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- Hw5

Prob 1 Let's find first $P(X > t)$:

$$P(X > t) = P(\text{No arrival in } [0, t])$$
$$= e^{-\lambda t} \frac{(\lambda t)^0}{0!} = e^{-\lambda t}$$

then the CDF of X for $x > 0$ is given by

$$F_X(x) = 1 - P(X > x) = 1 - e^{-\lambda x}$$

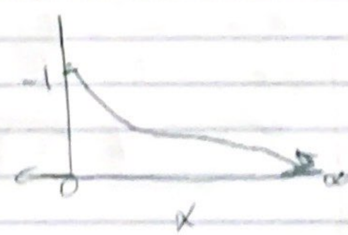
which is the CDF of Exponential (λ). the time between the ~~first~~ k^{th} and $k+1$'th customer is Exponential (λ).

Qs 2 Let x: Amt of time spouses shop for anniversary cards $x \sim$ exponential (mean = 6).

$\therefore$ Its pdf is $f(x) = \frac{1}{6} e^{-x/6}$, $x > 0$

$= 0$ o.w

Probability of density curve:



Qs 3 Let x: No. of days where travelers airline ticket

$x \sim$ exponential (mean = 10)

Its pdf is $f(x) = \frac{1}{10} e^{-x/10}$, $x > 0$

$= 0$ o.w

$\therefore$ the cdf is given by $F(x) = P(X \le x) = 1 - e^{-x/10}$

The prob of travelers will buy a ticket fewer than 8 days in advance =

$P(X < 8) = F_X(8) = 1 - e^{-8/10} = \boxed{0.5507}$

QS 4

Let X: the length of time the computer part lasts.

$X \sim$ exponential (mean = 8)

$\therefore$ Pdf is $f(x) = \frac{1}{8} e^{-x/8}$, $x > 0$

= 0 O/W

CDF is $F(x) = 1 - e^{-x/8}$, $x > 0$, the prob that a computer part lasts more than 6 years

$P(X > 6) = 1 - P(X \leq 6) = 1 - [F_X(6)]$

$= 1 - [1 - e^{-6/8}]$

$P(X > 6) = e^{-6/8} = 0.4724.$

QS 5

a) to find $P(X \leq 2, Y \leq 4)$

$= P_{XY}(1,2) + P_{XY}(1,4) + P_{XY}(2,2) + P_{XY}(2,4)$

$= \frac{1}{12} + \frac{1}{24} + \frac{1}{6} + \frac{1}{12} = \frac{3}{8}$

(b) $R_X = \{1, 2, 3\}$ and $R_Y = \{2, 4, 5\}$

$$P_X = \begin{cases} \frac{1}{6} & x = 1 \\ \frac{3}{8} & x = 2 \\ \frac{11}{24} & x = 3 \\ 0 & \text{otherwise} \end{cases} \qquad P_Y(y) = \begin{cases} \frac{1}{2} & y = 2 \\ \frac{1}{4} & y = 4 \\ \frac{1}{4} & y = 5 \\ 0 & \text{otherwise} \end{cases}$$

c) Using the formula of conditional Prob

$$P(Y=2 \mid X=1) = \frac{P(X=1, Y=2)}{P(X=1)}$$

$$= \frac{P_{XY}(1,2)}{P_X(1)} = \frac{\frac{1}{12}}{\frac{1}{6}} = \frac{1}{2}$$

d) $P(X=2, Y=2) = \frac{1}{6} \neq P(X=2)P(Y=2)$

$= \frac{3}{16}$, so as a result, we can say that

X and Y are not independent.

Part (b) a) joint pdf of X and Y, $f_{XY}(x,y) = \begin{cases} c(x+\frac{1}{2}y) & 0 \le x, \ldots \\ 0 & \text{otherwise} \end{cases}$

given ↗

Qs 2

Since total Prob is equal to 1, i.e $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dy\,dx = 1$

$\Rightarrow \int_{x=0}^{2}\int_{y=0}^{1} c(x+\frac{1}{4})\,dx = 1 \Rightarrow c\left[\frac{x^2}{2} + \frac{x}{4}\right]_0^2 = 1$

$\Rightarrow c = \frac{2}{5}$

(b) marginal pdf of X, $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 \frac{2}{5}(x+\frac{1}{2}y)\,dy$

$= \frac{2}{5}\left[xy + \frac{y^2}{4}\right]_0^1 = \frac{2}{5}(x+\frac{1}{4}),\ 0 \le x \le 2$

(c) Prob that observed radiation exceeds the limit 1.6 w/kg

$= P(X > 1.6) = \int_{1.6}^{2} \frac{2}{5}(x+\frac{1}{4})\,dx = \frac{2}{5}\left[x^2 + \frac{x}{4}\right]_{1.6}^{2}$

$= 0.328$

(d) marginal pdf of Y, $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

$= \int_0^2 \frac{2}{5}\left(x + \frac{1}{2}y\right) dx$

$= \frac{2}{5}\left(\frac{x^2}{x} + \frac{xy}{2}\right]_0^2 = \frac{2}{5}(2+y),\quad 0 \le y \le 1$

