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Valuation of adopters based on the Bass model for a new product

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ABSTRACT

This paper shows that a hazard function can be additively decomposed, and with the additively decomposed hazard function, it shows that one can identify the others' effects which come from interactions between potential adopters and the adopters who bought at every adoption time interval. Also, from the additive decomposition of the discrete Bass hazard function, it derives the discrete Bass model which is consistent with the continuous version of Bass model. Furthermore, the coefficients of the derived discrete Bass model are expressed as the functions of coefficients of the continuous Bass model. Based on the derived discrete Bass model, thus, this paper presents a valuation scheme (calculator) for adopter groups which are classified by every observation time. The valuation calculator allows managers to calculate adopters' values only with the estimates of the continuous Bass model.

1. Introduction

Since the introduction of the Bass (1969), the marketing literature on the diffusion of innovations continues to expand. However, we believe that there may still be some useful properties of the Bass model which have not been well extended and utilized by previous studies. Bass (1969) has introduced two versions of mathematical model of innovation diffusion processes (the continuous version and discrete-analog version), which is one of the most widely applied models in the marketing literature. The two versions of Bass model assume that diffusion processes are defined by the interaction mechanism between two adopter groups (Innovators and Imitators) in a social system. However, the discrete-analog Bass hazard (which is defined by the interaction mechanism between Innovators and Imitators) is not consistent with the continuous Bass hazard (which is defined by the interaction mechanism between the identical two groups), in the view that the adoption probability expressed by the discrete-analog Bass model is not identical to that by the continuous Bass model.

This paper proposes a mathematical representation that additively decomposes a discrete hazard function and, with the additively decomposed hazard function, shows that one can identify the others' effects which come from interactions between potential adopters and the adopters who bought at every adoption time interval. In addition, from the additive decomposition of the discrete Bass hazard function, it derives the discrete Bass model which is consistent with the continuous version of Bass model. Unlike the discrete-analog Bass hazard, the derived discrete Bass hazard is described by the interaction mechanism between various adopter groups which are classified by every discrete adoption time. We focus on the diffusion mechanism by the various adopter groups to identify different values of the adopter groups in this paper.

Measuring the value of word of mouth (or referrals) can provide highly valuable implications for customer management (especially in referral marketing campaigns) (see Pres et al., 2010; Kumar et al., 2013). Thus, several studies in the marketing literature have tried to identify the value of customers' social interactions in the diffusion process (e.g., Kumar et al., 2010, 2013; Ho et al., 2012; Libai et al., 2013; Haenlein and Libai, 2013). However, it is not easy for managers to identify customers who have greater impacts on the firms' future profits (i.e., referral behavior) and add more value to the firms through referral activities, due to difficulties in evaluating the referral values which differ from one customer to another. Therefore, to identify the different referral values by customers, managers should use a complex mathematical model and collect additional data to test the mathematical model.

The discrete version of the Bass model, which is derived in this paper, allows us to identify the coefficients of others' effects that are different across the adopter groups of which adoption times are different. With the different coefficients, one can identify the varying referral values of the adopter groups that are classified by every observation time. Thus, we present a simple and easy valuation calculator for adopters based on the derived discrete version of the Bass model. The calculator enables us to measure values of various adopter groups using estimates of the continuous Bass model (hereafter BM).

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2. The continuous Bass model and discrete Bass models

2.1. Inconsistency between two versions of the Bass model

Bass (1969) introduced two versions of his mathematical model of innovation diffusion processes (the continuous-time diffusion BM and discrete-analogue BM). The hazard function for the continuous BM is written by:

$$h(t)=\frac{F\prime(t)}{1-F(t)}=a+\beta \mathrm{F}(t), \tag{1}$$

where $F(t)$ represents the cumulative adoption probability at time t. In the continuous BM, a diffusion process is determined by the two types of forces. The coefficient α represents the effect of innovators and the coefficient β does the effect of imitators. The former is referred to as the external effect (which is independent from others in a diffusion system) and the latter is known as the internal effect (which is dependent on others in the system) (Lekvall and Wahlbin, 1973). The two effects are also referred to as 'self-motivation effect' and 'others effect', respectively (Ho et al., 2012). From the continuous hazard (Eq. (1)), the continuous Bass distribution is derived by:

$$F(t)\frac{1-e^{-(a+\beta)t}}{1+(\beta/a)e^{-(a+\beta)t}}. \tag{2}$$

On the other hand, the hazard of the discrete-analog BM (at time t) is represented by:

$$\lambda(t)=\frac{F(t)-F(t-1)}{1-F(t-1)}=p+qF(t-1), \tag{3}$$

where $F(t)$ and $F(t-1)$ represent the cumulative adoption probabilities at time t and time $t-1$, respectively. This formulation allows the current hazard (at t) to be a linear function of the lagged cumulative adoption probability (at $t-1$). The discrete-analog hazard is again written by:

$$\lambda(t)=\lambda(t-1)+q[F(t-1)-F(t-2)]. \tag{4}$$

It implies that the change of the hazard rate from time $t-1$ to time t is expressed only by the force that comes from the buyers who adopted in the previous time period ($t-1$ to $t-2$), i.e., Eq. (4) can be written by $\lambda(t)-\lambda(t-1)=q[F(t-1)-F(t-2)]$. The specification of the BM is similar to the linear learning model of Kuehn (1962) that specifies a consumer's choice probability at time t as a linear function of the consumer's previous choice probability. The coefficient p captures the consumer's intrinsic choice probability and the coefficient q captures the effects of state dependence or purchase carryover (Park and Seetharaman, 2005).

