Convex Optimization

Exercise 3 (100/500)

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Instructor: Athanasios P. Liavas

130/1000 points

1. (10) Let $\mathbf{a} \in \mathbb{R}^n$. Solve the KKT and compute the projection of $\mathbf{x}_0 \in \mathbb{R}^n$ onto set $\mathbb{S} := \{\mathbf{x} \in \mathbb{R}^n \,|\, \mathbf{a} \leq \mathbf{x}\}$.

2. (20) Let $\mathbb{S} := \{\mathbf{x} \in \mathbb{R}^2 \,|\, \|\mathbf{x}\|_2 \leq 1, x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1\}$. Solve the problem

$$\min_{\mathbf{x} \in \mathbb{S}} f_0(\mathbf{x}), \tag{1}$$

   with

   (a) $f_0(\mathbf{x}) = (x_1 - 2)^2 + (x_2 - 2)^2$,

   (b) $f_0(\mathbf{x}) = (x_1 + 2)^2 + (x_2 + 2)^2$.

3. (100) Consider the problem

$$\begin{aligned} &\min_{\mathbf{x} \in \mathbb{R}^n} \; f(\mathbf{x}) = -\textstyle\sum_{i=1}^n \log(x_i) \\ &\text{subject to } \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{2}$$

   where $\mathbf{A} \in \mathbb{R}_+^{p \times n}$, $\text{rank}(\mathbf{A}) = p$, $\mathbf{b} \in \mathbb{R}_+^p$.

   (a) Data generation: In order to guarantee feasibility, we generate $\mathbf{A}$ with i.i.d. elements, with $A_{i,j} \sim \mathcal{U}[0, 1]$, and $\mathbf{x}$ with i.i.d. elements, with $x_i \sim \mathcal{U}[0, 1]$, and set $\mathbf{b} = \mathbf{A}\mathbf{x}$.

   (b) (10) Solution 1: solve problem (2) via `cvx`.

   (c) (40) Solution 2: solve problem (2) via Newton, starting from a feasible point, as follows:

      i. compute a feasible point $\mathbf{x}_0$ via `cvx`;

      ii. implement the Newton algorithm, starting from $\mathbf{x}_0$.

   (d) (40) Solution 3: solve problem (2) via the primal-dual algorithm, starting from point $\mathbf{x}_0 = \mathbf{1}$.

   (e) (10) Solution 4: solve the problem dual to (2) via `cvx`, and then find the primal solution via the dual.