HW1_ME547_W22

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HW1_ME5...

ME547- Winter 2022

Homework # 1

Note that all figures need to be nicely captioned, and the axes need to be labeled and be readable.

Problem 1: Forward Kinematics and Workspace

We have a 4-DoF Robot manipulator which moves objects on a work unit. This manipulator has two revolute and two prismatic joints (Figure 1). Its base can be moved but have to be fixed to the ground before the robot starts working.

$$L_0 = 1m \;, \qquad L_1 = L_2 = 0.5m$$

$$d_1 \in [0.1 \quad 0.3]m, \; d_2 \in [0.1 \quad 0.3]m$$

$$\theta_1 \in [0^\circ \; 145^\circ], \qquad \theta_2 \in [-120^\circ \; 120^\circ]$$

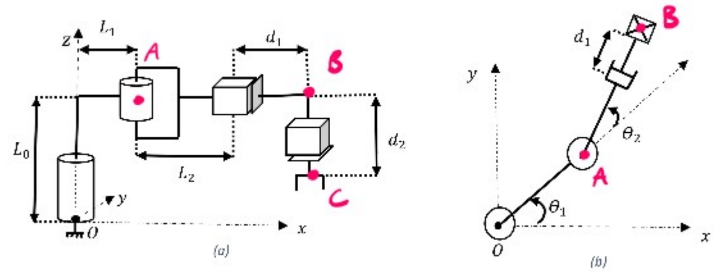


Figure 1. (a) 3D view (b) 2D top view of manipulator.

A) [30 pt] Derive the forward kinematic model. (3D position of end-effector as a function of θ_1, θ_2, d_1, and d_2). Note that the orientation of the end effector is always fixed. ?

B) [20 pt] Simulate the workspace (WS) of this robot in $x - y$ 2D space. Note that the 2D workspace is a projection of the actual workspace of this robot. Plot the WS in your solution report. (Append your code (MATLAB code) as a separate file, do not submit .zip or .rar).

This robot is supposed to move objects from location A=(1.7,1.3,0.8) to B=(-0.25,1,0.8) and from B to C=(0,0,0.8).

C) [10 pt] What should be the location and orientation of the robot base to enable the manipulator to perform the tasks?
Hint: Plot the three points (A, B, C) on the simulated 2D WS and find how much the robot base needs to be shifted from the origin and/or rotated with respect to the original orientation in the workstation (The rotation of the base can be modeled by a shift on θ_1)

Problem 2: Rotation Matrices

A) [20 pt] Determine the rotation matrix $\boldsymbol{R}$ that transforms the coordinate frame $\{O, x_A, y_A, z_A\}$ to the coordinate frame $\{O, x_B, y_B, z_B\}$ as shown in Figure 2.

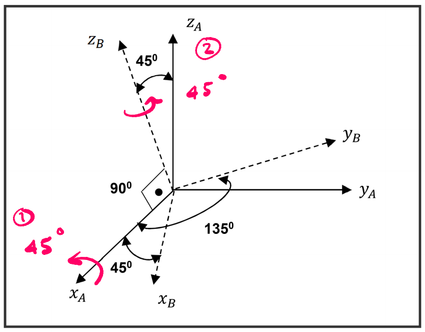


Figure 2. Coordinate frames {A} and {B}.

Hint 1: Find a cascade of two rotations.

Hint 2: x_A is orthogonal to both z_A and z_B.

B) [20 pt] Find the corresponding axis and angle of rotation.
Hint: rotation can be described as a single rotation of ϕ about an axis u.

1A) find P_c in terms of θ_1, θ_2, d_1 & d_2

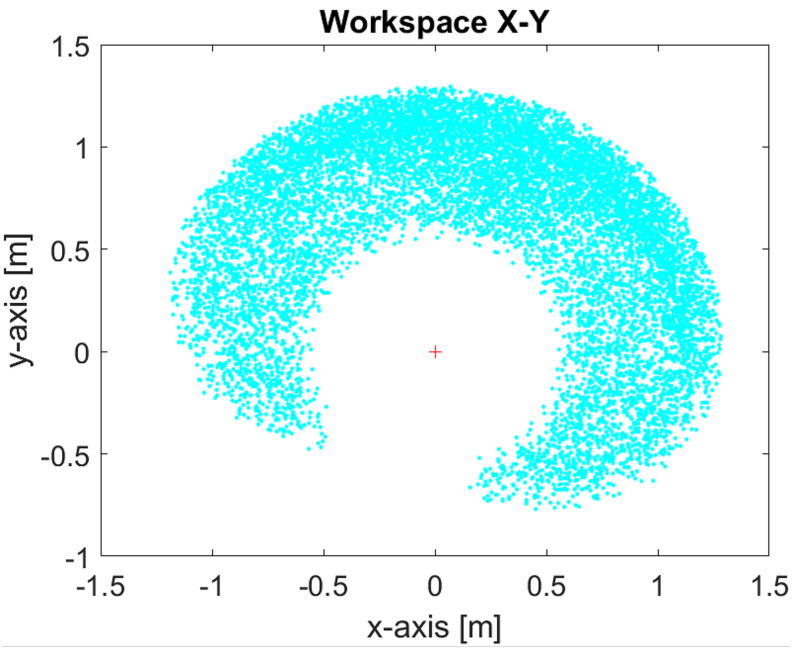
$$P_A = \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} = \begin{pmatrix} L_1 \cos\theta_1 \\ L_1 \sin\theta_1 \\ L_0 \end{pmatrix}$$

