solution i in the evolutionary process and give priority to copying solution i to EP. In line 10-14, the ineffective solutions of current S are deleted and left to the front S of the next iteration.

The relaxation of constraint generally impacts the convergence speed. A cooperative evolutionary mechanism, which utilizes the similarity of neighboring non-dominated solutions and contributes to the quality information transferring in neighboring subpopulations, is adopted to strengthen the convergence speed. For the ith subpopulation, we define the nearest T subpopulations as the neighborhood $B(i, T)$ which can be calculated by Equation 16.

$$B(i,T) = \begin{cases} \{sp(1), \ldots, sp(T+1)\}, & \textit{if } i - \lceil T/2 \rceil \leq 0 \\ \{sp(m-T), \ldots, sp(m)\}, & \textit{if } i + \lfloor T/2 \rfloor > m \\ \{sp(i - \lceil T/2 \rceil), \ldots, sp(i + \lfloor T/2 \rfloor)\}, & \textit{otherwise} \end{cases} \tag{16}$$

The evolutionary operators are implemented on the neighboring solutions, and the procedure is outlined in Algorithm 7.

Algorithm 7 Evolutionary Procedures of MOEA-CMWTA

Input: $EP = \{SEP_1, SEP_2, \ldots, SEP_m\}$: the effective population where SEP_i is the ith subpopulation; m: the number of weapons.

Output: Q: the offspring population.

for $i = 1$ to m **do**
if $|SEP_i| > 0$ **then**
Randomly select the individual x_1 and x_2 from SEP_i and $B(i, T)$ respectively;
Generate solutions y_1 and y_2 from x_1 and x_2 by evolutionary operators;
Detect and repair y_1 and y_2.
Copy y_1 and y_2 to corresponding subpopulations of Q.
end if
end for

D. COMPUTATIONAL COMPLEXITY

Let P denotes the size of the population; M denotes the number of weapons; N denotes the number of targets. The worst time complexity of one generation of proposed MOEA can be calculated according to the process: (1) The time complexity of population initialization is $O(MP)$. (2) The worst time complexity of the selection algorithm is $O(MP)$. (3) In the effective population, the time complexity of the evolutionary process is also $O(MP)$. (4) In the detection and repair algorithm applied on population, the worst time complexity of consistency repair is $O(MNP)$. The worst time complexity of damage threshold repair is $O(NP)$. The worst time complexity of preference assignment repair is $O(MNP)$. According to the operational rules of the symbol O, the worst time complexity of one generation in MOEA-CMWTA is $O(MNP)$.

E. PERFORMANCE METRICS

In MOPs, the major performance metrics are to measure the convergence and spacing. The former evaluates the proximity of obtained Pareto-optimal set to the true Pareto-optimal set, and the latter evaluates the uniformity of the non-dominated solutions. In CMWTA, since the non-dominated solutions follow the specific non-uniform distribution, the spacing metric is not employed to reflect the quality of the Pareto-optimal set. Generally, the calculation of convergence metric needs the true Pareto-optimal set which is hardly obtained in the CMWTA problem. Thus a variant of convergence metric is proposed to compare the algorithms solving MWTA. Let $P = \{p_1, p_2, \cdots, p_{|P|}\}$ be the approximate Pareto-optimal set coming from the historical non-dominated solutions of the comparison algorithms; $A = \{a_1, a_2, \cdots, a_{|A|}\}$ is the Pareto-optimal set found by a comparison algorithm. The convergence metric $C(A)$ is the normalized proximity of A to P.

$$d_i = \begin{cases} \dfrac{f_1(a_k) - f_1(p_i)}{f_1^{\max} - f_1^{\min}}, & \exists f_2(a_k) = f_2(p_i) \\ 1, & otherwise \end{cases} \tag{17}$$

$$C(A) \triangleq \frac{\sum_{i=1}^{|P|} d_i}{|P|} \tag{18}$$

Besides the quantitative evaluation of the convergence metric, the coverage metric[20] is employed for the qualitative evaluation.

$$I_C(A, B) \triangleq \frac{|\{b \in B; \exists a \in A : a \succeq b\}|}{|B|} \tag{19}$$

where $\succeq$ means weakly dominate; A, B are two approximate Pareto-optimal sets and the function I_C maps the ordered pair (A, B) to the interval [0, 1]. It shows that A is better than B when $I_C(A, B) > I_C(B, A)$.

IV. EXPERIMENT STUDIES

In this section, we compare MOEA-CMWTA with several state-of-the-art MOEAs, an MOEA/D for WTA, and approved constraint handling methods. All experiments were carried out in MATLAB R2016b environment on a PC with i7-2.5GHz CPU and 8GB memory.

A. COMPARISON ALGORITHMS AND CONSTRAINT HANDLINGS

To verify the performance of MOEA-CMWTA, we compare our algorithms with several MOEA frameworks and approved constraint handling approaches. The comparison frameworks are:

1) NSGA-II [61]. Alleviating the difficulties of NSGA, NSGA-II has better convergence and lower computational complexity by using a fast non-dominated sorting approach and an elitist selection operator. The reason why being a comparison algorithm is that NSGA-II performs rather good diversity in bio-objective instances.
2) SPEA2 [62]. As an improved version of SPEA, SPEA2 incorporates a revised fitness assignment strategy, a nearest neighbor density estimation technique and an enhanced archive truncation method. SPEA2 has a great advantage in distribution.
3) MOEA/D [63]. MOEA/D has achieved great success in the field of evolutionary MOPs based on decomposing a MOP into some scalar optimization subproblems and optimizes them simultaneously. MOEA/D performs well on multi-objective 0-1 knapsack problems, continuous multi-objective optimization problems and disparately-scaled objectives by using objective normalization.
4) MOEA/D-WTA [7]. For solving the classic MWTA, MOEA/D-WTA integrate a population initialization based on prior knowledge, a problem-specific selection, and a mating restriction. The experimental results show that MOEA/D-WTA outperforms several MOEA/D frameworks in terms of solving the point-to-point MWTA problem.

The comparison constraint handling approaches are:

1) Self-adaptive penalty (SP) function [44]. This method modified the objective function by an adaptive penalty function and a distance measure. Besides the superior performance, the advantage is simple to implement and does not need any parameter tuning.
2) Constrained-dominated principle (CDP) [46]. As the representative method of the modified dominance rule, the CDP compares the objectives and constraints separately. The infeasible solutions are guided toward the feasible region. This method performs well on real-code GAs and does not require any parameter setting.
3) ϵ-constraint handling (EC) method [53]. EC designs a dynamic relaxation on the constraints, which is in favor of maintaining diversity.
4) Stochastic ranking (SR) method [54]. SR adopts a probability parameter pf to decide if the comparison is to be based on objectives or constraints, which contributes to solving the COPs with the narrow feasible region.

In the practical composition of the frameworks and constraint handling approaches, the repair operator is also implemented on each algorithm. The parameters are set as follows:

1) Global settings: the population size $pop = 1200$; the max generation $maxgen = 100$; the crossover probability $p_c = 0.8$; the mutation probability $p_m = 0.2$; the crossover distribution index $\eta_c = 5$; the mutation distribution index $\eta_m = 5$;
2) Parameter setting in MOEA-CMWTA: the size of effective population $ep = 0.5pop$; the number of neighboring subpopulations $T = 2$;

TABLE 2. Parameters of the comparison algorithms.

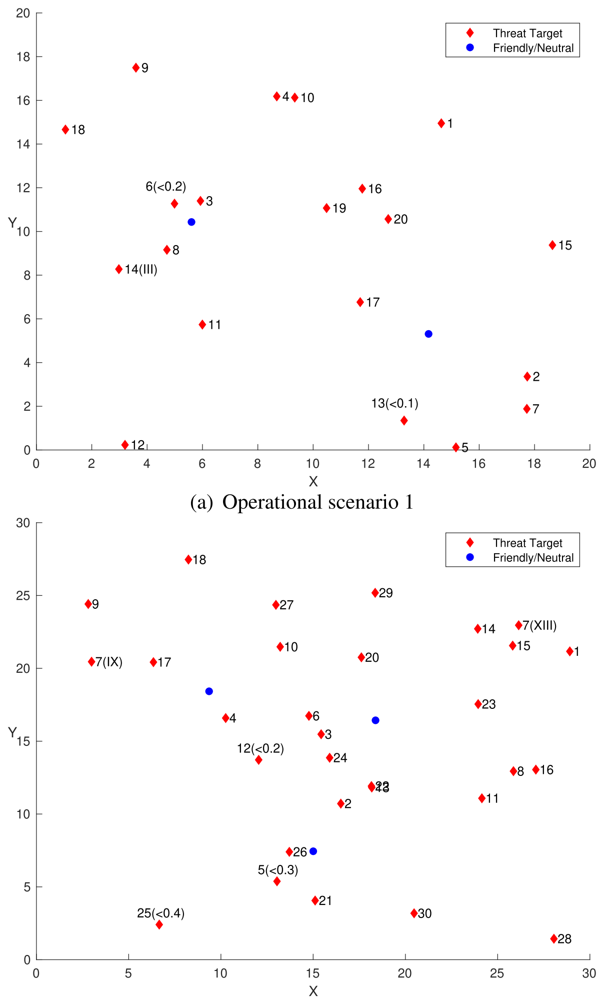
Parameter	MOEA-CMWTA	NSGA-II	SPEA2	MOEA/D	MOEA/D-WTA
pop	1200	1200	1200	1200	1200
$maxgen$	100	100	100	100	100
p_c/p_m	0.8/0.2	0.8/0.2	0.8/0.2	1/0.2	0.8/0.2
η_c/η_m	20/20	20/20	20/20	20/20	20/20
Repair Operator	✓	✓	✓	✓	✓
Elitist environment	Effective population/Mating pool	Mating pool	External archive/Mating pool	-	-
Size of elitist	600	600	1200/600	-	-
Neighborhood Index	2	-	35	10	3
Constraint Handling	Self-adaptive selection	CDP/EC/SP	CDP/EC/SP	CDP/EC/SR	CDP/EC/SP

3) Parameter setting in NSGA-II: the size of the mating pool is set to $0.5pop$;
4) Parameter setting in SPEA2: the size of the archive $\overline{pop} = 1200$; the nearest neighbor index k is set to 49 by $k = \sqrt{pop + \overline{pop}}$.
5) Parameter setting in MOEA/D-WTA:
6) Parameter setting in MOEA/D: the number of the neighbor weight vectors is set to 10; the decomposition strategy is the Tchebycheff approach.
7) Parameter setting in EC: $T_c = 80$; $cp = 2$; $theta = 0.05pop$;
8) Parameter setting in SR: $P_f = 0.45$.