However, the discrete-analog BM is not consistent with the continuous BM because the adoption probability expressed by the continuous BM is not identical to that by the discrete-analog BM. Eq. (3) or (4) can be written by:

$$\frac{F(t)-F(t-1)}{1-F(t-1)}=\frac{F(t-1)-F(t-2)}{1-F(t-2)}+q[F(t-1)-F(t-2)]. \tag{5}$$

From Eq. (5), the cumulative adoption probability (at time t) of the discrete-analog BM can be written by:

$$F(t)=F(t-1)+\left[\frac{F(t-1)-F(t-2)}{1-F(t-2)}+q[F(t-1)-F(t-2)]\right][1-F(t-1)]. \tag{6}$$

However, it is not possible to find the constant coefficient q which satisfies Eq. (6) for all discrete times where $F(t)$ and $F(t-1)$ represent the continuous Bass distributions at time t and time $t-1$, respectively.

2.2. The discrete BM corresponding to the continuous BM

The continuous hazard function can be written by

$$h(t)=[h(t)-h(t-1)]+[h(t-1)-h(t-2)]+\ldots+[h(2)-h(1)]+h(1), \tag{7}$$

where $h(t)=\frac{f(t)}{1-F(t)}$ and $f(t)=F'(t)$.

On the other hand, the discrete-time hazard, which is exactly corresponding to the continuous hazard function, is written by:

$$\lambda(t)=[\lambda(t)-\lambda(t-1)]+[\lambda(t-1)-\lambda(t-2)]+\ldots+[\lambda(2)-\lambda(1)]+\lambda(1), \tag{8}$$

$$\text{where } \lambda(t)=\frac{F(t)-F(t-1)}{1-F(t-1)}. \tag{9}$$

As shown in Eq. (10), the discrete-time hazard at time t can be interpreted as an approximation of the continuous hazard at that time.

$$h(t)=\frac{f(t)}{1-F(t)}\cong\frac{F(t)-F(t-1)}{1-F(t-1)}=\lambda(t). \tag{10}$$

Note that

$$h(t)=\frac{f(t)}{1-F(t)}=\lim_{\delta\to 0}\frac{F(t)-F(t-\delta)}{1-F(t-\delta)}=\lambda(t). \tag{11}$$

The discrete-time distribution (corresponding to the discrete-time hazard, i.e., Eqs. (8) and (9)) is identical to the continuous-time distribution (corresponding to the continuous-time hazard, i.e., Eq. 7) at every discrete time (see Appendix A).

It is very common in longitudinal data sets to have aggregate information about the timing of adopters at regular intervals of time. That is, researchers frequently estimate a continuous distribution $F(t)$ based on adoptions observed at regular discrete intervals of time (monthly adoption data, quarterly adoption data or yearly adoption data). In those cases, however, the distribution that researchers recover based on such adoption data is not a continuous distribution (corresponding to the continuous hazard) but its discrete-time distribution (corresponding to the discrete hazard). In other words, the recovered continuous-time distribution is empirically identical to its discrete-time distribution as shown in Appendix A.

Here, we start with the representation that additively decomposes a discrete hazard (i.e., Eqs. (8) and (9)). Note that in the representation a discrete-time distribution is defined by its corresponding discrete hazard function. According to the linear learning process of BM (i.e., Eq. (4)), we assume that the change of the hazard rate from time $t-1$ to time t is expressed by the force that comes from the buyers who adopted in the previous time period. Unlike Eq. (4), however, we assume that the coefficient of $F(t-1)-F(t-2)$ is not constant to derive the discrete model which is consistent with the continuous model. Then, the discrete hazard (at time t) is mathematically represented by:

$$\lambda(t)=\lambda(t-1)+q_{t-1}[F(t-1)-F(t-2)]. \tag{12}$$

From Eqs. (8) and (12) we can derive Eq. (13).

$$\lambda(t)=\lambda(1)+q_1[F(1)-F(0)]+q_2[F(2)-F(1)]+\ldots+q_{t-1}[F(t-1)-F(t-2)], \tag{13}$$

because $\lambda(t)-\lambda(t-1)=q_{t-1}[F(t-1)-F(t-2)]$ in Eq. (12). The terms $q_i[F(i)-F(i-1)]$'s represent the others' effects which come from interactions between potential adopters and the adopters who bought at time interval $(i-1, i)$. Thus, the coefficients q_i's ($i=1,2,3,\ldots$) can be interpreted as the effect of others who adopt at each time interval. Eq. (13) implies that the discrete hazard is defined by the interaction mechanism of adopter groups which are classified by every observation time (or it is expressed by the learning process from the various adopter groups). Note that the diffusion mechanism of the additively decomposed discrete hazard function is defined by the interaction mechanism between Innovators and Imitators (or it is expressed by the learning process from the previous adopters).

One can identify the coefficients of the discrete hazard (Eq. (12) or (13)) with the discrete-time distribution $F(t)$ (corresponding to the discrete-time hazard, i.e., Eq. 9). Note that as shown in Appendix A continuous-time distribution is identical to its discrete-time distribution at every discrete time.