$$P_B = \begin{pmatrix} x_B \\ y_B \\ z_B \end{pmatrix} = \begin{pmatrix} x_A + (L_2 + d_1)\cos(\theta_1 + \theta_2) \\ y_A + (L_2 + d_1)\sin(\theta_1 + \theta_2) \\ z_A \end{pmatrix}$$

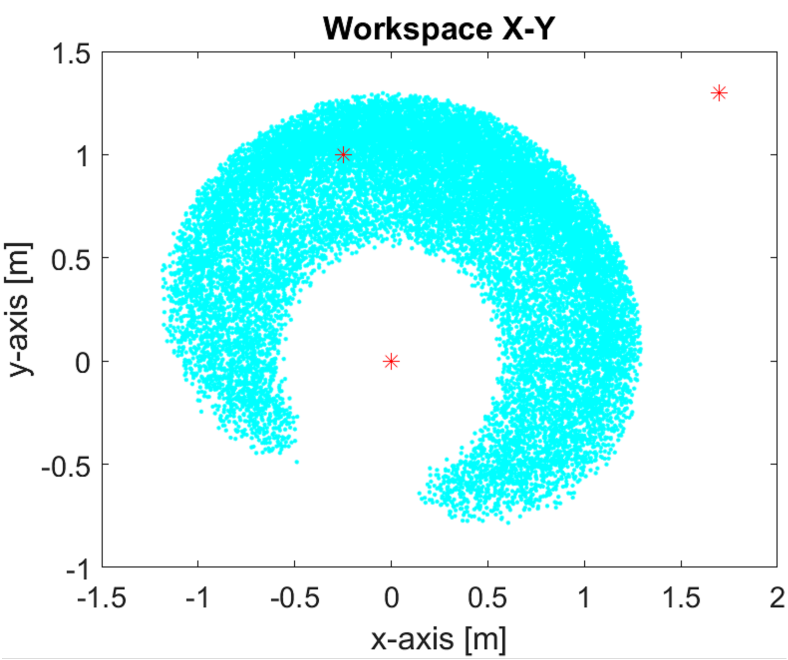
$$p_c = \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \begin{pmatrix} x_s \\ y_s \\ z_s - d_2 \end{pmatrix} = \begin{pmatrix} L_1 \cos\theta_1 + (L_2 + d_1)\cos(\theta_1 + \theta_2) \\ L_1 \sin\theta_1 + (L_2 + d_1)\sin(\theta_1 + \theta_2) \\ L_0 - d_2 \end{pmatrix}$$

$$\theta = \theta_1 + \theta_2$$

1B) attached code is modified from code posted on LEARN



1C)



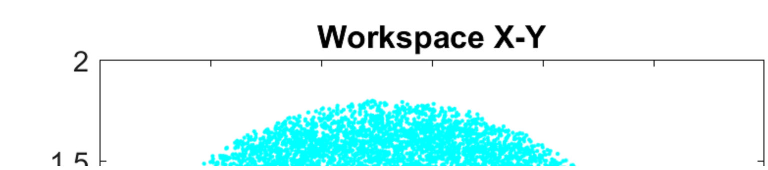
robot base shift & rotation:

$\psi = 30°$

$x_s = 0.8$ m

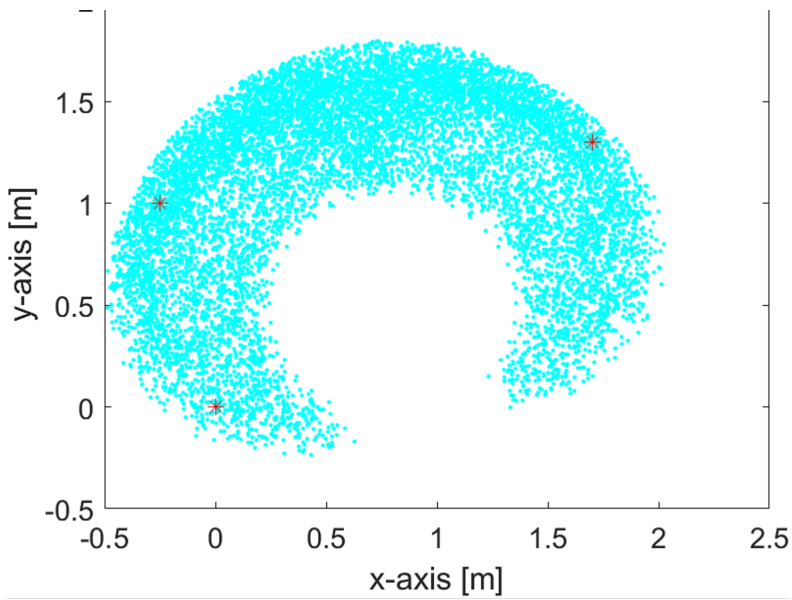
$y_s = 0.5$ m

$z_s = 0$ m



base shift & rotation modeled as:

$\theta_1 \in [\,0° + \psi \quad 145° + \psi\,]$



$\theta_1 \in [\ \ldots\ ]$

$$p_c = \begin{pmatrix} x_c + x_s \\ y_c + y_s \\ z_c + 0 \end{pmatrix}$$

2A) order of rotation (about rotated axis)

45° about x

45° about z'

$$(R^A_B)_m = R_x(\alpha)\, R_{z'}(\gamma) = [R]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix} \begin{bmatrix} C\gamma & -S\gamma & 0 \\ S\gamma & C\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left\{ \begin{array}{l} \gamma = \alpha = 45^\circ \\ C\gamma = S\gamma = \sqrt{2}/2 \\ C\alpha = S\alpha = \sqrt{2}/2 \end{array} \right.$$

$$[R] = \begin{bmatrix} C\gamma & -S\gamma & 0 \\ C\alpha S\gamma & C\alpha C\gamma & -S\alpha \\ S\alpha S\gamma & S\alpha C\gamma & C\alpha \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 1/2 & 1/2 & -\sqrt{2}/2 \\ 1/2 & 1/2 & \sqrt{2}/2 \end{bmatrix}$$

2B)

$$[R]\,\vec{u} = \lambda_R \vec{u} \qquad \lambda_R = 1\ ,\ \cos\phi + i\sin\phi\ ,\ \cos\phi - i\sin\phi$$

$\vec{u}$?

$$\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 1/2 & 1/2 & -\sqrt{2}/2 \\ 1/2 & 1/2 & \sqrt{2}/2 \end{bmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = 1 \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{2}/2\, u_x - \sqrt{2}/2\, u_y \\ 1/2\, u_x + 1/2\, u_y - \sqrt{2}/2\, u_z \\ 1/2\, u_x + 1/2\, u_y + \sqrt{2}/2\, u_z \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

$$\left(\frac{2}{2} - \frac{\sqrt{2}}{2}\right) u_x = -\frac{\sqrt{2}}{2} u_y$$

$$(1 - \sqrt{2}) u_x = u_y$$

$$\vec{u} = \begin{pmatrix} u_x \\ (1-\sqrt{2}) u_x \\ u_x \end{pmatrix}$$

$$\tfrac{1}{2} u_x + \tfrac{1}{2}(1 - \sqrt{2}) u_x - \frac{\sqrt{2}}{2} u_z = u_z$$

$$\left(1 - \frac{\sqrt{2}}{2}\right) u_x = \left(1 - \frac{\sqrt{2}}{2}\right) u_z$$

$$u_x = u_z$$

$$\| \vec{u} \| = \sqrt{u_x^2 + (1-\sqrt{2})^2 u_x^2 + u_x^2} = 1$$

$$= 2 u_x^2 \; (1 - 2\sqrt{2} + 2) u_x^2 = 1$$

$$u_x = \sqrt{\frac{1}{5 - 2\sqrt{2}}} = 0.6786$$

$$u_y = (1 - \sqrt{2})\, 0.6786 = -0.2811$$

$$\vec{u} = \begin{pmatrix} 0.6786 \\ -0.2811 \\ 0.6786 \end{pmatrix}$$

ϕ?

$$\text{trace}(R) = \sum_{i=1}^{3} \lambda_i$$

$$\frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2} = (1) + (\cos\phi + i\sin\phi) + (\cos\phi - i\sin\phi)$$

$$\frac{2\sqrt{2} + 1}{2} = 1 + 2\cos\phi$$

$$\phi = \cos^{-1}\left(\frac{2\sqrt{2} - 1}{4}\right) = 62.8^\circ$$