- Hw 2

- Dom Arishi

① 4 Coffee Types:

3 Sizes

2 opt for cream or no cream

2 " for sugar or " sugar

2 " " milk or no milk

So; total of ways $= 4 \times 3 \times 2 \times 2 \times 2 = \boxed{96}$

② :: The order in this case matter because we want to see how many ways;

$${}_{12}P_8 = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = \boxed{19958400}$$

③ g: 20 Black Phones

30 white "

:: Employee takes 10 randomly.

ⓐ $P(4\ \text{Black}) = \dfrac{({}_{20}C_4)({}_{30}C_6)}{({}_{50}C_{10})} = \boxed{0.280}$

ⓑ $P(\text{less than } 3\ \text{Black}) =$

$({}_{20}C_2)({}_{30}C_8) + ({}_{20}C_1)({}_{30}C_9) + ({}_{20}C_0)({}_{30}C_{10})$

$\boxed{= 0.139}$

⑦ g: 50 students

choosing 15 out of 50

the Prob is the complement of Prof choosing not choosing me or Joe.

$P(\text{not choosing me or Joe}) = 1 - \dfrac{{}_{48}C_{15}}{{}_{50}C_{15}} = \boxed{0.857}$

(8) No of letters = 13

# of repeated letters with count:

a = 2 s = 4 t = 2

So, number of words = $\frac{13!}{(2! \times 4! \times 2!)}$ = $\boxed{64864800}$

---

(9) Using Binomial Law:

(a) P(observe 8 heads and tail) = $20C_8 \, p^8 (1-p)^{12}$

$= 125970 \, p^8 (1-p)^{12}$

(b) P(observe & heads & tails)

$20C_9 \, p^9 (1-p)^{11} + 20C_{10} \, p^{10} (1-p)^{10} +$

$20_{11} \, p^{11} (1-p)^9 = 167960 \, p^{9} (1-p)^{11} + 184756 \, p^{10} (1-p)^{10}$

$+ 167960 \, p^{11} (1-p)^9$

---

(10) For Sensor (0,0) to send message to the Sensor at (20,10) It will have send to any 20 horizontal sensors and any 10 vertical Sensors, plus this is a permutation of 30 objects (steps) out of 20 are of one kind and 10 are of other kind.

$= \frac{30!}{20! \times 10!} = 30045015$

(14) $$S = \frac{(15)!}{(5!)(5!)(5!)} \times \frac{1}{3!}$$

$S = 126126$

let us assume the # of ways in which hannah and sarah together. so if we think of it as grouping 13 tables remaining in group of 5, 5 and 3 and adding Hannah and Sarah to the group with 3 table

So:
$$= \frac{(13)!}{(5!)\times(5!)\times(3!)} \times \frac{1}{2!} = 36036$$

$\therefore$ the Prob $= \frac{36036}{126126} = \frac{2}{7} \approx \boxed{0.2857}$

(13) the Prob of getting a head in the first coin is $\frac{1}{2}$ and the second is $\frac{1}{3}$:

C_1 = 1st coin is choosen

C_2 = 2nd coin is choosen

$$P(C_1) = P(C_2) = \frac{1}{2}$$

$P(H|C_1) = \frac{1}{2}$

$\frac{P(H \cap C_1)}{(H C_1)} = \frac{1}{2}$ $\quad P(H \cap C_1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Similarly

$P(H|C_2)\ \frac{1}{3}$

$\frac{P(H \cap C_2)}{P(C_2)} = \frac{1}{3}$ $\quad P(H \cap C_2) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

the Prob of head

$P(H) = P(H|C_1) + P(H, C_2) = \frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12} = \boxed{0.4166}$

(b) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= \left\{\binom{5}{3}(0.33)^3(1-0.33)^2\right\} + \left\{\binom{5}{4}(0.33)^4(1-0.33)^1\right\}$

$+ \left\{\binom{5}{5}(0.33)^5(1-0.33)^0\right\}$

$= (10 \times 0.0359 \times 0.4489) + (5 \times 0.0118 \times 0.67) + 0.0039$

$= \boxed{0.2049}$

---

(17) P(2 red marbles) = P(1st bag) × P(2 red marbles | 1st bag) + P(2nd bag) × P(2 marbles | 2nd bag)

Baye's Rule

$$P(B|A) = \frac{\frac{6C_2 \cdot 10C_3}{16C_5}}{\frac{6C_2 \cdot 10C_3}{16C_5} + \frac{6C_2 \cdot 15C_3}{21C_5}}$$