By Eq. (9),

$$p=\lambda(1)=\frac{F(1)-F(0)}{1-F(0)}=F(1). \tag{14}$$

In addition, because $\lambda(t)-\lambda(t-1)=q_{t-1}[F(t-1)-F(t-2)]$ in Eq. (12),

$$\begin{aligned} q_{t-1}&=\frac{\lambda(t)-\lambda(t-1)}{F(t-1)-F(t-2)}\\ &=\left[\frac{F(t)-F(t-1)}{1-F(t-1)}-\frac{F(t-1)-F(t-2)}{1-F(t-2)}\right][F(t-1)-F(t-2)]^{-1}\quad \text{for } t=1,2,\ldots. \end{aligned} \tag{15}$$

If the coefficients in Eq. (12) or (13) are expressed by Eqs. (14) and (15), the adoption probability expressed by Eq. (12) or (13) is identical to that by a continuous distribution. For example, Eq. (12) can be written by:

$$\frac{F(t)-F(t-1)}{1-F(t-1)}=\frac{F(t-1)-F(t-2)}{1-F(t-2)}+q_{t-1}[F(t-1)-F(t-2)]. \tag{16}$$

Thus, the cumulative adoption probability (at time t) can be written by:

$$\begin{aligned} F(t)&=F(t-1)+\left[\frac{F(t-1)-F(t-2)}{1-F(t-2)}+q_{t-1}[F(t-1)-F(t-2)]\right]\\ &\quad\times[1-F(t-1)], \end{aligned} \tag{17}$$

where $q_{t-1}=[\frac{F(t)-F(t-1)}{1-F(t-1)}-\frac{F(t-1)-F(t-2)}{1-F(t-2)}][F(t-1)-F(t-2)]^{-1}$.

Through simplified Eq. (17), one can confirm that the adoption probability expressed by Eq. (12) is identical to that by a continuous distribution for all discrete times.

Various distributions can be used to investigate the additively decomposed discrete hazard that follows the linear learning process of the BM (Eq. (12) or (13)) because the additive decomposition of a discrete hazard function is independent of types of distributions. However, we see that the discrete hazard corresponding to a continuous distribution determines the pattern of coefficients q_i's (i = 1,2,3,…), because the coefficients are determined by the differences between hazard rates at a time and its previous time. More specifically, the differences [i.e., $\lambda(i)-\lambda(i-1)$'s] which are derived from a flexible distribution may monotonically decrease or monotonically increase (or non-monotonically increase and then decrease or non-monotonically decrease and then increase) over time, furthermore, the effects from previous adopters (i.e., q_i for $i=1,2,3,\ldots$) may have negative values if the hazard function [i.e., $\lambda(i)$] monotonically decreases (or non-monotonically increases and then decreases) over time. In other words, one can guarantee the non-negative effects from previous adopters only when one adopts a distribution of which the hazard function is monotonically increasing over time. The positive effects from previous adopters are widely accepted in the literature and are empirically supported (e.g., Easingwood et al., 1983; Van den Bulte and Joshi, 2007; Ho et al., 2012). This implies that one needs justifications on various patterns of coefficients q_i's if one wants to use a flexible distribution.

On the other hand, the hazard function corresponding to the Bass distribution is monotonically increasing over time. By this property, the q_i's, which are derived from the Bass distribution, are always non-negative and decrease over time. This implies that every previous adopter positively influences potential adopters, and early adopters are always more influential on potential adopters than late ones when the Bass distribution is adopted. Note that Eq. (18) represents the coefficients of Eq. (13) in which the discrete-time Bass distribution (corresponding to the continuous Bass distribution) is adopted.

$$\begin{aligned} p&=F(1)=\frac{1-e^{-(\alpha+\beta)}}{1+(\beta/\alpha)e^{-(\alpha+\beta)}}\quad \textit{and}\\ q_i&=\frac{\frac{\beta}{\alpha+\beta}\left(e^{(\alpha+\beta)}-1\right)}{\left[\frac{e^{-2(\alpha+\beta)}+(\beta/\alpha)e^{-(\alpha+\beta)(i+1)}}{1+(\beta/\alpha)e^{-(\alpha+\beta)(i+1)}}\right]^{-1}}\quad \textit{for}\ \ i=1,2,3,\ldots. \end{aligned} \tag{18}$$

The parameter q_i (for $i=1,2,3,\ldots$) is always greater than zero and is decreased over time (see Eq. 19).

$$\begin{aligned} \frac{dq_i}{di}&=-\frac{\alpha\beta\left(1-e^{(\alpha+\beta)}\right)\left(1-e^{2(\alpha+\beta)}\right)e^{(\alpha+\beta)(i-2)}}{\left(\beta+\alpha e^{(\alpha+\beta)i}\right)^2}\leq 0\quad \textit{and}\\ q_\infty&=\left(\frac{\beta}{\alpha+\beta}\right)\left(1-e^{-(\alpha+\beta)}\right)e^{-(\alpha+\beta)}\geq 0. \end{aligned} \tag{19}$$

In this paper, we focus on the discrete-time Bass distribution which is consistent with the continuous-time Bass distribution. One can obtain the coefficients of the discrete BM (which means Eq. (13) defined by the discrete-time Bass distribution) if the recovered Bass distribution is applied to the algebraic expression in Eqs. (14) and (15). In sum, if an adoption process at an instant time interval is explained by the interaction mechanism between two groups (Innovators and Imitators) (i.e., the continuous BM), then the adoption process at each discrete time interval can be explained by the interaction mechanism between various adopter groups which are classified by all possible discrete adoption times (i.e., the derived discrete BM). Because the coefficients of the derived discrete BM are expressed as the function of the Bass distribution, one can identify the coefficients based on the coefficients of the recovered Bass distribution, without additional efforts.