The detailed setting of the comparison algorithms are listed in Table 2.

B. OPERATIONAL SCENARIO

First, the scenario 1 of limited weapons, namely $m < n$, is defined and omitting the units for simplicity. As shown in Fig 2, the scenario scale is 20×20. There are six available weapons and 20 hostile targets displayed by red diamonds and marked by Arabic numerals. The inequalities after target 6 and 13 indicate the requirement of survival threshold. The Roman numeral III, after target 14, indicates the preference assignment of weapon 3. There are also two friendly/neural targets displayed by blue dots. Let the lethal radius of available weapons be [2.5:0.5:5]. Then the scenario 2, which involves 30 weapons and 30 hostile targets, is designed to investigate the performance facing the saturated weapon resource and a more complex environment. The lethal radius of available weapons is [2.1:0.1:5]. The specific parameters of the two scenarios are shown in Table 3.



(a) Operational scenario 1

(b) Operational scenario 2

FIGURE 2. Diagram of two operational scenarios.

C. EXPERIMENTS ON MOEAS

In this section, we perform 13 compositions of the MOEA frameworks and constraint handlings on scenario 1. Then five algorithms, which preserve the convergence metric less than 0.1 and have better distribution, are compared on scenario 2.

1) PERFORMANCE ON SCENARIO 1

We run each algorithm on scenario 1 over 30 independent runs. The statistics of the Pareto-optimal sets are shown in Table 4. In Table 4, column 1 lists the comparison algorithms. Columns 2 to 6 show the size of the obtained Pareto-optimal sets. Column 7 gives the percentage of infeasible solutions in non-dominated solutions, and column 8 gives the mean value of constraint violation. Column 9 gives the mean value of the convergence metric of the Pareto-optimal set. Column 10 to 14 show the mean value of fitness of the optimal solution in each subpopulation, namely the operational effect of the optimal plan under different weapon consumption.

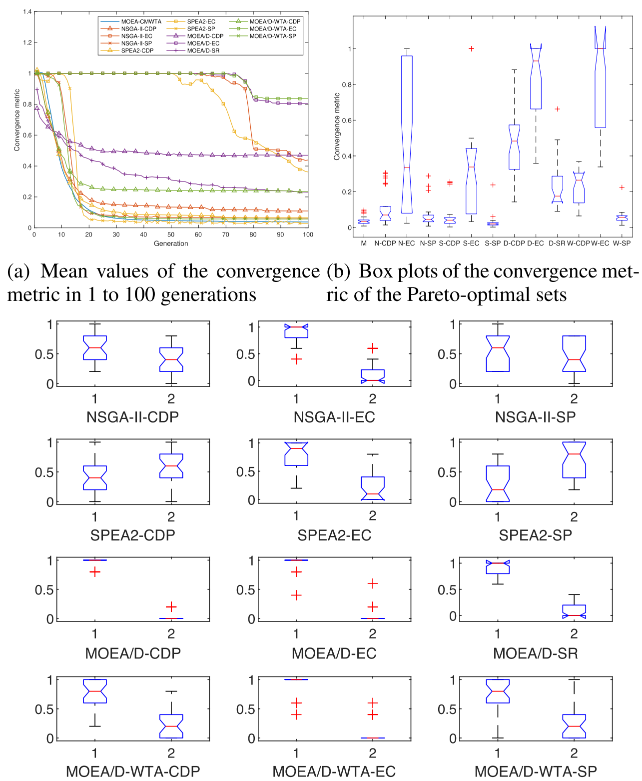
TABLE 3. Numerical values of two operational scenarios.

Scenario 1				Scenario 2							
Unit	Threat	Position	Remarks	Unit	Threat	Position	Remarks	Unit	Threat	Position	Remarks
F1	0	(5.60, 10.43)	No damage	F1	0	(18.38, 16.43)	No damage	H20	0.53	(17.61, 20.76)	
N1	0	(14.17, 5.31)	No damage	F2	0	(15.00, 7.44)	No damage	H21	0.66	(15.11, 4.06)	
H1	0.71	(14.63, 14.95)		N1	0	(9.36, 18.42)	No damage	H22	0.03	(18.16, 11.92)	
H2	0.05	(17.74, 3.36)		H1	0.05	(28.91, 21.16)		H23	0.57	(23.94, 17.55)	
H3	0.85	(5.93, 11.40)		H2	0.52	(16.50, 10.71)		H24	0.67	(15.89, 13.86)	
H4	0.85	(8.69, 16.19)		H3	0.39	(15.43, 15.48)		H25	0.56	(6.66,2.41)	sp<0.4
H5	0.61	(15.16, 0.12)		H4	0.99	(10.26, 16.59)		H26	0.66	(13.71, 7.40)	
H6	0.93	(4.99, 11.27)	sp<0.2	H5	0.43	(13.04, 5.38)	sp<0.3	H27	0.45	(12.98, 24.36)	
H7	0.88	(17.72, 1.88)		H6	0.71	(14.77, 16.74)		H28	0.12	(28.05, 1.44)	
H8	0.39	(4.72, 9.16)		H7	0.52	(2.99, 20.45)	W9-T7	H29	0.69	(18.36, 25.18)	
H9	0.43	(3.60, 17.49)		H8	0.24	(25.86, 12.94)		H30	0.95	(20.47, 3.19)	
H10	0.93	(9.34, 16.13)		H9	0.37	(2.81, 24.42)					
H11	0.11	(6.00, 5.74)		H10	0.17	(13.21, 21.48)					
H12	0.54	(3.20, 0.23)		H11	0.63	(24.15, 11.08)	sp<0.2				
H13	0.43	(13.29, 1.35)	sp<0.1	H12	0.16	(12.05, 13.72)					
H14	0.51	(2.98, 8.28)	W3-T14	H13	0.91	(18.18, 11.82)					
H15	0.29	(18.65, 9.38)		H14	0.90	(23.92, 22.72)					
H16	0.29	(11.77, 11.96)		H15	0.92	(25.81, 21.56)					
H17	0.21	(11.70, 6.77)		H16	0.26	(27.07, 13.05)					
H18	0.69	(1.05, 14.66)		H17	0.13	(6.34, 20.43)					
H19	0.83	(10.48, 11.07)		H18	0.04	(8.24, 27.47)					
H20	0.16	(12.71, 10.57)		H19	0.53	(26.14, 20.76)	W13-T19				

According to the prior information and simulation results, the size of the true Pareto-optimal set is 5, and the f_2 of the optimal non-dominated solutions is 2 to 6. As shown in Table 4, Only MOEA-CMWTA can guarantee the completeness in each run. NSGA-II-SP, SPEA2-CDP, SPEA2-SP, and MOEA/D-WTA-SP obtain the size of 4 in two, three, one, and one run respectively. The other algorithms perform worse on the distribution of PS size. In column 7 and 8, the drawbacks are concentrate on the algorithms with the EC method, and NSGA-II-SP also obtains one infeasible solution. MOEA-CMWTA and SPEA2-SP outperform on the convergence metric, which can guarantee the value of less than 0.05. MOEA/D-WTA-SP, NSGA-II-SP, and SPEA-CDP follow by 0.0570, 0.0636, and 0.0652 respectively. In mean values of fitness, SPEA2-SP performs better on the weapon consumption of 2, 5 and 6. NSGA-II-SP and SPEA2-CDP obtain optimal fitness as the weapon consumption is 3 and 4 respectively.

Based on the results of Table 4, the convergence metric is mainly related to the completeness of the Pareto-optimal set. The elaborate self-adaptive penalty function performs rather well in the frameworks of NSGA-II, SPEA2, and MOEA/D-WTA-SP, although it also can not guarantee the completeness of the Pareto-optimal set. MOEA-CMWTA is the only algorithm that can guarantee completeness in each run. Hence MOEA-CMWTA achieves 0.0379 on convergence metric, although it has no optimal values of fitness.

In terms of the complete Pareto-optimal set, the rigid search space may lead to the local optimum and incomplete Pareto-optimal set of CMWTA, such as NSGA-II-CDP, SPEA2-CDP, MOEA/D-CDP, MOEA/D-SR, and MOEA/D-WTA-SP. The frameworks of NSGA-II and SPEA2 utilize the non-dominated sorting algorithm with a strict



(a) Mean values of the convergence metric in 1 to 100 generations

(b) Box plots of the convergence metric of the Pareto-optimal sets

(c) Box plots of the coverage of the two sets obtained by MOEA-CMWTA and a comparison algorithm

FIGURE 3. Convergence metric and coverage of the Pareto-optimal sets obtained by the comparison algorithms solving scenario 1 over 30 independent runs.

selection principle. MOEA/D-WTA-SP selects the non-dominated solution of each weapon consumption forcibly. CDP implements a rigorous selection criterion which gives

TABLE 4. Statistics of the Pareto-optimal sets obtained by the comparison algorithms solving scenario 1 over 30 independent runs.

