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Exercise 4

①

i) = A

ii) = C. Because we have 6 different ways for the rest to stand at different places while the tallest one have 1 exactly one.

iii) = A, Exculsivity doesnt imply independence.

iv) = B

v) = C, $3 + 4 = 7$

vi) = B, $3^2 \cdot Var(X) = 9 * 7 = 63$

Q2 $P(B)$ = badmiton = 0,32

$P(T)$ = tennis = 0,60

$P(B \text{ and } T) = 0{,}10$

$P(B \cup T) = P(B) + P(T) - P(B \text{ and } T) = 0{,}32 + 0{,}60 - 0{,}10$

$= 0{,}82$

$P(B^c \cup T^c) = 1 - 0{,}82 = \boxed{0{,}18}$

Q3

$= (25+3-1)C(3-1) = (27)C(2) = \boxed{351}$

Q4 $P(K)$ = Knows ans = 0,75

$P(G)$ = Guess ans = 0,25

$P(C/G)$ = Correct when Guess = 0,2

$P(NC/G)$ = not correct when Guess = 0,8

∴ The Prob of Not correct = $P(NC) = P(G) \times P(NC/G)$

∴ the Prob that student guess given ans correctly =

$P(G/NC) = (P(NC/G) \times P(G)) / P(NC) = \boxed{1}$

Q5

a) total number of permutation of the letters

$= \frac{13!}{2!2!2!4!} = \boxed{32,432,400}$

b) considering 2 C's as 1, the total permutation

$= \frac{12!}{2!2!4!} \times 2! = \boxed{9,979,200}$

c) Required No. of permutation =

(total permutation) − (permutation with C's together)

$= 32,432,400 - 9,979,200 = \boxed{22,453,200}$

Q6

x_i = Number of cookies received by i-th child.

$i = 1,2,3,4,5$

$x_1 + x_2 + x_3 + x_4 + x_5 = 15$

$= \frac{(15+4)!}{4!\,15!} = \frac{19!}{4!\,15!} = \frac{\cancel{16}^{2} \times 17 \times \cancel{18}^{6} \times 19}{\cancel{24}} = 12 \times 17 \times 19$

$= 3876$

b) x_i # of cookies received by i-th child

now, $y_i = x_i - 1 \geq 0$

$\therefore x_1 + x_2 + x_3 + x_4 + x_5 = 15,\ x_i \geq 1$

$y_1 + y_2 + y_3 + y_4 + y_5 = 10,\ y_i \geq 0$

$= \frac{(10+4)!}{4!\,10!} = \frac{14!}{4!\,10!} = \frac{11 \times \cancel{12} \times 13 \times \cancel{14}^{7}}{\cancel{24}}$

$= 91 \times 11 = \boxed{1001}$

Q7

(a) # of ways choosing 4 out 10 = $C(10,4)$

$= \frac{10!}{4!6!} = \boxed{210}$

(b) Govid or Gopal has been included

No of ways $= C(8,2) = \frac{8!}{2!6!} = \boxed{28}$

(c) 3 chosen others has to be choosen from 8 remaing

$= C(2,1) \times C(8,3) = \frac{2!}{1!1!} \times \frac{8!}{3!5!} = \boxed{112}$

Q8

X = # of student who resister for Calc II

$X \sim Bin(n=28, p=0.9) \Rightarrow P(X=x) = \binom{28}{x}(0.9)^x(0.1)^{(28-x)}$

$\therefore P(\text{class will be overbooked}) = P(X=26 \text{ or } 27 \text{ or } 28)$

$= \binom{28}{26}(0.9)^{26}(0.1)^2 + \binom{28}{27}(0.9)^{27}(0.1) + \binom{28}{28}(0.9)^{28}(0.1)$

$= \boxed{0.4594}$

Q9

P(John passes test in 2nd attempt) $X \sim G(P=0.6)$

$P(X=x) = pq^{x-1}$

$P(X=2) = (0.6)(0.4)^{2-1} = \boxed{0.24}$

+P(Peter pass in 3rd attempt)

$P(X=3) = (0.6)(0.4)^{3-1} = \boxed{0.096}$

Q10 Using Negative Binomial distn

$X+K =$ No. of trial $= 8$, $K = 3$

$P(X=5) = \binom{X+K-1}{K-1} p^K (0.7)^5$

$= \binom{7}{2} (0.3)^2 (0.7)^5$

$= \boxed{0.3176}$

Q11 X = No. of hurricane in a given year | X = 2 hurricanes per year

given: Ave. No. of hurricane per year = 2

If $X \sim Poisson(\lambda)$, then $P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$, $x = 1, 0$

So, $P(X=3) = \frac{e^{-2}(2)^3}{3!} = \frac{4}{3} e^{-2} \approx \boxed{0.180447}$

Q2 a) $\sum_x P(X=x) = 1 \Rightarrow P(X=1) + P(X=2) + P(X=3) = 1$

$\Rightarrow 0.3 + a + 0.5 = 1 \Rightarrow \boxed{a = 0.2}$

b) $F_X(x) = P(X \leq x)$

$F_X(x) = \begin{cases} 0, & x < 1 \\ 0.3, & 1 \leq x < 2 \\ 0.5, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$

since $F_X(0) = P(X \leq 0) = 0$

$F_X(1) = P(X \leq 1) = P(X=1) = 0.3$

$F_X(2) = P(X \leq 2) = P(X=1) + P(X=2) = 0.5$

$F_X(3) = P(X=1) + P(X=2) + P(X=3) =$

$0.3 + 0.2 + 0.5 = 1$

$0.3 + 0.2$

(c) $f_X(2) = P(X \le 2) \cancel{P}(X=1) + P(X=2) = \boxed{0.5}$

(d) $E(X) = \sum_{x=1}^{3} P(X=x) = 1 \cdot P(X=1) + 2 \cdot P(X=2) 3 \cdot (X=3)$
$= 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.5 = \boxed{2.2}$

(e) $E(X)^2 = \sum_{x=1}^{3} x^2 P(X=x) = (1)^2 \cdot P(X=1) + (2)^2 \cdot (X=2)$
$+ (3)^2 (X=3) = \boxed{5.6}$

(f) $Var(X) = E(X)^2 - [E(X)]^2 = 5.6 - (2.2)^2$

$= \boxed{0.76}$