3. A calculator for valuation of adopters

It is well known that social influence plays a key role in the diffusion process of new products. Customers' social interactions create considerable social values in the diffusion process (Kumar et al., 2010, 2013; Ho et al., 2012; Libai et al., 2013; Haenlein and Libai, 2013). Thus, it is imperative to estimate a customer's influence value along with her purchase value because identifying a customer's overall value develops a deeper understanding of new product adoption and provides useful insights into how much resources should be allocated to each adopter category. Accordingly, Ho et al. (2012) decompose a customer value into her purchase and influence values by her type (either Influentials or Imitators) and by her time of adoption. They posit that a customer's value is the sum of her purchase and influence values in the context of the asymmetric influence model proposed by Van den Bulte and Joshi (2007). The influential value is referred to also as the indirect value or the word-of-moth value or the social value (Libai et al., 2013).

In this paper, we consider adopter groups (or categories) which are classified by every discrete adoption time and decompose an adopter category's value (CV) into the adopter group's purchase value (PV) and influence value (IV), and then present an alternative valuation scheme on the basis of the discrete version of BM (of which the adopter categories are classified by their adoption times). As aforementioned, under the proposed valuation scheme (calculator for valuation of adopters), one can calculate values of adopters only with estimates of the continuous Bass model without additional efforts.

From Eqs. (13) and (14) the adoption (or sales) at discrete time t where M denotes the market potential can be written by:

$$S(t) = p[1-F(t-1)]M + \sum_{i=1}^{t-1} q_i[F(i)-F(i-1)][1-F(t-1)]M \text{ for } t = 1,2,3,\ldots, \tag{20}$$

where $S(t) = [F(t) - F(t-1)]M$.

Let $PV(t)$ denote the sales of adopter category t (who adopt at discrete time t) which results from the self-motivation effect and $IV(i,t)$ denote the sales of adopter category t which results from the effect of category i on category t. Then, Eq. (20) can be represented once again as:

$$S(t) = PV(t) + \sum_{i=1}^{t-1} IV(i,t) \text{ and } \sum_{t=1}^{\infty} S(t) = M \text{ for } t = 1,2,3,\ldots, \tag{21}$$

where $PV(t) = p[1-F(t-1)]M$ and $IV(i,t) = q_i[F(i)-F(i-1)][1-F(t-1)]M$. This indicates that the sales of each category can be decomposed into the sales driven by self-motivation (or the effects of innovation) and other's influences (or the effects of imitation). Fig. 1 shows the adoption sources of each category where the number of all possible discrete adoption times is five. The arrow directed from category t (for t = 1,2,3,4,5) to its own category represents the sales which comes from the self-motivation effect of category t, whereas the arrow directed from category i (for $1 \le i \le t-1$) to category t represents the sales of category t which comes from the influence of category i on category t. The sales of category t can be expressed as all the arrows destined to category t.

Ho et al. (2012) state that the value of a customer in new durable product categories is the sum of her purchase and influence values when social contagion is prevalent. Following their concept, we define that the value of an adopter category (Category Value) is the sum of the sales purchased by adopters' self-motivation (Purchase Value) and the sales of other categories driven (or influenced) by the category (Influence Value). Thus, the arrow directed to its own category indicates Purchase Value (PV), whereas the arrow directed from category t to group j indicates Influence Value (IV), which is the effect of category t on category j. Thus, all the arrows started at category t can be interpreted as CV of category t, which is the sum of PV and IV of the category. Note that the first category has IVs which affect all the other categories, but the last category has no IV because the last category does not influence any other category. Formally, the value of category t can be defined by:

$$CV(t) = PV(t) + IV(t) \text{ for } t = 1,2,3,\ldots, \tag{22}$$

$$\text{where } PV(t) = p[1-F(t-1)]M \text{ and } IV(t) = \sum_{j=t+1}^{\infty} IV(t,j), \tag{23}$$

where $IV(t,j) = q_t[F(t) - F(t-1)][1 - F(j-1)]M$.