Algorithm	Size of PS					Infeasibility		Convergence metric	Mean values of the fitness under each weapon consumption				
	5	4	3	2	1	%	Mean		6	5	4	3	2
MOEA-CMWTA	**30**	0	0	0	0	0	0	0.03796948	1.18613171	1.46369502	1.99870344	3.05914387	6.94233816
NSGA-II-CDP	24	6	0	0	0	0	0	0.10947655	1.30337952	1.53691869	1.95034898	3.36504456	7.00916463
NSGA-II-EC	13	8	0	2	7	6.48	0.0703	0.43937094	1.61527547	1.86712760	2.50076797	3.95476551	7.01871626
NSGA-II-SP	28	2	0	0	0	0.68	0.0236	0.06361190	1.36574984	1.57919597	1.97083404	**2.95675853**	6.94092238
SPEA2-CDP	27	3	0	0	0	0	0	0.06528147	1.12439058	1.40071270	**1.92189134**	3.26156324	6.98320975
SPEA2-EC	17	13	0	0	0	18.98	0.1309	0.36172785	1.34995469	1.62569575	2.12809649	4.26761576	7.02359706
SPEA2-SP	29	1	0	0	0	0	0	**0.02849572**	**1.10894854**	**1.39538003**	1.92541419	2.95744218	**6.93929157**
MOEA/D-CDP	9	12	8	1	0	0	0	0.47007922	2.18499595	2.49026557	3.12380357	4.88918491	7.06989340
MOEA/D-EC	0	3	6	6	15	24.56	0.0706	0.80446787	1.89647577	2.20097431	2.57203922	4.92286555	7.37321712
MOEA/D-SR	21	7	1	1	0	0	0	0.23137060	1.70370318	2.00372769	2.45880462	3.79840971	7.02798235
MOEA/D-WTA-CDP	11	19	0	0	0	0	0	0.23426148	1.42194788	1.64150993	2.01752055	3.71960971	7.20513175
MOEA/D-WTA-EC	0	5	3	1	21	40.38	0.0682	0.83594250	1.74406824	2.08974885	2.41326216	4.65705256	-
MOEA/D-WTA-SP	29	1	0	0	0	0	0	0.05698250	1.39881516	1.60912535	2.00380805	2.96642146	6.94636257

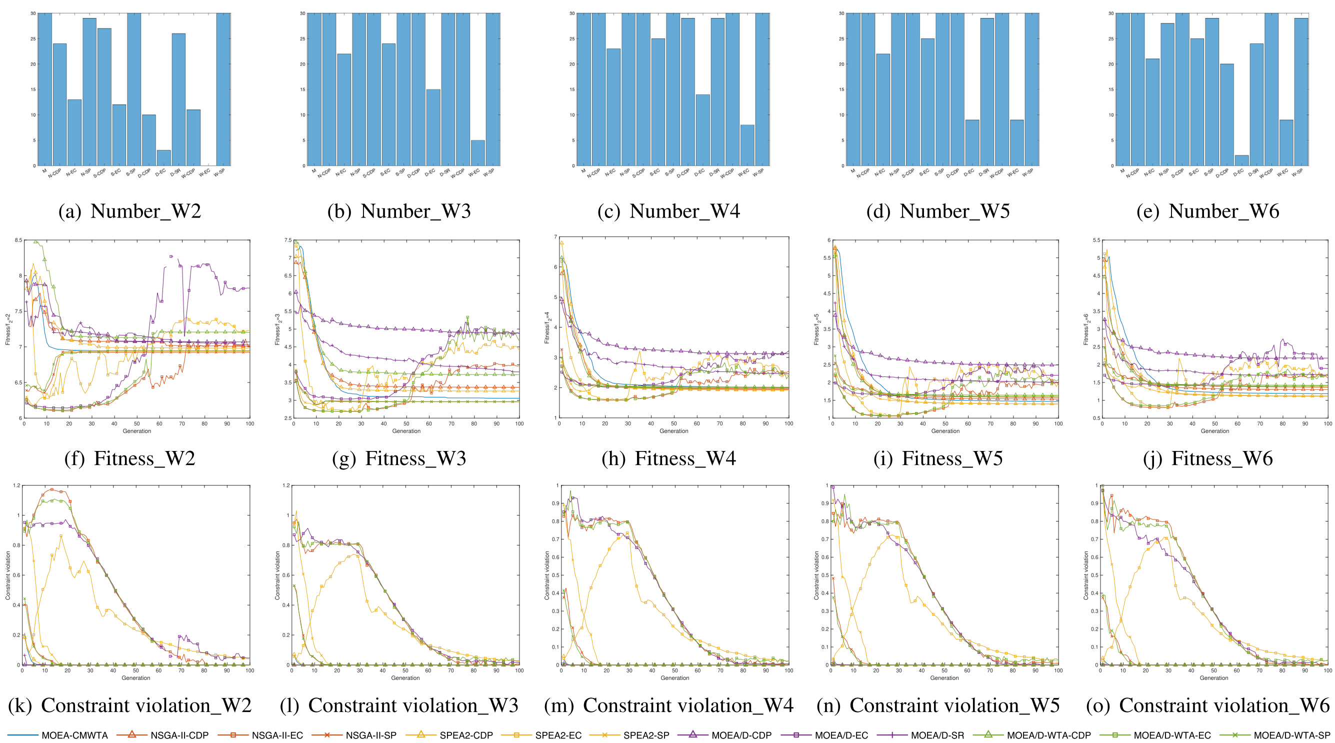
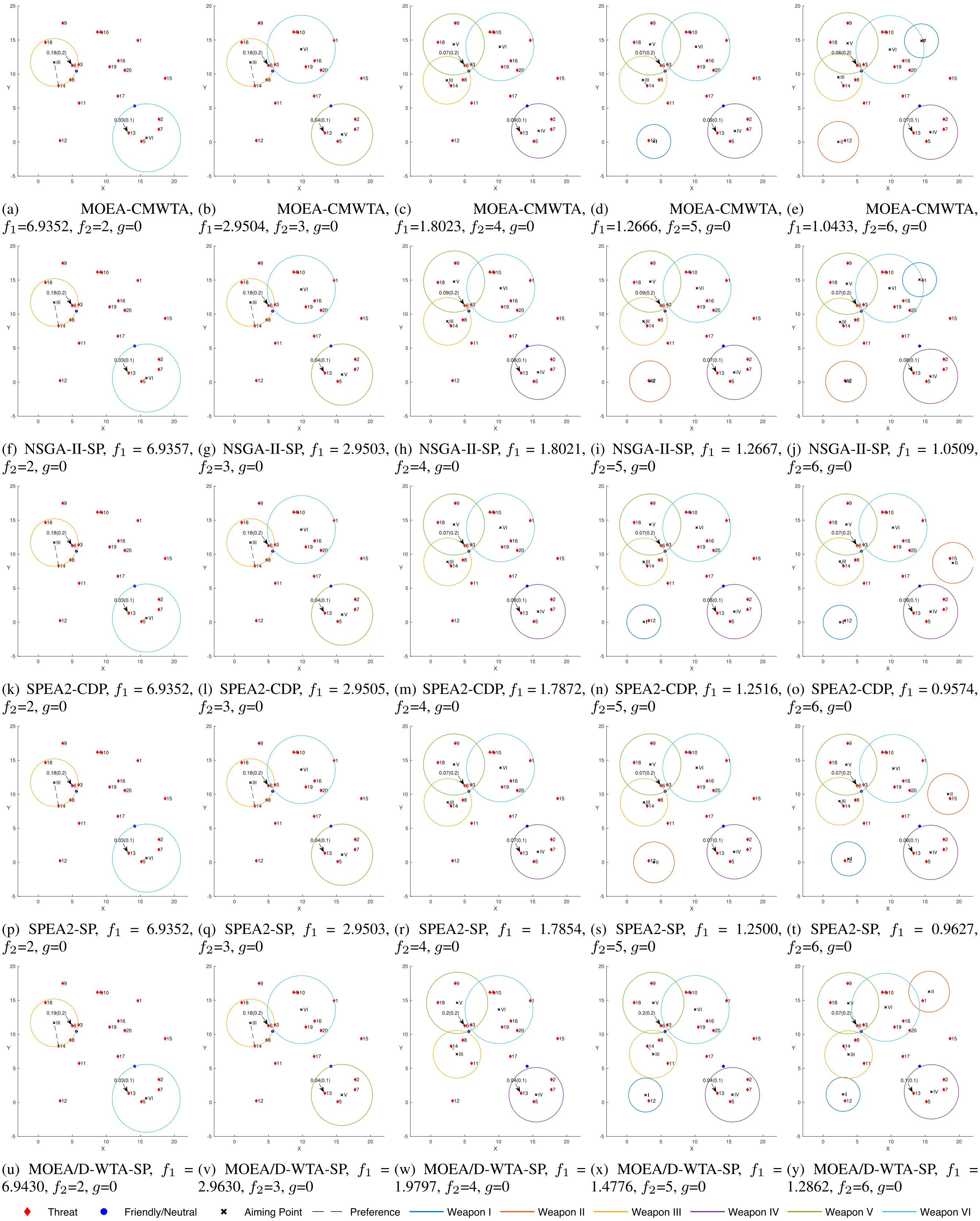


FIGURE 4. Solving scenario 1 and sorted by weapon consumption, the number of the feasible non-dominated solutions obtained by comparison algorithms over 30 independent runs; mean values of the fitness and constraint violation obtained by comparison algorithms over 3FourFitnessConstraint0 independent runs in 1 to 100 generations.

the non-dominated solutions with 0 constraint violation. Consequently, the composition of NSGA-II/SPEA2/MOEA/D-WTA and CDP has a forced ability to cleaning the infeasible solutions out and against the diversity of the population. MOEA/D transforms a piecewise continuous objective and an integer objective into a scalar optimization, and the search capability is further restricted by CDP. As shown in Table 4, NSGA-II-CDP and SPEA2-CDP concentrate on the size 5 and 4, and MOEA/D-CDP concentrates on the size 5 to 3. SR adopts a stochastic selection mechanism to decide the selection is based on objectives or constraints. In the latter case, SR equates to CDP. Therefore, MOEAD/D-SR performs better than MOEA/D-CDP on solving CMWTA. SR can ensure the feasibility of obtained sets but is not ideal for the completeness. The reason is that SR has no information to guide the search space such as SP.

