- Dom Arishi
- Hw3

5.2

a) $P(X \geq 10) = 1 - P(X < 10) = 1 - \sum_{x=0}^{9} b(x; 15, 0.4) =$

$1 - 0.9662 = 0.0338$

b) $P(3 \leq X \leq 8) = \sum_{x=3}^{8} b(x; 15, 0.4) = \sum_{x=0}^{8} b(x; 15, 0.4) -$

$\sum_{x=0}^{2} b(x; 15; 0.4) = 0.9050 - 0.0271 = 0.8779$

c) $P(X = 5) = b(5; 15, 0.4) = \sum_{x=0}^{5} b(x; 15, 0.4) - \sum_{x=0}^{4} b(x; 15, 0.4)$

$= 0.4032 - 0.2173 = 0.1859$

5.4 a) $b(3; 10, 0.3) = \sum_{x=0}^{3} b(x; 10, 0.3) - \sum_{x=0}^{2} b(x; 10, 0.3)$

$0.6496 - 0.3828 = 0.2668$

b) $P(X > 3) = 1 - 0.6496 = 0.3504$

5.2 $n = 12$, Prob of success $p = \frac{1}{2}$ so,

$q = 1 - p$
$= 1 - \frac{1}{2}$
$= \frac{1}{2}$

Let x be random variable which denote the number of people who heard the difference, the Prob is

$P(X = x) = \binom{n}{x} p^x q^{n-x}$, $x = 0, 1, 2, \ldots, n$

$= \binom{12}{x} \left(\frac{1}{2}\right)^x \left(1 - \frac{1}{2}\right)^{12-x}$,

so the Prob of three people claim to have heard the difference is $P(X = 3)$

$P(X=3) = \binom{12}{3}\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^{12-3}$

$= 220(0.125)(0.001953) = \boxed{0.0537}$

5.6 $P(X=x) = b(x;6,\frac{1}{2})$, where $x = 0,1,2,\dots,6$

$= \binom{6}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{6-x}$

$= \binom{6}{x}\left(\frac{1}{2}\right)^6$, where $x = 0,1,2,\dots,6$

(a) $P(2 \le X \le 5) = \{P(X=2)+P(X=3)+P(X=4)+P(X=5)\}$

$= \left\{\binom{6}{2}\left(\frac{1}{2}\right)^6 + \binom{6}{3}\left(\frac{1}{2}\right)^6 + \binom{6}{4}\left(\frac{1}{2}\right)^6 + \binom{6}{5}\left(\frac{1}{2}\right)^6\right\}$

$= \left(\frac{1}{2}\right)^6\left\{\binom{6}{2}+\binom{6}{3}+\binom{6}{5}\right\}$

$= (0.0156)(15+20+15+6)$

$\boxed{= 0.875}$

(b) $P(X<3) = P(X=0)+P(X=1)+P(X=2)$

$= \left\{\binom{6}{0}\left(\frac{1}{2}\right)^6 + \binom{6}{1}\left(\frac{1}{2}\right)^6 + \binom{6}{2}\left(\frac{1}{2}\right)^6\right\}$

$= \left(\frac{1}{2}\right)^6\left\{\binom{6}{0}+\binom{6}{1}+\binom{6}{2}\right\}$

$= \boxed{0.3432}$

5.7 (a) the prob of success in each trial is $P = 70$

and trial are independent.

$X \sim Bin(n,P)$ where $n=10$ and $P = 0.70$

the Prob mass function (P.m.F) is

$P(X = x) = b(x; 10, 0.70)$ where $x = 0, 1, 2, \ldots, 10$

$= \binom{10}{x} (0.70)^{10-x}$

$= \binom{10}{x} (0.70)^{x} (0.30)^{10-x}$, where $x = 0, 1, 2, \ldots, 10$

(a) $P(X < 5) = P(X \leq 4)$ [$\because X$ is discrete, $X < 5 \equiv X \leq 4$]

$= \sum_{x=0}^{4} b(x; 10, 0.70) = \boxed{0.0473}$

(b) $X \sim Bin(n, p)$, where $n = 20$ and $p = 0.70$

$\therefore$ the prob mass function (p.m.f) is

$P(X = x) = b(x; 20, 0.70)$ where $x = 0, 1, 2, \ldots 10$

$= \binom{20}{x} (0.70)^{x} (1 - 0.70)^{20-x}$

$= \binom{20}{x} (0.70)^{x} (0.30)^{20-x}$

So, $P(X < 10) = P(X \leq 9)$

$= \sum_{x=0}^{9} b(x; 20, 0.70) = \boxed{0.0171}$

5.12 $P\left(\begin{array}{c}\text{fewer than 4 from} \\ \text{the out of state}\end{array}\right) = P(X < 4)$