That is, the value of each category is expressed as the sales driven by the category. Note that the sum of all category values is identical to the market potential of a product category (see Eq. (24)).

$$CV = \sum_{t=1}^{\infty} CV(t) = \sum_{t=1}^{\infty} (PV(t) + IV(t)) = M. \tag{24}$$

On the other hand, the value of one adopter in adopter category t can be expressed by:

$$UCV(t) = \frac{CV(t)}{[F(t)-F(t-1)]M} \text{ for } t = 1,2,3,\ldots. \tag{25}$$

The present values of $PV(t)$, $IV(t)$, and $CV(t)$ at the time 0 [which are denoted by$NPV(t)$, $NIV(t)$, and $NCV(t)$] can be computed with the discount rate (i.e., e^{-rt}, where γ is the discount rate).

$$NPV(t) = PV(t)e^{-rt} \text{ and } NIV(t) = IV(t)e^{-rt} \text{ for } t = 1,2,3,\ldots. \tag{26}$$

Because $NCV(t) = NPV(t) + NIV(t)$, the present value of category t and present value of one adopter in category t are represented by:

$$NCV(t) = (PV(t) + IV(t))e^{-rt} \text{ and } \tag{27}$$

$$UCV(t) = \frac{NCV(t)}{[F(t)-F(t-1)]M} \text{ for } t = 1,2,3,\ldots.$$

4. Empirical study

In this empirical illustration, we first estimated parameters of the continuous BM using Srinivasan and Mason's (1986) NLS estimation technique for three products [Clothes Dryers (# of data points = 13), Room Air Conditioners (# of data points = 10), and Color TV (# of data points = 13)]. The estimates were α = .00944, β = .37478, and M (10^3) = 18,712 for Room Air Conditioners; they were α = .00512, β = .63546, and M (10^3) = 40,245 for Color TV; they are α = .01360, β = .32670, and M (10^3) = 16,497 for Clothes Dryers.

Then, we computed the coefficients of the discrete BM (Eqs. (14) and (15)) and calculated PVs, IVs, CVs, and UCVs (Eqs. (22), (23), and (25))

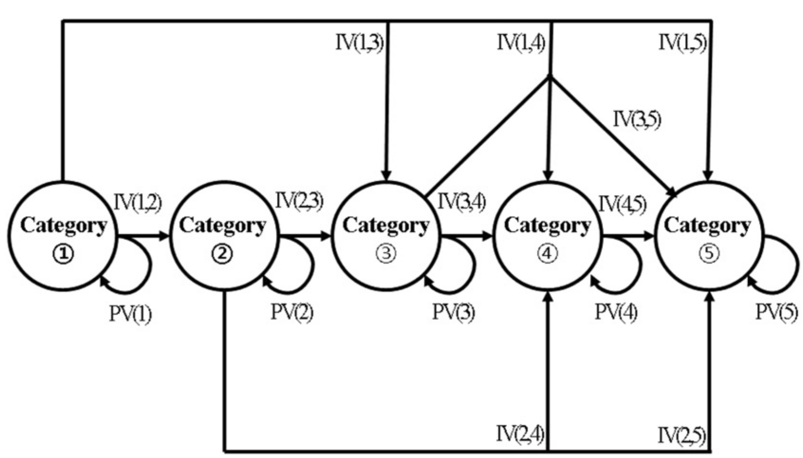

Fig. 1. Purchase values and influence values during five adoption times.

Table 1
Valuation for adopters of room air conditioners.

t (or i)	$[F(t)-F(t-1)]M$	q_i	$PV(t)$	$IV(t)$	$CV(t)$	$UCV(t)$
1	0.5212.95768	.44433	212.95768	888.47697	1101.43465	5.17208
2	304.08092	.43885	210.53411	1121.08196	1331.61607	4.37915
3	428.84226	.43126	207.07350	1373.86669	1580.94020	3.68653
4	594.32947	.42099	202.19304	1621.12652	1823.31957	3.06786
5	804.06041	.40754	195.42925	1822.40974	2017.83899	2.50956
6	1053.04550	.39065	186.27861	1928.00199	2114.28059	2.00778
7	1321.98575	.37054	174.29438	1894.89427	2069.18865	1.56521
8	1574.18529	.34805	159.24946	1709.71394	1868.96340	1.18726
9	1760.72080	.32459	141.33437	1404.10957	1545.44395	.87773
10	1836.61556	.30184	121.29641	1046.20147	1167.49789	.63568
11	1781.47263	.28127	100.39473	709.41706	809.81179	.45457
12	1610.23138	.26383	80.12060	441.63771	521.75832	.32403
13	1365.21106	.24983	61.79529	255.59584	317.39113	.23249
14	1096.05016	.23907	46.25845	139.44938	185.70783	.16943
15	842.06954	.23108	33.78480	72.68545	106.47025	.12644
16	625.32682	.22530	24.20159	36.61480	60.81639	.09726
17	452.75033	.22119	17.08503	17.99195	35.07698	.07748
18	321.82008	.21831	11.93249	8.68558	20.61807	.06407
19	225.77082	.21630	8.27000	4.14092	12.41092	.05497
20	156.93364	.21492	5.70061	1.95709	7.65770	.04880
21	108.38658	.21396	3.91462	.91938	4.83400	.04460
22	74.52586	.21331	2.68112	.43009	3.11121	.04175
23	51.08718	.21287	1.83297	.20061	2.03358	.03981
24	34.94677	.21256	1.25157	.09337	1.34495	.03849
25	23.87149	.21235	.85386	.04339	.89725	.03759
…	…	…	…	…	…	…
40	0.07547	.21190	.00269	.00000	.00269	.03568
Sum	18,712.27340	–	2212.53972	16,499.78325	18,712.32296	–

Note 1: $p = .01138$.

with the estimates of the continuous BM for the products. Tables 1, 2, and 3 show the estimates of parameters, *PV*s, *IV*s, *CV*s, and then *UCV*s for the three products where the discount factor is not considered. Because the discount factor is fixed as '0', the sum of categories' values for all adopter categories should be identical to the market potential. In such a case, the values (i.e., *PV*s, *IV*s, and *CV*s) represent the decomposed market potential. Note that $CV \cong M$ for the each product in the tables. This implies that the market potential is correctly decomposed into *CV*s by the calculator for valuation of adopters which is introduced by this paper.