The permissive search space causes the infeasibility and incomplete Pareto-optimal set of CMWTA, such as NSGA-II-EC, SPEA2-EC, MOEA/D-EC, and MOEA/D-WTA-EC. EC performs worst on not only convergence but also spacing. Four frameworks with EC can not solve the complete sets in more than half runs, and can not guarantee the feasible of non-dominated solutions. MOEA/D-WTA-EC even obtains one non-dominated solution in 21 runs and



(a) MOEA-CMWTA, f_1=6.9352, f_2=2, g=0 (b) MOEA-CMWTA, f_1=2.9504, f_2=3, g=0 (c) MOEA-CMWTA, f_1=1.8023, f_2=4, g=0 (d) MOEA-CMWTA, f_1=1.2666, f_2=5, g=0 (e) MOEA-CMWTA, f_1=1.0433, f_2=6, g=0

(f) NSGA-II-SP, f_1 = 6.9357, f_2=2, g=0 (g) NSGA-II-SP, f_1 = 2.9503, f_2=3, g=0 (h) NSGA-II-SP, f_1 = 1.8021, f_2=4, g=0 (i) NSGA-II-SP, f_1 = 1.2667, f_2=5, g=0 (j) NSGA-II-SP, f_1 = 1.0509, f_2=6, g=0

(k) SPEA2-CDP, f_1 = 6.9352, f_2=2, g=0 (l) SPEA2-CDP, f_1 = 2.9505, f_2=3, g=0 (m) SPEA2-CDP, f_1 = 1.7872, f_2=4, g=0 (n) SPEA2-CDP, f_1 = 1.2516, f_2=5, g=0 (o) SPEA2-CDP, f_1 = 0.9574, f_2=6, g=0

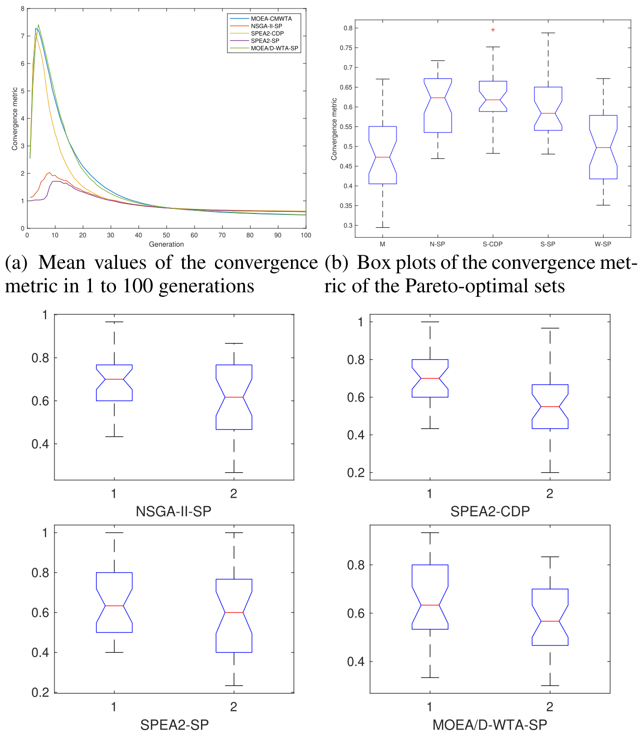
(p) SPEA2-SP, f_1 = 6.9352, f_2=2, g=0 (q) SPEA2-SP, f_1 = 2.9503, f_2=3, g=0 (r) SPEA2-SP, f_1 = 1.7854, f_2=4, g=0 (s) SPEA2-SP, f_1 = 1.2500, f_2=5, g=0 (t) SPEA2-SP, f_1 = 0.9627, f_2=6, g=0

(u) MOEA/D-WTA-SP, f_1 = 6.9430, f_2=2, g=0 (v) MOEA/D-WTA-SP, f_1 = 2.9630, f_2=3, g=0 (w) MOEA/D-WTA-SP, f_1 = 1.9797, f_2=4, g=0 (x) MOEA/D-WTA-SP, f_1 = 1.4776, f_2=5, g=0 (y) MOEA/D-WTA-SP, f_1 = 1.2862, f_2=6, g=0

FIGURE 5. The display of the optimal non-dominated solutions obtained by MOEA-CMWTA, NSGA-II-SP, SPEA2-CDP, SPEA2-SP, and MOEA/D-WTA-SP solving scenario 1 over 30 independent runs.

TABLE 5. **Statistics of the Pareto-optimal sets obtained by MOEA-CMWTA, NSGA-II-SP, SPEA2-CD, SPEA2-SP, and MOEA/D-WTA-SP solving scenario 2 over 30 independent runs.**

Algorithm	Size of PS					Infeasibility		Convergence metric
	30-26	25-21	20-16	15-11	≤10	%	Mean	
MOEA-CMWTA	14	16	0	0	0	0	0	**0.48631359**
NSGA-II-SP	0	2	23	5	0	0	0	0.60822153
SPEA2-CDP	0	0	21	9	0	0	0	0.62945846
SPEA2-SP	0	0	26	4	0	0	0	0.60270033
MOEA/D-WTA-SP	21	9	0	0	0	8.03	1.5387	0.49539967



(a) Mean values of the convergence metric in 1 to 100 generations (b) Box plots of the convergence metric of the Pareto-optimal sets

(c) Box plots of the coverage of the two sets obtained by MOEA-CMWTA and a comparison algorithm

FIGURE 6. **Convergence metric and coverage of the Pareto-optimal sets obtained by the comparison algorithms solving scenario 2 over 30 independent runs.**

contains 40.38% infeasible solutions. The reason is that EC loses control of the constraint violations of CMWTA.

The convergence metric and coverage of the obtained sets are presented in Figure 3. Let M denotes the Pareto-optimal sets obtained by MOEA-CMWTA, N denotes the sets obtained by NSGA-II, S denotes the sets obtained by SPEA2, D denotes the sets obtained by MOEA/D, and W denotes the sets obtained by MOEA/D-WTA. Figure 3(a) gives the dynamic performance of the mean values of the convergence metric in evolutionary generations. Figure 3(b) gives the box plots to illustrate the distribution of the convergence metric of the Pareto-optimal sets. The notches represent a robust estimate of the uncertainty about the medians for box-to-box comparison, and symbol + denotes outliers. To verify the proposed MOEA, Figure 3(c) exhibits the box plots of the coverage of the two sets obtained by MOEA-CMWTA and a comparison algorithm. In each plot, the left box represents the distribution of $I_C(M, *)$, and the right box represents the distribution of $I_C(*, M)$.

As shown in Figure 3(a) and 3(b), the comparison algorithms are located in several tiers. In the first tier, there are MOEA-CMWTA, NSGA-II-CDP/SP, SPEA2-CDP/SP, and MOEA/D-WTA-SP which can obtain the complete feasible non-dominated solutions in the most runs. In Figure 3(c), only SPEA2-SP is slightly better than MOEA-CMWTA. However, this gap may result from the repair operator used in all algorithms. The influence of the repair operator is investigated in the following experiment.

In view of the decision plan of C2, Figure 4 shows the results identified by different weapon consumption. The first row of the figures gives the number of feasible plans obtained in 30 runs. To evaluate the dynamic performance of algorithms, the latter two rows present the mean values of the fitness and constraint of the optimal plans in the evolutionary generations respectively.

As shown in Figure 4(a)-4(e), the challenge is to obtain the optimal plan with two or six weapons. Only MOEA-CMWTA can solve the optimal plans under all weapon consumption in 30 runs. SPEA2-SP and MOEA/D-WTA-SP solve the optimal plan of two weapons and six weapons in 29 runs respectively. Considering Figure 4(f)-4(o), the most plots of the comparison algorithms have the shocks since the algorithms can not maintain the non-dominated solution in every generation of each run. By contrast, MOEA-CMWTA is robust and has a stable decrease trend. The fitness of MOEA-CMWTA increases slightly in the early generations, and the constraint violation is suppressed to 0. In this process, the individuals are converted from infeasible solutions to feasible solutions. However, NSGA-II-SP, SPEA2-SP, and MOEA/D-WTA-SP perform opposite. The fitness drops down to a low level rapidly and the constraint violation has a sharp increase, then fitness is increasing with the decrease of constraint violation. The reason is that MOEA-CMWTA and SPEA2-CDP give priority to decreasing the constraint violation. Hence the fitness of MOEA-CMWTA and SPEA2-CDP has a temporary increase in early generations. The algorithms with SP optimize the modified objectives, which optimize the objective and constraints simultaneously. Hence MOEA-CMWTA

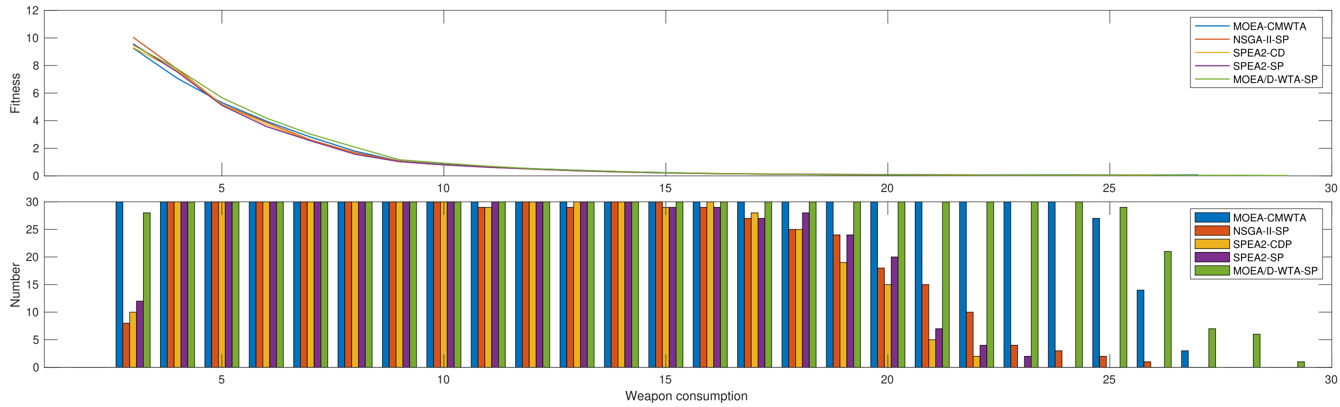


FIGURE 7. Mean values of the fitness and number of feasible non-dominated solutions obtained by MOEA-CMWTA, NSGA-II-SP, SPEA2-CD, SPEA2-SP, and MOEA/D-WTA-SP under different weapon consumption over 30 independent runs.

outperforms on convergence speed, which is a crucial advantage of real-time decision support in C2. As shown in Figure 4(k)-4(o), the constraint violation of MOEA-CMWTA can converge to 0 within 5 generations, and the others of the first tier eliminate the constraint violation at about 18th generation.

So far the proposed MOEA, NSGA-II-SP, SPEA2-CDP, SPEA2-SP, and MOEA/D-WTA-SP are qualified solver for CMWTA. The sketch maps of the optimal plans, which are obtained by three algorithms over 30 runs, are displayed in Figure 5. In Figure 5, the roman number I to VI represent the aiming points of available weapon resources of which the lethal radii are displayed by specified colors. The targets with damage requirements are indicated by arrow wherein the first number represents the expected survival, and the number in parentheses represents the survival threshold. The weapon-target pair $i-j$ with preference assignment, that is $a_{ij} = 1$, is linked by the dotted line.

