Module-3 Dimension Analysis

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- Units such as meters, cubic centimeter, degrees Celsius, Joule are referred as dimensions. We can analyze and solve problems only if we set up these dimensions appropriately.

Example: 1

A van is 5.2 meter long. How long is this van in mm?

$$5.2\,m \times \left(\frac{?}{?}\right) = ?\,mm$$

this is the step-1 to solve the problem if have the appropriate conversion factors. The conversion factor for meter to millimeter: 1 *m* = 1000 *mm*

$$\frac{1\,m}{1000\,mm} \quad OR \quad \frac{1000\,mm}{1\,m}$$

Both of these conversion factors seem ok, but only the second one will cancel out the unit *meter* and give the result in *mm*.

$$5.2\,\cancel{m} \times \left(\frac{1000\,mm}{1\,\cancel{m}}\right) = 5200\,mm$$

Example: 2

An average adult has 5.2 L of blood. What is the volume of blood in m^3?

Volume in m^3 = 5.2 L $\times \frac{1000\ cm^3}{1\ L} \times \left(\frac{1 \times 10^{-2} m}{1\ cm}\right)^3$

$= 5.3 \times 10^{-3}\ m^3$.

Example: 3

The density of liquid nitrogen is 0.808 g/cm^3 at its boiling point (77 K). Convert the density units to kg/m^3.

We have to do two conversions for this problem: g to kg and cm^3 to m^3. It is a well-known fact that 1 kg = 1000 g and 1 m = 100 cm.

$$= \frac{0.808\ g}{1\ cm^3} \times \frac{1\ kg}{1000\ g} \times \frac{(100\ cm)^3}{1\ m^3} = 808 \text{ kg/m}^3.$$

Unit conversion error and a near disaster

A story and a problem:

Only July 23rd 1983 Air Canada's Boeing 767 scheduled to fly from Montreal to Edmonton. The fuel gauge in the aircraft was not working, and with a dipstick a mechanic found that 7682 L of fuel was left in the aircraft. To fly from Montreal to Edmonton the plane needed 22,300 kg of fuel. The pilot was provided with conversion factor of 1.77 by one of the mechanics. The pilot assumed that the factor was in metric units (kg/L), and after calculations, as requested, 4916 L of fuel was added. The flight took off and subsequently ran out of fuel, and fortunately landed safely in an old Canadian air force base without any loss to life. Later it was found that the conversion factor of 1.77 was in units of pounds per liter and not in kg/L. Note: Aircrafts can fly by gliding even if both engines fail, but by losing altitude rapidly.

Question: What volume of fuel in liters should have been added at Montreal to the aircraft?

Unit conversion error that cost a spacecraft and 168 million dollars

What happened in 1983 repeated again sixteen years later and this time on the planet Mars. The spacecraft, Mars Climate Orbiter (MCO), was launched by NASA on December 11th 1998, and on 23rd September 1999 the spacecraft was lost as it crashed on to the Mars surface. The investigation revealed that instead of entering an elliptical orbit at 250 km, the spacecraft entered Martian atmosphere at lower trajectory of about 56 km above Mars surface. This error occurred because NASA engineers did calculations in British Engineering Units but the spacecraft computer used SI units.

No calculations here, but you can realize that a simple arithmetic error can be very costly.

Practice Problems

1. Density of the element lithium is 5.34×10^2 kg/m^3. Convert the density to g/cm^3.

2. Calculate the mass of a sphere of gold with a radius of 10.0 cm; the density of gold is 19.3 g/cm^3.

3. The speed of sound in air is 343 m/s. Calculate this speed in miles/hour.

4. The surface area and average depth of the Pacific Ocean are 1.8×10^8 km^2 and 3.9×10^3 m. Calculate the volume of water in Pacific Ocean.

5. A sheet of aluminum foil has a total area of 1.000 ft^2 and a mass of 3.636 g. What is the thickness of the sheet in millimeters? Density of aluminum = 2.699 g/cm^3.

6. The radius of a copper atom is 1.3×10^{-10} m. How many times you can divide evenly a piece of 10-cm copper wire until it is reduced to two separate copper atoms? Assume that the copper atoms are lined up in a straight line in contact with one another.

7. Find the radius (r) of an aluminum cylinder that is 2.00 cm long and has a mass of 12.4 g. Use the density of aluminum given in the problem number-5.

8. The Avogadro's number 6.022×10^{23} equals one mole of any substance. If 23.0 g of sodium has 6.022×10^{23} atoms, how many atoms of sodium are there in 230.0 g of sodium?

9. If one mole of titanium has a mass of 181.0 g. How many atoms are there in 50.0 g of titanium?

10. A jogger runs a mile in 8.92 minutes. Calculate the speed in
(a) *in/s* (b) *m/min* (c) *km/h* (1 *mile* = 1609 *m*; 1 *in* = 2.54 *cm*)