One can see that q_i (for $i = 1,2,3,\ldots$) decreases with the increases in i as shown in Eq. (19). In addition, $PV(t)$ decreases with the increases in t, whereas $IV(t)$ and $CV(t)$ increase and then decrease with the increases in t. The results show that the proportions of *IV* in *CV* are more than 80% for the three products (88.2% for Room Air Conditioner; 94.2% for Color TV; 83.5% for Clothes Dryers). This implies that it is very important

Table 2
Valuation for adopters of Color TV.

t (or i)	$[F(t)-F(t-1)]M$	q_i	$PV(t)$	$IV(t)$	$CV(t)$	$UCV(t)$
1	286.69772	.87228	286.69772	1775.52113	2062.21885	7.19301
2	532.95394	.85685	284.65535	2788.80234	3073.45770	5.76684
3	973.20670	.82957	280.85872	4139.51364	4420.37237	4.54207
4	1720.56523	.78400	273.92584	5627.52120	5901.44704	3.42995
5	2874.74411	.71460	261.66896	6695.24688	6956.91584	2.42001
6	4377.97235	.62253	241.18999	6589.68748	6830.87747	1.56028
7	5808.18976	.52041	210.00240	5094.32369	5304.32610	.91325
8	6441.63619	.42740	168.62630	3020.80899	3189.43529	.49513
9	5867.30137	.35669	122.73767	1400.34373	1523.08140	.25959
10	4460.09171	.30998	80.94047	534.76910	615.70957	.13805
11	2946.42088	.28192	49.16788	178.84017	228.00805	.07738
12	1770.24773	.26601	28.17831	55.10240	83.28071	.04704
13	1003.56832	.25729	15.56751	16.19313	31.76063	.03165
14	550.27453	.25259	8.41833	4.63558	13.05392	.02372
15	296.21857	.25009	4.49831	1.30831	5.80663	.01960
16	157.87762	.24876	2.38813	.36646	2.75458	.01745
17	83.69868	.24806	1.26344	.10223	1.36567	.01632
18	44.24766	.24769	.66720	.02846	.69565	.01572
19	23.35680	.24750	.35199	.00791	.35990	.01541
20	12.31952	.24739	.18560	.00220	.18780	.01524
21	6.49521	.24734	.09784	.00061	.09845	.01516
22	3.42372	.24731	.05157	.00017	.05174	.01511
23	1.80448	.24730	.02718	.00005	.02722	.01509
24	0.95100	.24729	.01432	.00001	.01434	.01507
25	0.50118	.24728	.00755	.00000	.00755	.01507
…	…	…	…	…	…	…
35	.00083	.25306	.00001	.00000	.00001	.01506
Sum	40,245.32243	–	2322.19697	37,923.12589	40,245.32286	–

Note 1: $p = .00712$.

Table 3
Valuation for adopters of Clothes Dryers.

t (or i)	$[F(t)-F(t-1)]M$	q_i	$PV(t)$	$IV(t)$	$CV(t)$	$UCV(t)$
1	262.92411	.37458	262.92411	921.56156	1184.48567	4.50505
2	355.65043	.36928	258.73369	1099.68360	1358.41729	3.81953
3	474.26835	.36234	253.06543	1273.49769	1526.56311	3.21878
4	620.56474	.35348	245.50666	1420.77140	1666.27806	2.68510
5	792.14066	.34253	235.61626	1514.23418	1749.85044	2.20901
6	979.81600	.32949	222.99133	1527.90189	1750.89321	1.78696
7	1165.83331	.31468	207.37527	1446.88847	1654.26374	1.41895
8	1324.80398	.29870	188.79452	1276.54948	1465.34400	1.10608
9	1429.01196	.28241	167.68014	1044.47427	1212.15441	.84825
10	1457.33059	.26671	144.90491	791.76120	936.66611	.64273
11	1403.61838	.25241	121.67835	557.72060	679.39895	.48403
12	1279.37892	.24001	99.30784	367.40495	466.71279	.36480
13	1108.85605	.22972	78.91743	228.33075	307.24819	.27709
14	919.87673	.22150	61.24477	135.17259	196.41736	.21353
15	735.77672	.21511	46.58401	76.95846	123.54248	.16791
16	571.54214	.21026	34.85739	42.50033	77.35773	.13535
17	433.95246	.20663	25.74830	22.93199	48.68029	.11218
18	323.81161	.20397	18.83208	12.16015	30.99223	.09571
19	238.50711	.20202	13.67125	6.36586	20.03711	.08401
20	173.99947	.20061	9.86999	3.30135	13.17134	.07570
21	126.05325	.19960	7.09683	1.70043	8.79726	.06979
22	90.85637	.19887	5.08783	.87151	5.95934	.06559
23	65.24788	.19835	3.63978	.44507	4.08485	.06261
24	46.73420	.19797	2.59988	.22670	2.82657	.06048
25	33.41062	.19770	1.85504	.11524	1.97028	.05897
…	…	…	…	…	…	…
43	.07391	.19704	.00408	.00000	.00408	.05526
Sum	16,496.76594	–	2723.14950	13,773.64823	16,496.79774	–

Note 1: $p = .01594$.

for marketing managers to manage the influential values of potential adopters.