As shown in Figure 5, the solved plans can satisfy the operational requirements. The survival thresholds of target 6 and 13 are 0.2 and 0.1. However, the no-damage targets 1 and 2 are close to the hostile targets 6 and 13 respectively, and the target 14 with the preference assignment of weapon III is close to no-damage target 1. Constrained by this situation, when the weapon consumption is less than 6, the optimal plans obtained by three algorithms have high similarity. In the case of two weapons, the weapon V and VI cannot deal with the above constraints and only the weapon III meets both the preference assignment for target 14, the survival threshold of target 6 and the security evasion of target 1. Only the weapon IV or V can satisfy the maximum fitness based on the survival threshold of target 13 and the security evasion of target 2. The distinct difference emerges as the weapon consumption is six, and the optimal plan obtained by SPEA2-SP is more rational according to the fitness and the distribution of the aiming points.

In addition, MOEA-CMWTA has low computational complexity. As described in Section 3, the repair algorithm, which is adopted in all comparison algorithms, has the worst computational complexity of $O\,(MNP)$ where M is the number of weapons, N is the number of targets, and P is the population size. Regardless of the complexity of the repair algorithm, the worst computational complexity of MOEA-CMWTA is $O\,(MP)$. The worst computational complexity of NSGA-II and SPEA2 are $O\left(KP^2\right)$ and $O\left(P^3\right)$ respectively where K is the number of objectives. The worst computational complexity of MOEA/D and MOEA/D-WTA are $O\,(KPT)$ where T is the number of the weight vectors in the neighborhood of each weight vector. The number of objectives is 2 in the proposed CMWTA model.

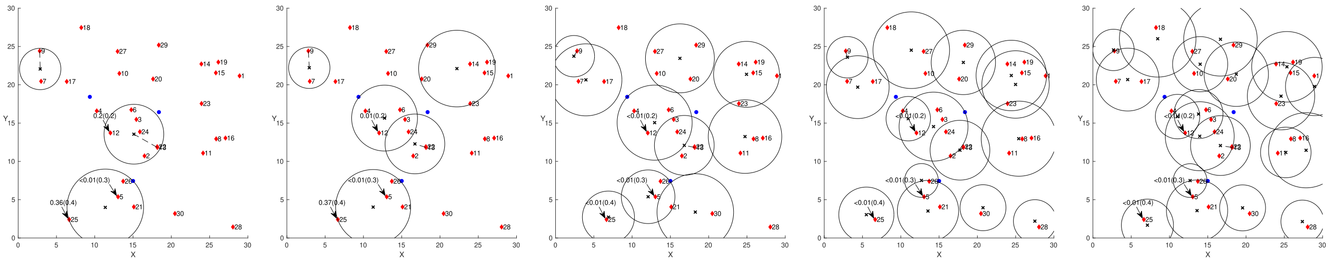
Based on the above analysis, we conclude SPEA2-SP outperforms on the values of convergence in the comparison algorithms. MOEA-CMWTA approaches to SPEA2-SP on convergence and has advantages in spacing, robust, real-time and computational complexity.

2) PERFORMANCE ON SCENARIO 2

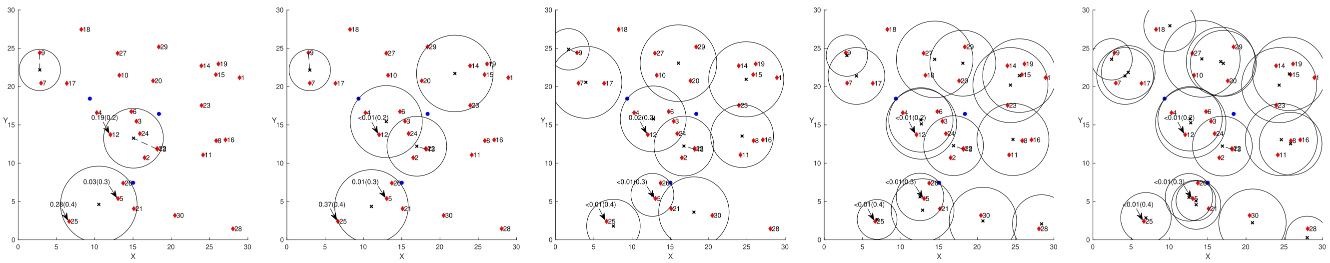
As shown in the above experiments, MOEA-CMWTA, NSGA-II-SP, SPEA2-CD, SPEA2-SP, and MOEA/D-WTA-SP have the convergence metric less than 0.1 and approximate complete Pareto set. These algorithms are run on scenario 2 over 30 independent runs.

The statistics of the obtained Pareto-optimal sets are shown in Table 5. Owing to the max weapon consumption is 30, we do not give the mean values of the fitness under each weapon consumption like Table 4. Instead, the mean values of the fitness are plotted in Figure 7. As shown in Table 5, MOEA/D-WTA-SP outperforms on completeness of the Pareto-optimal set but contains 8.03% infeasible non-dominated solutions and the mean value is 1.5387. MOEA-CMWTA gives better completeness of the Pareto-optimal set than three other algorithms and preserves the feasibility of the non-dominated solutions.

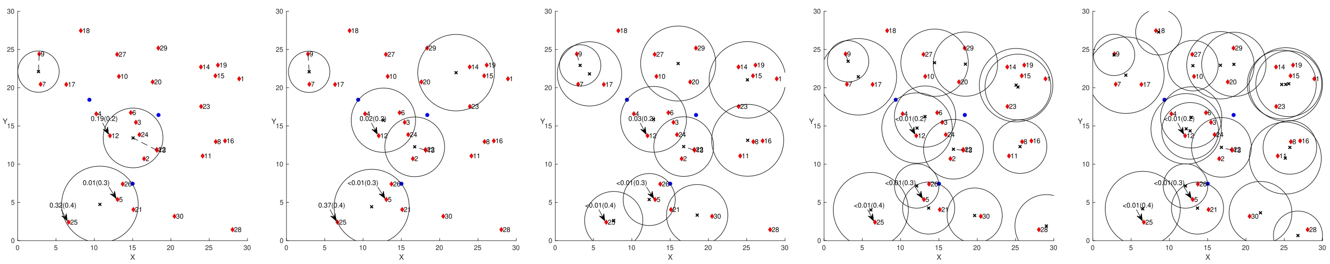
Figure 6 shows the convergence metric and coverage of the Pareto-optimal sets obtained by five algorithms. Considering Table 5 and Figure 6(a), only MOEA-CMWTA and MOEA/D-WTA-SP get the convergence metric less than 0.5. MOEA-CMWTA has a better convergence metric,



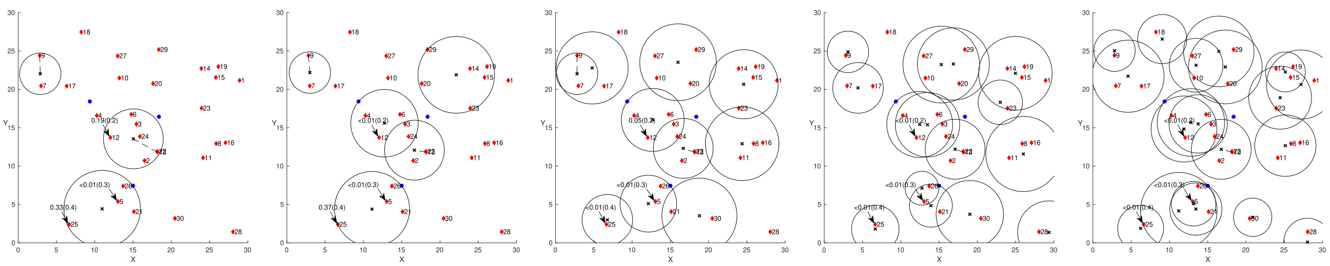
(a) MOEA-CMWTA, f_1=9.1895, f_2=3, g=0 (b) MOEA-CMWTA, f_1=4.6299, f_2=5, g=0 (c) MOEA-CMWTA, f_1=0.6963, f_2=10, g=0 (d) MOEA-CMWTA, f_1=0.1164, f_2=15, g=0 (e) MOEA-CMWTA, f_1=0.0118, f_2=20, g=0



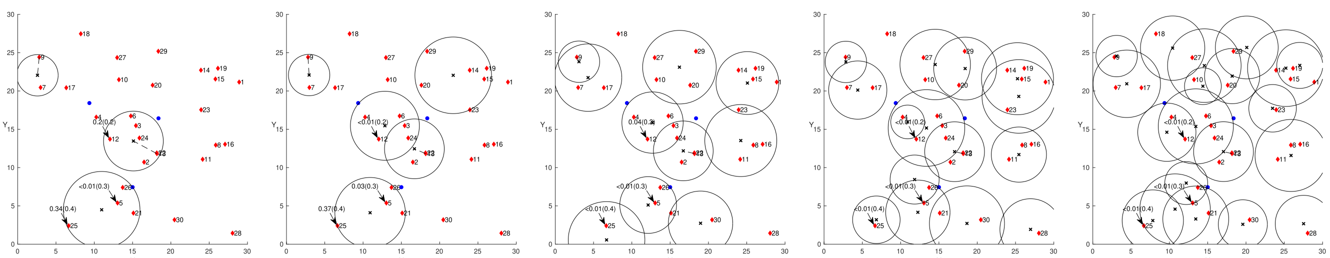
(f) NSGA-II-SP, f_1 = 9.2712, f_2=2, g=0 (g) NSGA-II-SP, f_1 = 4.6624, f_2=5, g=0 (h) NSGA-II-SP, f_1 = 0.6833, f_2=10, g=0 (i) NSGA-II-SP, f_1 = 0.1213, f_2=15, g=0 (j) NSGA-II-SP, f_1 = 0.0244, f_2=20, g=0



