- Dom Arishi
- Hw 4

$\boxed{4.2}$

a) $1 = \int_{-\infty}^{\infty} f_X(u)\,du$

$= \int_0^{\infty} ce^{-u}\,du$

$= c\left[-e^{-x}\right]_0^{\infty}$

$= c \Rightarrow c = 1$

b)

$F_X(x) = \int_0^x e^{-u}\,du = 1 - e^{-x}$

$= F_X(x) = \begin{cases} 1 - e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

c) $P(1 < X < 3) = F_X(3) - F_X(1) = [1 - e^{-3}]$
$- [1 - e^{-1}] = e^{-1} - e^{-3}$.

Using PDF,

$P(1 < X < 3) = \int_1^3 f_X(t)\,dt$

$= \int_1^3 e^{-t}\,dt$

$= e^{-1} - e^{-3}$

$\boxed{4.4}$ $EX = \int_{-\infty}^{\infty} x f_X(x)\,dx$

$= \int_0^1 x(2x)\,dx$

$= \int_0^1 2x^2\,dx$

$= \boxed{\frac{2}{3}}$

$\boxed{4.6}$

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$
$$= \int_{1}^{\infty} \frac{3}{x^3}\, dx$$
$$= \left[-\frac{3}{2} x^{-2}\right]_1^{\infty}$$
$$= \boxed{\frac{3}{2}}$$

Find EX^2 using LOTUS,

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dt$$
$$= \int_{1}^{\infty} \frac{3}{x^2}\, dx$$
$$= \left[-3x^{-1}\right]_1^{\infty} = \boxed{3}$$

So,

$$\text{var}(X) = EX^2 - (EX)^2 = 3 - \frac{9}{4} = \boxed{\frac{3}{4}}$$

$\boxed{\text{Problem 3}}$

$$P\left(X \le \frac{2}{3} \mid X > \frac{1}{3}\right) = \frac{P\left(\frac{1}{3} < X \le \frac{2}{3}\right)}{P\left(X > \frac{1}{3}\right)}$$
$$= \frac{\int_{\frac{1}{3}}^{\frac{2}{3}} 4x^3\, dx}{\int_{\frac{1}{3}}^{1} 4x^3\, dx} = \boxed{\frac{3}{16}}$$

$\boxed{\text{Problem 1}}$

Let, $X \to U[2,6] \Rightarrow a = 2$ and $b = 6$

CDF of X is given by,

$$F(x) = \frac{x-a}{b-a} = \frac{x-2}{6-2} = \frac{x-2}{4}$$

$$F(x) = 0;\ x < 2 \quad = \frac{x-2}{4};\ 2 \le x \le 6$$

$= 1 ; x > 6$

(b) $E(X) = \frac{a+b}{2} = \frac{2+6}{2} = \frac{8}{2} = \boxed{4}$

$\boxed{3.9}$

a) $P(0 < X < 1) = 1$

$P[0 < x < 1] = \frac{2}{5} \int_0^1 (x+2)\, dx$

$= \frac{2}{5} \left[\int_0^1 x\, dx + 2 \int_0^1 dx \right]$

$= \frac{2}{5} \left[\left(\frac{x^2}{2} \right)_0^1 + 2 (x)_0^1 \right]$

$= \frac{2}{5} \left[\frac{1}{2} + 2 \right] = \frac{2}{5} \left(\frac{5}{2} \right) = \boxed{1}$

(b) $P(\frac{1}{4} < x < \frac{1}{2}) = \frac{2}{5} \int_{\frac{1}{4}}^{\frac{1}{2}} (x+2)\, dx$

$= \frac{2}{5} \left(\frac{x^2}{2} \right]_{\frac{1}{4}}^{\frac{1}{2}} + 2 \left[\frac{1}{2} - \frac{1}{4} \right]$

$= \frac{2}{5} \left[\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \times \frac{1}{16} + 2 \cdot \frac{1}{4} \right]$