On the other hand, $UCV(t)$ decreases with the increases in t. In Room Air Conditioners $UCV(t=1) = 5.17208$ which implies that each adopter at observation time 1 has the value of more than five adopters. In contrast, $UCV(t=9) = .87773$ which implies that each adopter at observation time 9 has the value less than one adopter (note that $UCV(t)>1$ where $t<9$.). In Color TV $UCV(t=1) = 7.19301$ and $UCV(t)>1$ where $t<7$, whereas in Clothes Dryers $UCV(t=1) = 4.50505$ and $UCV(t)>1$ where $t<9$. Based on $UCV(t)$ (for all t) marketing managers can compare the relative values of adopter groups whose adoption times are different.

5. Discussion

Most firms periodically observe consumers' adoptions of their products or services and the Bass model is the most well-known, simple and easy tool to predict future adoptions based on an observed adoption data. Since Bass has introduced his model, many researchers have continued to extend it. Unlike previous research, we focus on the characteristics of the Bass model which are not well discovered and utilized. The distinct features of this paper can be summarized as follows. First, we introduce a mathematical representation that additively decomposes a hazard function and, from the additive decomposition of the discrete Bass hazard function, derive the discrete version of the Bass model which exactly corresponds to the continuous Bass model. The discrete-analog Bass model can identify the interaction mechanism only between two groups in a social system (Innovators and Imitators), whereas the derived discrete Bass model allows us to analyze the interaction mechanism between various adopter groups which are classified by every observation time. Second, we present a valuation scheme of adopters based on the derived discrete Bass model. We decompose the category values (CV) into the purchase value (PV) and influence value (IV), and present a calculator for PV, IV, and CV for each adopter category using the discrete Bass model. With the calculator for valuation of adopters presented in this paper, managers can calculate values of adopters who adopt at every observation time only with estimates of the continuous Bass model, because the coefficients of the discrete Bass model can be expressed as the function of the Bass distribution.

One may consider additive decompositions for various discrete hazard functions which exactly correspond to their continuous hazard function [such as Weibull distribution, Log-Logistic distribution, a generalized BM (Bass et al., 1994), a model with heterogeneity effects (Bemmaor and Lee, 2002), a model with dynamic market potential (Guseo and Guidolin, 2009), and so on]. However, it is necessary to justify various patterns of the effects from previous adopters if one wants to use a flexible distribution. For example, the effects from previous adopters (i.e., q_i for $i = 1,2,3,\ldots$) may be negative if one adopts a flexible distribution of which the hazard function is not monotonically increasing over time, consequently the negative effects may lead to negative values of adopters. Note that the positive effects from previous adopters are widely accepted and are also empirically supported in the literature (e.g., Easingwood et al., 1983; Van den Bulte and Joshi, 2007; Ho et al., 2012). We leave this issue for future research.

Appendix A

Let T be a nonnegative random variable representing the adoption time of an individual from a homogeneous population. The probability distribution of T can be specified in many ways. Two ways of which are widely used to express the adoption process: the survivor function (or the distribution) and the hazard function. The survivor function is defined for both discrete and continuous distributions as the probability that T is at least great as a value t; that is

$$S(t) = P(T \geq t) = 1-F(t), \quad 0<t<\infty. \tag{A.1}$$

If T is a continuous random variable then the hazard function can be written by:

$$h(t) = \frac{d\log[1-F(t)]}{dt} = \frac{f(t)}{1-F(t)}. \tag{A.2}$$

The corresponding continuous distribution is given by:

$$F(t) = 1 - \exp\left[-\int_0^t h(u)du\right]. \tag{A.3}$$

The distribution $F(t)$ represents an adoption process which is a monotone non-decreasing continuous function. Let $F(t_k)$ denote the discrete counterpart distribution which exactly corresponds to the continuous distribution $F(t)$ where $0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots$. In addition, let us every discrete time interval be equal: $t_k - t_{k-1} = t_1 - t_0$ for $k = 0,1,2,3,\ldots$. Then the hazard function corresponding to $F(t_k)$ is defined as follows.

$$\lambda(t_k) = \frac{F(t_k) - F(t_{k-1})}{1 - F(t_{k-1})}. \tag{A.4}$$

The corresponding $F(t_k)$, which is the discrete-time counterpart distribution of $F(t)$, is given by.

$$F(t_k) = 1 - \prod_{u \le k}(1 - \lambda(t_u)). \tag{A.5}$$

Note that $F(t) = F(t_k)$ for t satisfying $t = t_k$, which implies that the discrete-time counterpart distribution is identical to the continuous distribution at every discrete time.

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Update

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Erratum

Erratum to "Valuation of Adopters Based on the Bass Model for a New Product" [Technol. Forecast. Soc. Chang. 108 (2016) 53–69]

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Correction 1: The value of $[F(t) - F(t-1)]M$ in $t = 1$ (the first value in the second column) in Table 1 *should* be 212.95768 (currently the value shown is 0.5212.95768).