(k) SPEA2-CDP, f_1 = 9.2396, f_2=3, g=0 (l) SPEA2-CDP, f_1 = 4.6239, f_2=5, g=0 (m) SPEA2-CDP, f_1 = 0.6510, f_2=10, g=0 (n) SPEA2-CDP, f_1 = 0.1225, f_2=15, g=0 (o) SPEA2-CDP, f_1 = 0.0244, f_2=20, g=0



(p) SPEA2-SP, f_1 = 9.2205, f_2=3, g=0 (q) SPEA2-SP, f_1 = 4.6158, f_2=5, g=0 (r) SPEA2-SP, f_1 = 0.6659, f_2=10, g=0 (s) SPEA2-SP, f_1 = 0.0952, f_2=15, g=0 (t) SPEA2-SP, f_1 = 0.0310, f_2=20, g=0



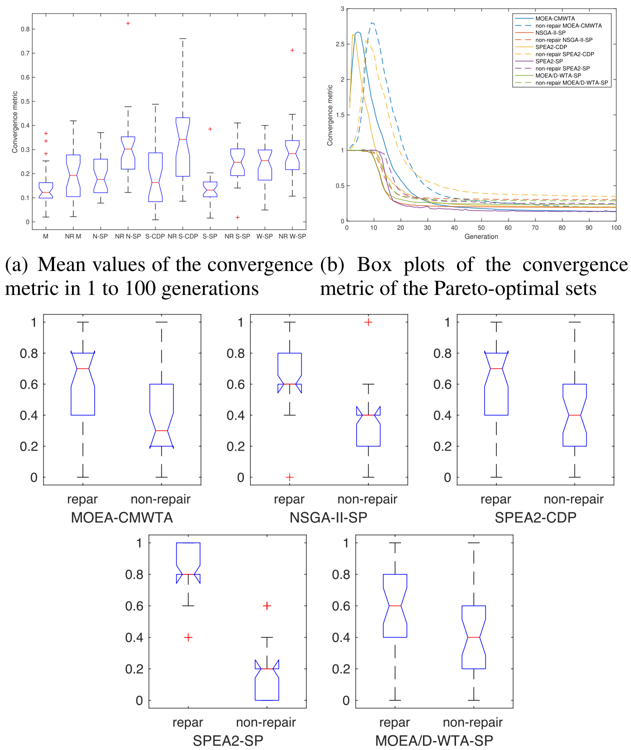
(u) MOEA/D-WTA-SP, f_1 = 9.2134, f_2=3, g=0 (v) MOEA/D-WTA-SP, f_1 = 4.7887, f_2=5, g=0 (w) MOEA/D-WTA-SP, f_1 = 0.7658, f_2=10, g=0 (x) MOEA/D-WTA-SP, f_1 = 0.1241, f_2=15, g=0 (y) MOEA/D-WTA-SP, f_1 = 0.0234, f_2=20, g=0

Threat Friendly/Neutral Aiming Point Preference

FIGURE 8. The display of the optimal non-dominated solutions obtained by MOEA-CMWTA, NSGA-II-SP, SPEA2-CDP, SPEA2-SP, and MOEA/D-WTA-SP solving scenario 2 over 30 independent runs.

TABLE 6. Statistics of the Pareto-optimal sets obtained by the repair and non-repair versions of MOEA-CMWTA, NSGA-II-SP, SPEA2-CDP, and SPEA2-SP in 30 independent runs.

Algorithm	Size of PS			Infeasibility		Convergence metric	Mean values of the fitness under each weapon consumption				
	5	4	≤ 3	%	Mean		6	5	4	3	2
MOEA-CMWTA	30	0	0	0	0	0.13634975	1.17603219	1.44166066	1.96665203	3.09438195	6.93925962
non-repair MOEA	30	0	0	0	0	0.19627163	1.19336561	1.50885716	2.06619803	3.40140200	6.95137814
NSGA-II-SP	29	1	0	0	0	0.19336857	1.35470202	1.56719467	2.01689722	3.00607847	6.94541942
non-repair NSGA-II-SP	28	2	0	1.35	0.0237	0.302151172	1.46038735	1.66435621	2.08163854	3.17693941	6.94899371
SPEA2-CDP	23	7	0	0	0	0.19973988	**1.16245224**	**1.42532121**	**1.93276840**	3.38133551	7.02025116
non-repair SPEA2-CDP	15	15	0	0	0	0.35030033	1.20908668	1.54244373	2.09709001	3.97316478	6.98121104
SPEA2-SP	29	1	0	0	0	**0.13297757**	1.16329986	1.44534160	1.97288129	**2.95712583**	6.94055130
non-repair SPEA2-SP	29	1	0	0	0	0.24659084	1.31757592	1.63701617	2.05484690	3.16806788	6.95294700
MOEA/D-WTA-SP	29	1	0	0	0	0.23335987	1.39881516	1.60912535	2.00380805	2.96642146	6.94636257
non-repair MOEA/D-WTA-SP	30	0	0	2	0.0568	0.28682654	1.4607794	1.65367074	2.05790502	3.05110879	**6.87574372**



(a) Mean values of the convergence metric in 1 to 100 generations

(b) Box plots of the convergence metric of the Pareto-optimal sets

(c) Box plots of the coverage of two sets obtained by the repair and non-repair version of each algorithm

FIGURE 9. Convergence metric and coverage obtained by the repair and non-repair version of MOEA-CMWTA, NSGA-II-SP, SPEA2-CDP and SPEA2-SP in 30 independent runs.

and MOEA/D-WTA-SP has a smaller overshoot. As shown in Figure 6(b), the convergence metric of MOEA-CMWTA and MOEA/D-WTA-SP also have better distribution than the others. Figure 6(c) illustrates that MOEA-CMWTA outperforms than four other algorithms on coverage.

In Figure 7, the upper figure shows the mean values of the Pareto-optimal fronts obtained by MOEA-CMWTA, NSGA-II-SP, SPEA2-CD, SPEA2-SP, and MOEA/D-WTA-SP. The lower figure gives the number of the non-dominated solutions obtained by five algorithms and sorted by weapon consumption.

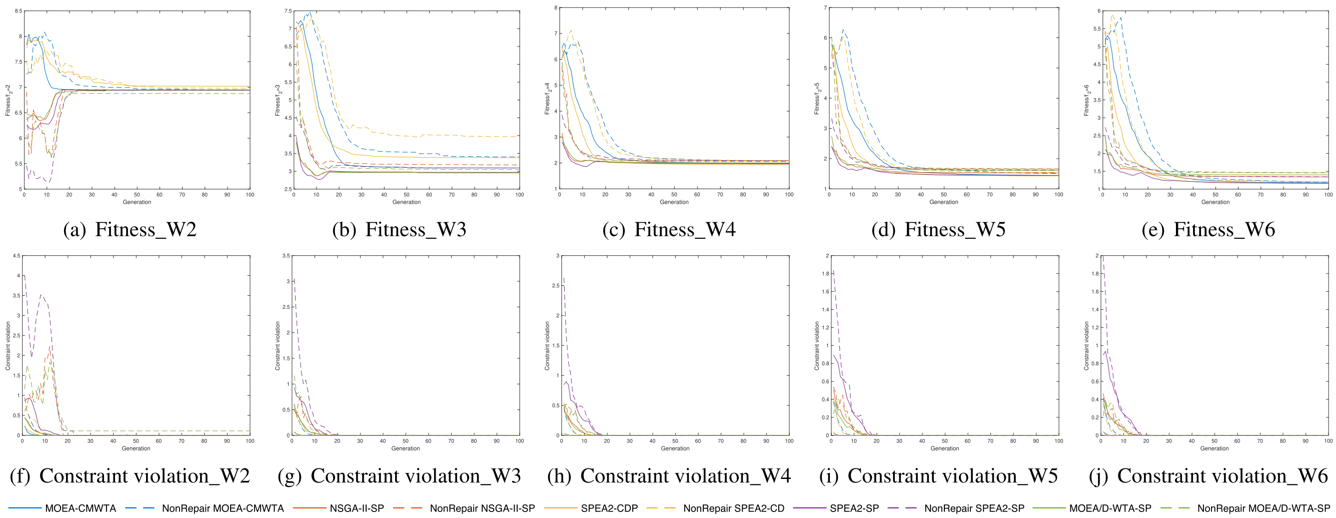
The mean values of the Pareto-optimal fronts are close in the upper figure. It is noteworthy that the more weapon consumption, the less benefit obtained. Integrating the lower figure, MOEA-CMWTA outperforms when weapon consumption is less than 25, and MOEA/D-WTA-SP is outstanding when weapon consumption is more than 24. However, as shown in the upper figure, fitness has little benefit when weapon consumption is more than 15. Besides, MOEA/D-WTA-SP gets no infeasible non-dominated solutions in scenario 1 possessing six weapons but obtains 8.03% infeasible solutions in scenario 2 possessing 30 weapons. Hence we deduce that the infeasible non-dominated solutions of MOEA/D-WTA-SP are concentrated in the solutions of middle or high weapon consumption. All the above, we conclude: (1) The rational number of max weapon resource is about 16 for scenario 2. (2) CMWTA-MOEA has more superiority in solving CMWTA.

Figure 8 shows the sketch maps of the optimal plans, which are obtained by five algorithms over 30 runs. As illustrated in Figure 8, the valuable targets can be covered by the optimal assignment when weapon consumption achieves 10.

D. EXPERIMENTS ON REPAIR OPERATOR

To verify the performance of the repair operator, we compare the repair and non-repair two versions of four MOEAs, namely MOEA-CMWTA, NSGA-II-SP, SPEA2-CDP, SPEA2-SP and MOEA/D-WTA-SP which are verified in the previous experiment. We perform 30 independent runs on each algorithm, and the statistics of obtained Pareto-optimal sets are shown in Table 6.