$= \frac{2}{5} \cdot \frac{4 - 1 + 16}{32} = \boxed{\frac{19}{80}}$

$\boxed{3.29}$

Verify that this is a valid density function

$\int_{-\infty}^{\infty} f(x)\, dx = \int_1^{\infty} 3x^{-4}\, dx$

$= \int_1^{\infty} \frac{3}{x^4}\, dx$

$$3\int_1^{\infty} \frac{1}{x^4}\,dx = 3\left[-\frac{1}{3x^3}\right]_1^{\infty}$$

$$= \left[-\frac{1}{x^3}\right]_1^{\infty} = -0 + \frac{1}{1} = \boxed{1}$$

As a result, this is a valid density function

(b) $F(x) = P[X \le x]$

$$= 3\int_1^x t^{-4}\,dt = \left[-\frac{1}{t^3}\right]_1^x$$

$$= 1 - \frac{1}{x^3} = 1 - x^{-3}$$

$F(x)$ is $F(x) = \begin{cases} 0 & \text{for } x < 1 \\ 1 - x^{-3} & \text{for } x \ge 1 \end{cases}$

(c) $P(X > 4) = 1 - P(X \le 4)$

$$= 1 - \int_1^4 f(x)\,dx$$

$$= 1 - \int_1^4 3x^{-4}\,dx$$

$$= 1 - \left[-\frac{1}{64} - (-1)\right]$$

$$= 1 - \left[\frac{-1 + 64}{64}\right] = 1 - \left[\frac{63}{64}\right]$$

$= 1 - 0,984375$

$= \boxed{0,0156}$ (4 decimal)

$\boxed{3.30}$

ⓐ $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-1}^{1} K(3-x^2)\,dx = 1$

$= K\int_{-1}^{1}(3-x^2)\,dx = 1$

$= K\left[3\int_{-1}^{1} dx - \int_{-1}^{1} x^2 \cdot dx\right] = 1$

$= K\left[3 \times x\Big|_{-1}^{1} - \frac{x^3}{3}\Big|_{-1}^{1}\right] = 1$

$= K\left[6 - \left(\frac{1}{3} + \frac{1}{3}\right)\right] = 1$

$= K\left[6 - \frac{2}{3}\right] = 1 \quad = \quad K\frac{18-2}{3} = 1$

$= \frac{16K}{3} = 1 \Rightarrow K = \boxed{\frac{3}{16}}$

The valid density func is:

$f(x) = \begin{cases} \frac{3}{16}(3-x^2), & -1 \le x \le 1 \\ 0 & ; \text{elsewhere} \end{cases}$

ⓑ $P(X < 0.5) = \int_{-1}^{0.5} f(x)\,dx = \frac{3}{16}\int_{-1}^{0.5}(3-x^2)\,dx$

$= \frac{3}{16}\left[3\int_{-1}^{\frac{1}{2}} dx - \int_{-1}^{\frac{1}{2}} x^2\,dx\right]$

$= \frac{3}{16}\left\{3\left[\frac{1}{2}+1\right] - \frac{x^3}{3}\Big|_{-1}^{\frac{1}{2}}\right\}$

$= \frac{3}{16}\left\{4.5 - \left[\frac{1}{24} + \frac{1}{3}\right]\right\}$

$= \frac{3}{16}\left\{\frac{9}{2} - \frac{1}{24} - \frac{1}{3}\right\} = \frac{3}{16} \times \frac{99}{24} = 0.7734$

$\boxed{\approx 77.34\%}$

(c) $P(|X| > 0.8) = 1 - P(|X| \le 0.8)$

$= 1 - P(-0.8 \le x \le 0.8) = 1 - \int_{-0.8}^{0.8} f(x)\,dx$

$= 1 - \int_{-0.8}^{0.8} \frac{3}{16}(3 - x^2)\,dx = 1 - \frac{3}{16}\left[3x - \frac{x^3}{3}\right]_{-0.8}^{0.8}$

$= 1 - \frac{3}{16}\left[\left\{3(0.8) - \frac{(0.8)^3}{3}\right\} - \left\{3(-0.8) - \frac{-(0.8)^3}{3}\right\}\right]$

$= 1 - \frac{3}{16}\{2.2293 + 2.2293 = 1 - 0.836 = \boxed{0.164}$

$\approx \boxed{16.4\ \%}$

$\boxed{3.32}$

(a) for a density func to be valid

$\int_0^1 f(y)\,dy = 1$

$\int_0^1 5(1-y)^4\,dy \Rightarrow 5 \cdot \frac{(1-y)^5}{(-1)5}\Big|_0^1 \Rightarrow -(y-1)^5\Big|_0^1$