Correction 2: The value of q_i in $t = 35$ (the last value in the third column) in Table 2 *should* be 0.24306 (currently the value shown is 0.25306). Thus, Tables 1 and 2 should be as follows:

Table 1
Valuation for adopters of room air conditioners.

t (or i)	$[F(t)-F(t-1)]M$	q_i	$PV(t)$	$IV(t)$	$CV(t)$	$UCV(t)$
1	212.95768	0.44433	212.95768	888.47697	1,101.43465	5.17208
2	304.08092	0.43885	210.53411	1,121.08196	1,331.61607	4.37915
3	428.84226	0.43126	207.07350	1,373.86669	1,580.94020	3.68653
4	594.32947	0.42099	202.19304	1,621.12652	1,823.31957	3.06786
5	804.06041	0.40754	195.42925	1,822.40974	2,017.83899	2.50956
6	1,053.04550	0.39065	186.27861	1,928.00199	2,114.28059	2.00778
7	1,321.98575	0.37054	174.29438	1,894.89427	2,069.18865	1.56521
8	1,574.18529	0.34805	159.24946	1,709.71394	1,868.96340	1.18726
9	1,760.72080	0.32459	141.33437	1,404.10957	1,545.44395	0.87773
10	1,836.61556	0.30184	121.29641	1,046.20147	1,167.49789	0.63568
11	1,781.47263	0.28127	100.39473	709.41706	809.81179	0.45457
12	1,610.23138	0.26383	80.12060	441.63771	521.75832	0.32403
13	1,365.21106	0.24983	61.79529	255.59584	317.39113	0.23249
14	1,096.05016	0.23907	46.25845	139.44938	185.70783	0.16943
15	842.06954	0.23108	33.78480	72.68545	106.47025	0.12644
16	625.32682	0.22530	24.20159	36.61480	60.81639	0.09726
17	452.75033	0.22119	17.08503	17.99195	35.07698	0.07748
18	321.82008	0.21831	11.93249	8.68558	20.61807	0.06407
19	225.77082	0.21630	8.27000	4.14092	12.41092	0.05497
20	156.93364	0.21492	5.70061	1.95709	7.65770	0.04880
21	108.38658	0.21396	3.91462	0.91938	4.83400	0.04460
22	74.52586	0.21331	2.68112	0.43009	3.11121	0.04175
23	51.08718	0.21287	1.83297	0.20061	2.03358	0.03981
24	34.94677	0.21256	1.25157	0.09337	1.34495	0.03849
25	23.87149	0.21235	0.85386	0.04339	0.89725	0.03759
…	…	…	…	…	…	…
40	0.07547	0.21190	0.00269	0.00000	0.00269	0.03568
Sum	18,712.27340	–	2,212.53972	16,499.78325	18,712.32296	–

Note 1: $p = 0.01138$.

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Table 2
Valuation for adopters of color TV.

t (or i)	$[F(t)-F(t-1)]M$	q_i	$PV(t)$	$IV(t)$	$CV(t)$	$UCV(t)$
1	286.69772	0.87228	286.69772	1,775.52113	2,062.21885	7.19301
2	532.95394	0.85685	284.65535	2,788.80234	3,073.45770	5.76684
3	973.20670	0.82957	280.85872	4,139.51364	4,420.37237	4.54207
4	1,720.56523	0.78400	273.92584	5,627.52120	5,901.44704	3.42995
5	2,874.74411	0.71460	261.66896	6,695.24688	6,956.91584	2.42001
6	4,377.97235	0.62253	241.18999	6,589.68748	6,830.87747	1.56028
7	5,808.18976	0.52041	210.00240	5,094.32369	5,304.32610	0.91325
8	6,441.63619	0.42740	168.62630	3,020.80899	3,189.43529	0.49513
9	5,867.30137	0.35669	122.73767	1,400.34373	1,523.08140	0.25959
10	4,460.09171	0.30998	80.94047	534.76910	615.70957	0.13805
11	2,946.42088	0.28192	49.16788	178.84017	228.00805	0.07738
12	1,770.24773	0.26601	28.17831	55.10240	83.28071	0.04704
13	1,003.56832	0.25729	15.56751	16.19313	31.76063	0.03165
14	550.27453	0.25259	8.41833	4.63558	13.05392	0.02372
15	296.21857	0.25009	4.49831	1.30831	5.80663	0.01960
16	157.87762	0.24876	2.38813	0.36646	2.75458	0.01745
17	83.69868	0.24806	1.26344	0.10223	1.36567	0.01632
18	44.24766	0.24769	0.66720	0.02846	0.69565	0.01572
19	23.35680	0.24750	0.35199	0.00791	0.35990	0.01541
20	12.31952	0.24739	0.18560	0.00220	0.18780	0.01524
21	6.49521	0.24734	0.09784	0.00061	0.09845	0.01516
22	3.42372	0.24731	0.05157	0.00017	0.05174	0.01511
23	1.80448	0.24730	0.02718	0.00005	0.02722	0.01509
24	0.95100	0.24729	0.01432	0.00001	0.01434	0.01507
25	0.50118	0.24728	0.00755	0.00000	0.00755	0.01507
⋯	⋯	⋯	⋯	⋯	⋯	⋯
35	0.00083	0.24306	0.00001	0.00000	0.00001	0.01506
Sum	40,245.32243	–	2,322.19697	37,923.12589	40,245.32286	–

Note 1: $p = 0.00712$.