As shown in Table 6, only two versions of MOEA-CMWTA and non-repair MOEA/D-WTA-SP can guarantee the completeness and feasibility of the Pareto-optimal sets. However, the percentage of the infeasible solutions is 2% and the mean values is 0.0568. The repair operator has a slight improvement on the completeness and feasibility of NSGA-II-SP. The repair operator has a great improvement on the completeness of SPEA2-CDP but has no influence on the completeness of SPEA2-SP. Comparing the convergence metric, the repair version of each algorithm performs better than the non-repair version. In terms of the fitness,



(a) Fitness_W2 (b) Fitness_W3 (c) Fitness_W4 (d) Fitness_W5 (e) Fitness_W6

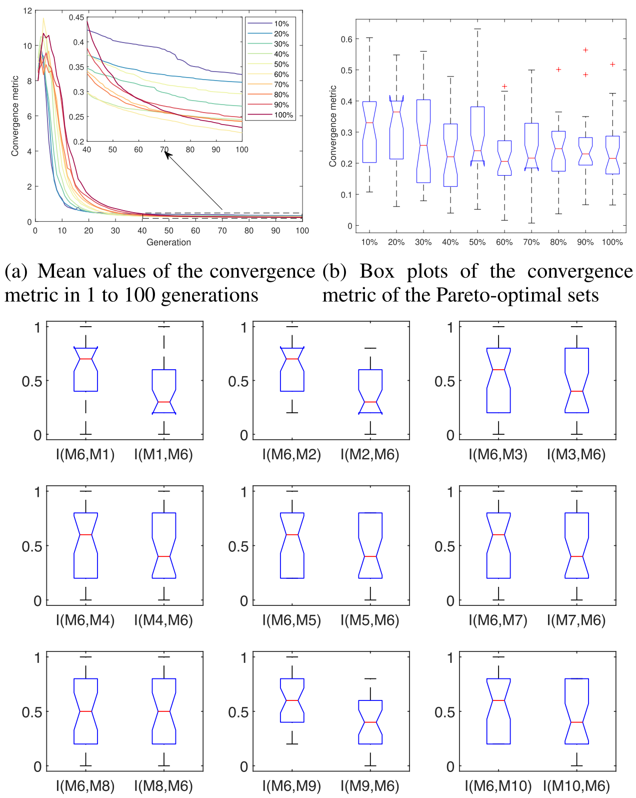
(f) Constraint violation_W2 (g) Constraint violation_W3 (h) Constraint violation_W4 (i) Constraint violation_W5 (j) Constraint violation_W6

FIGURE 10. Mean values of the fitness and constraint violation obtained by the repair and non-repair version of MOEA-CMWTA, NSGA-II-SP, SPEA2-CDP and SPEA2-SP over 30 independent runs in 1 to 100 generations.

MOEA-CMWTA almost obtains the suboptimal fitness of each subpopulation. SPEA2-CDP obtains the optimal mean values of SP_4 to SP_6 but it has an indifferent convergence metric owing to the incompleteness. SPEA2-SP performs better in solving the non-dominated solutions of SP_3 and SP_6. MOEA/D-WTA-SP has the optimal fitness of SP_2 but is related to the infeasible solutions. Besides, the most absent solutions are in SP_2.

The convergence metric and coverage of the obtained sets are shown in Figure 9. Figure 9(a) shows the distribution of the convergence metric obtained in 30 independent runs. Figure 9(b) shows the mean values of the convergence metric over 30 independent runs in 1 to 100 generations. Figure 9(c) exhibits the box plots of the coverage metric obtained by two versions of each algorithm. The left plot shows the distribution of the repair version dominating the non-repair version. The right plot shows the distribution of the non-repair version dominating the repair version. For each algorithm in Figure 9, the repair version has a significant improvement compared to the non-repair version.

In Figure 10, the figures of the first row plot the mean values of fitness of optimal plans during the iteration process, which are identified by weapon consumption. Similarly, the figures of second-row give the mean values of constraint violation. As shown in Figure 10, the repair version performs better than the non-repair version on not only fitness but also convergence speed. Owing to the narrower feasible region of SP_2, this difference is distinct in Figure 10(a). In view of the feasible decision of C2, MOEA-CMWTA and SPEA2-CDP have an advantage in real-time.



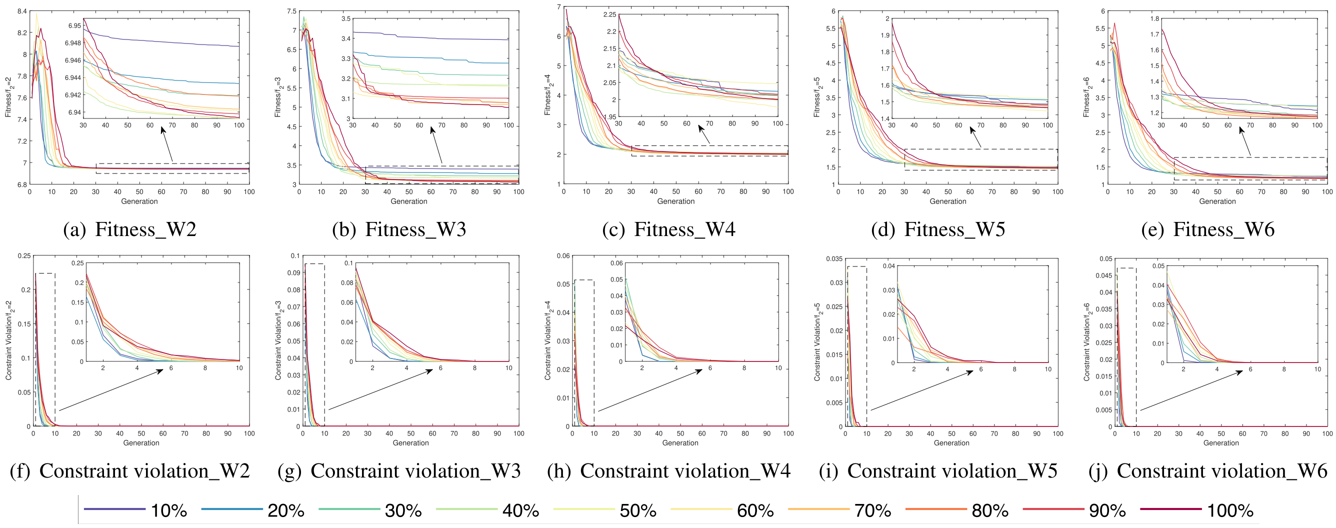
(a) Mean values of the convergence metric in 1 to 100 generations (b) Box plots of the convergence metric of the Pareto-optimal sets

(c) Box plots of the coverage of the two sets obtained by the case of 60% and the other cases

FIGURE 11. Convergence metric and coverage obtained by MOEA-CMWTA of different *ep* in 30 independent runs.

E. SENSITIVITY ANALYSIS

The parameters of MOEA-CMWTA can be divided into two categories. One is the parameters of the evolutionary framework, such as p_c, p_m, η_c and η_m. Another is the unique parameters embedded in MOEA-CMWTA, such as the size of the effective population ep and the number of neighboring subpopulation T. In this paper, the former parameters adopt the proposed settings in extensive research. The sensitivity of the unique parameters is analyzed by the following experiments.



(a) Fitness_W2 (b) Fitness_W3 (c) Fitness_W4 (d) Fitness_W5 (e) Fitness_W6

(f) Constraint violation_W2 (g) Constraint violation_W3 (h) Constraint violation_W4 (i) Constraint violation_W5 (j) Constraint violation_W6

FIGURE 12. Mean values of the fitness and constraint violation obtained by MOEA-CMWTA with different *ep* over 30 independent runs in 1 to 100 generations.

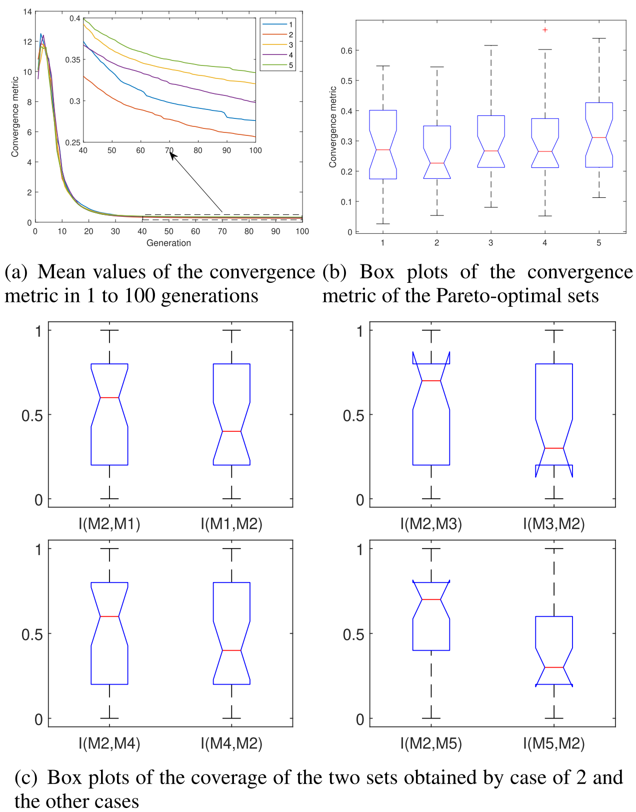
1) SIZE OF THE EFFECTIVE POPULATION

In the above experiments, the size of the effective population is set to $pop/2$ empirically. In order to investigate the influence of *ep*, we set $ep = K \cdot pop$ where $K = [0.1 : 0.1 : 1]$ is to control the ratio of *ep* to *pop*. For each *ep*, we perform 30 independent runs on MOEA-CMWTA solving scenario 1, and the results are shown in Table 7, Figure 11 and 12.

As shown in Table 7, the different *ep* do not destroy the completeness and feasibility of the Pareto-optimal sets. According to the convergence metric, the cases of more than 50% perform better than the others. In the column of fitness, the optimal mean values are concentrated in the cases of more than 50%. The case of 60% outperforms on the mean values of the convergence metric and the fitness of SP_4 and SP_5.

As shown in Figure 11(a), the cases of less than 60% show the faster convergence speed and the worse convergence metric. According to the drawing of partial enlargement, the case of 60% has a better convergence metric and the medium convergence speed. The cases of 70%, 80%, and 100% show a similar performance. In Figure 11(b), the cases of no more than 50% have a wide distribution. The convergence metric can be stabilized less than 0.2 when *ep* is more than $0.5pop$. It is worth noting that the case of $K = 0.6$ is also outstanding in not only value but also distribution. To verify the superiority of the case of 60%, Figure 11(c) gives the box plots of the coverage of the two sets obtained by the case of 60% and the others in 30 independent runs. The case of 60% outperforms the others except the case of 80%. Considering the convergence metric and fitness, the case of 60% performs better than the case of 80%.