$\int f(y)\,dy = \boxed{1}$

(b) $P(y < 0.1) = 5\int_0^{0.1}(1-y)^4\,dy = \frac{5}{-5}(1-y)^5\Big|_0^{0.1}$

$P(y < 0.1) = 1 - (0.9)^5 = 0.4095$

(c) $P(y > 0.5) = \int_{0.5}^{1} 5(1-y)^4\,dy$

$P(y > 0.5) = (-1)(1-y)^5\Big|_{0.5}^{1} = (0.5)^5$

$P(y > 0.5) = 0.0313$

Ex 6.8 $x_1 = 778$ and $x_2 = 834$ are

$$z_1 = \frac{778-800}{40} = -0.55 \text{ and } z_2 = \frac{834-800}{40} = 0.85$$

Therefore,

$P(778 < X < 834) = P(-0.55 < Z < 0.85) = P(Z < 0.85)$

$- P(Z < -0.55) = 0.8023 - 0.2912 = \underline{\underline{0.5111}}$

Ex 6.13 $x = \sigma z + \mu$. An area of .12, corresponding to the func of student receiving, we require z value that leaves 0.12 of the area to the right and, hence an area of 0.88 to the left; $P(Z < 1.18)$ has the closest value to 0.88, the desired z value is 1.18

$$x = 7 \times 1.18 + 74 = 82.26$$

* The lowest A is 83, the higest B is 82.

6.3 (a) $f(x) = \frac{1}{B-A} = \frac{1}{10-7} = 3, (7 \le x \le 10)$

So, $P(X \le 8.8) = \int_7^{8.8} \frac{1}{3}\, dx$

$= \frac{1}{3}\int_7^{8.8} dx = \frac{1}{3}[x]_7^{8.8} = \frac{8.8-7}{3} = \boxed{0.6}$

(b) $P(7.4 \le X \le 9.5) = \int_{7.4}^{9.5} \frac{1}{3}\, dx$

$= \frac{1}{3}\int_{7.4}^{9.5} dx = \frac{1}{3}[x]_{7.4}^{9.5} = \frac{9.5-7.4}{3} = \boxed{0.7}$

(c) $P(X \geq 8.5) = \int_{8.5}^{10} \frac{1}{3}\,dx$

$= \frac{1}{3}\int_{8.5}^{8/10.8} dx = \frac{1}{3}[x]_{8.5}^{8/10.8} = \frac{10-8.5}{3}$

$= \boxed{0.5}$

6.4 (a) $P(X > 7) = \int_{7}^{10} \frac{1}{10}\,dx$

$\boxed{\begin{array}{l} a=0 \\ b=10 \end{array}}$

$= \left(\frac{x}{10}\right)_{7}^{10} = \frac{10-7}{10} = \frac{3}{10} = \boxed{0.3}$

(b) $P(2 < X < 7) = \int_{2}^{7} \frac{1}{10}\,dx$

$= \frac{1}{10}\int_{2}^{7} 1\,dx = \frac{1}{10}(x)_{2}^{7}$

$= \frac{7-2}{10} = \frac{5}{10} = \boxed{0.5}$

6.11 (a) $z = \frac{x-\mu}{\sigma} = \frac{224-200}{15} = \boxed{1.6}$

So,

$P(X > 224) = P(z > 1.6)$
$= 1 - P(z \geq 1.6)$
$= 1 - 0.9452 = 0.0548$

(b) $z = \frac{x_1-\mu}{\sigma} = \frac{191-200}{15} = -0.6$

$z = \frac{x_2-\mu}{\sigma} = \frac{209-200}{15} = 0.6$

$P(-0.6 < X < 0.6) = P(-0.6 < Z < 0.6)$

$= P(Z < 0.6) - P(Z < -0.6)$

$= 0.7257 - 0.2743 = \boxed{0.4515}$

(c) $Z = \frac{x-\mu}{\sigma} = \frac{230-200}{15} = 2$

1000×0.0228

$= 22.75 \approx 23$

$P(X > 230) = P(Z > 2)$

$= 1 - P(Z > 2) = 1 - 0.9772 = \boxed{0.0228}$

(d) $P(X < x) = 0.25$

$P\left(Z < \frac{x-\mu}{\sigma}\right) = 0.25$

$P(Z < -0.68) = 0.25$

$\frac{x-\mu}{\sigma} = -0.68$

$x = -0.68 \cdot \sigma + \mu$

$x = -0.68 \cdot 15 + 200 = 189.95$