$= P(X \leq 3)$

$= P(X = 0) + \ldots + P(X = 3)$

$= \binom{9}{0} (0.25)^{0} (1 - 0.25)^{9-0} + \ldots + \binom{9}{3} (0.25)^{3} (1 - 0.25)^{9-3}$

$= 0.07508 + \ldots + 0.23360 = \boxed{0.8343}$

5.9 Using the hyper geometric distribution with

$n = 5, N = 40, k = 3$ and $x = 1$

$= h(1; 40, 5, 3) = \frac{\binom{3}{1}\binom{37}{4}}{\binom{40}{5}} = \boxed{0.3011}$

this plan is not desirable since it detects a bad lot (3 defective) only about 30% of the time

5.32 $K = 3, N = 10, n = 4$

$X \sim \text{Hypergeom}(N = 10, K = 3, n = 4)$

The prob density function is

$f(x) = \frac{\binom{3}{x}\binom{7}{4-x}}{\binom{10}{4}} \quad x \in \{\max(0, 4+3-10), \ldots, \min(4,3)\}$

$f(x) = \frac{\binom{3}{x}\binom{7}{4-x}}{\binom{10}{4}}, \quad x \in \{\max(0,-3), \ldots, \min(4,3)\}$

$f(x) = \frac{\binom{3}{x}\binom{7}{4-x}}{\binom{10}{4}}, \quad 0 \leq x \leq 3$

(a) $P(X = 0) = f(0)$

$= \frac{\binom{3}{0}\binom{7}{4-0}}{\binom{10}{4}} = \frac{(1)(35)}{210} \approx \boxed{0.1667}$

(b) $P(X \leq 2) = f(0) + f(1) + f(2)$

$$= \frac{\binom{3}{0}\binom{7}{4-0}}{\binom{10}{4}} + \frac{\binom{3}{1}\binom{7}{4-1}}{\binom{10}{4}} + \frac{\binom{3}{2}\binom{7}{4-2}}{\binom{10}{4}}$$

$$= \frac{(1)(35)}{210} + \frac{(3)(35)}{210} + \frac{(3)(21)}{210}$$

$$= \frac{1}{210}\{35 + 105 + 63\} \approx \boxed{0,9667}$$

5.46 $N = 10{,}000$ total number of voting resident considered

$X = 4000$ number of voting resident of alama are against new sale tax.

$$p = \frac{X}{N} = \frac{4000}{10{,}000} = 0{,}4$$

$$q = 1 - p = 1 - 0{,}4 = \underline{0{,}6}$$

So,

$$P(X \leq 7) = \sum_{x=0}^{7} P(X = x)$$

$$= P(X=0) + P(X=1) + \ldots + P(X=7)$$

$$= \binom{15}{0}(0{,}6)^{0}(0{,}4)^{15-0} + \binom{15}{1}(0{,}6)^{1}(0{,}4)^{15-1} + \ldots +$$

$$\binom{15}{7}(0{,}6)^{7}(0{,}4)^{15-7} = \boxed{0{,}2131}$$

5.15 Using geometric distribution with $x=5$ and $p=0.01$

$= g(5;0.01) = (0.01)(0.99)^4 = \boxed{0.0096}$

5.16 Using geometric distribution $x=5$ $p=0.05$ yield

$P(X=x) = g(5;0.05) = (0.05)(0.95)^4 = \boxed{0.041}$

5.17 Using Poisson distribution with $x=6$ and $\lambda t=4$

$$p(6;4) = \frac{e^{-4}4^6}{6!} = \sum_{x=0}^{6} p(x;4) - \sum_{x=0}^{5} p(x;4)$$

$= 0.8893 - 0.7851 = 0.1042$

5.18 X = number of tankers arriving each day

$$P(X>15) = 1 - P(X \leq 15) = 1 - \sum_{x=0}^{15} p(x;\lambda t) = 1 - 0.9513$$

$= 0.0487$

5.49 Let x be the number of persons to be interviewed to get 5 dogs owner.

$\therefore$ the prob of success trial is $p=0.3$
$\therefore$ the prob of failure in each trial is $q = 1-p = 0.7$
$k=5$ $p=0.3$
$\therefore P(X=10) = b^*(10;5;0.3) = \binom{10-1}{5-1}(0.3)^5(0.7)^{10-5}$

