(2) Bayes Theorem: $P(A|B) = \frac{P(A \wedge B)}{P(B)}$

$P(R) = 0{,}001$

$P(\text{not } R) = 1 - 0{,}001 = 0{,}999$

$P(R|T) = \frac{P(R \wedge T)}{P(T)}$

$= \frac{P(R \wedge T)}{P(R \wedge T) + P(\text{not } R \wedge T} = \frac{0{,}001 \times 0{,}98}{(0{,}001 \times 0{,}98) + (0{,}999 \times 0{,}05)}$

$\boxed{= 0{,}0192}$

(1) max number of combination are $= 5 \times 5 \times 5 \times 5 = 625$

but no two can visit Pair on the same day so the number is
$= 5 \times 4 \times 3 \times 2 = 120$

the Prob is $= \frac{120}{625} = 0{,}192$

(3) Let F = forecast is rain $\Rightarrow P(F) = 0{,}2$

Fn = " " no rain $\Rightarrow P(Fn) = 0{,}8$

R = rain occure.

Rn = no rain occure.

So give forecast is rain, $P(R|F) = 0.8$

given $\bar{F}$ no rain of actual rain $\Rightarrow P(R|F_n) = 0.1$

Let V = victor comes home early and forecast is Rain and actual rain occure $\Rightarrow P(V|(R \wedge F)) = 0.9$

also: $P(F \wedge R) = P(R|F)\,P(F)$
$= 0.8\,(0.2)$
$= 0.16$

(a) P(forecast is rain, it actually rain and victor comes home early) $= P((F \wedge R) \wedge V)$
$= P(V|(F \wedge R))\,P(F \wedge R)$
$= (0.9)(0.16)$
$= 0.144$

$P(A \wedge B) = P(B|A)\,P(A)$

(b) P(that it actually rain occures)
$= P(R)$
$= P(F)\,P(R|F) + P(F_n)\,P(R|F_n)$
$= (0.2)(0.8) + (0.8)(0.1)$
$= 0.24$

c) P(that forecast was Rain | rained on that)
$= P(F|R)$
$= \dfrac{P(F)\,P(R|F)}{P(F)\,P(R|F) + P(F_n)\,P(R|F_n)}$
$= \dfrac{(0.2)(0.8)}{(0.2)(0.8) + (0.8)(0.1)} = \dfrac{0.16}{0.24} = 0.6667$