(e) $P(1 < X < 1.6,\ Y > 0.2) = \int_{x=1}^{1.6}\int_{y=0.2}^{1} \frac{2}{5}\left(x + \frac{1}{2}y\right) dy\,dx = \int_1^{1.6} \frac{2}{5}(0.8x + 0.24)\,dx = \frac{2}{5}\left[0.4x^2 + 0.24x\right]_1^{1.6} = 0.3072.$

(f) $E(x) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^2 x \cdot \frac{2}{5}\left(x + \frac{1}{4}\right) dx = \frac{19}{15}$

$E(y) = \int_{-\infty}^{\infty} y f_Y(y)\,dy = \int_0^1 y \cdot \frac{2}{5}(2+y)\,dy = \frac{8}{15}$

$E(x^2) = \int_0^2 x^2 \cdot \frac{2}{5}\left(x + \frac{1}{4}\right) dx = \frac{28}{15}$

$E(y^2)\ \int_0^1 y^2 \cdot \frac{2}{5}(2+y)\,dy = \frac{11}{30}$

$Var(X) = E(x^2) - [E(x)]^2 = \frac{28}{15} - \left(\frac{19}{15}\right)^2 = \frac{59}{225}$

$var(Y) = E(y^2) - (E(y))^2 = \frac{11}{30} - \left(\frac{8}{15}\right)^2 = \frac{37}{450}$

$E(xy) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\,dy\,dx = \int_{x=0}^{2}\int_{y=0}^{1} xy \cdot \frac{2}{5}\left(x + \frac{1}{2}y\right) dy\,dx = \frac{2}{3}$

$Cov(x,y) = E(xy) - E(x)\cdot E(y) = \frac{2}{3} - \frac{19}{15}\cdot\frac{8}{15} = -\frac{2}{225}$

correlation btw x, y $= \frac{-2/225}{\sqrt{\frac{59}{225}}\cdot\sqrt{\frac{37}{450}}} = \rho(x,y) \approx \boxed{-0.0605}$

Qs 1 a) $P(X=2, Y \geq 1) = P(X=2, Y=1) + P(X=2, Y=2)$

$= 0.05 + 0.05 = 0.10$

b) $P(X=1) = P(X=1, Y=0) + P(X=1, Y=1) + P(X=1, Y=2)$

$= 0.15 + 0.1 + 0.2 = 0.45$

$P(X=2) = P(X=2, Y=0) + P(X=2, Y=1) + P(X=2, Y=2)$

$= 0.10 + 0.05 + 0.05 = 0.2$

$P(X=3) = P(X=3, Y=0) + P(X=3, Y=1) + P(X=3, Y=2)$

$= 0 + 0.25 + 0.10 = 0.35$

marginal pdf of X

x	1	2	3
$P(x)$	0.45	0.2	0.35

c) $P(Y=0) = P(X=1, Y=0) + P(X=2, Y=0) + P(X=3, Y=0)$

$= 0.15 + 0.10 + 0 = 0.25$

$P(Y=1) = P(X=1, Y=1) + P(X=2, Y=1) + P(X=3, Y=1) =$

$0.10 + 0.05 + 0.25 = 0.45$

$P(Y=2) = P(X=1, Y=2) + P(X=2, Y=2) + P(X=3, Y=2)$

$0.20 + 0.05 + 0.10 = 0.35$

Y	1	2	3
$P(Y)$	0,25	0,40	0,35

(d) $P(Y \geq 1 | X \leq 2) = \frac{P(Y \geq 1 \cap X \leq 2)}{P(X \leq 2)}$

$= P(Y=1, X=1) + P(Y=1, X=2) + P(Y=2, X=1)$

$P(Y=2, X=2) = 0,1 + 0,05 + 0,20 + 0,05$

$= 0,4$

$P(X \leq 2) = P(X=1) + P(X=2) = 0,45 + 0,2$

$= 0,65$

$P(Y \geq 1 / X \leq 2) = \frac{0,4}{0,65} = 0,6154$

(e) For independence, $P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$

must be true for each cell ~~in the table~~ in contingency table

$P(Y=0) = 0,15 + 0,1 + 0 = 0,25$

$P(X=1) = 0,15 + 0,1 + 0,2 = 0,45$

$P(X=1, Y=0) = 0,15$

so,

since $P(X=1) \times P(Y=0) = 0,45 \times 0,25 = 0,1125$ is not equal to $P(X=1, Y=0)$; therefore events X and Y are not independent.

(F) $E(X) = \sum x P(x, y) = 1,9000$

$E(X)^2 = \sum x^2 P(x,y) = 4,4000$

$E(Y) = \sum y P(x,y) = 1,1000$

$E(y)^2 = \sum y^2 P(x,y) = 1,8000$

$var(X) = E(X)^2 - (E(X))^2 = 0,7900$

$var(y) = E(y)^2 - (E(y))^2 = 0,5900$

$SD(x) = var(x) = 0,8888$

$SD(Y) = var(y) = 0,7681$

$E(XY) = \sum xy P(x,y) = 2,1500$

$cov(x,y) = E(XY) - E(X) \times E(Y) = 0,0600$

correlation btw $X, Y = cov(X,Y) / (SD(x) \times SD(y))$

$= 0,0879.$