$= \boxed{0.0515}$

5.50 X = gettig the third head on the seventh flip. $n=7$, $x=3$, $p=0.50$

$P(X=3) = \binom{7-1}{3-1}(0.50)^3(1-0.50)^{7-3}$

$= \binom{6}{2} \times 0.50^3 \times 0.50^4 = \boxed{0.1172}$

(b) X = gettig the first head on the fourth time.
$n=4$, $x=1$ $p, 0.50$

$P(X=1) = \binom{4-1}{1-1}(0.50)^1(1-0.50)^{4-1}$

$= \binom{3}{0} \times 0.50^1 \times 0.50^3 = \boxed{0.0625}$

5.55 Prob of success in each trial $(P) = 0.7$
Prob of a failure in each trial is $q = 1-p = 0.3$

(a) $K=1$, $p=0.7$
$P(X=x) = b(x, K, p)$. $X = K, K+1, K+2 \ldots$

$= \binom{x-1}{K-1} p^K q^{x-K}$

So, $P(X=3)$

$\therefore P(X=3) = b(3, 1, 0.7) = \binom{2}{0}(0.7)^1(0.3)^2$

$= \boxed{0.0630}$

(b) $P(X<4) = P(1 \leq X \leq 3) = \sum_{x=1}^{3} b^*(1; 1, 0.7) + b(2, 1, 0.7)$

$B(3, 1, 0.7) = \binom{0}{0}(0.7)^1(0.3)^0 + \binom{1}{0}(0.7)^1(0.3)^1 + \binom{2}{0}(0.7)^1(0.3)^2$

$\boxed{= 0.9730}$

$\mu = 2$

5.57 ⓐ $P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= 1 - [e^{-2} + 2 \times e^{-2} + \frac{2^2 e^{-2}}{2} + \frac{2^3 \times e^{-2}}{3!}$

$= 1 - [0,1353 + 0,2707 + 0,2707 + 0,1804]$

$= \boxed{0,1429}$

ⓑ $P(X=0)$

$P(X=0) = \frac{2^0 e^{-2}}{0!} = \boxed{0,1353}$

5.73 ⓐ X = number of emergency Patients in an Hour

ave number of Patient in an hour $\lambda = 5$, $t = 1$

$\lambda t = 5$ so,

$P(X = x) = P(x; \lambda t)$ where $= 0, 1, 2, 3, \ldots = e^{-\lambda t} \frac{(\lambda t)^x}{x!}$

$P(X \geq 10) = 1 - P(X \leq 10) = 1 - \sum_{x=0}^{10} P(x; 5) = 1 - 0,986305$

$= \boxed{0,0137}$

ⓑ $P(X > 20) = 1 - P(X \leq 20) = 1 - \sum_{x=0}^{20} P(x; 15) = 1 - 0,917029$

$= 0,0830$

5
14 nb

Prob 1

(a) $Range(X) = \{0, 1, 2\}$

(b) $P(X \geq 1.5) = P(X = 2)$

$= \frac{1}{6} = \boxed{0.1667}$

(c) $P(0 < X < 2) = P(X = 1)$

$= \frac{1}{3} = \boxed{0.3333}$

(d) $P(X = 0 | X < 2) = \frac{P(X = 0 \cap X < 2)}{P(X < 2)}$

$= \frac{P(X=0)}{P(X=0) + P(X=1)}$

$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} = \frac{6}{10} = \frac{3}{5} = \boxed{0.6}$

Prob 5 $P(X=x) = \binom{n}{x}(p)^x(1-p)^{n-x}$

The Prob mass function for the binomial is obtained by substituting $n = 50$, $p = 0.50$ in $\binom{n}{x}p^x(1-p)^{n-x}$

So,

$P(X > 30) = P(X = 31 + \ldots\ldots P(X = 50)$

$= \binom{50}{31}\left(\frac{1}{2}\right)^{31}\left(1 - \frac{1}{2}\right)^{50-31} + \ldots\ldots$

$+ \binom{50}{50}\left(\frac{1}{2}\right)^{50}\left(1 - \frac{1}{2}\right)^{50-50}$

$= \boxed{0.0593}$

Prob 14 $E(X) = \sum_{x=-\infty}^{\infty} x P_X(x) = 1 \times 0.5 \times 2 \times 0.3 \times + 3 \cdot 1\ 0.2 + 0$

$= \underline{1.7}$

$E(X^2) = \sum_{x=-\infty}^{\infty} x^2 P_X(x) = 1^2 \times 0.5 + 2^2 \times 0.3 + 3^2 \times 0.2 + 0 =$

$0.5 + 1.2 + 1.8 + 0 = \underline{3.5}$

$V(X) = E(X^2) - E(X)^2 = 3.5 - 2.89 = \underline{0.61}$

$SD(X) = \sqrt{V(X)} = \sqrt{0.61} = \underline{0.78}$