In Figure 12, the algorithms of different *ep* have close fitness in subpopulations except for SP_3, and the constraint violation can converge to 0 within 10 generations. According to the drawing of partial enlargement, the algorithms of smaller *ep* have the faster convergence speed but no advantage of the fitness.



(a) Mean values of the convergence metric in 1 to 100 generations (b) Box plots of the convergence metric of the Pareto-optimal sets

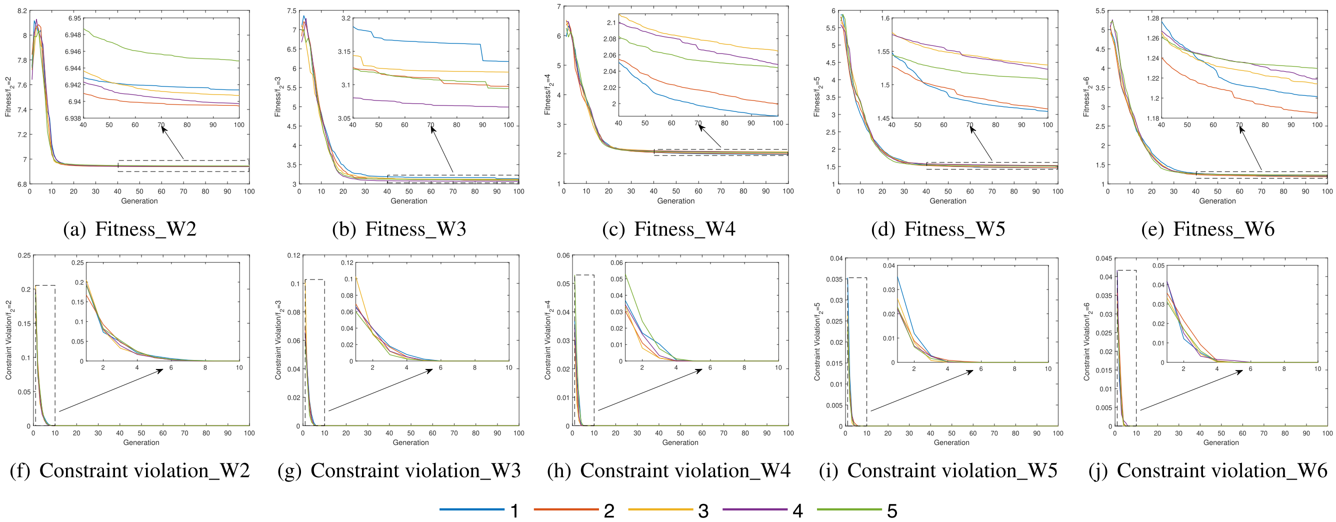
(c) Box plots of the coverage of the two sets obtained by case of 2 and the other cases

FIGURE 13. Convergence metric and coverage obtained by MOEA-CMWTA of different *T* in 30 independent runs.

Based on the above results, we conclude that MOEA-CMWTA is easy to trap in local optimum when *ep* is less than $0.5pop$, and the performance of MOEA-CMWTA has a stable improvement when *ep* is no less than $0.6pop$.

TABLE 7. Statistics of the Pareto-optimal sets obtained by MOEA-CMWTA of different *ep*.

ep/*pop*	Size of PS			Infeasibility		Convergence metric	Mean values of the fitness under each weapon consumption				
	5	4	≤3	%	Mean		6	5	4	3	2
10%	30	0	0	0	0	0.33446657	1.21447880	1.48586760	2.01186965	3.39263121	6.94757704
20%	30	0	0	0	0	0.31877602	1.24142924	1.51150917	2.02408727	3.27627889	6.94328087
30%	30	0	0	0	0	0.27066128	1.18399445	1.47469405	2.01375536	3.21545763	6.94185768
40%	30	0	0	0	0	0.24387855	1.18373603	1.46407986	1.99858421	3.16632432	**6.93925577**
50%	30	0	0	0	0	0.29567391	1.23445383	1.52052677	2.04619240	3.16061797	6.94017686
60%	30	0	0	0	0	**0.21745812**	1.17074824	**1.44483357**	**1.97810385**	3.07934652	6.93926312
70%	30	0	0	0	0	0.23982810	1.17083386	1.47488734	2.01065088	3.06919557	6.94033815
80%	30	0	0	0	0	0.24038300	**1.16247176**	1.46590194	2.00143067	3.07787333	6.94184384
90%	30	0	0	0	0	0.24825370	1.18322718	1.48054343	2.01555349	3.10142079	6.93978306
100%	30	0	0	0	0	0.22798895	1.17427134	1.46403321	1.99938791	**3.05513795**	6.93936234



(a) Fitness_W2 (b) Fitness_W3 (c) Fitness_W4 (d) Fitness_W5 (e) Fitness_W6

(f) Constraint violation_W2 (g) Constraint violation_W3 (h) Constraint violation_W4 (i) Constraint violation_W5 (j) Constraint violation_W6

FIGURE 14. Mean values of the fitness and constraint violation obtained by MOEA-CMWTA of different *T* over 30 independent runs in 1 to 100 runs.

TABLE 8. Statistics of the Pareto-optimal sets obtained by MOEA-CMWTA of different *T*.

T	Size of PS			Constraint violation		Convergence metric	Fitness of the non-dominated solutions in subpopulations				
	5	4	≤3	Mean	Percent		6	5	4	3	2
1	30	0	0	0	0	0.27603206	1.20139948	**1.45999678**	**1.98448357**	3.13479126	6.94137854
2	30	0	0	0	0	**0.25638917**	**1.18510670**	1.46381112	1.99916427	3.09733389	**6.93944496**
3	30	0	0	0	0	0.32050908	1.21422058	1.52954825	2.06463783	3.11885818	6.94071071
4	30	0	0	0	0	0.29794719	1.21889626	1.52310418	2.04819926	**3.06653831**	6.93971087
5	30	0	0	0	0	0.33394151	1.22963115	1.50817569	2.04403889	3.09418464	6.94484723

However, a larger *ep* has an impact on convergence speed. Hence we suggest *ep* be set to 0.6*pop*.

2) THE NUMBER OF NEIGHBORING SUBPOPULATIONS

In MOEA-CMWTA of the previous experiments, the number of neighboring subpopulations is set to 2 tentatively. We set $T = 1, 2, \ldots, 5$ to investigate the sensitivity of T on MOEA-CMWTA. The neighboring subpopulations are calculated by Eq. 16. For each T, we perform 30 independent runs on MOEA-CMWTA solving scenario 1, and the results are shown in Table 8, Figure 13 and 14.

As shown in Table 8, the different T have no influence on completeness and constraint violation of the Pareto-optimal sets. The cases of 1 and 2 perform better than the others on convergence metric. The optimal mean values of fitness are also concentrated in the cases of 1 and 2. The case of 1 performs better in SP_4 and SP_5, and the case of 2 has an advantage in SP_2 and SP_6.

As shown in Fig. 13(a), the case of 2 outperforms on the convergence metric and has a medium convergence speed. As shown in Fig. 13(b), the case of 2 outperforms and the case of 5 performs worst. The other cases are relatively close to the median. To verify the outstanding of the case of 2, Fig. 13(c) shows the box plots of the coverage of two sets obtained by the case of 2 and the others in 30 independent runs. The non-dominated solutions obtained by the case of 2 has a high ratio of dominating the non-dominated solutions obtained by the other cases. As shown in the box plots of the case 2 and case 4, the case of 2 has more advantage than the other box plots, which demonstrates the validity of the neighbor evolutionary strategy on solving CMWTA.

In Fig. 14(a)-14(j), the number of neighboring subpopulations has a slighter influence than *ep* on the dynamic performance. In Fig. 14(f)-14(j), the constraint violation of each subpopulation can converge to 0 in the first ten generations, and the convergence speeds are close.

Based on the above results, we conclude that the small size of neighboring subpopulations improves MOEA-CMWTA slightly, and the number of neighboring subpopulations is set to 2 reasonably. The possible reason is that the case of 1 and 2 take advantage of the local information. The case of 1 performs worse than the case of 2 on diversity because the case of 1 only operates with the pre-subpopulation. When the number of neighboring subpopulations increases, the evolutionary process is reducing to be operated in the entire population and can not utilize the similar information of neighboring non-dominated solutions.

V. CONCLUSIONS AND FUTURE WORK

The WTA problem is less studied based on a specific operational environment. For the low efficiency-cost ratio of the point-to-point WTA model facing swarming targets, this paper formulates a constrained multi-objective weapon target assignment problem. The CMWTA model can be viewed as an attack pattern based on overlapping target clusters if the threat assessment can cluster the targets by physical position. The objective of the CMWTA model is minimizing the expected survival probability of targets and weapon consumption by considering unit type, lethal radius, damage effect and so forth. The constraints are derived from the actual operational requirements of security evasion, survival threshold, and preference assignment. The non-dominated solutions of CMWTA are solved to be the candidates for the decision-maker.

Owing to the characteristics of the CMWTA model, the major MOEAs solving CMWTA have two challenges: (1) Handling the multi-constraint. (2) Obtaining the complete Pareto-optimal set. Therefore we present a novel multi-objective evolutionary algorithm. An elitist selection algorithm and a neighbor evolutionary strategy are proposed to guarantee the completeness and convergence of the Pareto-optimal set. A repair mechanism is proposed to improve the quality of infeasible solutions. A measurement of constraint violation is designed to evaluate the infeasible solutions.

In the experimental section, we compared our approaches with several state-of-the-art MOEAs and approved constraint handling methods. A variant of the convergence metric is introduced to evaluate the algorithm solving MWTA. From the experimental results, we conclude that MOEA-CMWTA outperforms on convergence and spacing by preserving the diversity of the population. The repair operator further strengthens the searchability of the algorithms. In the sensitivity analysis of parameters of MOEA-CMWTA, the size of the effective population, which is suggested 60% of *pop*, has a moderate improvement on the Pareto-optimal set. The number of neighboring subpopulations has a slight advantage and is suggested to 2.

We noticed that MOEA/D solves the classic WTA problem efficiently and has low computational complexity. MOEA/D also has the advantage of using information among neighboring subproblems. Hence we think that MOEA/D has the potentiality of solving CMWTA. How to design an appropriate decomposition strategy, which is based on constraints handling, is the future challenge for us.

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