$= 0.551$

---

(19) let assume $y = x - 1$

so $(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1)(y_5+1) = 100$

$y_1 + y_2 + y_3 + y_4 + y_5 = 95$

such that $y_i \{0, 1, 2, 3, 4, \ldots\}$

$95 + 5 - 1 = 99$

$5 - 1 = 4$

$99C4 = 3764376$

2.17 $n_1 n_2 n_3 n_4 = (1)(5)(4)(3) = 60$

$n_1 n_2 n_3 n_4 = (2)(4)(4)(3) = 96$

since these cases are mutually exclusive then the total number of even four-digit = $60 + 96 = \boxed{156}$

2.18 Since the award are distinguishable, it's permutation problem, the total number of sample points

$= {}_{25}P_3 = \frac{25!}{(25-3)!} = \frac{25!}{22!} = 25 \times 24 \times 23 = 13,800$

2.19

a) the total of choices of offices, with out any restriction

$${}_{50}P_2 = \frac{50!}{48!} = (50)(49) = 2450$$

b) two cases;

i: A selected as the president, which yields 49 possible outcomes.

ii): officers are selected from the remaining 49 people without A which has the number of choices 2352, so the total number of choices $49 + 2352 = 2401$

c) number of selection btw B and C serve together is 2, the number of selection when both B and C are not choosen is ${}_{48}P_2 = 2256$, therefore, the total number of choises in this case is $2 + 2256 = 2258$

selection

(D) Number of when D serve as officer but not E

is $(2)(48) = 96$, where 2 is the position D can take and 48 is the # of selection of other officers from remaining people except E.

the # of selection E serve as officer but not D is also, $(2)(48) = 96$

both E and D not choosen is ${}_{42}P_2 = 2256$

total # of Choices $= (2)(96) + 2256 = \boxed{2448}$

2.20)

total # of arrangement $= \dfrac{10!}{1!\,2!\,4!\,3!} = 12{,}600$

the number of Partition $= \binom{5}{4,1} = \dfrac{5!}{4!\,1!} = 5.$

2.21) total number of possible Partition

$\binom{7}{3,2,2} = \dfrac{7!}{3!\,2!\,2!} = 210$

2.33) a) On each 5 Qs can be ans in 4 ways

$r = 5$ and $n_i = 4$

$4 \times 4 \times 4 \times 4 \times 4 = 4^5 = \boxed{1024}$

b) we 3 in correct ans so $r = 5$ $n_i = 3$

$= 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$

2.37 G - B - G - B - G - B - G - B - G -

5 girls and 5 slot for them so, 5! ways

same with boys they have 4 slots so, 4! ways

$5! \times 4! = 120 \times 24 = 2880$ ways

① a) Choosing 3 Republican from 50 Republican

is $_{50}C_3 = \frac{50 \times 49 \times 48}{3 \times 2 \times 1} = \frac{50 \times 49 \times 8}{1} = 19,600$ ways

choosing 4 Democrates from 40 Democrats

is $_{40}C_4 = \frac{40 \times 39 \times 38 \times 37}{4 \times 3 \times 2 \times 1} = 10 \times 13 \times 19 \times 37 = 91,390$ ways

so, # of ways to selecting exactly 3 R and 4 D is $_{50}C_3$

$\times {}_{40}C_4 = (19,600)(91,390) = 1,791,244,000$ ways

b)

D (50)	R (40)	No, of ways
7	0	$_{50}C_7 \cdot {}_{40}C_0$
6	1	$_{50}C_6 \cdot {}_{40}C_1$
5	2	$_{50}C_5 \cdot {}_{40}C_2$
4	3	$_{50}C_4 \cdot {}_{40}C_3$

total No of way selecting majority of Democrats

$= ({}_{50}C_7 \times {}_{40}C_0) + ({}_{50}C_6 \times {}_{40}C_1) + ({}_{50}C_5 \times {}_{40}C_2) + ({}_{50}C_4 \times {}_{40}C_3)$

$= 4,663,509,200$ ways

② total No of tringles (including equilaterals)

$15C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} = 455$

For equilateral tringle out of 15 = 15 - 3 = 12 dots won't be connected and 12 / 3 = 4 dot won't be btw two connecting dots

So following connecting dots will for equilateral tringles (possible cases)

1 --- 6 --- 11
2 --- 7 ... 12
3 --- 8 --- 13
4 --- 9 --- 14
5 --- 10 --- 15

So, there're 5 equilateral tringles therefore, 455 - 5 = $\boxed{450}$

③ So, each Prof must sit btw 2 students, they can't be seated at either the seats at the end of the row, each prof has 7 seats they can choose from & must be at least 1 seat apart, this 9 is equivalent to choosing 3 seats from a row of 5 seats with no restriction, therefore,

$= \binom{5}{3} \times 3! = \boxed{60}$